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Abstract

The models in the literature on exchange rate target zones imply a non-linear time series model for the exchange rate. We show how the parameters of such models can be estimated and develop Maximum Likelihood and Method of Simulated Moments estimators for the basic target zone model. The Maximum Likelihood estimator is based on a computationally attractive approximation to the exact predictive density of the continuous time model. Monte Carlo experiments are used to assess the properties of this estimator. In the empirical part we estimate the model with data on recent EMS exchange rates. Non-linearities appear to be significant for three out of six series. The target zone model is not able to explain the full observed kurtosis and conditional heteroskedasticity of the exchange rate returns.

1. Introduction

In recent years, target zone arrangements for exchange rates have become very important in Europe. The EMS has extended to cover nearly all EC countries, and the Nordic countries have self-imposed target zones for their currencies. In such target zones, monetary authorities promise to keep the exchange rate within a prespecified band. A large and growing theoretical literature of exchange rate determination in target zones has developed, based on the seminal model of Krugman (1988, 1991). The crucial observation in these papers is that the target zone influences the expectations of future spot prices because the central banks will intervene if the exchange rate deviates too much from its central parity. Hence, in a forward looking model of exchange rate determination, the presence of a target zone has an impact on the exchange rate itself, even if there are currently no interventions. One of the most striking implications of the Krugman model, and nearly all other theoretical models, is the non-linear relation between fundamentals and exchange rates.

Despite the number of theoretical papers on exchange rate target zones, empirical work in this area is not very well developed. In fact, most of the current empirical exchange rate literature uses non-parametric methods to test for non-linearities in univariate stochastic process of the exchange rate or in the relation between economic fundamentals and the exchange rate. Using a target zone model, when appropriate, as an alternative to the linear model may give a more powerful test against non-linearities than a non-parametric approach. There are some papers that attempt to estimate and test target zone models, but the results of these papers are not conclusive about the effect that target zones have on exchange rate behaviour.

In this paper, we develop several methods for estimating the Krugman target zone model. For the basic target zone model the likelihood function is known, though quite complicated, so efficient estimation and testing based on the method of Maximum Likelihood is possible. However, for almost any extension of the model that is present in the literature, the likelihood function is not known, so other estimators are needed. An alternative class of estimators is given by the Generalised Method of Moments. A recent development in GMM estimation is the Method of Simulated Moments, where the model is simulated to compute the moments of the theoretical distribution. Although the computational burden of simulating artificial data and computing their moments is high, simulated moments estimators are very well suited for applications in target zone models, because in these models the stochastic process generating fundamentals and exchange rates is explicitly specified, and simulation of the model is conceptually straightforward.

In the empirical part of this paper the Krugman target zone model is estimated and tested on EMS exchange rate data. The samples are taken from a period where the EMS has shown no realignments; the absence of such realignments is an important assumption in the model. The results indicate that there are significant non-linearities in the exchange rate processes of the Belgian Franc, the French Franc and the Danish Kroner, possibly caused by the impact of the band on expectations of future variables. For the three other currencies, however, there seem to be no non-linearities at all. For the Guilder and the Lira this result may be explainded by the presence of an implicit band that is narrower than the official EMS band. Furthermore, the model is tested by comparing some moments of the theoretical distribution with the sample moments. The test is an application of the moment test of Newey (1985). The tests reveal that the Krugman target zone model is misspecified for the data under consideration: the model is not capable of explaining the full magnitude of the observed conditional heteroskedasticity and leptokurtosis in exchange rate returns. Finally, a comparison with US dollar exchange rates is made. In line with most previous empirical research, we do not detect any non-linearities in these series.

The organisation of the paper is as follows. Section 2 reviews the basic target zone model briefly, and section 3 gives an overview of the empirical literature on testing for non-linearities in exchange rates and on estimating target zone models. Section 4 discusses Maximum Likelihood estimation, and section 5 Method of Simulated Moments estimation of target zone models. Section 6 presents the empirical results and section 7 concludes the paper. The appendices give details on the computation of the estimators and tests.

