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ABSTRACT

This paper supposes that an individual cares about his own wealth not only directly but also via the relative standing this wealth induces in the wealth distribution. The implications of this second effect for risk–taking are investigated in particular. Such a model indeed provides a very natural explanation of the "concave–convex–concave" utility described by Friedman and Savage. The status interaction effect here involves an externality, the full treatment of which requires a discussion of Pareto efficiency. It is shown that a certain limited kind of gambling is needed in order to attain the entire utility possibility frontier and, indeed, that it is not true that any deterministic wealth distribution is always Pareto–efficient. It is now shown that banning individually rational gambling will generally raise welfare when the original distribution of wealth is Pareto efficient. It is further shown that such gambling may well be Pareto inefficient. The appropriate tax/subsidy has the property that poorer individuals tend to be taxed whereas richer individuals tend to be subsidized.

1. INTRODUCTION AND SUMMARY

The notion that an individual might care not only about his own wealth but also about his relative standing in the distribution of wealth has a long but checkered history. (An early exponent of such ideas was, of course, Veblen, 1899.) It is indeed a notion which most economists reject, albeit usually without explicit comment. (A notable and articulate exception, however, is Frank, 1985. See also the references cited on pp. 33–34.) It is, however, a notion with substantial intuitive appeal and it seems useful to ascertain its consequences before coming to a final judgement as to its merits.

The present paper is concerned to derive the consequences of such valuation of status for risk taking. What seems to be the sharpest possible model is adopted here. This assumes that ordinal rank in the wealth distribution enters von Neumann–Morgenstern utility as an argument in addition to wealth itself. Thus higher wealth increases utility directly but also indirectly via higher status. In the interests of both simplicity and drama, it is assumed that all individuals are identical and that utility is concave in wealth alone.

Section 2 presents an immediate positive prediction of the model— that it provides a natural explanation of the phenomenon addressed by Friedman and Savage (1948). Although the typical individual here has a utility function which is concave in wealth alone, he will, nonetheless, engage in fair bets if winning entails moving up in status sufficiently more rapidly than losing entails moving down. It can be shown, for example, that a typical distribution of wealth, combined with separable utility, suffices to explain the "concave–convex–concave" utility described by Friedman and Savage. This formulation has the property that the class of risk–preferring individuals would vary in terms of its absolute wealth from country to country or over time, as is presumably realistic.

The above discussion raises the question of which distributions of wealth might be stable in the sense that no individual has an incentive to take any fair bet. It is immediate that stability is entailed by a density function which decreases sufficiently rapidly. On the other hand, instability must equally be entailed by a density function which increases sufficiently rapidly. Distributions of wealth described by continuous distribution functions which approximate an equal distribution are then inevitably unstable. This naturally arouses the suspicion that an externality is present. The individual risk preferences here are indeed inappropriate from a social viewpoint. For example, an increase in the rank of a given individual can only be accomodated at the cost of a lowering in rank of other individuals and there is assumed to be no compensation paid to them for this effect.

Less immediate properties of the model then require the investigation of the Pareto efficiency of various allocations of wealth across individuals. This is carried out in Section 3. There are, perhaps, some surprises. If the cross–partial between wealth and status is positive, for example, an unequal distribution of wealth maximizes the unweighted integral of utility. (Such a welfare criterion is referred to here as Bentham–Harsanyi welfare, or BH welfare for short.) Furthermore, although sufficiently unequal deterministic distributions of wealth are Pareto efficient, sufficiently equal distributions can be Pareto dominated by suitably chosen gambles. Indeed gambles over wealth and implied status assignment within the BH optimal distribution are also Pareto efficient. If, on the other hand, the cross–partial between wealth and status is non–positive, an equal distribution of wealth is BH optimal. Now all deterministic distributions of wealth are Pareto efficient. However, gambles purely over status within the equal distribution are also Pareto efficient.

Section 4 provides welfare and efficiency analyses of individually rational gambling. It is first shown that the existence of the externality implies that such gambling will decrease BH welfare when the initial distribution of wealth is Pareto efficient. The existence of the externality is, furthermore, shown to imply that the following more fundamental possiblility can arise. Suppose that the initial distribution of wealth is Pareto efficient but includes a region where the density is increasing rapidly enough that some individuals are risk–preferring. In this case, a gamble can be arranged which is consistent with the incentives of the gamblers and with the incentives of the suppliers, but which is, nevertheless, Pareto–inefficient, due to the uncompensated status interaction effect.

The final result of Section 4 is to show how a tax/subsidy scheme might be employed in order that individual incentives towards risk would be socially appropriate. It is shown that a suitable tax/subsidy has the property that the after–tax/subsidy marginal utility of wealth is the partial derivative of utility with respect to wealth, so that the tax/subsidy nets out the status effect, to first–order. It is further shown that there is a tendency for poorer individuals to be taxed on fair bets but for richer individuals to be subsidized, given that the tax/subsidy is actually paid. (Ted Bergstrom suggests that this might be the reason that universities have departments of finance rather than departments of bingo.) This tax/subsidy ensures that the Pareto inefficient gambles above will not occur.

It is suggested that the following strategy for a first reading of the present paper might be optimal. First read Section 2.1. Skim Section 2.2. Omit Section 3, with the exception of Figures 1 and 2 which should be loosely intelligible even in the absence of Propositions 1 through 4. In Section 4, take it on faith that Pareto efficient initial distributions can characterized as claimed. Finally, then, read Section 4.1, skim Section 4.2 and read Section 4.3.

2. IMMEDIATE POSITIVE IMPLICATIONS

2.1. Friedman and Savage Revisited

The phenomenon which Friedman and Savage (1948) wish to explain is the simultaneous occurence of preference for risk, as evidenced by gambling, and aversion to risk, as evidenced by purchase of insurance. The resolution they propose is that the von Neumann–Morgenstern utility be concave over an initial range of wealth, but be then convex over a subsequent range of wealth. If observations concerning the structure of prizes of lotteries are also to be explained, there must also be a final range of wealth over which utility is concave. It is usually accounted to be a defect of this theory that the intermediate range of wealth over which utility is convex should be tied to the population distribution over wealth in an apparently ad hoc fashion. If individuals are supposed to be concerned with status, however, such a tie–in effect may arise naturally.

Individuals are taken to have identical von Neumann Morgenstern utility functions given by the twice continuously differentiable function:

$$U(w,N)$$

where w is the wealth of the individual and N is his status. Status here is taken to be the ranking of the individual in the wealth distribution, counting from the bottom. If this distribution is described by a density function, this ranking is just the cumulative distribution function. (Section 3 describes a more general mechanism for determining status even when there are sets of positive measure with equal wealth.) The following restrictions are imposed throughout on utility:

$$U_w(w,N) > 0,\ U_N(w,N) > 0,\ U_{ww}(w,N) < 0, \quad \forall\ w,\ N. \tag{U}$$

The first two of these are self–explanatory. The third will be shown to rule out neither the possibility of individually rational gambling nor the social desirability of all gambling, but still serves the interests of simplicity.

Such a status motive might have an evolutionary origin. For example, the caveman with the largest dead mammoth might have won dinner with the cavewoman having the most refined table manners, with less prodigious efforts rewarded with accordingly less congenial dinner companions. This would have led to a concern about standing in the distribution of mammoth kills transcending their inherent delectability. Indeed, if any analogous reward system based on wealth exists in these more refined times, perhaps still for intertwining the sexes, such a concern now is perfectly rational.

It is assumed here that what is given initially is a distribution of population over wealth. This is described by a cumulative distribution function, F(.), with a continuously differentiable density f(.) so that

$$F(w) = \int_0^w f(w)dw, \text{ and } F(W) = 1,$$

where the support of the population distribution over wealth is $[0,W]$ and the total population has size unity. It is innocuous, indeed, to assume that $f(w) > 0$ on $(0,W)$, given the support assumption. It is convenient here to take the view that the independent variable is the wealth of the individual. (In the next section, when a normative view is adopted, it will be appropriate to switch to the name of the individual as the independent variable.)

