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Abstract

We consider a cyclic polling system in which the server is allowed to rest at a queue when the entire system is empty. The performance of such a system is analyzed by means of the power-series algorithm (PSA), a tool for the numerical evaluation and optimization of a broad class of queueing models. The applicability of the PSA is extended towards polling systems in which the server is allowed to rest at a queue, instead of having to move along the queues, when the system is empty. Numerical examples will show that this extension may lead to considerable improvements of the performance of the system. Moreover, we consider the question at which queues the server should rest when the system is empty; a heuristic approximation method is proposed and tested.

1. Introduction

A polling system consists of a number of queues, attended to by a single server. Polling systems are widely used to model computer and communication systems and are also applicable in the areas of manufacturing and maintenance (cf. [12, 14]). In the polling literature the server is usually assumed to be switching when not serving, particularly when there are no customers in the system, because it is assumed that the server has no global information on the status of the queues. In computer-network and telecommunication applications the overhead involved to control the server and the buffer contents is believed to outweigh the possible benefits from resting. The purpose of the present paper is to develop a means for quantifying the benefits from resting. In applications, this gain can then be compared with the loss due to an increased overhead. Moreover, it is interesting in its own right to gain insight into the performance of systems in which the server is allowed to rest. Throughout, systems in which the server is allowed to rest at a queue when the system is empty will be referred to as dormant server systems, as opposed to non-dormant server systems, in which the server keeps on moving along the queues when the system is empty.

Polling systems in which the server is allowed to rest at a queue are generally hard to analyze by means of mathematical techniques, except for some special cases (cf. [6–8]). Consequently, there is a need for numerical algorithms to gain insight into the behavior of such systems. In this paper we focus on the use of the PSA, a numerical tool for evaluation and optimization of the performance of a broad class of multi-queue models, such as a number of polling models (cf. [1–3, 10]). The basic idea of the PSA is the transformation of the non-recursively solvable (infinite) set of balance equations into an, in principle, recursively solvable set of equations by adding one dimension to the state space. This transformation is realized by means of power-

series expansions of the state probabilities as functions of the load in light traffic. In this paper the applicability of the PSA is extended towards polling systems in which the server is allowed to rest at a queue when the system is empty. We will consider a model in which the server visits the queues in cyclic order and at each visit of the server to a queue the customers are served according to the so-called Bernoulli service discipline (cf. [11]).
In the vast majority of papers about polling systems the server is assumed to be either serving or switching. As an exception, for a two-queue model with exhaustive service at both queues, Eisenberg [8] considers a system with either alternating or strict priority, in which the server rests at a queue when the system is empty. Liu, Nain and Towsley [13] show that, in order to minimize the amount of unfinished work in the system at any time, the server should neither switch nor rest when at a non-empty queue; moreover, they show that for symmetrical systems the server should rest at a queue when the entire system is empty. Borst [6] shows that for cyclic polling systems with globally gated service in which the server is allowed to rest only at one particular queue, the waiting times are smaller (in the increasing convex ordering sense) than in the non-dormant server case. A pseudo-conservation law for cyclic polling systems with mixtures of exhaustive, gated, 1-limited and globally gated service is derived in [7].

The rest of the paper is organized as follows. In Section 2 the model is described in detail. In Section 3 the global balance equations of the system are formulated and in Section 4 a complete computation scheme to derive the coefficients of the power-series expansions of the state-probabilities is obtained. In Section 5 the extension of the PSA is used to gain insight into the improvements of the performance that can be made by allowing the server to rest; numerical examples will show that the performance of the system may be strongly improved, especially in light and medium loaded systems in which the switch-over times are considerable. Apart from the question whether it makes sense at all to allow the server to rest at a queue, in Section 6 we consider the problem at *which* queues the server should rest when the system is empty; a simple and fast-to-evaluate heuristic approximation method to solve this optimization problem is proposed and tested.

2. Model description

Consider a polling system consisting of s queues, $Q_1, \dots, Q_s$, each queue having infinite capacity. The queues are attended to by a single server, who visits the queues in a cyclic order. Customers arrive at the queues according to independent Poisson processes; the arrival rate at Q_i is denoted by λ_i, $i=1,\dots,s$. The service times of the customers at Q_i are assumed to have a Coxian distribution consisting of Ψ_i^1 exponential phases; with probability $\pi_i^{1,\varphi}$ a service is composed of phases φ, $\varphi-1,\dots,1$, $\varphi=1,\dots,\Psi_i^1$, and the transition rate from phase ψ is $\mu_i^{1,\psi}$, $\psi=1,\dots,\Psi_i^1$, $i=1,\dots,s$. Let

$\beta^{(k)}=(\beta_1^{(k)},\ldots,\beta_s^{(k)})$ be the vector of k-th moments of the service time distributions, $k=1,2,\ldots$. The times needed by the server to move from Q_{i-1} to Q_i are assumed to be Coxian distributed with parameters Ψ_i^0 , $\mu_i^{0,\varphi}$, $\pi_i^{0,\varphi}$, $\varphi=1,\ldots,\Psi_i^0$, which are defined in a similar way as those of the service time distributions, $i=1,\ldots,s$; let $\sigma^{(k)}$ be the vector of k-th moments of the switch-over time distributions and let σ_k be the k-th moment of the total switch-over time per cycle of the server along the queues, $k=1,2,\ldots$.

It is assumed that all interarrival times, service times and switch-over times are mutually independent. At each queue the customers are served on first-in-first-out basis.

The server is allowed to rest at a queue when the entire system is empty. More specifically, we allow the server to rest at *some* of the queues; throughout, the set of indices corresponding to these queues, referred to as the dormant set, is denoted by D.