2. A monetary model of exchange rate determination in a target zone

In this section we derive the basic target zone model of Krugman (1991) from a simple two country monetary model of exchange rate determination. The basic equations of the model are

(1a) $m - p = \varphi y - \alpha i$ (LM)

(1b) $m^* - p^* = \varphi y^* - \alpha i^*$ (LM*)

(1c) $e = p - p^*$ (PPP)

(1d) $E(\dot{e}) = i - i^*$ (UIP)

where m is the money supply, p the price level, y real national income, i the nominal interest rate, e the nominal exchange rate (expressed as domestic currency per unit of foreign currency) and $E(\dot{e})$ the expected (instantaneous) rate of depreciation. Foreign variables are denoted with a *. All variables, except the interest rates, are in logarithms. Equations (1a) and (1b) are the domestic and foreign money market equilibrium schedules, whose parameters are assumed to be equal. (1c) is the relative purchasing power parity condition. Finally, (1d) is the Uncovered Interest Parity condition, which makes the interest differential equal to the expected rate of depreciation.

The reduced form expression for the exchange rate is derived by subtracting (1b) from (1a) and substituting (1c) for the price differential and (1d) for the interest differential, which gives

$$(2) \qquad e = (m-m^*) - \varphi(y-y^*) + \alpha E(\dot{e})$$

Defining the economic **fundamental** as $f(t) \equiv (m-m^*) - \varphi(y-y^*)$ one can rewrite the logarithm of the exchange rate, e(t), as a function of the economic fundamental and its own expected rate of change

$$(3) \qquad e(t) = f(t) + \alpha \cdot E_t(de(t))/dt.$$

Excluding 'bubble' solutions, this implies that e(t) is the present discounted value of all expected future fundamental values

$$(4) \qquad e(t) = \alpha^{-1} \int_{\tau=0}^{\infty} \exp(-\tau/\alpha) E_t[f(t+\tau)] d\tau.$$

In order to compute the exchange rate from (4) we have to make some assumptions on the stochastic process driving the fundamental, f(t). In this paper we shall assume that, except for occasional interventions, the fundamental follows a Brownian motion with constant drift, μ, and variance, σ^2,

(5) $$df(t) = \mu dt + \sigma dW(t)$$

where W(t) is the Wiener process with standard normal increments over the unit time interval. The general solution for the exchange rate, given by Svensson (1991), is

(6) $$e(t) = G(f(t)) = f(t) + \alpha\mu + A_1\exp(\lambda_1 f(t)) + A_2\exp(\lambda_2 f(t))$$

where λ_1 and λ_2 are the roots of the characteristic equation

(7) $$\tfrac{1}{2}\alpha\sigma^2\lambda^2 + \alpha\mu\cdot\lambda = 1$$

and A_1 and A_2 are constants that are to be determined from the economic model. If the relevant exchange rate regime is a free float without interventions, the solution of the model is linear

(8) $$e(t) = G(f(t)) = f(t) + \alpha\mu$$

On the other hand, if there is a fully credible target zone with infenitesimal interventions at the margin only, the solution to the model is non-linear and has the S-shaped form depicted in figure 1. The relevant boundary conditions for the target zone model are the 'smooth pasting' conditions, which state that the derivative of $G(f(t))$ is zero at the margins of the target zone. In that case A_1 and A_2 are implicitly determined by the smooth pasting conditions and are functions of (α, μ, σ^2)[1]. Note that some of the parameters of the model have time dimensions: μ and σ^2 have dimension $[\text{time}]^{-1}$, whereas α has dimension [time]. Therefore, $\alpha\mu$, $\alpha\sigma^2$, μ/σ^2 and A_1, A_2, λ_1 and λ_2 have no time dimension.

The non-linear relation between exchange rate and fundamental in a target zone implies that the stochastic process of the exchange rate is non-linear, even if the stochastic process for the fundamental is linear. The only case where the solution of the exchange rate in a credible target zone is linear is when $\alpha=0$. In that case the presence of a target zone has no effect on the exchange rate, except at the margins when there are interventions. This gives a natural test against non-linearities by testing H_0: $\alpha=0$. Not rejecting the hypothesis of linearity implies that the target zone has not been credible and the exchange rate has behaved according to the linear free float solution.

