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DOMINATED STRATEGIES AND COMMON KNOWLEDGE*

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HEADNOTE

This paper establishes five results. First, the common knowledge of admissibility is not equivalent to iterated admissibility. Second, there exist games in which the common knowledge of admissibility does not uniquely determine which strategies should be eliminated on admissibility grounds. Third, there exist games in which assuming that admissibility is common knowledge yields a contradiction. Fourth, admissibility can be common knowledge without the players knowing the choice sets implied by this knowledge. Finally, difficulties with the common knowledge of admissibility can arise even in games in which each player has a unique dominant strategy.

KEYWORDS: Dominated Strategies, Common Knowledge, Admissibility, Iterated Admissibility

JEL Classification number 026

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DOMINATED STRATEGIES AND COMMON KNOWLEDGE

I. Introduction

The prescription that players in a game should avoid playing weakly dominated strategies (should play "admissible" strategies) is one of the most basic tenets of game theory.[1] It is widely recognized, however, that if players in a game are assumed to not choose dominated strategies, and this is common knowledge, then one derives implications which are potentially much stronger than simply removing dominated strategies from the game.[2] In particular, if player 1 knows that others will not play dominated strategies, then 1 will presumably shun not only strategies which are dominated but also those which become dominated once the dominated strategies of others have been eliminated from consideration. Further iterations of this type of reasoning are possible, yielding the iterated elimination of weakly dominated strategies (or iterated admissibility). Iterated admissibility thus appears to be an implication of the common knowledge of admissibility and has been taken to be equivalent to the common knowledge of admissibility (e.g., Rath (1988)).[3]

The point of departure for this paper is the observation that some intuitive puzzles appear when applying iterated admissibility. First, it is well known that the order in which dominated strategies are eliminated can affect the outcome of the process. Second, cases arise in which agents

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[1]Kohlberg and Mertens (1986) provide a discussion of admissibility. Dekel and Fudenberg (1987) observe that the (iterated) deletion of weakly dominated strategies "clearly incorporates the intuitive objectives of rationality postulates".

[2]The concept of common knowledge was popularized by Aumann (1976). See Binmore and Brandenburger (1988) and Tan and Werlang (1985) for discussions of common knowledge.

[3]Iterated admissibility is examined in Luce and Raiffa (1957) and Moulin (1986). These treatments do not discuss the common knowledge of admissibility. If the outcome of iterated admissibility yields a set of strategies over which players' payoffs are invariant, then Moulin identifies the game as dominance solvable.

eliminate strategies on the strength of the presence of opponents' strategies which are themselves subsequently eliminated. Finally, the process appears to initially call for agents to assume that opponents may play any of their strategies but to subsequently assume that opponents will certainly *not* play some strategies. These anomalies suggest that a more careful examination of what it means to assume that admissibility is common knowledge would be useful.

This paper begins this examination. We embed a game in a framework in which the common knowledge of admissibility can be explicitly modeled and its implications derived. We establish five results.

First, the common knowledge of admissibility is not equivalent to iterated admissibility. Games exist in which iterated admissibility eliminates more strategies than can be justified by an appeal to common knowledge of admissibility as well as games in which iterated admissibility eliminates fewer strategies than does common knowledge of admissibility.

Second, there exist games in which assuming that admissibility is common knowledge does not provide players with sufficient information to determine which strategies should be eliminated on admissibility grounds. Instead, multiplicity or coordination problems arise. It is important to note that the difficulty is *not* that the common knowledge of admissibility fails to eliminate sufficient strategies to reduce strategy sets to singletons, but rather that iterated admissibility does not provide sufficient information to determine which (if any) strategies should be eliminated.

Third, there exist games in which assuming that admissibility is common knowledge yields a contradiction. Admissibility can thus be inconsistent with common knowledge, calling into question two of the seemingly most basic concepts in game theory.

Fourth, suppose that admissibility is common knowledge and that this implies that player 1 chooses from a strategy set given by Z_1 and player 2 chooses from a set given by Z_2. Does it necessarily follow that player 1 knows Z_2 and player 2 knows Z_1? If so, we say that the implications of common knowledge of admissibility are known. We show that admissibility can be common knowledge without the implications of common knowledge of admissibility being known.

Finally, we compare the common knowledge of admissibility with familiar admissibility notions. We show that if both players in a game have dominant strategies, then it is always possible for admissibility to be common knowledge in that game. However, the common knowledge of admissibility need not imply that players choose only their dominant strategies, and games exist in which dominant strategies are the sole implications of common knowledge of admissibility but this cannot be known by the players. We also attempt to extend these results to games with slightly less structure by examining the common knowledge of admissibility in dominance solvable games. It may be impossible for admissibility to be common knowledge in such a game.

The common knowledge of admissibility is thus a paradoxical construction. These results are not surprising. Admissibility and common knowledge are concepts which readily conflict. The usual motivation for admissibility is that one cannot be entirely certain as to an opponent's strategy choice and hence should choose only "safe" or admissible strategies. The implication of common knowledge is that in certain circumstances players can *know* something about opponents' choices, where knowing is a degree of certainty surpassing even that of a probability-one belief. It is then to be expected that these concepts clash, and we have merely made this expectation explicit.[4]

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[4]The only potential surprise in our results is that the common knowledge of admissibility may not select unique dominant strategies (or dominance solvable outcomes) when they exist. Notice, however, that we are working with weak rather than strict dominance, so that even with unique dominant strategies, alternative best replies may exist. Common knowledge arguments can then cause these alternative best replies to be essentially equivalent to the dominant strategy, given what is known about opponents, causing the outcome of common knowledge of admissibility to yield more than simply the dominant strategy.

It is useful to examine our notion of common knowledge of admissibility. Our formulation of common knowledge is standard. In particular, we say that an event A is common knowledge if each agent knows A, each agent knows each agent knows A, each agent knows each agent knows each agent knows A, and so on. To further verify that our formulation of common knowledge is not problematic, we show that it is equivalent to Aumann's (1976) definition involving agents' information partitions. Our paradoxical findings thus cannot be traced to difficulties with the definition of common knowledge.

Attention then turns to our definition of admissibility. We say that if player 1 applies admissibility, then 1 chooses from the set of strategies which are not weakly dominated given the set of possible strategies for player 2. This is again the standard definition. We impose no further conditions, with all implications being derived from the assumption that this definition of admissibility is common knowledge.

The only potential difficulty here is the presumption that player 1's choice set consists of *all* of 1's undominated strategies. Why not allow player 1's choice set to be a subset of 1's undominated strategies? Players would then still respect dominance considerations while some of the paradoxes we discover would be eliminated. For example, our finding that the common knowledge of admissibility is an inconsistent concept in some games would no longer hold.

Two responses arise. First, not all of our paradoxes would be resolved. For example, multiplicity problems would still arise. Second, if we allow players to choose from sets Z_1 and Z_2, with each set containing a subset of the player's undominated strategies but not containing all undominated strategies, then admissibility (and the common knowledge of admissibility) provides an explanation for why the elements of Z_1 and Z_2 are included in players' choice sets, but the exclusion of some of the elements not contained in Z_1 and Z_2 must be motivated by some other considerations. We accordingly have not identified the implications of admissibility, but rather have identified the implications of admissibility coupled with some additional criterion. We would like to identify the implications of admissibility *only*, and hence must examine the choice sets that remain after admissibility and no other considerations have been applied. The relevant choice sets then include all undominated strategies.

Is it an interesting question to examine the implications of admissibility only? The answer is clearly yes. For example, it has often been noted (Kohlberg and Mertens (1986)) that equilibrium concepts such as properness perform well in all respects except admissibility calculations. In particular, the set of proper equilibria can be affected by the deletion of a dominated strategy from a game. One possible response is to construct a two-stage procedure. In the first step, the common knowledge of admissibility is applied to possibly eliminate some strategies. The second step then consists of the application of a solution concept such as properness to the resulting strategy sets.[5] The first step ensures that the resulting equilibrium concept exhibits desirable admissibility properties. In order to implement such a procedure, the first stage must identify *all* of each player's strategies which are not eliminated by dominance considerations. Hence, our admissibility calculations are the appropriate first stage.