The number of customers that is served during a visit of the server to a queue is determined by a so-called Bernoulli schedule, i.e., a vector $\mathbf{q}=(q_1,\ldots,q_s)$ of probabilities which are used as follows. When the server arrives at a non-empty queue, at least one customer at that queue is served; when at a service completion epoch at Q_h that queue is not empty, then with probability q_h another customer at that queue is served. Note that the case $q_h=0$ corresponds to the 1-limited service discipline and the case $q_h=1$ to the exhaustive service strategy. When the server arrives at Q_h while it is empty or when the server has just emptied Q_h , the behavior of the server depends on whether or not the entire system is empty at that instant. If the system is non-empty at that particular instant or if $h\notin D$, then the server proceeds to the next queue and otherwise, the server rests at Q_h and waits for the next customer to arrive. As soon as a customer arrives at the system, the server immediately resumes its activities. If the first arriving customer arrives at Q_h , then that customer is taken into service immediately; otherwise, the server starts moving to the next queue.

The offered load ρ_i to Q_i and the total offered load ρ to the system are defined by

$$\rho_i := \lambda_i \beta_i^{(1)}, \quad i=1,\ldots,s, \quad \rho := \sum_{i=1}^{s} \rho_i . \tag{1}$$

Because ρ will be used as a variable in the PSA, we define the following quantity

$$a_i := \lambda_i / \rho , \tag{2}$$

referred to as the relative arrival rate at Q_i , $i=1,\ldots,s$. Let $a:=(a_1,\ldots,a_s)$.

Necessary and sufficient conditions for polling systems in the non-dormant server case have been derived in [9]. Because in cases in which the ergodicity becomes critical the opportunity of resting arises with a probability which tends to 0, we suspect that the same conditions are necessary and sufficient in the dormant server case. For the present model these conditions read:

$$\rho\left[1 + \sigma_1 a_i (1 - q_i)\right] < 1 , \quad i=1,\ldots,s . \tag{3}$$

In the sequel we will assume that condition (3) is satisfied and that the system is in steady-state.

Throughout this paper we will need the notion of the following index set. For a given D we partition the set $\{1, \ldots, s\}$ into the subsets $\{U_D(i)\}_{i \in D}$, defined by

$$U_D(i) := \{i\} \cup \{1 \le h \le s \mid j \notin D \; (j = h, \ldots, i-1)\} \quad (i \in D); \tag{4}$$

here, the indices j should be read as j mod s if j>s. In words, for $i \in D$, $U_D(i)$ corresponds to the set of queues h for which Q_i is the first queue in D that is visited after the server has started to move from Q_{h-1} to Q_h.

3. Balance equations

For the case of a non-dormant server, the global balance equations for the present model are given in [3]. In order to extend these balance equations towards the dormant server case, we adopt the same notation as in [3]. Denote by $\mathbf{N}=(N_1, \ldots, N_s)$ the joint queue length vector. In order to transform the queue length process into a Markov process we introduce a triple (H, Z, Φ) of supplementary variables. Here, H will denote the index of the queue which is being served or is being switched to or at which the server is dormant; the variable Z will indicate whether the server is dormant (Z= −1) or switching (Z=0) or serving (Z=1); the variable Φ, which is only defined in the cases Z=0 and Z=1, will indicate the actual phase number of either the switch-over time or the service time.

We define the state probabilities as follows: for $\mathbf{n} \in \mathbb{N}^s$, h=1,...,s, ζ=0,1, $\varphi = 1, \ldots, \Psi_h^\zeta$,

$$p(\boldsymbol{n}, h, \zeta, \varphi) := Pr\{(N, H, Z, \Phi) = (\boldsymbol{n}, h, \zeta, \varphi)\}; \tag{5}$$

and for $h \in D$,

$$p(\mathbf{0}, h, -1) := Pr\{(N, H, Z) = (\mathbf{0}, h, -1)\}, \tag{6}$$

i.e., the probability that the server is dormant at Q_h ($h \in D$).
The balance equations read: for $\mathbf{n} \in \mathbb{N}^s$, $\mathbf{n} \neq \mathbf{0}$, h=1,...,s, $\varphi = 1, \ldots, \Psi_h^0$,

$$\begin{aligned}
&\left[\rho \sum_{j=1}^{s} a_j + \mu_h^{0,\varphi}\right] p(\boldsymbol{n}, h, 0, \varphi) = \rho \sum_{j=1}^{s} a_j p(\boldsymbol{n} - \boldsymbol{e}_j, h, 0, \varphi) I\{n_j > 0\} \\
&+ \mu_h^{0,\varphi+1} p(\boldsymbol{n}, h, 0, \varphi+1) I\{\varphi < \Psi_h^0\} + \mu_{h-1}^{0,1} \pi_h^{0,\varphi} p(\boldsymbol{n}, h-1, 0, 1) I\{n_{h-1} = 0\} \\
&+ \mu_{h-1}^{1,1} \pi_h^{0,\varphi} p(\boldsymbol{n} + \boldsymbol{e}_{h-1}, h-1, 1, 1)\left[1 - q_{h-1} I\{n_{h-1} > 0\}\right] \\
&+ \rho\, \pi_h^{0,\varphi} \sum_{\substack{j=1 \\ j \neq h-1}}^{s} a_j p(\mathbf{0}, h-1, -1) I\{\boldsymbol{n} = \boldsymbol{e}_j\} I\{h-1 \in D\};
\end{aligned} \tag{7}$$

here, the first term on the right hand side indicates an arrival of a customer into the system; the second term indicates a phase transition of the switch-over time from Q_{h-1} to Q_h; the third term indicates an arrival of the server at Q_{h-1}, which is empty at that particular instant, so that the server directly proceeds to Q_h ; the fourth term indicates a service completion at Q_{h-1}; the fifth term indicates that a customer arrives at Q_j ($j \neq h-1$) when the system is empty and the server is dormant at Q_{h-1}.