Each individual's von Neumann–Morgenstern utility can now be given in a "reduced form" by the continously twice–differentiable function of wealth:

$$V(w) = U(w,F(w)),$$

The function V(.) incorporates individual attitudes to risk when status can vary. Indeed

$$V'(w) = U_w(w,F) + U_N(w,F).f(w) > 0$$

and

$$V''(w) = U_{ww}(w,F) + 2U_{wN}(w,F).f(w) + U_{NN}(w,F).f(w)^2 + U_N(w,F).f'(w) \quad \text{(I)}$$

It is then inevitable that V(.) will be locally convex (concave) whenever the population density function is increasing (decreasing) rapidly enough, regardless of the signs of the other terms. For example, suppose that utility is given by the simple additively separable function:

$$U(w,N) = u(w) + \beta N;\ u'(w) > 0,\ u''(w) < 0,\ \forall\ w > 0;\ u''(0) = -\infty,\ u'(0) = \infty.$$

In this case,

$$V''(w) = u''(w) + \beta.f'(w).$$

If the distribution of population over wealth is unimodal, so that for some $m > 0$,

$$f'(w) \begin{cases} > 0, & \text{for } w \in [0,m) \\ = 0, & \text{for } w = m \\ < 0, & \text{for } w > m \end{cases} \quad \text{(D)}$$

it follows that there is an initial and a final range of wealth such that V is concave. In addition, if β is large enough, there will exist an intermediate range of wealth in which V is convex. That is, such a von Neumann–Morgenstern utility function can yield the risk attitudes described by Friedman and Savage. Further, any range of wealth over which V is convex is a subset of [0,m), the range of wealth over which the population density is increasing. That is, the range over which risk preference is exhibited is restricted to the lower half of the distribution of population over wealth. (See Clotfelter and Cook, 1987, for data suggesting that poorer individuals tend, at least, to spend a higher fraction of their wealth on lotteries.) That gambling be somehow tied to the distribution of population over wealth would seem likely to best describe data comparing countries or different periods.

2.2 Stability. Instability of Equal Distribution.

The above discussion of private risk–taking raises the issue of which distributions of population over wealth might be "stable" in the sense of creating no private incentives to take fair bets. Such stable distributions could be expected to comprise the set of long–run equilibria when the distribution evolves as a result of individuals who are risk–preferring taking fair bets. (The explicit dynamics here seem likely to be complicated and are not treated here.) Clearly, given the expression for $V''(w)$ in (I) of Section 2.1, stability will always obtain if $f'(w)$ is small enough. For example, if

$$U_{wN}(w,N) \leq 0 \text{ and } U_{NN}(w,N) \leq 0, \quad \text{for all } w,\ N,$$

it is sufficient that $f'(w) \leq 0$ in order that $V''(w) < 0$ everywhere. In general, distributions of wealth which are "pyramidal" in structure are likely to be stable in this sense. Such a structure is, perhaps, typical in feudal societies.

There is no guarantee that *any* Pareto efficient distribution is stable in this sense, as is examined in detail in Section 4.2. As a simple example, consider an equal distribution of wealth. If the cross–derivative between status and wealth is non–positive, indeed, Proposition 2 Section 3 will show that the Bentham–Harsanyi welfare optimum entails such equal wealth. This is not strictly characterized by a density function for the distribution of population over wealth, of course. However, if a twice continuously differentiable distribution of population were even to approach such equality, it is inevitable that $f'(w)$ would become arbitrarily large somewhere and gambling would occur.

Such an equal distribution is, in this sense, inevitably unstable.

The underlying reason for such results is the presence of an externality. That is, if a particular individual moves up in the wealth distribution, his enhanced status is obtained at the expense of lower status for other individuals, but this interaction is assumed to be mediated by no market mechanism. Similarly, if the particular individual moves down in the wealth distribution, he obtains no compensation for increasing the status of others. It is necessary to turn to a detailed examination of efficiency in order to derive the characteristics of such an externality.

3. THE EFFICIENCY OF ALLOCATIONS OF WEALTH AND STATUS

The independent variable in all the other sections of this paper is taken to be wealth. However, it is convenient to discuss efficiency from a formally different point of view. Notice first that a cumulative distribution function $F \in C^2[0,W]$ with a density function strictly positive on $(0,W)$ induces a unique distribution, $\hat{w}$, of wealth over status as the inverse function of F:

$$\hat{w}(F(w)) = w, \forall w \in [0,W]; \; F(\hat{w}(N)) = N, \forall N \in [0,1]; \; \frac{d\hat{w}}{dN} = 1/f(\hat{w}(N)) > 0, \forall N \in (0,1) \quad (T)$$

so that, indeed, $\hat{w} \in C^2[0,1]$. By a change of variable,

$$\int \hat{w}(x)dx = \int_0^W wf(w)dw = w^*$$

where w^* is the total (and average) level of wealth. (All integrals here are over [0,1] unless otherwise indicated.) Thus each well–behaved distribution of population of wealth of the type used elsewhere in the paper induces a well–behaved distribution of wealth over status. Indeed, it is desirable to consider here a more general class of distributions of wealth over status than those induced in this way. This permits, for example, the possibility of equality of wealth to be treated carefully. It is also desirable to ensure that the Pareto efficiency of a given initial well–behaved distribution of wealth is not an artifact of an overly restricted class of redistributions. Consider then the following set of functions:

Given status levels in [0,1], assign wealth levels as

$$w:[0,1] \to R^+, \ w(N_1) \geq w(N_2) \ \forall N_1, N_2 \in [0,1], N_1 \geq N_2.$$

(M) It is then immediate that higher wealth must be associated with higher status, as required. It follows that w is measurable and bounded by w(1). Given total wealth available is w*,

$$\int w(N)dN = w^*.$$

Note how the above scheme permits one individual to have higher status than another even though their wealth levels are equal. Since an arbitrarily slight difference in wealth can lead to a given change in status, continuity requires this.

It is appropriate here to consider the name of an individual as the fundamental independent variable. It is assumed that individuals are named in accordance with their initial status in the initial well–behaved distribution of wealth. (See the statements of Propositions 1 to 4 below.) It is necessary to consider reallocations of status, since these will arise from reallocations of wealth. These reallocations of status should, again, be general, in order to obtain Pareto efficiency in a strong sense.

Take then a continuum of individuals, with a typical individual named

$$x \in [0,1].$$

Suppose that, altogether, individuals are *first* assigned status levels and *then* that these status levels are assigned wealth levels, by means of a function from the set M above. These status assignments comprise the set:

Names from the unit interval are assigned status levels, also in the unit interval, as

(R)
$$N : [0,1] \to [0,1]$$

where N is measurable and Lebesgue measure–preserving so that

$\forall A \in \mathcal{B}[0,1]$, $N^{-1}(A) \in \mathcal{B}[0,1]$, and $\lambda\{N^{-1}(A)\} = \lambda(A)$

where $\mathcal{B}[0,1]$ is the set of Borel subsets of [0,1] and λ is Lebesgue measure. (See Dunford and Schwartz, 1958, p. 667.)

Note how a function N from this set R induces a reallocation of initial status, when considered in conjunction with the identification of initial status and name. Note also how the above complete two part mechanism assigning wealth and status facilitates analysis of the effect of rearranging individuals within a given wealth distribution.