Section II of this paper provides some necessary definitions. Section III constructs the formal model of common knowledge of admissibility. Section IV derives the main results. Section V discusses the implications of these results and their connection to the literature.

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[5]Kohlberg and Mertens (1986) consider such a procedure, but reject it because a cell of a game's payoff matrix can be replaced with a constant-sum game that has a value matching that payoffs of the cell but which disrupts the dominance calculations of the initial stage. We do not address this difficulty (though it might be solved by a second-stage solution concept which is not "fooled" by constant-sum games, instead replacing them by their value and returning the game to the first stage). Instead, we consider a prior problem: For a fixed game, what is the outcome of the first-stage admissibility calculation? Hence, we will be concerned with whether this procedure is well defined rather than with its desirability.

II. Definitions.

We restrict attention to finite, two-player normal-form games of complete information. Let the players be denoted 1 and 2; pure strategy sets, S_1 and S_2; pure strategies s_1 and s_2; mixed strategy sets, Δ_1 and Δ_2; mixed strategies δ_1 and δ_2; and payoff functions, π_1 and π_2. Letting $N = \{1,2\}$, $S = S_1 \times S_2$, and $\pi = (\pi_1,\pi_2)$, we represent the game as $(N,S,\pi) \equiv G$.

We now define:

Definition 1. Let $X_1 \subseteq \Delta_1$. Then a strategy δ_1 is **weakly dominated in X_1 given X_2** if $\delta_1 \in X_1$ and if there exists $\delta_1' \in \Delta_1$ such that

$$\pi_1(\delta_1',\delta_2) \geq \pi_1(\delta_1,\delta_2) \qquad \forall\, \delta_2 \in X_2 \tag{1}$$

with strict inequality holding for some $\delta_2 \in X_2$. Strategy δ_1 is **strictly dominated in X_1 given X_2** if strict inequality holds in (1) for all δ_2.

Let $D_1(X_1,X_2)$ be the subset of the strategies in X_1 which are not weakly dominated in X_1 given X_2 and let $D_2(X_1,X_2)$ be analogous. Now construct a sequence $\theta_i(t)$, $t = 1,\ldots,T$, $i=1,2$, by

$$\theta_i(1) = \Delta_i \tag{2}$$

$$\theta_i(t) = D_i(\theta_1(t-1),\theta_2(t-1)), \qquad t = 2,\ldots,T \tag{3}$$

where T is chosen large enough that

$$\theta_i(T) = \theta_i(T-1). \tag{4}$$

Because the game is finite, such a T exists. Let $\overline{D}_i(\Delta_1,\Delta_2) \equiv \theta_i(T)$. Then:

Definition 2. The set of **admissible strategies** for player 1 (player 2 is analogous), given X_2, is given by

$$D_1(\Delta_1,X_2). \tag{5}$$

The set of **iterated admissible strategies** for player i is given by

$$\bar{D}_1(\Delta_1, \Delta_2). \tag{6}$$

If weak domination is replaced by strict domination in definition 2 then we obtain strict admissibility and strict iterated admissibility.

Admissibility requires that weakly dominated strategies not be played. Iterated admissibility, also referred to as the iterated removal of weakly dominated strategies, requires that a sequence of moves be made in which weakly dominated strategies (given the remaining strategy sets) are removed at each step.[6] The process continues until no further removals can be made, at which point the remaining strategy sets are referred to as the sets of iterated admissible strategies. In two-player games, the outcome of *strict* iterated admissibility is the set of rationalizable strategies (Bernheim (1984), Pearce (1984)).

III. Common Knowledge of Admissibility

This section constructs a model in which we can examine the common knowledge of admissibility.

(III.1) Modal Logic

We require a model rich enough to capture what players know, what inferences they can draw from their knowledge, and what it means for players to apply admissibility. Hence, we require a system of epistemic modal logic. We work with the system commonly referred to as S5. This system has its origins in Hintikka (1962); it is discussed in Chellas (1980) and Snyder (1971).

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[6]Our definition of iterated admissibility requires that at each step, each player eliminates *all* dominated strategies. One easily conceives of alternative formulations in which players eliminate only some dominated strategies at each step. It is well known that the outcome of iterated admissibility can depend upon the order of eliminations.

Let L be a formal language, often called the "object" language, which includes the logical operators or constants $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (disjunction) $\Rightarrow$ (implication), $\Leftrightarrow$ (equivalence), $\oplus$ (tautology), $\otimes$ (contradiction), K_1, K_2, P_1, and P_2; the auxiliary symbols "(", ")", and ","; and countable numbers of statement constants, predicate constants, function constants, and variables. In our application, the variables will be used to represent subsets of strategy sets. The function constants we will require include D_1 and D_2, where D_1 associates, with the sets X_1 and X_2, the set $D_1(X_1,X_2)$ of strategies which are undominated in X_1 given X_2. The predicate constant we will require is the equal sign =. Statement constants allow the language to include propositions. The two required in our applications are D_1 and D_2, where D_1 is the statement that player i chooses only admissible strategies. The four operators K_1, K_2, P_1, and P_2 are modal operators and are to be interpreted as "player 1 knows," "player 2 knows," "player 1 thinks it is possible," and "player 2 thinks it is possible." From this basic vocabulary sentences are formed according to familiar rules.

A model (sometimes called a metalanguage) is a structure $\mathbf{M} \equiv \{W, \{M^w\}_{w \in W}, R_1, R_2\}$, where W is a set of possible worlds, M^w is a set of sentences in language L which are taken to be true in world w, and R_i is an equivalence relation on W. Intuitively, R_i captures the information agent i has about the state of the world. Definition 3 makes this intuition formal.

We now let w be a world in W and let "$\mathbf{M}, w \models \phi$" be interpreted as "proposition ϕ is true in world w of model $\mathbf{M}$." To make this precise, we offer:

Definition 3.

$$\mathbf{M}, w \models \phi \quad \text{iff} \quad \phi \in M^w \tag{7}$$

$$\mathbf{M}, w \models \oplus \tag{8}$$

$$\neg\ [\mathbf{M}, w \models \otimes] \tag{9}$$

$$\mathbf{M}, w \models \neg A \quad \text{iff} \quad \neg\ [\mathbf{M}, w \models A] \tag{10}$$

$$M,w \models A \wedge B \quad \text{iff} \quad [(M,w \models A) \wedge (M,w \models B)] \tag{11}$$

$$M,w \models A \vee B \quad \text{iff} \quad [(M,w \models A) \vee (M,w \models B)] \tag{12}$$

$$M,w \models A \Rightarrow B \quad \text{iff} \quad [(M,w \models A) \Rightarrow (M,w \models B)] \tag{13}$$

$$M,w \models A \Leftrightarrow B \quad \text{iff} \quad [(M,w \models A) \Leftrightarrow (M,w \models B)] \tag{14}$$

$$M,w \models K_1A \quad \text{iff} \quad [(M,w' \models A \quad \forall\ w' \in W \text{ s.t. } w'R_1w)] \tag{15}$$

$$M,w \models P_1A \quad \text{iff} \quad [(\exists w' \in W \text{ s.t. } w'R_1w \text{ and } M,w' \models A)]. \tag{16}$$

Statements (7)-(14) are straightforward. (8)-(9) save us from logical nonsense by ensuring that tautologies are true and contradictions false. (10)-(14) ensure that the logical operations of negation, conjunction, disjunction, implication and equivalence work in familiar ways. (15) indicates that player i knows A at world w if A is true in each world w' for which $w'R_1w$. Intuitively, $\{w' : w'R_1w\}$ is the set of possible worlds for i given w, and i knows something at w if it is true in all possible worlds given w. (16) indicates that i considers A to be possible at w if there exists some possible world (given w) at which A is true.