Similarly, one may verify that we have, for $h=1,\dots,s$, $\varphi=1,\dots,\Psi_h^0$,

$$\left[\rho\sum_{j=1}^{s} a_j + \mu_h^{0,\varphi}\right] p(\mathbf{0},h,0,\varphi) = \mu_h^{0,\varphi+1} p(\mathbf{0},h,0,\varphi+1) I\{\varphi<\Psi_h^0\}$$
$$+ \left[\mu_{h-1}^{0,1}\pi_h^{0,\varphi} p(\mathbf{0},h-1,0,1) + \mu_{h-1}^{1,1}\pi_h^{0,\varphi} p(\mathbf{e}_{h-1},h-1,1,1)\right] I\{h \notin D\}; \tag{8}$$

and for $\mathbf{n}\in\mathbb{N}^s$, $h=1,\dots,s$, $\varphi=1,\dots,\Psi_h^1$, $n_h>0$,

$$\left[\rho\sum_{j=1}^{s} a_j + \mu_h^{1,\varphi}\right] p(\mathbf{n},h,1,\varphi) = \rho\sum_{j=1}^{s} a_j p(\mathbf{n}-\mathbf{e}_j,h,1,\varphi) I\{n_j>0\}$$
$$+ \mu_h^{1,\varphi+1} p(\mathbf{n},h,1,\varphi+1) I\{\varphi<\Psi_h^1\} + \mu_h^{0,1}\pi_h^{1,\varphi} p(\mathbf{n},h,0,1) \tag{9}$$
$$+ q_h\mu_h^{1,1}\pi_h^{1,\varphi} p(\mathbf{n}+\mathbf{e}_h,h,1,1) + \rho a_h \pi_h^{1,\varphi} p(\mathbf{0},h,-1) I\{\mathbf{n}=\mathbf{e}_h\} I\{h\in D\};$$

and for $h\in D$,

$$\rho\sum_{j=1}^{s} a_j p(\mathbf{0},h,-1) = \mu_h^{1,1} p(\mathbf{e}_h,h,1,1) + \mu_h^{0,1} p(\mathbf{0},h,0,1). \tag{10}$$

Because the server cannot be serving at an empty queue we have for $h=1,\dots,s$, $\varphi=1,\dots,\Psi_h^1$,

$$p(\mathbf{n},h,1,\varphi) = 0, \quad \textit{if} \quad n_h = 0, \tag{11}$$

and, according to the law of total probability, we have

$$\sum_{h\in D} p(\mathbf{0},h,-1) + \sum_{\mathbf{n}\in\mathbb{N}^s}\sum_{h=1}^{s}\sum_{\zeta=0}^{1}\sum_{\varphi=1}^{\Psi_h^\zeta} p(\mathbf{n},h,\zeta,\varphi) = 1. \tag{12}$$

4. The computation scheme

The basic idea of the PSA is the expression of the state probabilities as power series in the offered load ρ. For the non-dormant server case a complete computation scheme to calculate all coefficients for the state probabilities is given in [3]. In this section we extend this computation scheme to the dormant server case.

The approach in [3] relies on the following property: for $\mathbf{n}\in\mathbb{N}^s$, $h=1,\dots,s$, $\zeta=0,1$, $\varphi=1,\dots,\Psi_h^\zeta$,

$$p(\mathbf{n},h,\zeta,\varphi) = O(\rho^{n_1+\dots+n_s}), \quad (\rho \downarrow 0). \tag{13}$$

Moreover, we have for $h\in D$,

$$p(\mathbf{0},h,-1) = O(1), \quad (\rho\downarrow 0). \tag{14}$$

Based on properties (13) and (14), we introduce the following power-series expansions for the state probabilities: for $\mathbf{n}\in\mathbb{N}^s$, $h=1,\dots,s$, $\zeta=0,1$, $\varphi=1,\dots,\Psi_h^\zeta$,

$$p(\boldsymbol{n},h,\zeta,\varphi) = \rho^{n_1+\cdots+n_s} \sum_{k=0}^{\infty} \rho^k b(k;\boldsymbol{n},h,\zeta,\varphi); \tag{15}$$

and for $h \in D$,

$$p(\boldsymbol{0},h,-1) = \sum_{k=0}^{\infty} \rho^k b(k;\boldsymbol{0},h,-1). \tag{16}$$

Substituting (15) and (16) into the balance equations (7), (8), (9) and (10) and equating corresponding powers of ρ yields a set of linear relations between the coefficients of the power series. For simplicity of the discussion, for $\zeta=0$ we explicitly distinguish the cases $\mathbf{n}=\mathbf{0}$, $\mathbf{n}=\mathbf{e}_i$, $i=1,\dots,s$ (cf. (18) and (19) below). The relations read as follows: for $\mathbf{n}\in\mathbb{N}^s$, $\mathbf{n}\neq\mathbf{0}$, $\mathbf{n}\neq\mathbf{e}_i$ ($i=1,\dots,s$), $h=1,\dots,s$, $\varphi=1,\dots,\Psi_h^0$, $k=0,1,\dots$,

$$\begin{aligned}
\mu_h^{0,\varphi} b(k;\boldsymbol{n},h,0,\varphi) &= \mu_h^{0,\varphi+1} b(k;\boldsymbol{n},h,0,\varphi+1) I\{\varphi<\Psi_h^0\} \\
&+ \sum_{j=1}^{s} a_j \left[b(k;\boldsymbol{n}-\boldsymbol{e}_j,h,0,\varphi) I\{n_j>0\} - b(k-1;\boldsymbol{n},h,0,\varphi) I\{k>0\} \right] \\
&+ \mu_{h-1}^{1,1} \pi_h^{0,\varphi} b(k-1;\boldsymbol{n}+\boldsymbol{e}_{h-1},h-1,1,1) I\{k>0\} \left[1 - q_{h-1} I\{n_{h-1}>0\} \right] \\
&+ \mu_{h-1}^{0,1} \pi_h^{0,\varphi} b(k;\boldsymbol{n},h-1,0,1) I\{n_{h-1}=0\};
\end{aligned} \tag{17}$$