3. Empirical work on target zone models

Recently, considerable attention has been paid in the literature to non-linear exchange rate models. Most of this literature is purely theoretical, but there are some papers that empirically test for non-linearities in exchange rates. The tests range from very general

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[1] The margins on the fundamental, $\bar{f}$ and $\underline{f}$, must be determined from the endpoint conditions $G(\bar{f})=\bar{e}$ and $G(\underline{f})=\underline{e}$ where $\bar{e}$ and $\underline{e}$ are the upper and lower limit of the target zone.

non-parametric approaches to testing against well-specified alternatives. In this section we briefly review this literature.

Diebold and Nason (1990) try to improve upon the forcastability of a linear autoregressive time-series model for exchange rates by using a non-parametric Locally Weighted Least Squares predictor. For a sample of weekly US dollar exchange rates from 1973 to 1987, the non-parametric technique produces better within-sample predictions, but the out-of-sample performance is not better than that of the linear models. Their conclusion is that non-linearities have not been important for the exchange rates under consideration.

The work of Meese and Rose (1990, 1991) is similar in spirit, but their models include a set of 'fundamental' determinants of the exchange rate, such as interest rates, money supplies and output. The aim of their research is twofold: firstly, to examine whether such structural models predict the exchange rate better than the random walk and secondly, to test for non-linearities in the relation between fundamentals and exchange rates. The technique used is a non-parametric Locally Weighted Regression method and a so-called Alternating Conditional Expectations method. Again, using dollar exchange rates, the conclusions are negative: there are no significant non-linearities and the predictive performance of structural models is not better than that of the random walk.

Flood, Rose and Mathieson (FRM, 1990) use a simple model to examine the presence of non-linear effects in EMS and dollar exchange rates. The starting point is the forward looking exchange rate equation $e(t)=f(t)+\alpha\cdot E[de(t)/dt]$. Assuming uncovered interest rate parity, the expected depreciation equals the instantaneous interest rate differential. Thus replacing $E[de(t)/dt]$ by a very short term (two days in FRM) interset rate differential gives a measure of the fundamental $\hat{f}(t)=e(t)-\alpha\cdot[i(t)-i(t)^*]$. The only thing that is unknown in this equation is the parameter α. FRM do not estimate α, but rather assume that its value is 0.1, and claim that choosing other values does not change the results very much. FRM have two tests for non-linearities. The first 'test' is performed by looking at the graphs of e(t) versus $\hat{f}(t)$. Remarkably few non-linearities seem to be present in the data, and if the relation appears to be non-linear, the shape is not quite the S-shaped type predicted by the Krugman model. The second test is a regression of e(t) on $\hat{f}(t)$ and two exponential terms, similar to those in equation (6). Surprisingly, the parameters of the exponential terms are often significantly different from zero, but the parameter values are not those predicted by the theoretical model.

Another class of tests, more specifically focused on the implications of target zone models, is proposed by Svensson (1990b). Assuming uncovered interest parity holds, so that there are no risk premia, every interest rate differential reflects an expected depreciation or appreciation. Svensson's test for credibility of the target zone checks whether the observed interest rate differentials for a certain maturity are consistent with the exchange rate remaining within the target zone. The conlusion for Swedish data is that the Swedish target zone is not fully credible. The idea of using interest rate differentials to predict realingnment risk is taken up by Rose and Svensson (1991). They use a model that allows for mean reversion within the band and for stochastic realingnments. For the French Franc/Deutsche Mark rate they find significant mean reversion within the band, but it appears difficult to predict actual realignments accurately.

A different approach to target zone modelling is taken by Pesaran and Samiei (1991). They incorporate rational expectations in a discrete-time model with a limited dependent variable, and derive an implicit solution for the expectations variable. Although their model has current expectations instead of future expectations, the model also implies an S-shaped relation between exchange rate and expected fundamental. The relation is however not deterministic but stochastic[2]. The results show that the target zone model fits the data of the Deutsche Mark/French Franc exchange rate during the EMS period much better than a model that doesn't take the presence of a band on this exchange rate into account.

