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A Comment on Shaked and Sutton's Model of Vertical Product Differentiation

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May 6, 1996

Abstract

In a duopoly model with vertical differentiation, it is assumed that two firms play a two-stage non-cooperative game, first quality-then-price, and there is a feasible quality spectrum from which two firms can choose for their product selection. A taxonomy of the degree of product differentiation is pursued. We demonstrate that there exists a unique subgame perfect equilibrium in pure strategies. This equilibrium exhibits maximum product differentiation if the quality of the 'outside good' is sufficiently low or if both the quality of the outside good is sufficiently high and the difference between it and the lowest feasible quality is sufficiently large; otherwise if the quality of the outside good is sufficiently high but the difference between it and the lowest feasible quality is sufficiently small, then this equilibrium exhibits some degree of product differentiation.

Keywords: *quality-then-price game, vertically differentiated market, product differentiation.*

1 Introduction

In their pathbreaking paper, Shaked and Sutton (1982) demonstrate how the existence of quality differences relaxes price competition between competing firms, so that profits are positive in equilibrium. Quality differences are formalized in terms of a framework for preferences due to Gabszewicz and Thisse (1979) in which individuals with identical preferences may, nevertheless, choose different goods because their respective marginal utilities of income differ.

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While Shaked and Sutton (1982) focus on relaxing price competition through product differentiation, the uniqueness of this subgame perfect equilibrium is not shown, and most importantly, the degree of product differentiation is not analyzed there. Donnenfeld and Weber (1992) solve the degree of product differentiation problem by introducing entry into Shaked and Sutton's (1982) model. They consider a slightly modified Shaked and Sutton (1982) model, in which every thing remains except that $Q \equiv [q_m, q_M]$ is assumed to be the feasible quality interval, the quality of outside good is set zero, and a consumer is identified by the his or her income, which is uniformly distributed over the interval $[0, I]$. They show that there exists a unique equilibrium, at which

> "... the first two firms to enter the industry, select the extreme qualities that are technologically feasible, thus exhibiting maximal product differentiation among incumbents".

Therefore

> "... the threat of later entry further increases the degree of product differentiation."

We focus on duopoly case in this paper, and extend Shaked and Sutton's discussion to give an explicit proof of the existence of a unique subgame perfect equilibrium. We extend the work of Shaked and Sutton (1982) by the following main results. First, we show that the subgame perfect equilibrium not only exists, but is also unique. Secondly, the unique subgame perfect equilibrium exhibits maximum product differentiation even in a model without entrants provided that the quality of the outside good is sufficiently low or both the quality of the outside good is sufficiently high and the difference between it and the lowest feasible quality is sufficiently large; otherwise if the quality of the outside good is sufficiently large but the difference between it and the lowest feasible quality is sufficiently small this equilibrium exhibits some degree of product differentiation. This specifies the result of Shaked and Sutton (1982), in which it is demonstrated that at any subgame perfect equilibrium one firm chooses the highest available quality while the other firm chooses an available quality somewhere between the two quality extremes. Finally, we show that Shaked and Sutton's proof of the existence of subgame perfect equilibria is incomplete[1], in the sense that they do not include the monopoly case into their consideration when they derive the equilibrium prices at the second stage of the game.

[1]To put it in another way, we argue that without exclusion of the case in which a subgame perfect equilibrium of the two-stage game results in a monopoly and creates a higher monopoly profit than any one of the duopoly profits, a subgame perfect equilibrium derived from the assumption that two firms exist and segment the market as did in Shaked and Sutton (1982) may not be true.

The remainder of the paper is organized as follows. The model is described in Section 2. In Section 3 we give a complete proof of the existence of the unique subgame perfect equilibrium for the quality-then-price game and a taxonomy of the degree of product differentiation in this equilibrium. Section 4 concludes.

2 The model

We employ a variant of the Shaked and Sutton (1982) model. Suppose some good can be produced in a continuous range of quality levels, represented by a technologically feasible interval $Q = [q_m, q_M]$, where $0 \leq q_0 < q_m < q_M < +\infty$, q_0 being the quality of the outside good, q_m being the lowest possible quality level and q_M being the highest one. We differ from Shaked and Sutton by using a lower bound q_m of the feasible quality interval which is independent of the quality of the outside good while they use the latter as the lower bound of the quality interval. There are two firms in the industry, each producing a single quality at zero costs. The firms play a two-stage game, first quality, then price, and compete for consumers by offering packages of price and quality $(p_i, q_i), i \in I = \{1, 2\}$. With $q_1 = q_2$, Bertrand competition results zero prices and profits for both firms, and this is obviously not an interesting case. So, we assume away it in this paper, and let $q_1 < q_2$. The prices are in terms of the numeraire good.