for $i=1,\dots,s$, $h=1,\dots,s$, $\varphi=1,\dots,\Psi_h^0$, $k=0,1,\dots$,

$$\begin{aligned}
\mu_h^{0,\varphi} b(k;\boldsymbol{e}_i,h,0,\varphi) &= \mu_h^{0,\varphi+1} b(k;\boldsymbol{e}_j,h,0,\varphi+1) I\{\varphi<\Psi_h^0\} \\
&+ a_i b(k;\boldsymbol{0},h,0,\varphi) - \sum_{j=1}^{s} a_j b(k-1;\boldsymbol{e}_i,h,0,\varphi) I\{k>0\} \\
&+ \mu_{h-1}^{1,1} \pi_h^{0,\varphi} b(k-1;\boldsymbol{e}_i+\boldsymbol{e}_{h-1},h-1,1,1) I\{k>0\} \left[1 - q_{h-1} I\{h-1=i\} \right] \\
&+ \Big[\mu_{h-1}^{0,1} \pi_h^{0,\varphi} b(k;\boldsymbol{e}_i,h-1,0,1) \\
&\qquad + \pi_h^{0,\varphi} a_i b(k;\boldsymbol{0},h-1,-1) I\{h-1\in D\} \Big] I\{h-1\neq i\};
\end{aligned} \tag{18}$$

for $h=1,\dots,s$, $\varphi=1,\dots,\Psi_h^0$, $k=0,1,\dots$,

$$\begin{aligned}
\mu_h^{0,\varphi} b(k;\boldsymbol{0},h,0,\varphi) &= \mu_h^{0,\varphi+1} b(k;\boldsymbol{0},h,0,\varphi+1) I\{\varphi<\Psi_h^0\} \\
&- \sum_{j=1}^{s} a_j b(k-1;\boldsymbol{0},h,0,\varphi) I\{k>0\} \\
&+ \Big[\mu_{h-1}^{0,1} \pi_h^{0,\varphi} b(k;\boldsymbol{0},h-1,0,1) \\
&\qquad + \mu_{h-1}^{1,1} \pi_h^{0,\varphi} b(k-1;\boldsymbol{e}_{h-1},h-1,1,1) I\{k>0\} \Big] I\{h-1\notin D\};
\end{aligned} \tag{19}$$

for $\mathbf{n}\in\mathbb{N}^s$, $h=1,\dots,s$, $\varphi=1,\dots,\Psi_h^1$, $n_h>0$, $k=0,1,\dots$,

$$\begin{aligned}
\mu_h^{1,\varphi} b(k;\boldsymbol{n},h,1,\varphi) &= \mu_h^{1,\varphi+1} b(k;\boldsymbol{n},h,1,\varphi+1) I\{\varphi<\Psi_h^1\} \\
&+ \mu_h^{0,1}\pi_h^{1,\varphi} b(k;\boldsymbol{n},h,0,1) + q_h \mu_h^{1,1}\pi_h^{1,\varphi} b(k-1;\boldsymbol{n}+\boldsymbol{e}_h,h,1,1) I\{k>0\} \\
&+ \sum_{j=1}^{s} a_j \left[b(k;\boldsymbol{n}-\boldsymbol{e}_j,h,1,\varphi) I\{n_j>0\} - b(k-1;\boldsymbol{n},h,1,\varphi) I\{k>0\} \right] \\
&+ a_h \pi_h^{1,\varphi} b(k;\boldsymbol{0},h,-1) I\{\boldsymbol{n}=\boldsymbol{e}_h\} I\{h\in D\};
\end{aligned} \tag{20}$$

and for $h\in D$, $k=0,1,\ldots$,

$$\sum_{j=1}^{s} a_j b(k;\boldsymbol{0},h,-1) = \mu_h^{1,1} b(k;\boldsymbol{e}_h,h,1,1) + \mu_h^{0,1} b(k+1;\boldsymbol{0},h,0,1). \tag{21}$$

For $\mathbf{n}\neq\mathbf{0}$ and $\mathbf{n}\neq\mathbf{e}_i$ $(i=1,\ldots,s)$, relations (17) and (20) express the coefficients $b(k;\mathbf{n},h,\zeta,\varphi)$ in terms of lower order with respect to the partial ordering $\prec$ defined in [3]. Hence, the only states that require further attention are the states with $\zeta=0$ and either $\mathbf{n}=\mathbf{0}$ or $\mathbf{n}=\mathbf{e}_i$ $(i=1,\ldots,s)$, the states with $\zeta=-1$, and the states with $\zeta=1$ and $\mathbf{n}=\mathbf{e}_i$ $(i=1,\ldots,s)$.

Consider the states with $\zeta=0$ and $\mathbf{n}=\mathbf{0}$ and let k be fixed, $k=0,1,\ldots$. It follows from (19) that if $D\neq\emptyset$, then according to the ordering $\prec$ in [3], the coefficients $b(k;\mathbf{0},h,0,\varphi)$, $h=1,\ldots,s$, $\varphi=1,\ldots,\Psi_h^0$, can be calculated recursively, starting with a coefficient $b(k;\mathbf{0},h,0,\Psi_h^0)$ for which $h-1\in D$. If $D=\emptyset$ then a set of s linear equations has to be solved to compute the coefficients $b(k;\mathbf{0},h,0,\varphi)$, $h=1,\ldots,s$, $\varphi=1,\ldots,\Psi_h^0$ (cf. [3]).