The following is then the set of feasible deterministic utility profiles:

$$F = \{ Y \in L_2[0,1] \mid Y(x) = U(w(N(x)),N(x)) \text{ for some } w \in M \text{ and } N \in R\}$$

(It is convenient to use $L_2[0,1]$ here in order to be able to employ the "projection theorem" which requires a Hilbert space setting.) It is necessary to allow gambles here, so recall that the convex hull of F is

$$co(F) = \{ \sum_{i=1}^{n} \pi_i Y_i, \text{ for some n, some } \pi_i \geq 0, \sum_{i=1}^{n} \pi_i = 1, \text{ some } Y_i \in F, i = 1,...,n\}$$

In general, such a convex hull need not be closed. However, it is simpler to confine attention to such finite gambles and it will be shown directly that the welfare functionals below attain the relevant maxima on co(F).

The set of Pareto preferred profiles to a given profile Y, say, can be defined as

$$P(Y) = \{\bar{Y} \in L_2[0,1] \mid \bar{Y}(x) \geq Y(x) \text{ a.e. } x \in [0,1], \bar{Y}(x) > Y(x) \text{ on a set of positive measure}\}$$

which is clearly also convex.

The class of welfare functionals used here mainly as mathematical props are linear, given as:

$$S{:}L_2[0,1] \to R, \quad S(Y) = \int \alpha(x)Y(x)dx \text{ where } \alpha \in L_2[0,1], \alpha(x) > 0 \text{ a.e. } x \in [0,1].$$

(Note that $S(Y) = c$, where $c \in R$, is the equation of a hyperplane in $L_2[0,1]$. See Luenberger, 1969, p. 129.) The case where $\alpha(x) \equiv 1$ will be referred to here as Bentham–Harsanyi welfare, or BH welfare for short. The associated functional will be denoted by H.

The "welfare" problem is then

$$\max_{Y \in co(F)} S(Y).$$

Lemma 1 of the Appendix shows that Pareto efficiency is implied by the maximization of *any* such linear functional. (The Appendix also discusses briefly the technical difficulties involved in showing that a Pareto efficient allocation must maximize *some* such linear functional.)

The following definition is a convenient shorthand in what follows:

$$\hat{\alpha}(x) = 1/U_w(\hat{w}(x),x), \text{ so that}$$

$$\hat{\alpha}'(x) = -[U_{ww}(\hat{w}(x),x)\hat{w}'(x) + U_{wN}(\hat{w}(x),x)]/[U_w(\hat{w}(x),x)]^2 \quad \text{(A)}$$

$$\text{for all } \hat{w} \in M \cap C^1[0,1], \text{ so that } \hat{w}'(x) \geq 0.$$

Proposition 1. Pareto Efficiency of Deterministic Wealth Allocations.

Suppose, for simplicity, that $\hat{w} \in M \cap C^1[0,1]$. Then $N(x) \equiv x$ and $\hat{w}$ are Pareto efficient in co(F) if $\hat{\alpha}'(x) \geq 0$, for all $x \in [0,1]$. On the other hand, if $\hat{\alpha}'(x) < 0$, but $\hat{w}'(x) > 0$, for x in some interval, then $N(x) \equiv x$ and $\hat{w}$ are Pareto inefficient in co(F).

Proof See Appendix. This proof considers a welfare functional S(Y) which uses weights $\hat{\alpha}(x)$ as above. The proof first shows that $N(x) \equiv x$ maximizes S(Y) over $N \in R$, for *any* $w \in M$. It is then straightforward to complete the proof of Pareto efficiency by showing that $\hat{w}$ maximizes S(Y) over $w \in M$, given $N(x) \equiv x$. The proof of Pareto inefficiency is by construction of a gamble which dominates the given allocation. This gamble has one outcome which simply reverses status for individuals in the appropriate range. This is taken in conjunction with a perturbation of the original distribution of wealth. (See Figure 1 below.)

The following two propositions consider the role of the cross–partial of wealth and status and exemplify the general results of Proposition 1. The BH welfare optimum is also obtained. For simplicity, the cross–partial is taken to be uniform in sign.

Proposition 2. Pareto Efficiency and Welfare–– Positive Cross–Partial.

Suppose that

$$U_{wN}(w,N) > 0, \text{ for all } w, N \quad \text{(PC)}$$

Suppose, again, that $\hat{w} \in M \cap C^1[0,1]$. In this case $\hat{\alpha}'(x) \geq 0$, for all $x \in [0,1]$ if and only if $\hat{w}$

is a "sufficiently unequal" distribution of wealth over status, in this sense. Then $N(x) \equiv x$ and such $\hat{w}$ are Pareto efficient in co(F), by Proposition 1. There exists an essentially unique unequal distribution of wealth maximizing BH welfare. All gambles over status assignments within this fixed distribution are also Pareto efficient in co(F). On the other hand, $\hat{\alpha}'(x) < 0$ for all $x \in [0,1]$ if and only if $\hat{w}$ is a "sufficiently equal" distribution of wealth, again in this sense. If, in addition, $w'(x) > 0$ for any $x \in [0,1]$, $N(x) \equiv x$ and $\hat{w}$ are Pareto inefficient in co(F), by Proposition 1.

Proof See Appendix. Figure 1 represents the utility possibility set for the case where (PC) holds and there are just two individuals. This diagram captures the full intuition of the continuum case, even though it might have seemed *a priori* that the non–convexity due to status would somehow loom larger with a small number of agents.

INSERT FIGURE 1 HERE

The above results are reminiscent of welfare analysis of the standard monocentric model of a city. (See Mirrlees, 1972, for example. Mirrlees, however, rules out gambles as being unrealistic in an urban locational model.) The next set of results reflect the possibility of a kind of "corner solution" which seems not to have an urban economics counterpart.

Proposition 3. Pareto Efficiency and Welfare–– Non–Positive Cross–Partial.

Suppose now that

$$U_{wN}(w,N) \leq 0, \ \forall \ w, N. \qquad \text{(NC)}$$

In this case, all distributions of wealth $\hat{w}$ in $M \cap C^1[0,1]$ entail $\hat{\alpha}'(x) \geq 0$, for all $x \in [0,1]$. Hence $N(x) \equiv x$ and such $\hat{w}$ are Pareto efficient outcomes in co(F), by Proposition 1. The BH welfare functional is essentially uniquely maximized by an equal distribution of wealth. All gambles over status, given this equal level of wealth, are also Pareto efficient.

Proof See Appendix. Figure 2 represents the utility possibility set in the case that (NC) holds and there are two individuals, and, again, this diagram captures the intuition for the general case.

INSERT FIGURE 2 HERE

The final result of this section concerns the circumstances in which gambling will lead to Pareto inefficiency. The next section shows that such gambling will quite possibly be consistent with individuals' private attitudes to risk.

Proposition 4. Pareto Inefficiency of a Class of Gambles.

Suppose that $\hat{w} \in M \cap C^1[0,1]$ is such that $\hat{w}'(x) > 0$, $\hat{\alpha}'(x) \geq 0$, for all $x \in [0,1]$. Hence $\hat{w}$ and $N(x) \equiv x$ are Pareto efficient by Proposition 1. Suppose that $Y \in co(F)$ is Lipshitz continuous on [0,1]. Suppose, finally, that the wealth distributions over status involved in the construction of the finite gamble Y are, with positive probability, essentially distinct from the original distribution $\hat{w}$.

It follows that the gamble in co(F) which yields expected utility

$$\pi U(\hat{w}(x),x) + (1-\pi)Y(x), \quad \text{for each } x \in [0,1],$$

is strictly Pareto dominated in co(F) for all $\pi \in (0,1)$.

Proof See Appendix. Figure 2 gives the geometric intuition for this result when (NC) holds. The intuition when (PC) holds is similar.

The results of this section set the stage for the more positive discussion of the next.

4. WELFARE AND EFFICIENCY EFFECTS OF PRIVATE GAMBLING

Note, as a remark that applies throughout Section 4, that whether or not there exist risk–preferring individuals is logically independent of whether or not the initial distribution is Pareto efficient. This follows from Section 3 since the function $\hat{\alpha}'(x)$, from (A), does not depend on $f'(\hat{w}(x))$, given (T), whereas the sign of $\hat{\alpha}'(x)$ determines Pareto efficiency, as in Proposition 1. It is also assumed throughout that Pareto efficiency of each distribution is evaluated in conjunction with the status assignment $N(x) \equiv x$, as in the statement of Proposition 1.