(III.2) Admissibility and Common Knowledge

We now introduce admissibility into our model. First, we fix a game $(N,S,\pi) \equiv G$. We then associate with each world α a pair of variables $Z_1^\alpha \subseteq \Delta_1$ and $Z_2^\alpha \subseteq \Delta_2$, where these identify the choice sets of players 1 and 2. We assume that $\alpha R_1 \beta \Rightarrow Z_1^\alpha = Z_1^\beta$, which (given (15)) is equivalent to assuming that players know their own choice sets. This knowledge is essential to a theory of how agents play games.

Let D_1^α be interpreted as the statement that player i applies admissibility in world α. Formally we assume that in each world α the following hold:

$$D_1^\alpha \Leftrightarrow Z_1^\alpha = D_1(\Delta_1, \bigcup_{\beta R_1 \alpha} Z_2^\beta) \tag{17}$$

$$D_2^\alpha \Leftrightarrow Z_2^\alpha = D_2(\bigcup_{\beta R_2 \alpha} Z_2^\beta, \Delta_1). \tag{18}$$

Condition (17) states that *if* player 1 applies admissibility in world α, then player 1's choice set consists of those strategies which are not weakly dominated given the set of possible strategies played by player 2, which is $\bigcup_{\beta R_1 \alpha} Z_2^{\beta}$ (cf. (16)). Notice that these are straight-forward definitions of admissibility. Notice also that these conditions define admissibility but make no statement about whether players apply admissibility when choosing strategies. Hence, we must offer:

Definition 4. Admissibility holds in world α (or the players apply admissibility in α) if D_1^{α} and D_2^{α} are true in α (i.e., $D_1^{\alpha}, D_2^{\alpha} \in M^{\alpha}$).

We now introduce common knowledge. In particular:

Definition 5. Admissibility is common knowledge in α if the following are true in α:

$$\begin{array}{cc}
D_1^{\alpha} & D_2^{\alpha} \\
K_1 D_2^{\alpha} & K_2 D_1^{\alpha} \\
K_1 K_2 D_1^{\alpha} & K_2 K_1 D_2^{\alpha} \\
K_1 K_2 K_1 D_2^{\alpha} & K_2 K_1 K_2 D_1^{\alpha}. \\
\vdots & \vdots
\end{array} \tag{19}$$

This is again a standard formulation, defining an event to be common knowledge if everyone knows it, everyone knows everyone knows it, everyone knows every one knows everyone knows it, and so on.

It will be helpful to explore the properties of this formulation of common knowledge of admissibility. We offer two results. First, we develop an equivalent statement of (19), i.e., an equivalent statement of what it means for admissibility to be common knowledge.

Lemma 1. The collection of statements in (19) holds, and hence admissibility is common knowledge, in world α if and only if:

$$Z_1^\alpha = D_1(\Delta_1, \bigcup_{\beta R_1 \alpha} Z_2^\beta) \tag{20}$$

$$Z_1^\alpha = D_1\left[\Delta_1, \bigcup_{\beta R_1\alpha} D_2(\bigcup_{\beta' R_2 \beta} Z_1^{\beta'}, \Delta_2)\right] \tag{21}$$

$$Z_1^\alpha = D_1\left(\Delta_1, \bigcup_{\beta R_1\alpha} D_2\left[\bigcup_{\beta' R_2\beta} D_1(\Delta_1, \bigcup_{\beta'' R_1 \beta'} Z_2^{\beta''}), \Delta_2\right]\right) \tag{22}$$

$$Z_1^\alpha = D_1\left(\Delta_1, \bigcup_{\beta R_1\alpha} D_2\left(\bigcup_{\beta' R_2\beta} D_1\left[\Delta_1, \bigcup_{\beta'' R_1\beta'} D_2(\bigcup_{\beta^* R_2 B''} Z_1^{\beta^*}, \Delta_2)\right], \Delta_2\right)\right) \tag{23}$$

$$\vdots$$

$$Z_2^\alpha = D_2(\bigcup_{\beta R_2\alpha} Z_1^\beta, \Delta_2) \tag{24}$$

$$Z_2^\alpha = D_2\left[\bigcup_{\beta R_2\alpha} D_1(\Delta_1, \bigcup_{\beta' R_1\beta} Z_2^{\beta'}), \Delta_2\right] \tag{25}$$

$$Z_2^\alpha = D_2^\alpha\left(\bigcup_{\beta R_2\alpha} D_1\left[\Delta_1, \bigcup_{\beta' R_1\beta} D_2(\bigcup_{\beta'' R_2\beta'} Z_1^{\beta''}, \Delta_2)\right], \Delta_2\right) \tag{26}$$

$$Z_2^\alpha = D_2\left(\bigcup_{\beta R_2\alpha} D_1\left(\Delta_1, \bigcup_{\beta' R_1\beta} D_2\left[\bigcup_{\beta'' R_2\beta'} D_1(\Delta_1, \bigcup_{\beta^* R_1\beta''} Z_2^{\beta^*}), \Delta_2\right]\right), \Delta_2\right) \tag{27}$$

$$\vdots$$

Proof. (20) duplicates the consequent of (17), so D_1^α and (17) give (20). Consider (21). From (15), $K_1 D_2^\alpha$ is equivalent to the statement that the consequent of (18) holds for all β with $\beta R_1 \alpha$. Substitution into (20) then yields (21). Similarly, (24) duplicates the consequent of (18) and hence follows from D_2^α and (18). From (15), $K_2 D_1^\alpha$ is equivalent to the statement that the consequent of (17) holds for all β with $\beta R_2 \alpha$. Substitution in (24) then yields (25). $K_1 K_2 D_1^\alpha$ and $K_2 K_1 D_2^\alpha$ can now be used, again with (15), to establish that (25) and (21) hold for all $\beta R_1 \alpha$ and $\beta R_2 \alpha$. Substitution into (20) and (24) gives (22) and (26). Iteration of this argument yields the result. ∎

We can next provide a check that our formulation of common knowledge is standard. Let $\mathbf{R}_1$ and $\mathbf{R}_2$ be the partitions of W induced by R_1 and R_2. Let $\mathcal{R}$ be the finest common coarsening (meet) of $\mathbf{R}_1$ and $\mathbf{R}_2$. Let a typical element of $\mathcal{R}$ be denoted $\imath$.

Lemma 2. Let $\alpha \in \imath \in \mathcal{R}$. Then admissibility is common knowledge at α if and only if D_1^β and D_2^β hold for all $\beta \in \imath$ or, equivalently, for all $\beta \in \imath$:

$$Z_1^\beta = D_1(\Delta_1, \bigcup_{\beta' R_1 \beta} Z_2^{\beta'}) \tag{28}$$

$$Z_2^\beta = D_2(\bigcup_{\beta' R_2 \beta} Z_1^{\beta'}, \Delta_2). \tag{29}$$

Proof. $\boxed{\text{If}}$ Let (28)-(29) hold. We derive (20)-(27). First, letting $\beta = \alpha$ in (28)-(29) gives (20) and (24). Noting that (28)-(29) hold for all β with $\beta R_1 \alpha$, and substituting, then gives (21) and (25). Iteration of this argument gives the remaining conditions. $\boxed{\text{Only if}}$ Let (19) hold. Then the repeated application of (15) gives (28)-(29). ∎

To see the implications of Lemma 2, note that it immediately yields:

Corollary 1. Admissibility is common knowledge at world α if and only if admissibility holds at each world in the element of the meet of $\mathbf{R}_1$ and $\mathbf{R}_2$ containing α.

This shows that the concept of common knowledge used in our formulation of common knowledge of admissibility is equivalent to the well-known formulation of Aumann (1976).

(III.3) Necessary and Sufficient Conditions

Lemmas 1 and 2 provide conditions for admissibility to be common knowledge, given by (20)-(27) or (28)-(29). These conditions are intuitive but are not especially easy to work with. It will be very helpful to derive conditions which can be expressed solely in terms of the structure of the game

G and which are necessary and sufficient for the ability to construct a model (given G) with a world in which admissibility is common knowledge. This section derives such conditions.