As for the coefficients $b(k;\mathbf{0},h,-1)$, $h\in D$, $k=0,1,\ldots$, substituting (21) into (19) and (20) leads to the following set of linear equations for the coefficients $b(k;\mathbf{0},h,-1)$, $h\in D$:

$$\left(\sum_{i=1}^{s} a_i\right) b(k;\boldsymbol{0},h,-1) = \sum_{j\in D}\left(\sum_{i\in U_D(h)} a_i\right) b(k;\boldsymbol{0},j,-1) + y(k;h), \tag{22}$$

where $y(0;h):=0$, $h\in D$, and for $h\in D$, $k=1,2,\ldots$,

$$\begin{aligned}
y(k;h) = &-\left(\sum_{j=1}^{s} a_j\right) \sum_{i\in U_D(h)} \left\{ \sum_{\varphi=1}^{\Psi_i^1} b(k-1;e_i,i,1,\varphi) + \sum_{j=1}^{s}\sum_{\varphi=1}^{\Psi_j^0} b(k-1;e_i,j,0,\varphi) \right\} \\
&+ \sum_{i\in U_D(h)} \sum_{j=1}^{s} \mu_j^{1,1} b(k-1;e_i+e_j,j,1,1) \\
&+ \left(\sum_{i\in U_D(h)} a_i\right) \sum_{j=1}^{s}\sum_{\varphi=1}^{\Psi_j^0} b(k;\boldsymbol{0},j,0,\varphi) - \left(\sum_{j=1}^{s} a_j\right) \sum_{i\in U_D(h)} \sum_{\varphi=1}^{\Psi_i^0} b(k;\boldsymbol{0},i,0,\varphi).
\end{aligned} \tag{23}$$

However, one may verify, by summing over $h \in D$, that the set of equations (22) is dependent. An additional linear equation follows from the law of total probability (12). Substituting (15) and (16) into (12) implies:

$$\sum_{h \in D} b(k;\mathbf{0},h,-1) = Y(k), \tag{24}$$

where $Y(0):=1$ and for $k=1,2,\ldots$,

$$Y(k) = -\sum_{0<n_1+\ldots+n_s\le k} \sum_{h=1}^{s} \sum_{\zeta=0}^{1} \sum_{\varphi=1}^{\Psi_h^\zeta} b(k-n_1-\ldots-n_s;\mathbf{n},h,\zeta,\varphi) - \sum_{h=1}^{s} \sum_{\varphi=1}^{\Psi_h^0} b(k;\mathbf{0},h,0,\varphi). \tag{25}$$

Now, in order to obtain the coefficients $b(k;\mathbf{0},h,-1)$, $h \in D$, from the set of equations (22) (and (24)), the terms in $y(k;h)$ have to be known; that is, for each $h \in D$, $k=0,1,\ldots$, the terms $b(k;0,h,-1)$ have to be of higher order than all terms in (23). Moreover, once the coefficients $b(k;\mathbf{0},h,-1)$, $h \in D$, are known, the coefficients $b(k;\mathbf{e}_l,h,0,\varphi)$, $l,h=1,\ldots,s$, $\varphi=1,\ldots,\Psi_h^0$, and the coefficients $b(k;\mathbf{e}h,h,1,\varphi)$, $h=1,\ldots,s$, $\varphi=1,\ldots,\Psi_h^1$, can be computed recursively according to (18) and (20) respectively, $k=0,1,\ldots$.

In practice, limitations on storage capacity and computation time restrict the number of coefficients that can be computed. The reader is referred to [3] for a discussion on the complexity of the PSA. Here, it will be assumed that the coefficients of the power series are computed up to a given power M_{max} of ρ.

Summarizing, the coefficients can be computed according to the following computation scheme:

step 1: $M:=0$;

step 2: determine $b(M;\mathbf{0},h,0,\varphi)$, $h=1,\ldots,s$, $\varphi=1,\ldots,\Psi_h^0$, according to (19);

step 3: for all $(k;\mathbf{n},h,\zeta,\varphi)$ for which $k + n_1 + \cdots + n_s = M+1$, $\mathbf{n}\neq\mathbf{0}$, $\mathbf{n}\neq\mathbf{e}_i$ $(i=1,\ldots,s)$, determine $b(k;\mathbf{n},h,0,\varphi)$, $h=1,\ldots,s$, $\varphi=\Psi_h^0,\ldots,1$, according to (17) and $b(k;\mathbf{n},h,1,\varphi)$, $h=1,\ldots,s$, $\varphi=\Psi_h^1,\ldots,1$, according to (20), in increasing order of $(k;\mathbf{n},h,\zeta,\varphi)$ w.r.t. $\prec$;

step 4: determine $b(M;\mathbf{0},h,-1)$, $h \in D$, according to (22) and (24);

step 5: determine $b(M;\mathbf{e}_l,h,0,\varphi)$, h, $l=1,\ldots,s$, $\varphi=\Psi_h^0,\ldots,1$, according to (18);

step 6: determine $b(M;\mathbf{e}_h,h,1,\varphi)$, $h=1,\ldots,s$, $\varphi=\Psi_h^1,\ldots,1$, according to (20);

step 7: $M:=M+1$;

step 8: if $M<M_{max}$ then return to step 2; otherwise, STOP.

Remarks:

For the case $D=\varnothing$, step 4 in the computation scheme vanishes. It is readily verified that the resulting computation scheme corresponds to the one given in [3].

For the case $k=0$, it follows from (22), (24), $Y(0)=1$ and $y(0;h)=0$ $(h=1,\ldots,s)$ that for $h \in D$,

$$b(0;0,h,-1) = \frac{\sum_{i \in U_D(h)} a_i}{\sum_{i=1}^{s} a_i} = \frac{\sum_{i \in U_D(h)} \lambda_i}{\sum_{i=1}^{s} \lambda_i}, \tag{26}$$

which corresponds to the light-traffic limit of the system.