A continuum of consumers is identical in taste but differs in income. Income t is uniformly distributed on an interval $[a, b]$ where $0 < a < b < +\infty$.

Consumers make indivisible and mutually exclusive purchases from the interval of qualities Q, in the sense that a consumer either makes no purchase, or else buys exactly one unit of the product from either suppliers. If a consumer with income t buys one unit of the commodity from firm $i \in I$ with quality $q_i \in Q$ at price p_i, his utility is given by

$$U_i(t, q_i, p_i) = q_i(t - p_i)$$

where $t - p_i$ is the consumer's disposable income devoted to the consumption of the numeraire good after the purchase of the differentiated good of quality q_i. Each consumer buys from the firm by maximizing his or her utility. If a consumer does not buy his or her utility is given by consuming the outside good, i.e., $U_0(t, q_0, 0) = q_0 t$. This specification of the consumers' utility functions implies that individuals with higher income enjoy quality improvement more than low income consumers. The market area of the product of firm $i \neq j \in I$ at qualities q_i and q_j and at prices p_i and p_j is therefore given by

$$M_i(q_i, q_j, p_i, p_j) = \{t \in [a, b] \mid U_i(t, q_i, p_i) \geq max\{0, U_j(t, q_j, p_j)\}\},$$

i.e., the set of consumers that prefer to buy from firm i.

At qualities q_i and q_j and at prices p_i and p_j, $i \neq j$, and $i,j \in I$, the demand $D_i(q_i, q_j, p_i, p_j)$ for the commodity of firm $i \in I$ is equal to

$$D_i(q_i, q_j, p_i, p_j) = \int_{M_i(q_i, q_j, p_i, p_j)} dt.$$

In Figure 1 we give an interpretation of the market segmentation between Firm 1 and Firm 2 in case $p_1 \leq p_2$ and $q_0 = 0$, where $t_{12} \in [a, b]$ denotes the marginal consumer who is indifferent between buying from firm 1 and buying from firm 2. For a concrete and complete description of the firms' demand functions we refer to the Appendix.

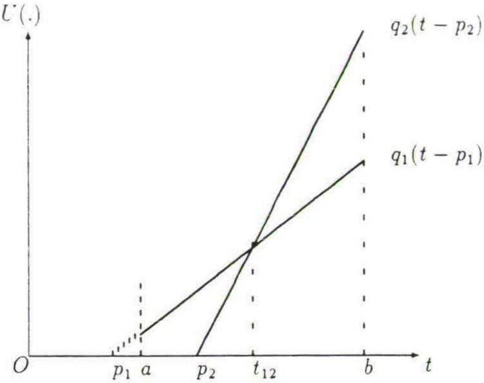


Figure 1: $p_1 \leq a \leq p_2$ and $q_0 = 0$

The corresponding profits are

$$\pi_1(q_1, q_2, p_1, p_2) = p_1 D_1(p_1, p_2, q_1, q_2)$$

for firm 1 and

$$\pi_2(q_1, q_2, p_1, p_2) = p_2 D_2(p_1, p_2, q_1, q_2)$$

for firm 2.

3 Quality and price competition

It has been shown in Shaked and Sutton (1982) that for given product quality specification, if the income distribution interval $[a, b]$ satisfies the condition $2a < b < 4a$, then of any N firms offering distinct products, exactly two will have positive market shares at the equilibrium of price competition. Moreover, at equilibrium, the market is covered, i.e., no consumer stays out of the market.

Our work in this section is to extend the above result. We focus on the duopoly case, and explicitly prove that under the assumption of $2a < b < 4a$ there is a unique subgame perfect equilibrium, at which the two firms maximally differentiate their products if the quality of the outside good is sufficiently low or if both the quality of the outside good is sufficiently high and the difference between it and the lowest feasible quality is sufficiently large; otherwise if the quality of the outside good is sufficiently large but the difference between it and the lowest feasible quality is sufficiently small some degree of product differentiation arises.