We begin, in Theorem 1, with conditions on a game which are both necessary and sufficient for the ability to construct a model with a world in which admissibility is common knowledge. The requirement that these conditions be both necessary and sufficient causes them to be complicated, and the statement and proof of Theorem 1 are tedious. Our subsequent results will follow from a simpler sufficient condition (presented in Theorem 2) and necessary condition (Theorem 3). Readers who are less interested in technical details may thus want to skip to Theorem 2 on page 16 (though Theorem 1 is used in the proof of Theorem 3).

Theorem 1. A model exists with a world α at which admissibility is common knowledge if and only if there exist sequences

$$Z_{111} \qquad \{Z_{22k}\}_{k=1}^{N_{22}} \qquad \{Z_{13k}\}_{k=1}^{N_{13}} \qquad \{Z_{24k}\}_{k=1}^{N_{24}} \qquad \{Z_{15k}\}_{k=1}^{N_{15}} \qquad \ldots \tag{30}$$

$$Z_{211} \qquad \{Z_{12k}\}_{k=1}^{N_{12}} \qquad \{Z_{23k}\}_{k=1}^{N_{23}} \qquad \{Z_{14k}\}_{k=1}^{N_{14}} \qquad \{Z_{25k}\}_{k=1}^{N_{25}} \qquad \ldots \tag{31}$$

with

$$Z_{1jk} \subseteq \Delta_1$$

and sequences

$$\{I_{1nk}\}_{n=1}^{\infty}{}_{k=1}^{N_{1n}}$$

$$\{I_{2nk}\}_{n=1}^{\infty}{}_{k=1}^{N_{2n}}$$

with

$$I_{ink} \subseteq \{1,\ldots,N_{jn+1}\} \tag{32}$$

$$\bigcup_{k=1}^{n_{in}} I_{ink} = \{1,\ldots,N_{jn+1}\} \tag{33}$$

such that

$$Z_{1nk} = D_1(\Delta_1, \bigcup_{k'\in I_{1nk}} Z_{2n+1k'}) \tag{34}$$

$$Z_{2nk} = D_1(\bigcup_{k'\in I_{2nk}} Z_{1n+1k'}, \Delta_2) \tag{35}$$

$$k' \in I_{1nk} \Rightarrow Z_{1nk} \in \{Z_{1n+2k''} : k'' \in I_{2n+1k'}\} \tag{36}$$

$$k' \in I_{2nk} \Rightarrow Z_{2nk} \in \{Z_{2n+2k''} : k'' \in I_{1n+1k'}\} \tag{37}$$

$$Z_{211} \in \{Z_{21k} : k \in I_{111}\} \tag{38}$$

$$Z_{111} \in \{Z_{11k} : k \in Z_{211}\} \tag{39}$$

Let us first intuitively describe (30)-(39). When constructing our model in which admissibility will be common knowledge at world α, we will take $Z_1^\alpha = Z_{111}$ and $Z_2^\alpha = Z_{211}$. (30) then requires that we find a sequence of sets, where this sequence begins with Z_1^α, then includes a collection of strategy sets for player 2, then a collection for player 1, then a collection for player 2, and so on. Two relationships must hold between these collections of sets. First, (34)-(35) indicate that Z_1^α must be 1's set of undominated strategies given that 2 chooses from $\bigcup_{k=1}^{N_{22}} Z_{22k}$. Similarly, each Z_{22k} must be the set of 2's undominated strategies given that 1 chooses from the union of some subset of the sets in $\{Z_{13k}\}_{k=1}^{N_{13}}$. Each Z_{13k} must in turn be 1's undominated strategies given that 2 chooses from the union of some subset of the sets in $\{Z_{24k}\}_{k=1}^{n_{24}}$, and so on. Second, (36)-(37) require the following. If Z_{13k} (for example) is the set of 1's undominated strategies given that 2 chooses from $\bigcup_{k\in I_{13k}} Z_{24k'}$, then Z_{13k} must appear in each of the collections of player 1 strategy sets which generate the various $Z_{24k'}$. (31) gives a similar sequence starting with a set Z_2^α. (38)-(39) link these sequence together, indicating that Z_1^α must be one of the sets generating Z_2^α and vice versa.

Theorem 1 states that iff sequences satisfying (30)-(39) exist, then we can construct a model in which (20)-(27) hold. From the structure of (20)-(27) it is no surprise that we need sequences of sets linked together by relationships like (34)-(35). Why must (36)-(37) hold? To see this, let

$$Z_{13k} = \bigcup_{k'\in I_{13k}} Z_{24k'}$$

When constructing our model in which admissibility is common knowledge, Z_{13k} will be player 1's choice set in some world γ and the sets in $\{Z_{24k'} : k'\in I_{13k}\}$ will be the choice sets of player 2 in those worlds which player 1 cannot distinguish from γ. Fix a $Z_{24k'}$. This is then the value of Z_2 in some world, say β, with $\beta R_1 \gamma$. Now if admissibility is to be common knowledge then $Z_{24k'}$ must be 2's undominated strategies given that 1 chooses from $\bigcup_{\beta' R_2 \beta} Z_1^{\beta'}$. But $Z_1^{\beta} = Z_{13k}^{\gamma}$ (since $\beta R_1 \gamma$ and $Z_1^{\gamma} = Z_{13k}$). Then Z_{13k} must be one of the sets generating $Z_{24k'}$, as ensured by (36)-(37).

We now prove Theorem 1.

Proof. $\boxed{\text{If}}$ Let (30)-(39) hold. We show that a model satisfying (20)-(27) at some world α can be constructed. First, we set

$$Z_1^{\alpha} = Z_{111}$$
$$Z_2^{\alpha} = Z_{211}.$$

Now create a collection of N_{22} worlds $\beta_1,\ldots,\beta_{n_{22}}$, with $\beta_i R_1 \alpha$ and $Z_2^{\beta_i} = Z_{22i}$, $i = 1,\ldots,N_{22}$. Let no other worlds satisfy $\beta R_1 \alpha$. From (38)-(39), there exists a world $\beta_{1'}$ for which $Z_2^{\beta_{1'}} = Z_2^{\alpha}$ and which can be renamed so that $\beta_{1'} = \alpha$. (34)-(35) then give (20). Next, for each β_i create a collection of worlds $\beta'_{i1},\ldots,\beta'_{in'}$ with $n' = |I_{22i}|$ and with $\beta'_{ij} R_2 \beta_i$. Let no other worlds satisfy $\beta R_2 \beta_i$. Let $Z_{13k} = \beta'_{ik}$ for each $k \in I_{22i}$. From (36)-(37), one of β'_{ij} can be taken to be identical to β_i, and (34)-(35) now give (21). Iteration of the argument yields the result.

$\boxed{\text{Only If}}$ Let (28)-(29) hold. Then we can construct sequences satisfying (30)-(39). Let

$$Z_{111} = Z_1^{\alpha}$$
$$Z_{211} = Z_2^{\alpha}.$$

Then let

$$\{Z_{22k}\}_{k=1}^{N_{22}} = \{Z_2^\beta : \beta R_1 \alpha\}$$

$$\{Z_{12k}\}_{k=1}^{N_{12}} = \{Z_1^\beta : \beta R_2 \alpha\}$$

and

$$I_{111} = \{1,\ldots,N_{22}\}$$

$$I_{211} = \{1,\ldots,N_{12}\}.$$

Then (38)-(39) hold by construction and (28)-(29) ensure (34)-(35). Now let

$$\{Z_{13k}\}_{k=1}^{N_{13}} = \{Z_1^{\beta'} : \beta' R_2 \beta \text{ for some } \beta R_2 \alpha\}$$

$$\{Z_{23k}\}_{k=1}^{N_{23}} = \{Z_2^{\beta'} : \beta' R_2 \beta \text{ for some } \beta R_1 \alpha\}$$

and let

$$I_{22k} = \{k: Z_{13k} = Z_1^{\beta'} \text{ for some } \beta' R_2 \beta \text{ with } Z_2^\beta = Z_{22k}\}$$

$$I_{12k} = \{k: Z_{23k} = Z_2^{\beta'} \text{ for some } \beta' R_1 \beta \text{ with } Z_1^\beta = Z_{12k}\}.$$

Then (28)-(29) again ensure (34)-(35). Fix Z_{22k}. By construction, I_{24k} includes Z_1^β for a value of β for which $Z_2^\beta = Z_{22k}$ and for which $Z_1^\beta = Z_{111}$. This gives (36). (37) is similarly verified. The iteration of this technique gives the result. ∎

The conditions given by (30)-(39) satisfy our requirement that they refer only to the structure of the game G, but are complicated. We can develop a convenient simpler sufficient condition for admissibility to be common knowledge.