5. Numerical examples

In this section the extension of the PSA, discussed in Sections 3 and 4, is used to gain insight into the performance of systems in which the server is allowed to rest at a queue when the system is empty. We will give some rough guidelines on the qualitative behavior of the system performance, illustrated by numerical examples. Throughout, we take as performance measure the following average waiting time function:

$$C(D) := \sum_{i=1}^{s} c_i E W_i, \tag{27}$$

i.e., an arbitrary weighted sum of the steady-state mean waiting times at the various queues. Here, the weights are assumed to be non-negative. Let $c=(c_1, ..., c_s)$.
For ease of the discussion, in the numerical examples discussed in this section, we will take $c_i := \lambda_i / (\Sigma_{j=1}^{s} \lambda_j)$, $i=1,...,s$, so that C(D) can be interpreted as the mean waiting time of an arbitrary customer arriving at the system. Throughout the rest of this paper, q_i will be taken to be 1 for all i, i.e., all queues are served exhaustively, and the service times and switch-over times will be assumed to be exponentially distributed, unless indicated otherwise. Denote by D^* the subset of $\{1,...,s\}$ which minimizes (27) over all subsets D of $\{1,...,s\}$. Throughout this section, the symbol "%" indicates the relative improvement which can be made by allowing the server to rest, defined by

$$\frac{C(\varnothing) - C(D^*)}{C(\varnothing)} \times 100\,\%. \tag{28}$$

Note that $C(\varnothing)$ is the average waiting time function (27) in the non-dormant server case.

In order to examine the influence of the offered load to the system on the improvements of the performance of the system that can be made by allowing the server to rest, $C(\varnothing)$ and $C(D^*)$ have been computed for various values of the offered load for a model with the following set of parameters: s=3; all arrival rates are equal; $\beta^{(1)}=(1, 1, 1)$; $\sigma^{(1)}=(r, r, r)$; all switch-over times are Coxian distributed with squared coefficient of variation α. For all these models, we have $D^*=\{1, 2, 3\}$ (cf. [13]). Tables 5.1 and 5.2 show the waiting cost (27) for various values of ρ and α.

Tables 5.1 and 5.2 show that the performance of the system can be strongly improved by allowing the server to rest when the entire system is empty. Moreover, it is shown that the relative decrease of the waiting cost decreases when the offered load to the system is increased. The latter is due to the fact that the opportunity of resting occurs less frequently when the offered load is increased.

In order to examine the influence of the mean switch-over times on the relative improvements that can be made by resting, we have computed the performance of a system with the following set of parameters: s=3; a=(0.50,0.25,0.25); $\beta^{(1)}$=(1, 1, 1); $\sigma^{(1)}$=(2r, r, r), for various values of ρ and r. Tables 5.3 and 5.4 show the performance measures for **q**=(0, 0, 0) and **q**=(1, 1, 1) respectively. Tables 5.3 and 5.4 illustrate that when the mean switch-over times are increased, starting with σ_1(=4r)=0, the relative improvement of the performance increases for low values of σ_1 and decreases for higher values of σ_1 . This phenomenon is due to the trade-off between the fact that possible improvements would increase for increasing values of σ_1 on the one hand and the fact that the opportunity of resting occurs less frequently for increasing values of σ_1 on the other hand.

In order to examine the effect of the variability of the switch-over times, consider the same model as in Table 5.4, with r=0.25, so that $\sigma^{(1)}$=(0.50,0.25,0.25). Moreover, we assume that the switch-over times to move from Q_2 to Q_3 and from Q_3 to Q_1 are exponentially distributed and that the times needed by the server to move from Q_1 to Q_2 are Coxian distributed with squared coefficient of variation α. Table 5.5 shows the values of the performance measures for various values of ρ and α. Table 5.5 illustrates that the benefit of the opportunity of resting increases considerably with increasing variability of the switch-over times.

Finally, we examine the effect of the asymmetry in the arrival rates. To this end, consider the polling model with the following set of parameters: s=3; a=(γ/(γ+2),1/(γ+2),1/(γ+2)), so that the ratios between the arrival rates are γ:1:1; $\beta^{(1)}$=(1, 1, 1); all service times are exponentially distributed; $\sigma^{(1)}$=(0.50,0.25,0.25); the switch-over times from Q_1 to Q_2 are Coxian distributed with squared coefficient of variation 4.00; all other switch-over times are exponentially distributed. Table 5.6 shows the performance of the system for various values of ρ and γ. Table 5.6 illustrates that the benefit of the opportunity of resting increases with increasing asymmetry of the arrival process.

We emphasize that, in practice, the variability of the switch-over times will generally be smaller than in the examples considered in this section. However, we have constructed the examples such that the various effects are illustrated clearly. In general, the benefits of the possibility of resting decrease with decreasing variability of the switch-over times (cf. Table 5.5 and [7]).

6. Optimization

In this section we consider the question at which queues the server should be allowed to rest when the system is empty or, in other words, the problem of determining an optimal dormant set D*. This problem might be solved numerically either by formulating the problem as a semi-Markov decision problem (cf. [7] (Remark 4.1)) or by complete enumeration of the performance measure for all 2^s subsets of D. However, the time and memory requirements of both approaches increase exponentially with the number of queues, so that their use is restricted to fairly small systems. For this reason, we propose and test in this section a simple heuristic approximation, for which the time requirements are negligible and which yields fairly accurate results over a wide range of admissible system parameters. This heuristic approximation is based on the observation that the choice of an appropriate dormant set D is most critical in light-traffic systems. Motivated by this observation, we propose to approximate the optimal dormant set D* by