3.1 The case $q_0 = 0$

Lemma 1 *Suppose $q_0 = 0$ and $2a < b < 4a$. Then for any given quality specification q_1 and q_2 in Q of firm 1 and firm 2, respectively, the Nash-Bertrand equilibrium (p_1^N, p_2^N) at the second stage of the game is such that both firms are in the market and*

$$\begin{array}{lll} p_1^N = \frac{q_2-q_1}{3q_1}(b-2a) & \text{and } p_2^N = \frac{q_2-q_1}{3q_2}(2b-a) & \text{if } q_1/q_2 \geq \frac{b-2a}{b+a}; \\ p_1^N = a & \text{and } p_2^N = \frac{b(q_2-q_1)+aq_1}{2q_2} & \text{if } \frac{b-2a}{b+a} \geq q_1/q_2 \end{array}$$

Proof. The proof is divided into two parts.

(i) First we prove that for any given quality specification, any Nash-Bertrand equilibrium at the second stage of the game, if it exists, denoted by (p_1^N, p_2^N), can not happen at the case when one of the two firms stays out of the market.

It is straightforward to prove that firm 2 can not stay out of the market at (p_1^N, p_2^N), because firm 2 can always set a price $p_2' = p_1^N$ and take over the market from firm 1 or at worst share the market with firm 1. Next we need to prove that firm 1 can not stay out of the market at (p_1^N, p_2^N), for which we follow a graphical proof and distinct between three cases:

<u>case 1</u>: $p_2^N < p_1^N$. Then $D_1(q_1, q_2, p_1^N, p_2^N) = 0$, and so $\pi_1(q_1, q_2, p_1^N, p_2^N) = 0$.

From (a) of Figure 2, it is found that if $p_2^N < a$, then firm 2 has an incentive to deviate from p_2^N, because by charging an infinitesimal higher price, its demand is not affected, and consequently its profit is increased. Similarly from (b) of Figure 2, it is found that if $p_2^N \geq a$, then firm 1 has an incentive to deviate from p_1^N by setting its price at p_1' with $0 < p_1' < a$, because then firm 1 gains a positive market share.

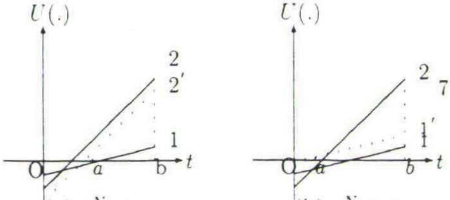
case 2: $p_2^N = p_1^N$. Then $D_1(q_1, q_2, p_1^N, p_2^N) = 0$, and so $\pi_1(q_1, q_2, p_1^N, p_2^N) = 0$.

From (a) of Figure 3 it is found that if $p_2^N \geq a$, then firm 1 has an incentive to deviate by setting its price at p_1' with $0 < p_1' < a$, because then firm 1 gains a positive market share. Similarly from (b) of Figure 3, it is found that if $p_2^N < a$, firm 2 has an incentive to deviate, because by charging an infinitesimal higher price, its demand is not affected, and consequently its profit is increased.

case 3: $p_2^N > p_1^N$. Then firm 1 stays out of the market if and only if $t_{12} \leq a$, and so $\pi_1(q_1, q_2, p_1^N, p_2^N) = 0$.

From (a) of Figure 4 it is found that if $t_{12} < a$, then firm 2 has an incentive to deviate, because by charging an infinitesimal higher price, its demand is not affected, and consequently its profit is increased. If $t_{12} = a$ and $p_1^N > 0$, then firm 1 has an incentive to deviate, because then firm 1 gains a positive market share, so a positive profit. Otherwise if $t_{12} = a$ but $p_1^N = 0$, then firm 2 has an incentive to deviate. In fact, if firm 2 does not deviate, its profit π^{nd} is given by $\pi^{nd} = p_2^N(b - t_{12})$, where $t_{12} = a$ satisfies the equation $q_1(t_{12} - 0) = q_2(t_{12} - p_2^N)$. Solving the equation for p_2^N and then substituting p_2^N and t_{12} into the equation for π^{nd}, we derive the profit $\pi^{nd} = a(b-a)(q_2 - q_1)/q_2$ for firm 2 in case it does not deviate. But if firm 2 deviates by maximizing its profit $p_2(b - t_{12}')$, where t_{12}' is given by the equation $q_1(t_{12}' - 0) = q_2(t_{12}' - p_2)$, then its profit π^d equals $\pi^d = b^2(q_2 - q_1)/(4q_2)$. For $2a < b$, we have $\pi^d > \pi^{nd}$, so firm 2 has an incentive to deviate.

(ii) Secondly we prove that in case of a duopoly the Nash-Bertrand equilibrium exists and is as given in the lemma. This directly follows from the proof of lemma 2 of Shaked and Sutton (1982).