The models closest in spirit to this paper are Smith and Spencer (1990) and Lindberg and Söderlind (1991). Both papers use the Method of Simulated Moments to estimate the Krugman target zone model. Estimation of the model appears to be difficult; both papers report difficulties in finding an optimum of the criterion function. In section 5.3 of this paper it is argued that this is caused by a less than optimal choice of moments used to estimate the parameters. In section 5 we shall try to improve on their method by using other simulation schemes and a better choice of moments.

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[2] It should be noted that the Krugman model implies a deterministic relation between exchange rate and fundamental. However, the fundamental in the Krugman model may be partly unobserved, due to short run deviations from PPP, risk premia in the interest rate differentials and stochastic shocks to the money demand functions, so that the relation between exchange rates and observed fundamentals need not be deterministic.

4. Maximum Likelihood Estimation

If the exact statistical distribution of a sequence $\{e_1(\vartheta),\ldots,e_T(\vartheta)\}$ generated by an economic model is known, the most efficient way of statistical inference is via the likelihood function, defined as the joint density[3] of the observations $D(e_1,\ldots,e_T|E_0;\vartheta)$, where E_0 is the set of initial conditions.

In the target zone model it is convenient to rewrite the likelihood function in terms of the fundamentals as we have specified the data generating process in terms of f(t). The transformation $e(t) = G(f(t);\vartheta)$ is bijective for any ϑ, so we can apply a change of variables, $f_t \equiv G^{-1}(e_t;\vartheta)$, and obtain

$$(9)\qquad D(e_1,\ldots,e_T|E_0;\vartheta) = D(f_1,\ldots,f_T|F_0;\vartheta)\cdot\prod_t G'(f_t;\vartheta)^{-1}$$

The latter part of (9) is the Jacobian of the transformation. Rewriting the joint density as a product of conditional densities by the prediction error decomposition gives

$$(10)\qquad D(f_1,\ldots,f_T|F_0;\vartheta) = [\prod_{t=2}^{T} D(f_t|F_{t-1};\vartheta)]\cdot D(f_1|F_0;\vartheta)$$

where F_{t-1} denotes the initial conditions plus all observations up to and including the $t-1^{th}$. The usefulness of the above decomposition follows from what Harrison (1985, p.81) calls the **strong markov property** of a regulated Brownian motion. The property is that, given f(t), the history $f(t-\tau)$, $\tau>0$, is irrelevant for the distribution of

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[3] The symbol D is used to denote any (joint) density function.

the future of the process, $f(t+\tau)$, $\tau>0$. This property allows us to condition the distribution of f_t on the previous observation only, so we can replace F_{t-1} in (10) by f_{t-1} and obtain

$$(11) \qquad D(f_1,\ldots,f_T|f_0;\vartheta) = [\prod_{t=2}^{T} D(f_t|f_{t-1};\vartheta)]\cdot D(f_1|F_0;\vartheta).$$

The unconditional likelihood function is obtained when $D(f_1)$ is the marginal distribution of the fundamental. Alternatively, when the first observation e_1 is included in the initial conditions, $D(f_1|F_0;\vartheta)$ disappears from (11) and $G'(e_1)$ must be removed from the Jacobian.

The predictive distribution function of the *one-sided* regulated Brownian motion with regulation at lower bound $\underline{f}$ is given in Harrison (1985, p.49). The expression is

$$(12) \qquad P(f|f_{t-s}) = \Phi\left(\frac{f-f_{t-s}-\mu s}{\sigma\sqrt{s}}\right) - e^{\tau(f-\underline{f})}\Phi\left(\frac{2\underline{f}-f-f_{t-s}-\mu s}{\sigma\sqrt{s}}\right), \qquad \tau\equiv 2\mu/\sigma^2$$

The first part of this function is the usual normal distribution function, whereas the second part represents the probability that $f(t)$ is regulated at the lower bound in the time interval $(t-s,t]$. The marginal distribution is obtained by letting s go to infinity which yields

$$(13) \qquad P(f) = \begin{cases} 1-e^{\tau(f-\underline{f})} & \text{if } \tau<0 \\ 0 & \text{if } \tau\geq 0 \end{cases}$$

If the target zone were one-sided, one could apply these distribution functions directly to obtain the likelihood function. However, most actual target zones are two-sided, so we need the conditional density or distribution function of a *two-sided* regulated Brownian motion. Svensson (1990a) derives this density, but the formula is quite complicated and contains an infinite summation, which makes actual computation very time-consuming. For completeness, the function is given in Appendix A.