4.1 Welfare Effects

It is now shown that simply banning individually rational gambling, as introduced in Section 2, could not lower Bentham–Harsanyi welfare given a Pareto efficient initial deterministic distribution of wealth. Of course, such a ban would generally reduce the expected utility of the erstwhile gamblers and then would not be Pareto improving. BH welfare is simply chosen here as a salient example.

Set

$$H = \int_0^W U(w,F(w))f(w)dw$$

where the notation is as in Section 2. (This is the obvious analogue of the additive criterion proposed by Harsanyi, 1955, with the additional requirement of symmetry over the identical individuals.) Suppose that the approximately $f(\bar{w})\delta$ individuals having initial wealth in the interval $[\bar{w},\bar{w}+\delta]$ engage in a fair bet, so that final wealth is in the interval $[\tilde{w},\tilde{w}+\delta]$, where $E\tilde{w} = \bar{w}$. (It is not hard to show how small approximately fair bets can be supplied by the total population with the judicious use of risk "discounts" and premiums. These can be shown to have effects on H which are of second order in δ. The next subsection carries out such a construction for a more delicate situation.) The effects on H concern the utility of individuals who gamble and that of individuals whose status is affected by the gamble. Thus the change in H, for each possible value of $\tilde{w}$, is given to first-order in δ as

$$\frac{\Delta H(\tilde{w})}{f(\bar{w})\delta} = -U(\bar{w},F(\bar{w})) + U(\tilde{w},F(\tilde{w})) - \int_{\bar{w}}^{\tilde{w}} U_N(w,F(w))f(w)dw, \quad \text{where } E\tilde{w} = \bar{w}.$$

The third term here is the uncompensated effect of the externality. Integrating by parts,

$$\Delta H(\tilde{w}) = \pi(\tilde{w})f(\bar{w})\delta, \text{ say, where } \pi(\tilde{w}) = \int_{\bar{w}}^{\tilde{w}} U_w(w,F(w))dw$$

The following relationship between π and $\hat{\alpha}$, as in (A) of Section 3, is then immediate:

$$\pi(\bar{w}) = 0, \ \pi'(\tilde{w}) = U_w(\tilde{w},F(\tilde{w})) = 1/\hat{\alpha}(F(\tilde{w})) > 0,$$

$$\text{sign } \pi''(\tilde{w}) = -\text{sign } \hat{\alpha}'(F(\tilde{w})).$$

Clearly, then, Jensen's inequality implies that, to first order in δ,

$$E\Delta H(\tilde{w}) \leq 0, \text{ for all bets } \tilde{w}, \text{ if } \hat{\alpha}'(F(\tilde{w})) \geq 0, \text{ for all } \tilde{w} \geq 0\,;$$

$$E\Delta H(\tilde{w}) > 0, \text{ for small enough bets } \tilde{w}, \text{ if } \hat{\alpha}'(F(\overline{w})) < 0.$$

Suppose the initial distribution is Pareto efficient, as in Proposition 1, so that $\hat{\alpha}'(x) \geq 0$, for all $x \in [0,1]$. The intuitive reason that such a gamble then generally cannot increase expected welfare is as follows. Such Pareto efficient distributions are those which are no more equal than the BH optimum. The gamble here yields an outcome which is still more unequal in a straightforward mean–preserving spread sense, and so generally decreases welfare. On the other hand, Proposition 1 also shows that, if $\hat{\alpha}'(x)$ is negative anywhere, then the associated twice continuously differentiable distribution is Pareto inefficient. Small gambles in the same region will now increase welfare. The intuitive reason for the difference is that Pareto inefficient distributions are locally too equal rather than too unequal.

4.2. Inefficient Private Gambling

This subsection constructs a class of examples in which Pareto inefficient gambling, of the type presented in Proposition 4, is consistent with individual attitudes to risk. Demonstrating that such inefficiency is possible is the heart of the matter in characterizing an externality. Accordingly, some care is taken in showing how small approximately fair bets can be supplied.

Suppose, indeed, that the approximately $f(\overline{w})\delta$ individuals in the wealth interval $[\overline{w}-\delta^2,\overline{w}+\delta+\delta^2]$ are strictly risk–preferring. There must then exist a fair bet as follows. For each dollar bet, the gross return is $\tilde{b}$, say, where $E(\tilde{b}) = 1$. All these individuals will take as much of this fair bet as possible. There must indeed exist a risk "discount", $\overline{r}$, say, such that the bet which yields final wealth $w\tilde{b} - \overline{r}$ is preferred by all individuals in $[\overline{w},\overline{w}+\delta]$ to remaining at w for sure, for small enough $\overline{r} > 0$. In order to apply Proposition 4, it must be supposed that the random variable $\tilde{b}$ has a finite number of realizations. Indeed, for simplicity, assume there are just two so that

$\tilde{b} = b_1 > 1$, with probability $\pi \in (0,1)$, and $\tilde{b} = b_2 < 1$, with probability $1-\pi \in (0,1)$.

It is also required that expected utility, $Y(x)$, say, of each individual, x, is Lipshitz in x.

This can be accomplished as follows. Define an exogenous deterministic "participation" function as

$$p:[0,W] \to [0,1] \text{ such that}$$
$$p(w) = 1 \ \forall w \in [\overline{w},\overline{w}+\delta], \ p(w) = 0 \ \forall \ w \notin (\overline{w}-\delta^2, \overline{w}+\delta+\delta^2),$$

which is piecewise linear, as in Figure 3. Define also a risk discount function $r(w)$, say, such that

$$r(w) = 0 \ \forall w \notin (\overline{w},\overline{w}+\delta); \quad r(\overline{w}+\delta/2) = \overline{r}$$

which is also piecewise linear as in Figure 3.

INSERT FIGURE 3 HERE

It follows that all individuals in the initial wealth interval $[\overline{w}-\delta^2,\overline{w}+\delta+\delta^2]$ would take the bet yielding final wealth as the random variable

$$(1-p(w))w + p(w)w\tilde{b} - r(w) = w + p(w)w(\tilde{b}-1) - r(w)$$

rather than remain at w for sure. Assume now that δ is small enough that the functions

$$w_1(w) = w + p(w)w(b_1-1) - r(w) \ \text{ and } \ w_2(w) = w + p(w)w(b_2-1) - r(w)$$

are of the form sketched in Figure 3. In particular, the derivative of each of these functions has the sign as indicated and this is bounded away from zero, wherever it exists. The final wealth of an individual of initial name and status x is then

$$w_i(\hat{w}(x)) \quad i = 1, 2$$

where $\hat{w}$ is the initial distribution of wealth over status function, $\hat{w} \in C^2[0,1]$, $\hat{w}'(x) > 0$, for all $x \in [0,1]$. The functions $w_i(\hat{w}(.))$, i=1,2, are clearly Lipshitz. The final status of an individual with initial name and status x is given as the continuous function

$$N_i(x) = \lambda\{z \mid w_i(\hat{w}(z)) \leq w_i(\hat{w}(x)) \} \quad \text{for } i = 1, 2,$$

where λ is Lebesgue measure. Given the functions w_i as constructed, it follows that the set involved here consists of at most three closed subintervals of [0,1]. Each endpoint of such a subinterval corresponds to a value of x with the same image under N_i. These endpoints are of the form

$$x_j(w_i(\hat{w}(x))) \ \text{ for } j = 1,\ldots,J, \text{ and } i = 1, 2, \text{ where } \ x_j'(w_i) = [w_i'(\hat{w}(x_j)).\hat{w}'(x_j)]^{-1}$$

for all except a finite number of values of x. The derivative of w_i, where it exists, is bounded away from zero, so that the derivative of each of these endpoints with respect to final wealth is bounded, where it exists. Recalling that each $w_i(\hat{w}(.))$ i=1,2 is Lipshitz, it follows that N_i is also Lipshitz. Hence Y(.) is Lipshitz, as required.