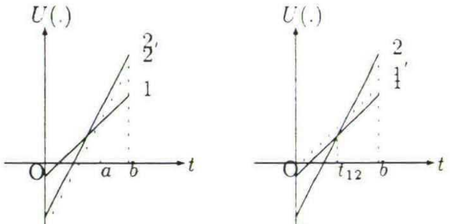


(a) $p_2^N < a$ (b) $p_2^N \geq a$

Figure 2. $p_2^N < p_1^N$

(a) $p_2^N \geq a$ (b) $p_2^N < a$

Figure 3. $p_2^N = p_1^N$



(a) $t_{12} < a$ (b) $t_{12} = a$

Figure 4. $p_2^N > p_1^N$ □

Consider the profit of firm i, $i \in \{1, 2\}$, at the first stage conditional on the quality specification q_i, $i \in \{1, 2\}$. From Lemma 1 we can calculate the firms' profits associated with the equilibrium prices as follows

$$\pi_1^N(q_1, q_2) = \frac{q_2 - q_1}{9q_1}(b - 2a)^2 \quad \text{and} \quad \pi_2^N(q_1, q_2) = \frac{q_2 - q_1}{9q_2}(2b - a)^2$$

if $q_1/q_2 \geq \frac{b-2a}{b+a}$; otherwise,

$$\pi_1^N(q_1, q_2) = \frac{a}{2}(b - a - \frac{aq_2}{q_2 - q_1}) \quad \text{and} \quad \pi_2^N(q_1, q_2) = \frac{b(q_2 - q_1) + aq_1}{4q_2}(b + a\frac{q_1}{q_2 - q_1})$$

From now on let $\underline{\alpha} = q_m/q_M$ and $\alpha = q_1/q_2$, so $0 < \underline{\alpha} \leq \alpha \leq 1$. Then α denotes the degree of product differentiation. The firm's profit associated with the equilibrium prices can be rewritten as follows:

If $\alpha \geq \frac{b-2a}{b+a}$, then

$$\pi_1^N(\alpha) = \frac{(1-\alpha)(b-2a)^2}{9\alpha} \quad (1)$$

$$\pi_2^N(\alpha) = \frac{(1-\alpha)(2b-a)^2}{9}. \quad (2)$$

Otherwise if $\frac{b-2a}{b+a} \geq \alpha$, then

$$\pi_1^N(\alpha) = \frac{a}{2}(b-a-\frac{a}{1-\alpha}) \quad (3)$$

$$\pi_2^N(\alpha) = \frac{1}{4}(b(1-\alpha)+a\alpha)(b-a+\frac{a}{1-\alpha}). \quad (4)$$

To establish our proposition we first claim the following

Lemma 2 *The firms' reduced profit function forms satisfy*

$\frac{\partial \pi_i^N(\alpha)}{\partial \alpha} < 0$ *for every* $\alpha \in (\underline{\alpha}, 1]$ *and* $i \in \{1,2\}$.

Proof.

Differentiating the firms' reduced profit function forms in qualities of equations (1), (2) and (3) with respect to α, it is straightforward to prove that $\frac{\partial \pi_1^N(\alpha)}{\partial \alpha} < 0$ for all α and $\frac{\partial \pi_2^N(\alpha)}{\partial \alpha} < 0$ for $\alpha \geq \frac{b-2a}{b+a}$. Then we are left with proving that $\frac{\partial \pi_2^N(\alpha)}{\partial \alpha} < 0$ for $\frac{b-2a}{b+a} \geq \alpha \geq \underline{\alpha}$. Differentiation of $\pi_2^N(\alpha)$ in (4) with respect to α yields

$$\frac{\partial \pi_2^N(\alpha)}{\partial \alpha} = \frac{1}{4}[(a-b)(b+\frac{a\alpha}{1-\alpha})+(b(1-\alpha)+a\alpha)\frac{a}{(1-\alpha)^2}] = -\frac{1}{4}[(b-a)(b+\frac{a\alpha(2-\alpha)}{(1-\alpha)^2})-\frac{ab}{(1-\alpha)^2}].$$

To prove that $\frac{\partial \pi_2^N(\alpha)}{\partial \alpha} < 0$, it is equivalent to prove that $(b-a)(b+\frac{a\alpha(2-\alpha)}{(1-\alpha)^2}) > \frac{ab}{(1-\alpha)^2}$, which requires that $\alpha < \frac{2b-3a}{2(b-a)}$. But $\alpha \leq \frac{b-2a}{b+a}$, and for $b > 2a$ we have $\frac{2b-3a}{2(b-a)} > \frac{b-2a}{b+a}$, so $\frac{\partial \pi_2^N(\alpha)}{\partial \alpha} < 0$.