$$D^*(app) := \lim_{\rho\downarrow 0} D^*, \tag{29}$$

i.e., the light-traffic limit of D*. Denote by π_h the light-traffic limit of the probability that the server is dormant at Q_h , h=1,...,s; here, the limits are taken in such a way that the ratios between the arrival rates are kept fixed (cf. (2)). Note that $\pi_h=b(0;\mathbf{0},h,-1)$, $h\in D$ (cf. (26)).
Then the light-traffic limit of the waiting cost function (27) can be expressed as follows: for $D\neq\varnothing$,

$$\lim_{\rho\downarrow 0} C(D) = \sum_{i=1}^{s} c_i \left[\sum_{\substack{h\in D\\ h\neq i}} \pi_h \sum_{k=h+1}^{i} \sigma_k^{(1)} \right] = \sum_{h\in D} \pi_h \left[\sum_{\substack{i=1\\ i\neq h}}^{s} c_i \sum_{k=h+1}^{i} \sigma_k^{(1)} \right] =: \sum_{h\in D} \pi_h \gamma_h ; \tag{30}$$

and

$$\lim_{\rho\downarrow 0} C(\varnothing) = \frac{\sigma_2}{2\sigma_1} \sum_{i=1}^{s} c_i ; \tag{31}$$

here, indices k should be replaced by k mod s if k>s. Using the definition of γ_h (h=1,...,s) in (30), we define

$$S := \{ i\in 1,\dots,s \mid \gamma_i \le \gamma_j \, (j=1,\dots,s) \}. \tag{32}$$

Then one may verify that S indeed minimizes (30) (and (31)) over all subsets of {1,...,s}; to this end, it should also be noted that (31) can not be strictly smaller than the minimal value of (30). Therefore, we propose to approximate the optimal dormant set D* by D*(app):=S.

Remarks:

The index set S in (32) can be directly obtained by calculating γ_h , h=1,...,s, and selecting the indices h which yield the minimal value of γ_h . The time requirements to obtain D*(app)=S are negligible.

The set which minimizes the light-traffic limit of C(D) is not neccessarily uniquely determined; in fact, any non-empty subset of S, yields the optimal value in (30). However, we propose to approximate D* by S, rather than by any proper non-empty subset of S, because in the former case the proposed approximation is in agreement with the fact that in symmetrical systems with exhaustive service at all queues, the mean amount of work in the system (i.e. $c_i \equiv \rho_i$) is minimized for D*={1,...,s} (cf. [13]).

In order to check the quality of the heuristic approach presented above, we have computed D* and D*(app) for a fairly wide range of feasible system parameters.

First, we consider a number of 3-queue models with the following set of parameters: a=(0.8,0.4, 0.4); $\beta^{(1)}$=(0.5,0.5,1.0); $\sigma^{(1)}$=(2r,r,½r); the switch-over times are all Coxian distributed with squared coefficient of variation α; the weights of the cost function are (5, 1, 1). Table 6.1 shows the results for various values of α and r; here, the relative error "%" is defined by

$$\frac{C(D^*(app)) - C(D^*)}{C(D^*)} \times 100\% . \tag{31}$$

One may verify that we have D*(app)={1} in all considered cases.

Consider also the following 6-queue model: a=(0.5,0.1,0.1,0.1,0.1,0.1); $\beta^{(1)}$=(1, 1, 1, 1, 1, 1); $\sigma^{(1)}$=(0.5,0.3,0.5,0.3,0.5,0.3); the switch-over times from Q_2 to Q_3 , from Q_4 to Q_5 and from Q_6 to Q_1 are Coxian distributed with squared coefficient of variation 10; all other switch-over times are exponentially distributed; c=(0.375,0.125,0.125,0.125,0.125,0.125). Table 6.2 shows the results for various values of ρ and for **q**=(0, 0, 0) and **q**=(1, 1, 1). Note that in this model we also have D*(app)={1}.

Remarks:

Note that expression (30) depends on the switch-over times only through their means and moreover, that (30) is also independent of the Bernoulli vector **q**. As a consequence, the same insensitivity properties also hold for the approximated optimum D*(app), defined in (29). However, as illustrated in Table 5.4, the optimal dormant set may indeed depend on the variability of the switch-over times (cf. Tables 6.1 and 6.2). Hence, the accuracy of the approximation may become poor when the variability of the switch-over times is very large. In practice, however, the variability of the switch-over times is generally smaller than in the examples discussed in this section.

In this section we have considered the problem of finding an optimal dormant set for given Bernoulli service disciplines at the queues. Alternatively, one may also consider the problem of determining an optimal set of Bernoulli parameters for a given dormant set. Properties of the

Bernoulli schedule which minimizes C(∅) (cf. (27)) are discussed in Blanc and van der Mei [4]. In order to determine an optimal set of Bernoulli parameters numerically, they propose to use the PSA, which has been extended towards the computation of derivatives of performance measures w.r.t. the Bernoulli parameters in [5]; we emphasize that the latter approach can be directly extended to the model discussed in this paper.

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	α=1.00			α=5.00			α=20.00		
ρ	C(∅)	C(D*)	%	C(∅)	C(D*)	%	C(∅)	C(D*)	%
0.10	1.04	0.69	34.1	2.17	0.81	65.5	5.92	0.95	83.9
0.30	1.52	1.21	20.1	2.64	1.42	46.2	6.39	2.10	67.2
0.50	2.38	2.13	10.2	3.50	2.52	28.1	7.25	3.74	48.5
0.70	4.38	4.21	3.9	5.50	4.79	13.0	9.25	6.62	28.4
0.90	14.38	14.29	0.6	15.50	15.12	2.5	19.25	17.83	7.4
0.95	29.38	29.32	0.2	30.50	30.24	0.9	34.25	33.31	2.7

Table 5.1 Comparison between dormant and non-dormant server systems; r=0.50.