Suppose the other side of the gamble is spread evenly across the total population. (Recall this has size unity.) Suppose that an individual with wealth w is paid a risk premium of s(w) so that he is indifferent to the gamble. (It is possible that $s(w) < 0$, if this individual is himself risk–preferring.) That is,

$$EV[w - \delta f(\overline{w})\overline{w}(\tilde{b}-1) + \text{a term in } \delta^2 + s(w)] = V(w),$$

which implies that s(w) is of second–order in δ. It needs to be shown that these required risk–premiums can be supplied from the risk–discounts paid by the gamblers. However, this follows immediately if δ is small enough because then

$$\int_0^W s(w)f(w)dw \leq \int_0^W r(w)f(w)dw = \bar{r}f(\overline{w})\delta/2 + \text{a term in } \delta^2.$$

Such a gamble is then consistent with all the pertinent private attitudes to risk. Note that the distribution of wealth over status after the gamble is clearly generally different from that before. Suppose the initial Pareto efficient distribution is as in the statement of Proposition 4. Proposition 4 then implies that a "compound gamble" involving this initial distribution and the gamble constructed above must be Pareto inefficient, if the probability of attaining the original distribution is positive but less than 1. This completes the description of the desired class of Pareto inefficient gambles.

4.3. Taxes and Subsidies.

Consider now the possibility of addressing the externality by means of a suitable tax/subsidy scheme. Suppose that the scheme treats the original deterministic distribution as a zero point and does not redistribute wealth in the absence of gambling. Consider indeed the following tax/subsidy on an individual of initial wealth $\overline{w}$

$$t(w) = \int_{\overline{w}}^{w} \frac{U_N(w, F(w))f(w)}{U_w(w, F(w))} dw, \text{ so that } t(\overline{w}) = 0$$

The numerator of the integrand here represents the rate of uncompensated loss of utility due to loss of status caused by an individual moving up in wealth. The denominator will be proved to be the after–tax/subsidy marginal utility of wealth for individuals affected by the move. (That is, assuming these individuals are taxed/subsidized in exactly the fashion to be derived here for the gambler.) Define the following

$R(w)$ = wealth retained by the individual when lottery pays w, so $R(\overline{w}) = \overline{w}$,

$t(R(w)) = T(w)$ = tax paid when final wealth attained is $R(w)$, so that

$w - t(R(w)) = R(w)$

It follows that

$$R'(w) = \frac{1}{1+t'(R(w))} = \frac{U_w(R, F(R))}{U_w(R, F(R)) + U_N(R, F(R))f(R)} \in (0,1),$$

dropping w as the argument of R(.) for expositional clarity. What determines whether an individual with initial wealth $\overline{w}$ will accept or reject a small such taxed/subsidized lottery is the function:

$$B(w) = V(R(w)).$$

Now

$$B'(w) = \{U_w(R,F(R)) + U_N(R,F(R))f(R)\}R'(w) = U_w(R,F(R)) = 1/\hat{\alpha}(F(R)),$$

where $\hat{\alpha}$ is as in (A) of Section 3. It follows that the after–tax marginal utility at $\overline{w}$ is indeed simply $U_w(\overline{w},F(\overline{w}))$, as claimed above. It also follows that

$$\text{sign } B''(w) = -\text{sign } \hat{\alpha}'(F(R(w))), \text{ for all } w \geq 0.$$

Hence all fair gambles are (without loss of generality) rejected when the initial distribution of wealth is Pareto efficient, as in Proposition 1, and so satisfies $\hat{\alpha}'(x) \geq 0$, for all x. On the other hand, if the initial distribution of wealth is not Pareto efficient as in Proposition 1, so that $\hat{\alpha}'(x) < 0$ for some x, it follows that some small fair gambles will be accepted. Hence the imposition of such a tax/subsidy, without loss of generality, rules out the Pareto inefficient gambles as constructed in Section 4.2, or indeed any generalization still involving the taking of fair bets given an initial Pareto efficient distribution.

It is interesting to derive the incidence of a gambling tax/subsidy on a small fair bet, although the above discussion proves that actual payments are not generally made when the initial distribution is Pareto efficient and the tax/subsidy is full as above. It is not hard to show that

$$T''(w) = \frac{t''(R)[1-R'(w)]}{[1+t'(R)]^2}, \text{ and } T''(\overline{w}) = \frac{t''(\overline{w})[1-R'(\overline{w})]}{[1+t'(\overline{w})]^2}$$

It now follows from the expression for t(w) that the sign of $T''(\overline{w})$ is then determined by the sign of $f'(\overline{w})$ if this is large in absolute value. That is, given the usual distribution of population over wealth, as in (D) of Section 2.1, there will be a tendency for $T''(\overline{w})$ to be positive for poorer individuals and negative for richer individuals. Clearly

$$E[T(\tilde{w})] \gtrless T[E\tilde{w}] = T[\overline{w}] = 0 \text{ according to } T''[\overline{w}] \gtrless 0,$$

for small fair bets with final wealth $\tilde{w}$. It follows that there is a tendency for poorer individuals to be taxed on small bets and for richer individuals to be subsidized. Clotfelter and Cook (1987) argue that the actual incidence of the implicit taxation associated with state lotteries is regressive. The present paper shows that such regressivity need not be as inappropriate as it might seem at first blush.

For example, suppose that utility is given as

$$U(w,N) = u(w) + \beta N, \text{ where } \beta > 0,\ u'(w) > 0,\ u''(w) < 0, \text{ for all } w \geq 0.$$

It follows that

$$t''(\overline{w}) = \frac{\beta f'(\overline{w})u'(\overline{w}) - \beta f(\overline{w})u''(\overline{w})}{[u'(\overline{w})]^2}$$

so that

$$t''(\overline{w}) \lesseqgtr 0 \text{ iff } \frac{u''(\overline{w})}{u'(\overline{w})} \gtrless \frac{f'(\overline{w})}{f(\overline{w})}$$

Hence, in particular, all individuals with wealth less than the mode, m, are taxed. On the other hand, individuals with wealth above the mode, m, will be subsidized when the elasticity of marginal utility with respect to wealth is less in absolute value than the elasticity of the density with respect to wealth.

5. CONCLUSIONS

As noted in Section 2.2, it might be of interest to model the dynamics of the evolution of the distribution when individuals, who want to do so, take gambles which the remainder of the population are willing to supply. This would seem likely to be a complex task in the present setting with a continuum of individuals.

Two restrictions imposed in the present paper were that that all individuals were identical and that utility was concave in wealth alone. These certainly serve the purpose of dramatizing the substantial difference that a concern with status might make. However, it would be useful to relax these assumptions.

Note finally that the observations here concerning gambling undoubtedly have counterparts concerning insurance. Perhaps, for example, richer individuals tend to be too risk–averse and then to insure too much. This analysis might well require exogenous wealth uncertainty and is left for future research.

6. APPENDIX

Recall the restrictions imposed on the utility function as in (U), the sets of status and wealth assignment functions, R and M, the set of utility profiles obtainable with finite gambles, co(F), and the definition of Pareto preference, P(Y), all as given in Section 3. Armed with these the following result is immediate.

Lemma 1

Suppose that $Y^* \in co(F)$ is a solution of

$$\underset{Y \in co(F)}{\text{Max}} \int \alpha(x)Y(x)dx$$

where $\alpha \in L_2[0,1]$ and $\alpha(x) > 0$ a.e. $x \in [0,1]$. It follows that Y^* is Pareto efficient in co(F).