□

Given Lemma 2, it is straightforward to establish the following

Proposition 1 *Suppose* $q_0 = 0$ *and* $2a < b < 4a$. *Then there exists a unique subgame perfect equilibrium when firms act non-cooperatively in first choosing qualities then prices, in which the two firms maximally differentiate their product specifications, so* $q_1^N = q_m$ *and* $q_2^N = q_M$.

Proof.

Consider the first stage of the quality-then-price game. The firms' reduced profit functions are given in equations (1) and (2) or (3) and (4). From Lemma 2 firm 2 always benefits by leaving its product quality from any given product quality of firm 1 as far as possible. So. $q_2^N = q_M$. Similarly from Lemma 2 firm 1 always benefits by keeping its product quality as far from any given product quality of firm 2 as possible, so $q_1^N = q_m$.

□

3.2 The case $q_0 > 0$

Lemma 3 *Suppose $q_0 > 0$ and $2a < b < 4a$. Then for any given quality specification q_1 and q_2 in Q of firm 1 and firm 2, respectively, the Nash-Bertrand equilibrium (p_1^N, p_2^N) at the second stage of the game is such that both firms are in the market and*

$$p_1^N = \frac{b-2a}{3}\frac{q_2-q_1}{q_1} \quad \text{and } p_2^N = \frac{2b-a}{3}\frac{q_2-q_1}{q_2} \qquad \text{if } \frac{q_2-q_0}{q_2-q_1} \geq \frac{b+a}{3a};$$
$$p_1^N = a\frac{q_1-q_0}{q_1} \quad \text{and } p_2^N = \frac{1}{2q_2}[b(q_2-q_1)+a(q_1-q_0)] \quad \text{otherwise.}$$

Proof.

The proof directely follows the proof of Lemma 1.

□

Given the Nash-Bertrand equilibrium (p_1^N, p_2^N) at the second stage of the game, we can calculate the firms' profits associated with the equilibrium prices as follows

$$\pi_1^N(q_1, q_2) = \frac{(b-2a)^2}{9}\frac{q_2-q_1}{q_1} \quad \text{and} \quad \pi_2^N(q_1, q_2) = \frac{(2b-a)^2}{9}\frac{q_2-q_1}{q_2}$$

if $\frac{q_2-q_0}{q_2-q_1} \geq \frac{b+a}{3a}$; otherwise,

$$\pi_1^N(q_1, q_2) = \frac{a}{2}\frac{q_1-q_0}{q_1}\left(b - a\frac{2q_2-q_1-q_0}{q_2-q_1}\right)$$

and

$$\pi_2^N(q_1, q_2) = \frac{1}{4q_2(q_2-q_1)}[b(q_2-q_1)+a(q_1-q_0)]^2.$$

From the above profit functions of firm 1 and firm 2 associated with the equilibrium prices it is straightforward to show through simple calculation that the following holds

$$\partial\pi_1^N(q_1, q_2)/\partial q_1 < 0$$

for any given $q_1, q_2 \in [q_m, q_M]$ satisfying $\frac{q_2-q_0}{q_2-q_1} \geq \frac{b+a}{3a}$, and

$$\partial\pi_2^N(q_1, q_2)/\partial q_2 > 0$$

for any given $q_1, q_2 \in [q_m, q_M]$ if $\frac{q_m}{q_M} \geq \frac{b-2a}{2b-a}$, i.e., the gap between the highest feasible quality and the lowest feasible quality is not 'too big'.

So, to focus on the degree of product differentiation in the case for $q_0 > 0$ is equivalent to focus on specifying the sign of $\partial\pi_1^N(q_1, q_2)/\partial q_1$ for any given $q_1, q_2 \in [q_m, q_M]$ satisfying $\frac{q_2-q_0}{q_2-q_1} \leq \frac{b+a}{3a}$. The following lemma gives a characterization of this sign.