Instead of using Svensson's formula we approximate the conditional distribution of a two-sided regulated Brownian by a weighted average of the conditional distributions of two one-sided regulated Brownian motions, regulated at the lower and at the upper bound, respectively. The weights are chosen such as to satisfy two conditions. First, the approximate conditional distribution must converge to the exact marginal distribution as the time between f(t) and the initial value f(0) goes to infinity. Second, the function must converge to the predictive distribution of a one-sided regulated Brownian motion if one of the bounds goes to infinity. The distribution function that satisfies these conditions is

$$(14) \qquad P(f|f_{t-s}) = \Phi\left(\frac{f-f_{t-s}-\mu s}{\sigma\sqrt{s}}\right) - (1-P(f))\Phi\left(\frac{2\underline{f}-f-f_{t-s}-\mu s}{\sigma\sqrt{s}}\right) + $$

$$+ P(f)(1-\Phi\left(\frac{2\bar{f}-f-f_{t-s}-\mu s}{\sigma\sqrt{s}}\right))$$

where

$$(15) \qquad P(f) = \frac{e^{\tau(f-\underline{f})}-1}{e^{\tau(\bar{f}-\underline{f})}-1} \text{ if } \tau \neq 0, \quad P(f) = \frac{f-\underline{f}}{\bar{f}-\underline{f}} \text{ if } \tau = 0$$

is the marginal or 'asymptotic' distribution of f(t), see Harrison (1985, p. 90). Numerical experimentation shows that the first derivative of this approximate distribution function gives a very accurate approximation to the exact density function from Appendix A.

Using the true density function or the approximation, we can compute the log-likelihood function as the sum of the logarithms of the conditional densities

$$(16) \qquad \ln L(\vartheta) = \sum_{t=1}^{T} \ln D(e_t|e_{t-1};\vartheta) = \sum_{t=1}^{T} \ln D(f_t|f_{t-1};\vartheta) - \ln G'(f_t;\vartheta)$$

The approximation of the exact density is so accurate that the Maximum Likelihood estimates on the real data in table 1 are the same whether the exact or the approximate density is used.

To test the accuracy and normality of the ML estimator in a reasonably large sample we picked a vector of 'true' parameter values and generated a number of artificial time series of exchange rates, using the simulation method of the target zone model described in Appendix B. Each simulated series contains 1000 observations. The true parameter values picked are $(\mu,\sigma^2,\alpha,\bar{e},\underline{e}) = (0,4,0.1,2.25,-2.25)$; these values imply first and second moments for the exchange rate that are comparable to those of actual EMS exchange rates. For each simulated series the parametervector was estimated by ML using the approximate likelihood function. In figures 2a-c, the empirical distribution functions of 100 estimates of μ, $\ln(\sigma^2)$ and $\ln(\alpha)$ are plotted against a normal distribution with mean equal to the true parameter value and variance equal to the variance of the estimates. The ML estimator seems to be normal with the correct mean, although there is a slight overestimation of the variance of the fundamental. The variance of the estimates of μ and $\ln(\alpha)$ is rather big, indicating that these parameters are not very precisely estimated.

According to Chernoff (1954), the Likelihood Ratio test of H_0: $\alpha=0$

against H_1: $\alpha>0$, the test for non-linearities, has a $\frac{1}{2}\chi_0^2 + \frac{1}{2}\chi_1^2$ distribution. This distribution function and the empirical distribution function computed from 100 Monte Carlo replications with true parameter values (0,4,0,2.25,-2.25) is shown in figure 3. The empirical distribution lies everywhere to the left of the theoretical one, so that using the critical values of Chernoff's distribution gives a conservative test.

5. Method of Moments Estimation of Target Zone models.

This section is divided into three subsections. The first subsection deals with Generalised Method of Moments estimation of the basic target zone model. The second subsection extends the method to cases where moments of the theoretical model are computed by means of simulation. The third subsection discusses the best choice of moments for practical application. Details on the computation of moments for the target zone model are given in Appendix B.