Proof Suppose not, so that there exists

$$\tilde{Y} \in P(Y^*) \cap co(F).$$

It is then clear that

$$\int \alpha(x) Y^*(x) dx < \int \alpha(x) \tilde{Y}(x) dx \qquad \square$$

Comment It is not trivial to establish a converse to the above result. (Such a converse might yield an alternative method of proving that certain allocations are not Pareto efficient.) The basic difficulty is that the set of feasible utility profiles, co(F), and each Pareto preferred set, P(Y), have empty interiors in $L_2[0,1]$, so that the separating hyperplane theorem cannot be applied. (See Luenberger, 1969, p. 133, for example.) Whereas Mas–Colell (1986) successfully establishes a separation result in the absence of this hypothesis, this was for a problem with an infinite number of commodities rather than an infinite number of agents. The additional restrictions on technology and preferences he imposed seem not to have immediate analogues here.

The following Lemma is useful in deriving the proof of Theorem 1 which characterizes an optimal solution for the status assignment function, N.

Lemma 2

Consider the linear programming problem:

$$\underset{\{x_{ij}\}}{\text{Max}} \sum_{i,j=1}^{n} a_i b_j x_{ij}$$

subject to

$$x_{ij} \geq 0, \ \sum_{i=1}^{n} x_{ij} = \sum_{j=1}^{n} x_{ij} = 1, \text{ for all } i,j,$$

where also

$$0 \leq a_1 \leq a_2 \leq \dots \leq a_n, \text{ and } 0 \leq b_1 \leq b_2 \leq \dots \leq b_n.$$

Then the following is always a solution:

$$x_{ij} = 0, \text{ for all } i \neq j$$
$$= 1, \text{ for all } i = j$$

Proof Suppose that i is the first index such that

$$x_{ii} \neq 1.$$

Clearly there exist j, k > i such that

$$x_{ij} > 0 \text{ and } x_{ki} > 0.$$

Take

$$\epsilon = \text{Min}\{x_{ij}, x_{ki}\} > 0$$

and define x′ as

$$x'_{ii} = x_{ii} + \epsilon,\ x'_{ij} = x_{ij} - \epsilon,\ x'_{ki} = x_{ki} - \epsilon,\ x'_{kj} = x_{kj} + \epsilon$$

and elsewhere equal to x. It is clear that the change induced in the objective function is then Δ, say, where

$$\Delta/\epsilon = a_i b_i - a_i b_j - a_k b_i + a_k b_j = (a_k - a_i)(b_j - b_i) \geq 0.$$

After a finite number of steps, this yields

$$x_{ii} = 1.$$

Then the entire process can be repeated, to obtain finally that

$$\sum_{i,j=1}^{n} a_i b_j x_{ij} \leq \sum_{i=1}^{n} a_i b_i \qquad \square.$$

The following characterization of an optimal status assignment is now immediate:

Theorem 1

Suppose that $\infty > \alpha(1)$, $\alpha(0) > 0$, $\alpha(x_1) \geq \alpha(x_2)$, for all $x_1, x_2 \in [0,1]$, $x_1 \geq x_2$. It follows that

$$\int \alpha(x) U(w(N(x)), N(x)) dx \leq \int \alpha(x) U(w(x), x) dx$$

for any $w \in M$ and $N \in R$, where the sets M and R were defined in Section 3. That is, if the weights $\hat{\alpha}(x)$ are non–decreasing in x, setting $N(x) \equiv x$ never decreases the welfare functional, regardless of $w \in M$.

Proof Define, for compactness of notation,

$$y(N) = U(w(N), N)$$

so that y(.) is then a strictly increasing function. Suppose, contrary to the assertion of the

Theorem, that there exists an $N \in R$ such that

$$\int \alpha(x)y(N(x))dx > \int \alpha(x)y(x)dx + \epsilon$$

for some $\epsilon > 0$. Consider the simple function

$$N_k(x) = i/k \ \text{ if } N(x) \in ((i-1)/k, i/k] = I_i^k, \text{ say, } i = 2,\dots,k \text{ and}$$
$$N_k(x) = 1/k \ \text{ if } N(x) \in [0,1/k] = I_1^k,$$

say. It follows that from the Lebesgue dominated convergence theorem that there exists an integer K such that

$$\int \alpha(x)y(N_k(x))dx > \int \alpha(x)y(x)dx + \epsilon/2, \text{ for all } k > K, \qquad (*)$$

since the functions N, y, and α are all bounded. Define now

$$A_i^k = N^{-1}(I_i^k) \text{ so that } \lambda(A_i^k) = \lambda(I_i^k) = 1/k, \text{ for } i = 1,\dots,k,$$

and the $\{A_i^k\}_{i=1}^k$ form a partition of $I = [0,1]$, given that N belongs to the set R. Define also

$$B_{ij}^k = A_i^k \cap I_j^k, \text{ and } \lambda_{ij} = \lambda(B_{ij}) \geq 0,$$

so that

$$\sum_{i=1}^{n} \lambda_{ij} = \sum_{j=1}^{n} \lambda_{ij} = 1/k, \quad \text{for all } i,j.$$

It follows that

$$\int_{B_{ij}^k} \alpha(x)dx \leq \alpha(j/k)\lambda_{ij}$$

so that the LHS of (*) satisfies

$$\text{LHS} = \sum_{i=1}^{k} y(i/k) \int_{A_i^k} \alpha(x)dx = \sum_{i=1}^{k} y(i/k) \sum_{j=1}^{k} \int_{B_{ij}^k} \alpha(x)dx \leq \sum_{i=1}^{k} y(i/k) \sum_{j=1}^{k} \alpha(j/k)\, \lambda_{ij}.$$

Applying Lemma 2, it follows then that

$$\text{LHS} \leq \sum_{i=1}^{k} (1/k)y(i/k)\alpha(i/k)$$

Since α and y are monotonic and bounded, their product is Riemann integrable. Hence

$$\lim_{k\to\infty} \sum_{i=1}^{k} (1/k)y(i/k)\alpha(i/k) = \int \alpha(x)y(x)dx > \int \alpha(x)y(x)dx + \epsilon/2. \qquad \square$$

The next result is required in the proofs of the Propositions to follow.

Lemma 3

Suppose that $\infty > \alpha(1)$, $\alpha(0) > 0$, $\alpha(x_1) \geq \alpha(x_2)$ for all $x_1, x_2 \in [0,1]$, $x_1 \geq x_2$. Any solution for w of the problem

$$\underset{Y \in co(F)}{\text{Max}} \int \alpha(x)Y(x)dx$$

is essentially unique in the class M.

Proof Suppose then that w_1 and w_2 belong to M and are essentially different. (That is, w_1 and w_2 differ on a set of positive measure.) Suppose that w_1 and w_2 are each used with positive probability in solutions of the above problem. It follows from Theorem 1 that w_1 and w_2 must both solve the following problem.

$$\underset{w \in M}{\text{Max}} \int \alpha(x)U(w(x),x)dx.$$

Note that

$$w(\epsilon,x) = \epsilon w_1(x) + (1-\epsilon)w_2(x)$$

also belongs to M. However, it is clear from Jensen's inequality, given the concavity of U in w, that w(1/2,x), for example, is a strict improvement as a solution over either w_1 or w_2, which is the desired contradiction.