Lemma 4 *Suppose $q_0 > 0$ and $2a < b < 4a$. Then for any given quality specification q_1 and q_2 of firm 1 and firm 2, respectively, satisfying $q_1, q_2 \in [q_m, q_M]$ and $\frac{q_2-q_0}{q_2-q_1} \leq \frac{b+a}{3a}$, there exists $\underline{q}, \overline{q} > 0$ with $\underline{q} < \overline{q}$ such that*

$$\partial\pi_1^N(q_1, q_2)/\partial q_1 < 0 \quad \text{if } q_0 < \underline{q},$$
$$\partial\pi_1^N(q_1, q_2)/\partial q_1 > 0 \quad \text{if } q_0 > \overline{q},$$

whereas for $q_0 \in [\underline{q}, \overline{q}]$ the sign of $\partial\pi_1^N(q_1, q_2)/\partial q_1$ is ambiguous.

Proof.

Given $q_1, q_2 \in [q_m, q_M]$ and $\frac{q_2-q_0}{q_2-q_1} \leq \frac{b+a}{3a}$, we have from the above derived profit function of firm 1 associated with its equilibrium price that

$$\pi_1^N(q_1, q_2) = \frac{a}{2}\frac{q_1-q_0}{q_1}(b - a\frac{2q_2-q_1-q_0}{q_2-q_1}) = \frac{a^2}{2}\frac{q_1-q_0}{q_1}(\frac{b}{a} + \frac{q_1+q_0-2q_2}{q_2-q_1}).$$

Then

$$\begin{aligned}\partial\pi_1^N(q_1, q_2)/\partial q_1 &= \frac{a^2}{2}[\frac{q_0}{q_1^2}(\frac{b}{a} + \frac{q_0+q_1-2q_2}{q_2-q_1}) + (1-\frac{q_0}{q_1})(\frac{q_2-q_1+q_0+q_1-2q_2}{(q_2-q_1)^2})] \\ &= \frac{a^2}{2q_1^2(q_2-q_1)^2}[q_1^2(\frac{b}{a}q_0 - q_2) + q_1((4-\frac{2b}{a})q_0q_2 - 2q_0^2) + q_2q_0^2 + (\frac{b}{a}-2)q_0q_2^2].\end{aligned}$$

Thus we have

$$\partial[\partial\pi_1^N(q_1, q_2)/\partial q_1|_{\frac{b}{a}=4}]/\partial q_0 > 0 \tag{5}$$

and

$$(\partial\pi_1^N(q_1, q_2)/\partial q_1|_{\frac{b}{a}=4})_{q_0=q_2} > 0. \tag{6}$$

But from section 3.1 we have

$$(\partial\pi_1^N(q_1, q_2)/\partial q_1|_{\frac{b}{a}=4})_{q_0=0} < 0. \tag{7}$$

So. from (5), (6) and (7) there exists a $\underline{q}$ such that $(\partial\pi_1^N(q_1,q_2)/\partial q_1)_{\frac{b}{a}=4} < 0$ for $q_0 < \underline{q}$ and $(\partial\pi_1^N(q_1,q_2)/\partial q_1)_{\frac{b}{a}=4} > 0$ for $q_0 > \underline{q}$. Since $\partial\pi_1^N(q_1,q_2)/\partial q_1$ is increasing in $\frac{b}{a}$, we have $\partial\pi_1^N(q_1,q_2)/\partial q_1 < 0$ for $q_0 < \underline{q}$ and $2a < b < 4a$.
Similarly, we can derive the follows

$$\partial[\partial\pi_1^N(q_1,q_2)/\partial q_1|_{\frac{b}{a}=2}]/\partial q_0 > 0 \tag{8}$$

and

$$(\partial\pi_1^N(q_1,q_2)/\partial q_1|_{\frac{b}{a}=2})_{q_0=q_2} > 0. \tag{9}$$

But from section 3.1 we have

$$(\partial\pi_1^N(q_1,q_2)/\partial q_1|_{\frac{b}{a}=2})_{q_0=0} < 0. \tag{10}$$

From (8), (9) and (10) there exists a $\overline{q}$ such that $\partial\pi_1^N(q_1,q_2)/\partial q_1)_{\frac{b}{a}=2} < 0$ for $q_0 < \overline{q} < 0$ and $(\partial\pi_1^N(q_1,q_2)/\partial q_1)_{\frac{b}{a}=2}$ for $q_0 > \overline{q}$. Since $\partial\pi_1^N(q_1,q_2)/\partial q_1$ is increasing in $\frac{b}{a}$, we have $\partial\pi_1^N(q_1,q_2)/\partial q_1 > 0$ for $q_0 < \overline{q}$ and $2a < b < 4a$.

It should be clear that for $q_0 \in [\underline{q},\overline{q}]$ the sign of $\partial\pi_1^N(q_1,q_2)/\partial q_1$ is ambiguous.