Theorem 2. A model can be constructed in which admissibility is common knowledge in world α if there exists a pair of sets $Z_1 \subseteq \Delta_1$ and $Z_2 \subseteq \Delta_2$ which satisfy:

$$Z_1 = D_1(\Delta_1, Z_2) \tag{40}$$

$$Z_2 = D_2(Z_1, \Delta_2). \tag{41}$$

Proof. Given a pair of sets Z_1 and Z_2 satisfying (40)-(41), we construct a model. Fix W, R_1 and R_2 and choose a world α. Let $\alpha \in \mathcal{r} \in \mathcal{R}$. For each $\beta \in$

$\mathcal{n}$, let

$$Z_1^\beta = Z_1$$
$$Z_2^\beta = Z_2.$$

Then (20)-(27) will obviously hold for each $\beta \in \mathcal{n}$ and hence admissibility will be common knowledge in α. ∎

A pair of sets satisfying (40)-(41) will be referred to as a **consistent pair**.

Remark 1. Notice that we have not established the uniqueness of consistent pairs for a game, and note that if (Z_1^α, Z_2^α) and (Z_1^β, Z_2^β) are consistent pairs, then $Z_1^\alpha \neq Z_1^\beta \;\Rightarrow\; Z_2^\alpha \neq Z_2^\beta$.

Examples 10-11 will show that the sufficient condition given by (40)-(41) is not necessary for admissibility to be common knowledge. We can find a relatively straightforward necessary condition:

Definition 6. A pair of sets $\{Z_{11},\ldots,Z_{1n(1)}\} \equiv \mathbf{Z_1}$ and $\{Z_{21},\ldots,Z_{2n(2)}\} \equiv \mathbf{Z_2}$, with $Z_{1i} \subseteq \Delta_1$ and $Z_{2j} \subseteq \Delta_2$, is a **generalized consistent pair** if $\mathbf{Z_1}$ and $\mathbf{Z_2}$ are nonempty and, for every $Z_{1i} \in \mathbf{Z_1}$, there is a subset θ_{1i} of $\mathbf{Z_2}$ such that

$$Z_{1i} = D_1(\Delta_1, \bigcup_{Z_{2k}\in\theta_{1i}} Z_{2k}) \tag{42}$$

and for every $Z_{2j} \in \mathbf{Z_2}$ there exists a subset θ_{2j} of $\mathbf{Z_1}$ such that

$$Z_{2j} = D_2(\bigcup_{Z_{1k}\in\theta_{2j}} Z_{1k}, \Delta_2) \tag{43}$$

and if

$$Z_{2k} \in \theta_{1i} \;\Rightarrow\; Z_{1i} \in \theta_{2k} \tag{44}$$

$$Z_{1k} \in \theta_{2j} \;\Rightarrow\; Z_{2j} \in \theta_{1k}. \tag{45}$$

A generalized consistent pair is a collection of strategy sets for player 1 and a similar collection for player 2 with the property that each set in player 1's collection is the set of player 1's undominated strategies given that player 2 chooses from some subcollection of player 2's choice sets. In addition, these subcollections must be linked by the conditions given in (44)-(45). A consistent pair is the special case in which each collection contains only a single set.

Theorem 3. Let a model exist in which admissibility is common knowledge for game G in world α. Then there exists a generalized pair of consistent sets for G.

Proof. Let (30)-(39) hold. Then define:

$$\mathbf{Z}_{1n} \equiv \{Z_{1nk}\}_{k=1}^{N_{1n}}$$

$$\mathbf{Z}_{2n} \equiv \{Z_{1n+1k}\}_{k=1}^{N_{2n+1}}$$

for n = 3,5,... . Then (36)-(37) ensure that $\mathbf{Z}_{1n}$ and $\mathbf{Z}_{2n}$ are ascending, i.e.,

$$\mathbf{Z}_{1n} \subseteq \mathbf{Z}_{1n+2}$$

$$\mathbf{Z}_{2n} \subseteq \mathbf{Z}_{2n+2}.$$

Because the game is finite, there exists an N such that

$$\mathbf{Z}_{1n} = \mathbf{Z}_{1m} \qquad \forall\ n,m \geq N$$

$$\mathbf{Z}_{2n} = \mathbf{Z}_{2m} \qquad \forall\ n,m \geq N.$$

Let

$$\mathbf{Z}_{1} \equiv \mathbf{Z}_{1N}$$

$$\mathbf{Z}_{2} \equiv \mathbf{Z}_{2N}.$$

(34)-(37) then ensure that $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are a generalized consistent pair. ∎

Finally, we turn to the question of what it means for players to know the outcome of common knowledge of admissibility.

Definition 7. The outcome of common knowledge of admissibility is **known at world** α if the common knowledge of admissibility holds at α and if the players know Z_1^{α} and Z_2^{α}, or, from (15):

$$Z_2^{\alpha} = \bigcup_{\beta R_1 \alpha} Z_2^{\beta} \tag{46}$$

$$Z_1^{\alpha} = \bigcup_{\beta R_2 \alpha} Z_1^{\beta}. \tag{47}$$

Theorem 4. Let common knowledge of admissibility hold in world α. Then the outcome of common knowledge of admissibility is known in world α if and only if (40)-(41) hold at α.

Proof. Substitution of (46)-(47) into (17)-(18) gives (40)-(41). ∎

(III.4) Examples

Much of our analysis will make us of the idea of a consistent pair, which is the sufficient condition for common knowledge of admissibility developed in Theorem 2. It is accordingly useful to illustrate (40)-(41).

Example 1. Consider the game given by

		2: L	2: R
	T	3,2	2,2
1	M	1,0	0,1
	B	0,1	1,0

Depending upon the order of eliminations, iterated admissibility can be used to give the pair {T},{L}, the pair {T},{R}, or the pair {T},Δ{L,R}. The unique pair of consistent sets given by {T} and Δ{L,R}. ∎

Example 2. Consider the matching coins game:

		2: L	2: R
1	T	1,-1	-1,1
	B	-1,1	1,-1

The unique consistent pair consists of Δ{T,B} and Δ{L,R}, or the entire strategy sets. ∎

Example 3. Consider the prisoners dilemma:

		2: L	2: R
1	T	2,2	0,3
	B	3,0	1,1

The unique consistent pair consists of {B} and {R}. ∎

IV. Implications

This section explores the implications of the common knowledge of admissibility.

(IV.1) Common Knowledge of Admissibility and Iterated Admissibility

We begin with the question of whether iterated admissibility is equivalent to the stipulation that admissibility is common knowledge. We answer this question negatively:

Theorem 5. Iterated Admissibility is not equivalent to the common knowledge of admissibility.

To prove this, it suffices to present an example in which the outcome of iterated admissibility differs from the outcome of common knowledge of admissibility. In particular, we can significantly simplify the analysis by noticing that it suffices to present examples of cases in which a consistent pair Z_1^α and Z_2^α exists which is not the outcome of iterated admissibility. We find it convenient to present two examples.

Example 4.