	α=1.00			α=5.00			α=20.00		
ρ	C(∅)	C(D*)	%	C(∅)	C(D*)	%	C(∅)	C(D*)	%
0.10	3.83	2.64	31.1	8.33	3.55	57.5	23.33	6.45	72.4
0.30	4.79	4.17	13.0	9.29	6.55	29.5	24.29	14.05	42.2
0.50	6.50	6.25	3.8	11.00	9.62	12.5	26.00	20.29	22.0
0.70	10.50	10.43	0.6	15.00	14.45	3.7	30.00	27.42	8.6
0.90	30.50	30.50	0.0	35.00	34.92	0.3	50.00	48.61	2.8
0.95	60.50	60.50	0.0	65.00	64.99	0.1	80.00	77.93	2.6

Table 5.2 Comparison between dormant and non-dormant server systems; r=2.00.

	ρ=0.30				ρ=0.70			
r	C(∅)	D*	C(D*)	%	C(∅)	D*	C(D*)	%
0.05	0.64	{1}	0.55	15.3	3.47	{1}	3.37	2.7
0.25	1.69	{1}	1.22	27.7	∞	–	∞	–
0.50	3.65	{1}	2.79	23.6	∞	–	∞	–
1.00	19.92	{1}	18.63	6.5	∞	–	∞	–
1.50	∞	–	∞	–	∞	–	∞	–
2.00	∞	–	∞	–	∞	–	∞	–

*Table 5.3 Influence of the mean switch-over times; **q**=(0, 0, 0).*

	ρ=0.30				ρ=0.70			
r	C(∅)	D*	C(D*)	%	C(∅)	D*	C(D*)	%
0.05	0.59	{1}	0.50	16.3	2.62	{1}	2.53	3.5
0.25	1.25	{1}	0.84	33.1	3.75	{1}	3.49	7.1
0.50	2.07	{1}	1.39	33.0	5.17	{1,2}	4.88	5.6
1.00	3.71	{1}	2.78	25.3	8.00	{1,2,3}	7.79	2.6
1.50	5.36	{1,2}	4.34	19.0	10.83	{1,2,3}	10.70	1.3
2.00	7.00	{1,2}	5.99	14.4	13.67	{1,2,3}	13.58	0.6

Table 5.4 Influence of the mean switch-over times; $\mathbf{q}=(1,\ 1,\ 1)$.

	ρ=0.30				ρ=0.70			
α	C(∅)	D*	C(D*)	%	C(∅)	D*	C(D*)	%
0.25	1.16	{1}	0.82	29.1	3.66	{1}	3.44	5.9
0.50	1.19	{1}	0.82	30.5	3.69	{1}	3.46	6.2
1.00	1.25	{1}	0.84	33.2	3.75	{1}	3.49	7.1
5.00	1.75	{1}	0.92	47.5	4.25	{1,2}	3.70	12.9
10.00	2.38	{1}	1.02	60.1	4.89	{1,2,3}	3.95	19.1
20.00	3.63	{1,2}	1.20	67.0	6.13	{1,2,3}	4.41	28.0

Table 5.5 Influence of the variability of the switch-over times.

	ρ=0.30				ρ=0.70			
γ	C(∅)	D*	C(D*)	%	C(∅)	D*	C(D*)	%
0.25	1.62	{2}	0.91	37.4	4.09	{2,3}	3.60	11.8
1.00	1.64	{1}	0.99	39.2	4.17	{1,2,3}	3.74	10.4
2.00	1.63	{1}	0.90	44.7	4.13	{1,2}	3.65	11.5
3.00	1.61	{1}	0.83	48.6	4.05	{1}	3.51	13.2
5.00	1.59	{1}	0.73	53.8	3.92	{1}	3.29	16.1
10.00	1.55	{1}	0.62	60.1	3.74	{1}	2.98	20.3

Table 5.6 Influence of the asymmetry in the arrival rates.

ρ	r	α	D*	C(D*)	C(D*(app))	%
0.1	0.01	0.25	{1}	0.58	0.58	0.0
0.1	0.01	10.00	{1}	0.59	0.59	0.0
0.1	0.15	0.25	{1}	1.18	1.18	0.0
0.1	0.15	10.00	{1}	1.51	1.51	0.0
0.1	0.50	0.25	{1}	3.18	3.18	0.0
0.1	0.50	10.00	{1}	6.63	6.63	0.0
0.5	0.01	0.25	{1}	5.05	5.05	0.0
0.5	0.01	10.00	{1}	5.06	5.06	0.0
0.5	0.15	0.25	{1}	7.13	7.13	0.0
0.5	0.15	10.00	{1,2}	8.80	8.94	1.6
0.5	0.50	0.25	{1}	13.98	13.98	0.0
0.5	0.50	10.00	{1,2,3}	25.85	27.64	6.9
0.8	0.01	0.25	{1}	19.84	19.84	0.0
0.8	0.01	10.00	{1}	19.86	19.86	0.0
0.8	0.15	0.25	{1}	25.52	25.52	0.0
0.8	0.15	10.00	{1,2,3}	28.49	28.85	1.3
0.8	0.50	0.25	{1,2}	41.18	41.22	0.1
0.8	0.50	10.00	{1,2}	60.25	61.19	1.6

Table 6.1 The accuracy of C(D(app)) versus C(D*); s=3.*

q=(0, 0, 0)					**q**=(1,1,1)			
ρ	D*	C(D*)	C(D*(app))	%	D*	C(D*)	C(D*(app))	%
0.1	{1}	1.48	1.48	0.0	{1}	1.32	1.32	0.0
0.3	{2,6}	5.99	5.99	0.1	{1,3–6}	2.61	2.67	2.3
0.5	–	∞	∞	–	{1–6}	4.44	4.52	1.7
0.7	–	∞	∞	–	{1–6}	8.15	8.21	0.7
0.8	–	∞	∞	–	{1–6}	12.51	12.55	0.3

Table 6.2 The accuracy of C(D(app)) versus C(D*); s=6.*

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