□

Proposition 2 *Suppose* $q_0 > 0, 2a < b < 4a$, *and* $\frac{3m}{q_M} \geq \frac{b-2a}{2b-a}$. *Then there exists a unique subgame perfect equilibrium in pure strategies when firms act non-cooperatively in first choosing qualities then prices. There exist also* $\underline{q}$, $\overline{q}$ *and* q_1^* *with* $\underline{q} < \overline{q}$ *and* $q_1^* = q_M - \frac{3a}{b+a}(q_M - q_0)$ *such that maximum product differentiation holds in this equilibrium if* $q_0 < \underline{q}$ *or if* $q_0 > \overline{q}$ *and* $q_m \geq q_1^*$; *and some degree of product differentiation is exhibited in this equilibrium with firm 1 locating at* q_1^* *and firm 2 locating at* q_M *if* $q_0 > \overline{q}$ *and* $q_m < q_1^*$.

Proof.

From the statement above and Lemma 4 it is straightforward that for any given feasible quality interval $[q_m, q_M]$, firm 2 always chooses q_M for its product selection. And for firm 1 there exist $\underline{q}$ and $\overline{q}$ such that if $q_0 < \underline{q}$ (see also Figure 5 (a), without loss of generality, lines are used instead of curves, the same for Figure 5 (b)), then the profit

$\pi_1^N(q_1, q_M)$ of firm 1 is always decreasing in q_1, so firm 1 chooses q_m; otherwise if $q_0 > \overline{q}$ (see also Figure 5 (b)), then the profit function $\pi_1^N(q_1, q_M)$ of firm 1 is increasing in q_1 satisfying $\frac{q_M - q_0}{q_M - q_1} \leq \frac{b+a}{3a}$ (called region 1) but decreasing in q_1 satisfying $\frac{q_M - q_0}{q_M - q_1} \leq \frac{b+a}{3a}$ (called region 2). So, the profit $\pi_1^N(q_1, q_M)$ of firm 1 is maximized at the intersection of its profit curves from region 1 and 2, defined by $\frac{q_M - q_0}{q_M - q_1} = \frac{b+a}{3a}$, i.e., $q_1^* = q_M - \frac{3a}{b+a}(q_M - q_0)$. If $q_m \geq q_1^*$, then the profit function $\pi_1^N(q_1, q_M)$ of firm 1 is also always decreasing in q_1, so firm 1 chooses q_m. Otherwise if $q_m < q_1^*$ then firm maximizes its profit by choosing q_1^*. In any case the equilibrium is unique and subgame perfect.