	Y_1	Y_2	Y_3	
X_1	2,4	5,4	-1,0	(iv)
X_2	3,4	2,4	-2,0	
X_3	1,2	0,0	2,2	(iv)
X_4	0,2	2,0	0,4	(ii)
		(i)	(iii)	

Regardless of assumptions made about order of eliminations, iterated admissibility eliminates strategies in the order shown (first i, then ii, and so on) to yield a unique outcome of $(\{X_2\},\{Y_1\})$. (In particular, if only one of strategies X_1 or X_3 is removed by player 1 at the fourth round, then the other strategy from this pair must be removed at the next round.) Common knowledge of admissibility, in contrast, yields a unique consistent pair of $\Delta\{X_1,X_2\}$ and $\Delta\{Y_1,Y_2\}$. The difference in these two outcomes reflects the fact that once a strategy such as Y_2 is eliminated by iterated admissibility, it cannot return even if the reason for its elimination has been purged. If admissibility is common knowledge, however, the knowledge that player 1 limits choices to $\{X_1\}$ provides player 2 with an admissibility-based reason to avoid Y_3 but not Y_2, yielding $\Delta\{Y_1,Y_2\}$ for player 2. Admissibility calculations then lead player 1 to reject X_3 and X_4 but not X_2, yielding $\Delta\{X_1,X_2\}$. We conclude that iterated admissibility and the common knowledge of admissibility are distinct concepts. ∎

Example 5. Consider the following game:

		2: L	C	R	
	T	3,5	2,1	2,0	
1	M	3,1	0,0	0,1	(i)
	B	0,-1	2,2	3,0	(iii)
			(iv)	(ii)	

Iterated admissibility eliminates strategies in the order shown, regardless of order considerations, to yield (T, L). The unique pair of consistent sets of this game is $\Delta\{T, M\}$ and $\{L\}$. The two concepts thus again diverge with payoff relevance. ■

In Examples 4-5, the outcome of common knowledge of admissibility is a superset of the outcomes of iterated admissibility. One might conjecture that this result holds in general. Examples 7-9 and Theorem 7 will show that it does not. Notice also the Examples 4-5 have only shown that the outcomes of common knowledge of admissibility and iterated admissibility *can* differ, in the sense that the unique consistent pair does not match the unique outcome of iterated admissibility. There remains the possibility that a sequence of sets exists satisfying the conditions of Theorem 1 in which Z_{111} and Z_{211} do not yield a consistent pair but do match the outcome of iterated admissibility. However, no such sequence exists for the games given in Examples 4-5. The verification of this is straightforward but tedious and is hence omitted.

(IV.2) Common Knowledge of Admissibility and Uniqueness

We now turn to a second property of the common knowledge of admissibility.

Theorem 6. There exist games in which the common knowledge of admissibility admits multiple solutions.

It is important to clarify what is meant by "multiple solutions." We are *not* referring to the fact that after dominance considerations have been applied, players may be left with sets from which to choose rather than singletons. Instead, multiplicity refers to the fact that simply assuming admissibility to be common knowledge may provide insufficient information to identify the pair of implied choices sets Z_1 and Z_2.

We again prove this by example. It suffices to exhibit games with multiple consistent pairs.

Example 6. Consider the following game:

		2: L	2: R
1	T	3,3	1,3
	B	3,1	0,0

There are two consistent pairs for this game, given by $\Delta\{T,B\}$ and $\{L\}$ and by $\{T\}$ and $\Delta\{L,R\}$ ∎

Example 7. The game given by

		2: L	2: C	2: R
	T	1,2	2,1	0,0
1	M	2,1	1,2	0,0
	B	0,0	0,0	1,1

has three consistent pairs, consisting of $(\Delta\{T,M\},\Delta\{L,C\})$, $(\{B\},\{R\})$, and the entire game. Notice that analogous result holds for a coordination game:

		2: L	2: R
1	T	1,1	0,0
	B	0,0	1,1

∎

A Nash equilibrium is strict if each player's equilibrium strategy gives a strictly larger payoff than all alternative strategies. This property allows us to identify immediately an obvious connection between strict Nash equilibria and consistent sets, stated in the following proposition. In the process, we generate a host of additional examples of games with multiple consistent sets: games with multiple strict Nash equilibria.

Theorem 7. If s^* is a strict Nash equilibrium, then letting $Z_1 = \{s_1^*\}$ yields a consistent pair.

These results ensure that one cannot first apply the common knowledge of admissibility and then apply a solution concept to the remaining sets.

Instead, there may be multiple outcomes consistent with admissibility, and the outcome of the solution concept may differ across these sets. The determination of the implications of admissibility man then fail to be independent of the subsequent solution concept. Equivalently, the outcome of the common knowledge of admissibility depends upon beliefs that may also be linked to the solution concept, destroying the independence between the two. The conclusion is that common knowledge of admissibility is no longer a coordination-free concept and hence loses a fundamental virtue.

The examples offered in proof of Theorem 6 indicate that in certain games, simple rules might be adopted to resolve the multiplicity inherent in the common knowledge of admissibility. However, these examples also make it clear that no obvious, general rule exists. If faced only with Example 6, one might advocate taking the intersection of the consistent pairs, but this will not work in the game given in Example 7 or games with multiple strict Nash equilibria, where the intersection is empty. Example 7 may suggest taking the largest consistent pair, but this will not work in the game given in Example 6 or again in many games with multiple strict Nash equilibria, where a largest pair does not exist. One might be tempted to resolve multiplicity problems by letting $\bar{Z}_1$ be the union of all sets Z_1 possessing the property that Z_1 is a member of a consistent pair and then taking $(\bar{Z}_1, \bar{Z}_2)$ as the outcome of common knowledge of admissibility.[7] However, applying this method to Example 6 shows that it has the unappealing property of failing admissibility!

(IV.3) Existence of Common Knowledge of Admissibility

We now turn to a third question. Is the common knowledge of admissibility a consistent concept, i.e., can one assume that admissibility is common knowledge without implying a contradiction? The answer is no:

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[7]This might be defended on the ground that player i could reasonably expect any of the possible consistent pairs to be the "right" one and hence should regard any element of player j's component of any consistent pair as a potential candidate for play. This reasoning exhibits some of the flavor of rationalizability.

Theorem 8. There exist games for which no generalized consistent pair exists.

From Theorem 3, the absence of a generalized consistent pair precludes the common knowledge of admissibility, so Theorem 8 implies that games exist in which one cannot consistently assert that admissibility is common knowledge. For the proof, it is again sufficient to offer an example:

Example 8. Consider the following game:

		2: L	R
1	T	1,1	1,0
	B	1,0	0,1

A pair of generalized consistent sets does not exist for this game. The difficulty here is that T dominates B for player 1. However, the two differ only if 2 plays R with positive probability, and R is strictly inferior iff 1 plays T. Admissibility then leads 1 to play T if there is any chance of 2 playing R, but 2 is then led to play only L, at which point admissibility now recommends T or B to 1, causing L and R to be the recommendation for player 2, and so on. To make this more precise, we first show that a consistent pair (Z_1, Z_2) does not exist. If Z_2 contains only L, then Z_1 must contain both T and B, in which case Z_2 must contain both L and R, a contradiction. Similarly, if $Z_2 = \{R\}$, then it must be that $Z_1 = \{T\}$, at which point $Z_2 = \{L\}$, a contradiction. Finally, If Z_2 contains any mixtures over L and R, then we must have $Z_1 = \{T\}$, in which case $Z_2 = \{L\}$, again a contradiction. Now extend this to the question of a generalized consistent pair. Z_1 can potentially contain $\{T\}$ and $\Delta\{T,B\}$ for player 1 and $\{L\}$ and $\Delta\{L,R\}$ for player 2, since these are the potential outcomes of the operators D_1 and D_2. Suppose $\Delta\{T,B\} \in Z_1$. Then the associated θ_1 (cf. Definition 6) must contain $\{L\}$. However, the θ_2 associated with $\{L\}$ can include only $\{T\}$ if (43) is to hold, a contradiction (since $\Delta\{T,B\}$ is then not contained in θ_2, falsifying (44)). Similarly, Z_2 cannot include $\Delta\{L,R\}$. Then we must have $Z_1 = \{T\}$ and $Z_2 = \{L\}$.