Proof of Proposition 1

Suppose that $\hat{\alpha}'(x) \geq 0$, for all $x \in [0,1]$. It will be shown that $N(x) \equiv x$ and $\hat{w}$ then solve the problem:

$$\underset{\substack{w \in M \\ N \in R}}{\text{Max}} \int \hat{\alpha}(x)U(w(N(x)),N(x))dx.$$

which implies that the given pair of functions also constitute a solution in co(F). It follows that $\hat{w}$ is Pareto efficient, by Lemma 1. Suppose indeed that $w \in M$ and $N \in R$ are any other feasible pair of functions. Define

$$w(\epsilon,x) = (1-\epsilon)\hat{w}(x) + \epsilon w(x)$$

which also clearly belongs to M. If

$$S(\epsilon) = \int \hat{\alpha}(x)U(w(\epsilon,x),x)dx$$

then it is easily shown that S(.) is twice continuously differentiable and

$$S'(0) = 0,\ \ S''(\epsilon) \leq 0,\ \ \text{for all } \epsilon \in [0,1],\ \ \text{so that}\ \ S(0) \geq S(1).$$

Theorem 1 can now be applied to yield

$$S(0) = \int \hat{\alpha}(x)\ U(\hat{w}(x),x)dx \geq S(1) = \int \hat{\alpha}(x)U(w(x),x)dx \geq \int \hat{\alpha}(x)U(w(N(x)),N(x))dx \quad \square$$

Suppose now that $\hat{\alpha}'(x) < 0$, but $\hat{w}'(x) > 0$, for all $x \in [a,b] \equiv I' \subset [0,1]$, where $a < b$. As a first step, consider the hyperplane which is the orthogonal complement of the subspace generated by $\hat{\alpha}$:

$$\{h \in L_2[a,b] \mid \int_{I'} \hat{\alpha}(x)h(x)dx = 0\} = L,$$

say. Consider also the element of $L_2[a,b]$ which represents the change in utility arising from a complete reversal of status within the distribution $\hat{w}$ on I':

$$U(\hat{w}(\sigma(x)),\sigma(x)) - U(\hat{w}(x),x) = \Delta U(x),\ \text{say, where } \sigma(x) = a + b - x,\ \text{for all } x \in [a,b].$$

It is clear that ΔU is a strictly decreasing function of x and that, by a change of variable

$$\int_{I'} \Delta U(x)dx = 0.$$

It follows that

$$\int_{I'} \hat{\alpha}(x)\Delta U(x)dx > 0$$

since this integral is the "covariance" between two strictly decreasing functions.

It follows from the "projection theorem" (Luenberger, 1969, pp. 51–53) that

$$\Delta U(x) = \mu\hat{\alpha}(x) + h(x)$$

for some $\mu > 0$ and $h \in L$. Indeed, since $\hat{\alpha}$ and ΔU are continuously differentiable, it follows that h is also.

Define then the following gamble in co(F). Leave all individuals $x \notin I'$ with wealth $\hat{w}(x)$ for sure. Otherwise, with probability $1-\epsilon$ assign an individual $x \in I'$ status x and wealth

$$\hat{w}(x) - \epsilon\hat{\alpha}(x)h(x) = \bar{w}(x),\ \text{say, where } \epsilon \text{ is such that}\ \ \bar{w}'(x) > 0,\ \text{for all } x \in [0,1].$$

Such a wealth distribution is feasible since $h \in L$. With remaining probability ϵ give each individual $x \in I'$ status $\sigma(x)$ and wealth $\hat{w}(\sigma(x))$. Hence expected utility for each individual $x \in I'$ is given by, say,

$$V(\epsilon,x) = (1-\epsilon)U[\hat{w}(x)-\epsilon\hat{\alpha}(x)h(x),x] + \epsilon U[\hat{w}(\sigma(x)),\sigma(x)]$$

so that

$$\begin{aligned} V_\epsilon(0,x) &= U[\hat{w}(\sigma(x)),\sigma(x)] - U[\hat{w}(x),x] - U_w[\hat{w}(x),x]\hat{\alpha}(x)h(x) \\ &= \Delta U(x) - h(x) = \mu\hat{\alpha}(x) > 0, \text{ for all } x \in [a,b]. \end{aligned}$$

Given that $V(\epsilon,x)$ is continuously twice differentiable, it is not hard to show that there must exist an $\bar{\epsilon}$ such that the above gamble yields a strict improvement in expected utility, for every individual in [a,b], for all $\epsilon \in [0,\bar{\epsilon}]$. (See Figure 1 for the geometric intuition for this construction, based on the two–person case.)

Proof of Proposition 2

All that remains to consider is Bentham–Harsanyi case where

$$\alpha(x) \equiv 1, \text{ for all } x \in [0,1].$$

Given Theorem 1 and Lemma 3, there clearly now exists an essentially unique optimal wealth distribution, w^H, say. Indeed, it is easily seen to satisfy

$$U_w(w^H(x),x) = \lambda, \text{ a.e., for some } \lambda > 0, \text{ so that } \frac{dw^H(x)}{dx} > 0, \text{ given (PC),}$$

assuming that $w^H(x) > 0$ a.e. Hence w^H is a necessarily unequal distribution of wealth. It can be shown that arbitrary $N \in R$ are now also optimal, by a proof similar to that of Theorem 1. Hence gambles over status within the given wealth distribution w^H are also Pareto efficient as claimed.

Proof of Proposition 3

Again consider the Bentham–Harsanyi case in which $\alpha(x) \equiv 1$. By Theorem 1, it is sufficient to consider

$$\underset{w \in M}{\mathrm{Max}} \int U(w(x),x)dx.$$

Suppose indeed that $w \in M$ is distinct from the constant function w^* on a set of positive measure. Define

$$w(\epsilon,x) = (1-\epsilon)w^* + \epsilon w(x) \in M \text{ and } S(\epsilon) = \int U(w(\epsilon,x),x)dx,$$

where S(.) is clearly twice continuously differentiable. It follows that

$$S'(0) = \int U_w(w^*,x)[w(x)-w^*]dx \leq 0,$$

since w(x) is a non–decreasing function, $U_w(w^*,x)$ is an non–increasing function and

$$\int w(x)dx = w^*,$$

so that S′(0) is the "covariance" between w(x) and $U_w(w^*,x)$. In addition,

$$S''(\epsilon) = \int U_{ww}(w(\epsilon,x),x)[w(x)-w^*]^2 dx < 0,$$

so that finally

$$\int U(w^*,x)dx > \int U(w(x),x)dx \qquad \square$$

Again, it can be shown that any N ϵ R is optimal here. Thus the equal wealth distribution can be combined with arbitrary gambles over status to yield further Pareto efficient outcomes.

Proof of Proposition 4

Define the hyperplane

$$L = \{h \in L_2[0,1] \mid \int \hat{\alpha}(x)h(x)dx = 0\}.$$

Given that Y ϵ co(F) and entails wealth distributions functions essentially distinct from $\hat{w}$, it follows from Theorem 1, Lemma 3 and Proposition 1 that, if

$$\Delta U(x) = Y(x) - U(\hat{w}(x),x), \text{ then } \int \hat{\alpha}(x)\Delta U(x)dx < 0.$$

From the "projection theorem" (Luenberger, 1969, pp.51–53), it follows that

$$\Delta U(x) = -\mu\hat{\alpha}(x) + h(x)$$

for some $\mu > 0$ and h ϵ L. Since $\hat{\alpha}$ and $\hat{w}$ are continuously differentiable, and Y is Lipshitz, h is Lipshitz. Since indeed, $\hat{w}'(x) > 0$, for all x ϵ [0,1], it is then possible to choose $\epsilon > 0$ small enough that

$$\bar{w}(x) = \hat{w}(x) + \epsilon\hat{\alpha}(x)h(x)/\pi \text{ is monotonically increasing, for all } x \in [0,1].$$

This is then a feasible redistribution of wealth over status, given that h ϵ L. Consider then the following perturbation of the original gamble:

$$V(\epsilon,x) = (\pi+\epsilon)U(\hat{w}(x)+\epsilon\hat{\alpha}(x)h(x)/\pi,x) + (1-\pi-\epsilon)Y(x).$$

It follows that

$$V_\epsilon(0,x) = U_w(\hat{w}(x),x)\hat{\alpha}(x)h(x) + U(\hat{w}(x),x) - Y(x)$$
$$= h(x) - \Delta U(x) = \mu\hat{\alpha}(x) > 0, \text{ for all } x \in [0,1].$$

Since $V(\epsilon,x)$ is continuously twice differentiable, it is not then hard to see that the constructed gamble is indeed Pareto preferred to the given gamble, for all small enough ϵ, as was to be shown. (See Figure 2.)