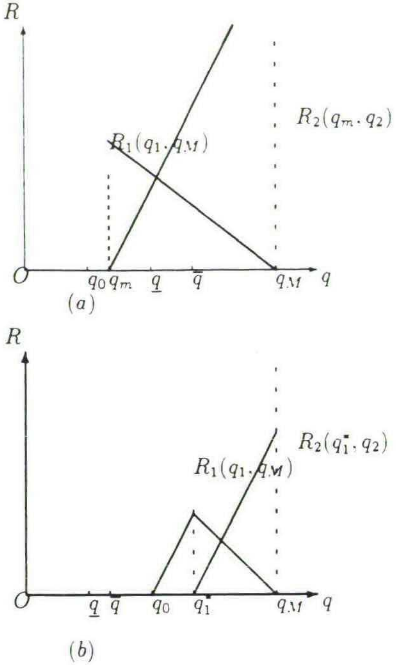


Figure 5

□

4 Conclusions

While Shaked and Sutton (1982) focus on relaxing price competition through product differentiation, the degree of product differentiation is not analyzed there. In this paper we focus on the degree of product differentiation, and demonstrate that firms maximally differentiate their products if the quality of the outside good is sufficiently low or if both the quality of the outside good is sufficiently high and the difference between it and the lowest feasible quality is sufficiently large; otherwise if the quality of the outside good is sufficiently high but the difference between it and the lowest feasible quality is sufficiently small this equilibrium exhibits some degree of product differentiation. Thus we give a taxonomy of product differentiation in Shaked and Sutton's framework in case $2a < b < 4a$. The reasons behind this product selection are quite intuitive. Because firm 2 enjoys higher quality advantage, it is always profitable for firm 2 to choose the highest possible quality. Therefore, we need only to consider the product selection of firm 1. If q_0 is sufficiently small, then price competition from the fall back good faced by firm 1 is very weak, and firm 1 finds it profitable to differentiate itself from firm 2 as far as possible, which yields maximum product differentiation. An alternative reason for the maximum product differentiation is found in case when q_0, the quality of the outside good, is sufficiently high, but the difference between q_0 and the lowest feasible quality is sufficiently large (in the sense that q_0 is on the left side of q_1^* while q_m is on the right side of q_1^*). Then price competition from the fall back good faced by firm 1 is offsetted by price competition of firm 2 from above and the isolation by q_1^* from the outside good (where q_1^* acts just like a fence which isolates price competition from the outside good). Finally, if the quality of the outside good is sufficiently high and the difference between it and the lowest feasible quality is not too large, price competition from the fall back good faced by firm 1 will outweigh price competition from firm 2 from above and the isolation from the outside good, thus forces firm 1 to the inside of the feasible quality interval, and some degree of product differentiation shows up. The above discussion in case $q_0 > 0$ crucially depends on the assumption that the gap between the highest feasible quality and the lowest quality is not too big.

Appendix

The demand functions of Firm 1 and Firm 2

We may distinguish three different types of indifferent consumers, namely a consumer being indifferent between buying from firm 1 and not buying at all, a consumer being indifferent between buying from firm 2 and not buying at all, and, finally, a consumer being indifferent between buying from firm 1 and buying from firm 2. We denote $t_{12} = \frac{p_2 q_2 - p_1 q_1}{q_2 - q_1}$ and $t_i = (\frac{q_i}{q_i - q_0})p_i$ for all $i \in I = \{1, 2\}$.

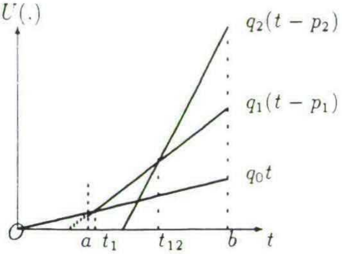


Figure 6 (a): $p_1 \leq a \leq p_2$, q_0 small

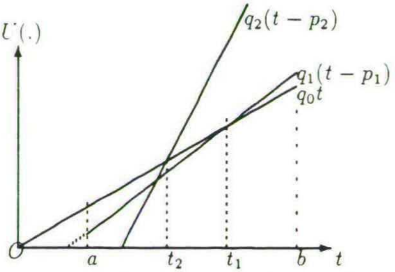


Figure 6 (b): $p_1 \leq a \leq p_2$, q_0 large

If q_0 is relatively small, as in Figure 6(a), there exists a consumer t_1 being indifferent between buying from firm 1 and not buying if $t_1 \geq a$, otherwise all consumers prefer to buy. Furthermore, there exists a consumer t_{12} being indifferent between buying from firm 1 and firm 2 if $a \leq t_{12} \leq b$.

If q_0 is relatively high, as in Figure 6(b), there does not exist a consumer t_1 being indifferent between buying from firm 1 and buying from firm 2. The reason is that all consumers prefer the outside option or the commodity of firm 2 to the commodity of firm 1. Clearly, there does exist a consumer t_2 being indifferent between buying from firm 2 and not buying if $t_2 \geq b$.

Consequently, the demand for the firms can be written as

$$D_1(q_1, q_2, p_1, p_2) = 0, \text{ for } p_1 \geq p_2.$$

and

$$D_1(q_1, q_2, p_1, p_2) = \begin{cases} t_{12} - t_1 & \text{if } a \leq t_1 \leq t_{12} \leq b \\ t_{12} - a & \text{if } t_1 \leq a \leq t_{12} \leq b \\ b - t_1 & \text{if } a \leq t_1 \leq b \leq t_{12} \quad \text{for } p_1 < p_2 \\ b - a & \text{if } t_1 \leq a, b \leq t_{12} \\ 0 & \text{otherwise,} \end{cases}$$

and

$$D_2(q_1, q_2, p_1, p_2) = \begin{cases} b - a & \text{if } a \geq t_2 \\ b - t_2 & \text{if } a \leq t_2 \leq b \quad \text{for } p_1 \geq p_2 \\ 0 & \text{otherwise,} \end{cases}$$

and

$$D_2(q_1, q_2, p_1, p_2) = \begin{cases} b - a & \text{if } a \geq max\{t_{12}, t_2\} \\ b - max\{t_{12}, t_2\} & \text{if } a \leq t_{12} \leq b \qquad \text{for } p_1 < p_2 \\ 0 & \text{otherwise.} \end{cases}$$

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