Then ({T},{L}) would have to constitute a consistent pair, which it does not, so there is no generalized consistent pair. ■

Example 9. Consider again the game given in

		2 L	C	R
	T	1,1	0,1	11,1
1	M	1,0	10,10	1,11
	B	1,11	11,1	10,10

Depending on order, iterated admissibility can be used to select (T,L), (B,L), or (T,R). This game has no generalized consistent pair. We again first examine the existence of a consistent pair. We must consider seven cases. First, let $Z_1 = \Delta\{T,M,B\}$. Then $Z_2 = D_2(Z_1,\Delta_2) = \Delta\{L,R\}$ and $D_1(\Delta_1,Z_2) = \{T\} \neq Z_1$, a contradiction. Similarly,

$$
\begin{array}{llll}
Z_1 = \Delta\{T,M\} & \Rightarrow Z_2 = \{R\} & \Rightarrow D_1(\Delta_1,Z_2) = \{T\} & \neq Z_1 \\
Z_1 = \Delta\{T,B\} & \Rightarrow Z_2 = \{L\} & \Rightarrow D_1(\Delta_1,Z_2) = \Delta\{T,M,B\} & \neq Z_1 \\
Z_1 = \Delta\{M,B\} & \Rightarrow Z_2 = \Delta\{L,R\} & \Rightarrow D_1(\Delta_1,Z_2) = \{T\} & \neq Z_1 \\
Z_1 = \{T\} & \Rightarrow Z_2 = \Delta\{L,C,R\} & \Rightarrow D_1(\Delta_1,Z_2) = \Delta\{T,B\} & \neq Z_1 \\
Z_1 = \{M\} & \Rightarrow Z_2 = \{R\} & \Rightarrow D_1(\Delta_1,Z_2) = \{T\} & \neq Z_1 \\
Z_1 = \{B\} & \Rightarrow Z_2 = \{L\} & \Rightarrow D_1(\Delta_1,Z_2) = \Delta\{T,M,B\} & \neq Z_1.
\end{array}
$$

In each case, $D_1(\Delta_1,Z_2) \neq Z_1$, yielding a contradiction and showing that there is no consistent pair. One similarly shows that there is no generalized consistent pair for this game. ■

The conclusion is clear. There exist games in which admissibility cannot be common knowledge. The assumptions of the application of admissibility and the common knowledge of its application yield logical inconsistencies.

(IV.4) Knowing the Outcome of Common Knowledge of Admissibility

One's initial impression might be that if it is common knowledge that players apply admissibility in world α, then the players must know the outcome of the common knowledge of admissibility in world α. However:

Theorem 9. There exist games in which a world α can arise in which it is common knowledge at α that players apply admissibility but the outcome of common knowledge of admissibility is not known in α.

Example 10 now proves Theorem 9. A similar phenomenon appears in Example 11.

Example 10. Consider the game:[8]

		L	C (2)	R
	T	1,1	1,1	2,1
1	M	1,1	0,0	3,1
	B	1,2	1,3	1,1

Then construct a model with three possible worlds, α, β, and γ, with

$$R_1 = \{\{\alpha,\beta\},\{\gamma\}\}$$

$$R_2 = \{\{\alpha,\gamma\},\{\beta\}\}$$

$$Z_1^\alpha = \Delta\{T,M\} \qquad Z_2^\alpha = \Delta\{L,C\}$$

$$Z_1^\beta = \Delta\{T,M\} \qquad Z_2^\beta = \Delta\{L,R\}$$

$$Z_1^\gamma = \Delta\{T,B\} \qquad Z_2^\gamma = \Delta\{L,C\}.$$

It is then easy to verify that (28)-(29) hold in every possible world, so that (by Lemma 2) admissibility is common knowledge in every world. However, player 1 does not know Z_2 in worlds α and β and player 2 does not know Z_1 in worlds α and γ, or, equivalently, (46)-(47) fail to hold, so that the players do not know the outcome of common knowledge of admissibility. ■

This result again raises problems for a decision procedure which first applies the common knowledge of admissibility and subsequently applies a solution concept. In order to compute the latter, it may be necessary for each player

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[8] I am grateful to Tilman Börgers for suggesting this example and prompting the analysis of this section. It is interesting to note that two consistent pairs for this game do exist, given by $(\Delta\{T,B\},\{C\})$ and $(\{M\},\Delta\{L,R\})$.

to know each of the players' choice sets implied by the common knowledge of admissibility. Example 10 shows that players may not have this knowledge.

(IV.5) Admissibility and Common Knowledge of Admissibility

This section examines the relationship between the common knowledge of admissibility and conventional admissibility. First, suppose each player has a dominant strategy. One might expect the common knowledge of admissibility to select these strategies.

Theorem 10.

(10.1) If a game contains unique dominant strategies for both players 1 and 2 then it contains a consistent pair, though this consistent pair need not be given by the unique dominant strategies.

(10.2) There exist games with the property that s_1 and s_2 are unique dominant strategies for players 1 and 2 and such that there exists a model and a world α with the common knowledge of admissibility holding at α, with $Z_1^\alpha = \{s_1\}$ and $Z_2^\alpha = \{s_2\}$, but with it being impossible for this to hold with the players knowing Z_1^α and Z_2^α.

(10.3) There exist games in which s_1 and s_2 are unique dominant strategies for players 1 and 2 in which it is impossible to achieve common knowledge of admissibility at world α with $Z_1^\alpha = \{s_1\}$ and $Z_2^\alpha = \{s_2\}$.

Proof. Example 11 proves (10.2) and the second statement in (10.1) while Example 12 proves (10.3). It then remains to consider (10.1). Let the strategies s_1 and s_2 be unique dominant strategies for player 1 and 2. Construct a following sequence of sets Z_{11}, Z_{22}, Z_{13}, Z_{24}, Z_{15}, Z_{26},... by:

$$Z_{11} = \{s_1\}$$
$$Z_{22} = D_2(Z_{11}, \Delta_2)$$

$$Z_{13} = D_1(\Delta_1, Z_{22})$$
$$Z_{24} = D_2(Z_{13}, \Delta_2)$$
$$\vdots$$

Now observe that the sets Z_{11}, Z_{13}, Z_{15}, ... are ascending ($Z_{1n} \subseteq Z_{1n+2}$) and Z_{22}, Z_{24}, Z_{26}, ... are descending ($Z_{2n} \supseteq Z_{2n+2}$). To verify this, note that we must have $Z_{11} \subseteq Z_{1n}$ for n = 3, 5, ... because $Z_{11} = \{s_1\}$ and s_1 is a dominant strategy. Next, let $s_2' \notin Z_{22}$, which holds if and only if

$$\pi_2(s_1, s_2') < \pi_2(s_1, s_2). \tag{48}$$

We then have that $s_1 \in Z_{1n}$ for odd n, that s_2 dominates s_2', and that (48) holds. These ensure that $s_2' \notin Z_{2n}$ for even n, and hence $Z_{2n} \subseteq Z_{22}$ for n = 4,6,.... Iteration of this argument gives the result. We next note that because the Z_{1i} and ascending and the Z_{2i} are descending, there must exist an n such that

$$Z_{1n} = Z_{1m} \qquad m = n+2,\ n+4,\ \ldots$$
$$Z_{2n+1} = Z_{2m} \qquad m = n+3,\ n+5,\ \ldots$$

It is then clear that (Z_{1n}, Z_{2n+1}) yields a consistent pair. ■

Example 11. Consider the game of example 6:

		2: L	2: R
1	T	3,3	1,3
	B	3,1	0,0

The strategies T and L are dominant for players 1 and 2. However, consistent pairs include ({T,B},{L}) and ({T},{L,R}) but exclude ({T},{L}). Hence, we can achieve common knowledge of admissibility at world α with Z_1^α = {T} and Z_2^α = {L} only if Z_1^α and Z_2^α are not known (cf. Theorem 4). To show that common knowledge of admissibility at world α with Z_1^α = {T} and Z_2^α = {L} can be achieved, construct a model with three worlds, denoted α, β, and γ. Let

$$R_1 = (\{\beta\}, \{\alpha, \gamma\})$$

$$R_2 = (\{\beta, \alpha\}, \{\gamma\})$$

$$Z_1^{\beta} = \Delta\{T, B\} \qquad Z_2^{\beta} = \{L\}$$

$$Z_1^{\alpha} = \{T\} \qquad Z_2^{\alpha} = \{L\}$$

$$Z_1^{\gamma} = \{T\} \qquad Z_2^{\gamma} = \Delta\{L, R\}.$$

Then admissibility is common knowledge in each world and in world α, $Z_1^{\alpha} = \{T\}$ and $Z_2^{\alpha} = \{L\}$. ■

Example 12. Consider

$$\begin{array}{cc|c|c|c|}
 & & & 2 & \\
 & & L & C & R \\
\hline
 & T & 1,1 & 0,1 & 0,0 \\
\cline{3-5}
1 & B & 0,1 & 0,0 & 0,1 \\
\hline
\end{array} \qquad (49)$$

The unique dominant strategies for the players are T and L. Notice that $\{T\}$ and $\Delta\{L, C\}$ constitute a consistent pair but $(\{T\}, \{L\})$ does not. Can we get common knowledge at some world α with $Z_2^{\alpha} = \{L\}$? If so, we must have

$$\bigcup_{\beta R_2 \alpha} Z_1^{\beta} = \Delta\{T, B\}.$$

Then there must exist some world β with strategy $B \in Z_1^{\beta}$ and with

$$Z_1^{\beta} = D_1(\Delta_1, \bigcup_{\beta' R_1 \beta} Z_2^{\beta'}).$$

This in turn can occur only if each $Z_2^{\beta'}$ with $\beta' R_1 \beta$ excludes L. However, from Lemma 2, L must be a member of Z_2 in each world in the element of the meet of R_1 and R_2 which contains α, since 2 applies admissibility in such worlds and L is dominant, yielding a contradiction. ■

We thus find that admissibility can always be made common knowledge in games in which each player has a unique dominant strategy, though this may require either that the outcome of common knowledge of admissibility contains more than simply the dominant strategies or that it contains only the dominant strategies but is not known.

The dominance considerations in (49) are straightforward, but the interplay between dominance and knowledge considerations is not straightforward. Dominance arguments may initially lead the players to {T} and {L}, but then the knowledge of these same arguments (in particular, 2's knowledge of {T}) leads 2 to $\Delta\{L,C\}$, and it is $(\{T\},\Delta\{L,C\})$ that emerges from common knowledge of admissibility. The dominance arguments that might appear to lead 2 to reject C are rendered irrelevant by knowledge considerations, with the interplay between dominance and knowledge considerations yielding $\Delta\{L,C\}$.

Can we generalize these results? We can consider games which are dominance solvable (cf. note 3).

Example 13. Consider

		2: L	R
1	T	2,2	1,1
	B	2,2	0,3

(50)

Common knowledge of admissibility again cannot obtain in this game. Dominance solvability for this game gives (T,L). ■

V. Discussion

This paper has examined the concept of the common knowledge of admissibility. Our first finding is that iterated admissibility is not equivalent to the common knowledge of admissibility and the differences may have significant payoff implications. There is no general, simple relationship between iterated admissibility and the common knowledge of admissibility.

In light of these findings we turn to the task of characterizing the implications of common knowledge of admissibility. Here, deeper results emerge. In some games, there are multiple pairs of strategy sets consistent with the presumption that admissibility is common knowledge. The common knowledge of admissibility thus yields ambiguous prescriptions. In other games, it is impossible to presume that admissibility is common knowledge without encountering logical inconsistencies. This calls into question two of the seemingly most basic assumptions in game theory. We also find that admissibility can be common knowledge without players knowing the implied choice sets. Finally, admissibility can always be made to be common knowledge in games where each player has a unique dominant strategy, but it may not always be possible to do this with the players knowing the outcome of common knowledge of admissibility and may not always be possible to have the common knowledge of admissibility select only the dominant strategies.

It is important to note that our inconsistency result appears because we require (17)-(18) to hold rather than

$$D_1^\alpha \quad \leftrightarrow \quad Z_1^\alpha \subseteq D_1(\Delta_1, \bigcup_{\beta R_1 \alpha} Z_2^\beta) \tag{51}$$

$$D_2^\alpha \quad \leftrightarrow \quad Z_2^\alpha \subseteq D_2(\bigcup_{\beta R_2 \alpha} Z_1^\beta, \Delta_2). \tag{52}$$

Conditions (51)-(52) require only a pair of sets with the property that once attention is limited to those sets, admissibility provides no further motivation for excluding strategies. Conditions (17)-(18) also require it to be the case that all excluded strategies are excluded because of admissibility. Condition (51), for example, requires a strategy to appear in Z_1 only if it is not dominated. Condition (17) strengthens the "only if" to "if and only if". Conditions (17)-(18) thus require the sets Z_1 and Z_2 to exhibit the admissibility analogues of Greenberg's (1990) internal consistency (the "only if" part) and external consistency (the "if" part) and hence form what Greenberg calls a consistent system.

If we were content to require only conditions (51)-(52), then some of our difficulties would be eliminated (though not all of them; multiplicity problems, for example, would persist). Why do we insist on (17)-(18)? We are interested in the implications of admissibility, and would like to determine the implications of applying *only* admissibility. If a pair of sets Z_1 and Z_2 exists satisfying (51)-(52) but not (17)-(18), then admissibility (and the common knowledge of admissibility) provides an explanation for why the elements of Z_1 and Z_2 are included in players' choice sets, but the exclusion of some of the elements not contained in Z_1 and Z_2 must be motivated by some other considerations. We accordingly have not identified the implications of admissibility, but rather have identified the implications of admissibility coupled with some additional criterion.

A comment on previous studies is now useful. Pearce (1984) and Bernheim (1984) examine the implications of assuming it to be common knowledge that players are rational. Tan and Werlang (1988) continue this line of inquiry. These studies find that the implications of the common knowledge of rationality are that players will employ the iterated elimination of strictly dominated strategies, or strict iterated admissibility. In two-player games, this yields the set of rationalizable strategies. Our work differs in that we begin directly with admissibility. We strengthen strict admissibility to weak admissiblity, but we then conduct the less severe test of examining the internal consistency of the common knowledge of (weak) admissibility rather than deriving admissibility properties as an implication of the common knowledge of rationality. Our finding that it may be impossible for admissibility to be common knowledge is reminiscent of Binmore's (1987-88) finding that perfect rationality is an inconsistent concept.[9]

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[9]Börgers (1989a) argues that the concepts of weak admissibility and common knowledge are inherently contradictory, since the former involves an implicit assumption that any of an opponents' strategies are possible while the latter yields cases in which it is *known* that some strategies will not be played. Börgers responds by constructing a model in which rationality is "approximately" common knowledge, finding that the implications of such a model to be that players will apply one round of admissibility and then apply strict iterated admissibility. Börgers (1989b) examines the possibility of designing efficient collective choice procedures with the usual Nash (or stronger) equilibrium assumption replaced by the assumption that players will not play dominated strategies.

Finally, notice that it is the combination of admissibility and common knowledge that yields difficulties. It remains an open question which is the best candidate for deletion from the model. The intuitive appeal of admissibility and the counter intuitive nature of many of the arguments associated with common knowledge suggest that perhaps the latter should be reconsidered. One approach along these lines is provided by Dekel and Fudenberg (1987), who presume that players are uncertain about opponents' payoffs. Strategies are selected by applying iterated admissibility to perturbed games and taking limits as the perturbations shrink. The implications for the original game are that players should apply one round of admissibility and then apply strict iterated admissibility. A somewhat similar prescription is provided by Pearce's (1984) cautious rationalizability, in which players at each step first iteratively eliminate strictly dominated strategies and then delete weakly dominated strategies. An alternative approach may be allowed by evolutionary arguments (e.g., Samuelson (1988)).

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