It is easily seen that the above proof can be generalized to cover a gamble Y which need have only one component Y_i, say, which need be only Lipshitz continuous from below in the sense that

$$Y_i(x_1) - Y_i(x_2) \geq -k(x_1 - x_2) \ \forall x_1, x_2 \in [0,1], x_1 \geq x_2, \text{ for some } k > 0.$$

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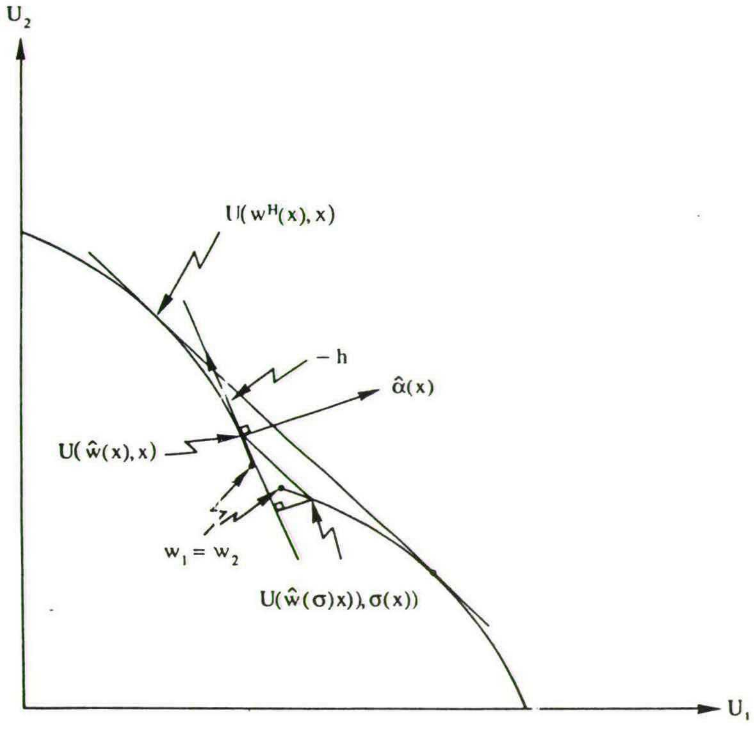

Figure 1. The Utility Possibility Set for Two Individuals.

Positive Cross–Partial Between Wealth and Status.

The Pareto Inefficiency of Certain Wealth Distributions.

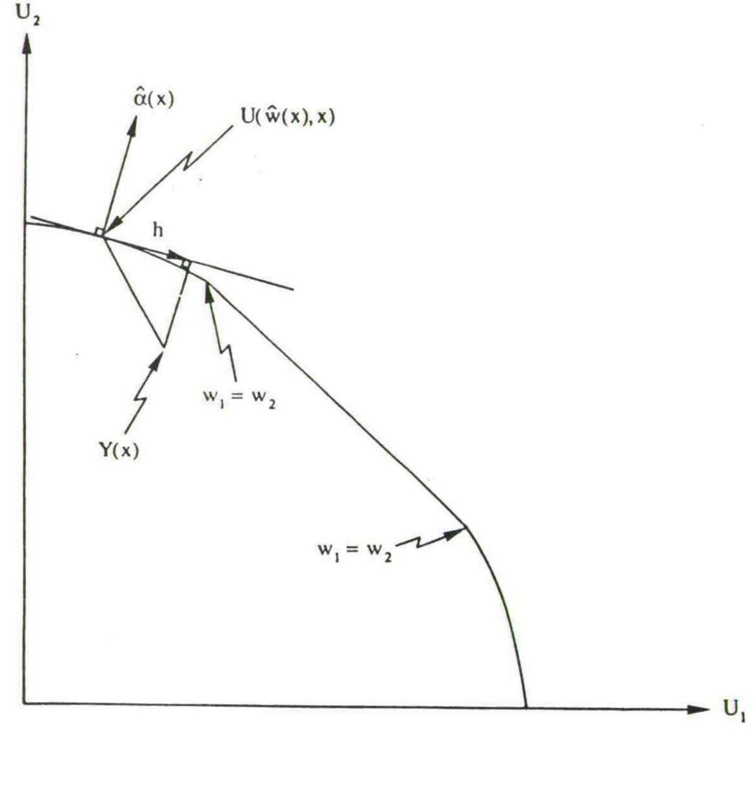

Figure 2. The Utility Possibility Set for Two Individuals.

Non–Positive Cross–Partial Between Wealth and Status.

The Pareto Inefficiency of Certain Gambles.

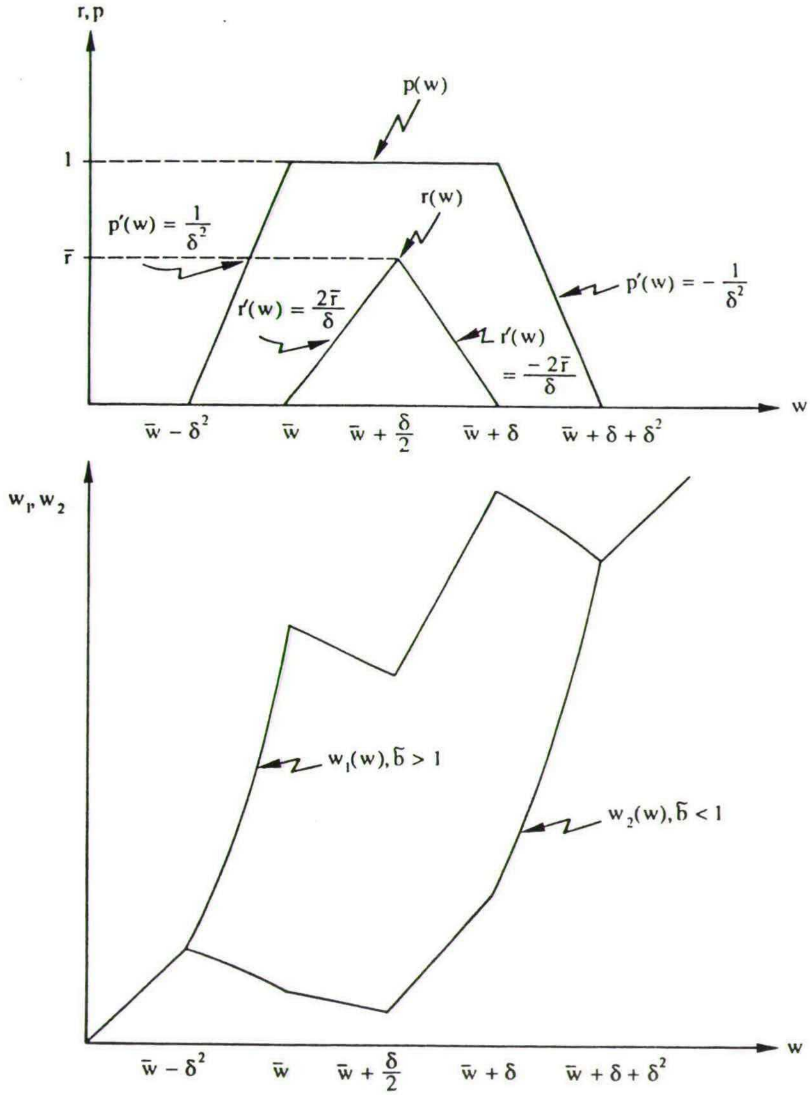

Figure 3. The Construction of a Pareto Inefficient Class of Gambles.

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