5.1 Generalized Method of Moments estimation.

An alternative to the method of Maximum Likelihood is the Generalised Method of Moments (GMM) developed by Hansen (1982). The principle of GMM estimation is to make some moments of the theoretical distribution of the exchange rate as close as possible to the corresponding moments of the sample $\{e_1,\dots,e_T\}$. In general, GMM estimators are consistent, but not efficient, because only the information in the moments chosen is used, not the information provided by the complete distribution of the data, because in general the vector of moments is not a sufficient statistic.

In the target zone model, unconditional population moments of the exchange rate distribution $E[h(e)] = \int h(e)dP(e)$ can in principle be computed by numerical integration of $h(e)$ over the marginal distribution of the exchange rate, $P(e)$, but this method has the problem that the marginal density $dP(e)/de$ goes to infinity at the bounds of the support $[\underline{e},\bar{e}]$ which may give numerical problems. A simple change-of-variables from exchange rate to fundamental solves these:

$$(17) \qquad E[h(e)] = \int_{\underline{e}}^{\bar{e}} h(e)dP(e) = \int_{\underline{f}}^{\bar{f}} h(G(f))dP(f)$$

where $P(f)$ is the marginal distribution of the fundamental presented in section 4. Computation of unconditional moments of first differences of the exchange rate involves the computation of the double integral

$$(18) \qquad E[h(\Delta e)] = \int\int h(\Delta e)p(f_1,f_0)df_1df_0$$

where $\Delta e \equiv G(f_1)-G(f_0)$ and $p(f_1,f_0)$ is the joint density of f_1 and f_0. Rewriting the joint density as the product of marginal and conditional density gives

$$(19) \qquad E[h(\Delta e)] = \int\left(\int h(\Delta e)p(f_1|f_0)df_1\right)p(f_0)df_0$$

One interpretation of the inner part of the integral is that it is the conditional expectation of $h(\Delta e)$ given f_0. Numerical integration of (19) is feasible, but we also want to compute autocovariances of Δe, which involve the computation of the threefold integral

$$E[h(\Delta e_k, \Delta e_1)] = \int\int\int h(\Delta e_k, \Delta e_1) dP(f_k, f_1, f_0) \tag{20}$$

Computing this integral by e.g. a trapezium rule over a three dimensional grid of points is not very attractive.

However, it is possible to compute (20) by Monte Carlo integration. Conceptually this is straightforward as it is easy to generate random drawings from the marginal distribution of the fundamental. The conditional expectations can be computed by simulating the model several periods forward with the methods described in Appendix B. This way of computing the moments of the target zone model is the basis of the Method of Simulated Moments, which we discuss in the next subsection.

5.2 Method of Simulated Moments Estimation.

Because in the basic target zone model the marginal distribution is known, and has a simple form, computation of the theoretical moments $E[h(e;\vartheta)]$ is possible by integrating $h(e;\vartheta)$ over the marginal distribution of the exchange rate. For more complicated target zone models the marginal distribution of fundamentals or exchange rates is usually unknown. However, the stochastic process that drives the exchange rate is usually explicitly specified, so that the population moments can be estimated by simulating the stochastic process. The parameter value that minimizes the distance between observed sample moments and simulated population moments is McFadden's (1989) Method of Simulated Moments (MSM) estimator.

In target zone models the moments from the stochastic process (model) can be computed by simulation in basically two ways. In the first way, one very long time series of exchange rates $\{\tilde{e}_i(\vartheta)\}_{i=1}^{N}$ is simulated from the model, with arbitrary initial value e_1. If the process is stationary and ergodic, the average of $h(\tilde{e}_i(\vartheta))$ converges to the unconditional population moment $E[h(e;\vartheta)]$. Alternatively, a number of series $\{\tilde{e}_{it}\}$ of length equal to the sample size are simulated and for each simulated series its 'sample' moments are computed. A consistent estimator of the population moments is then obtained by averaging the artificial sample moments over all simulated series. If the initial values $\tilde{e}_{i1}$ are drawn form the marginal distribution this gives an estimator of the unconditional population moments. If simulation of each series is started at the first observation, e_1, one obtains a form of conditional estimation, because then effectively the moments of the joint distribution of T observations, of which the first is e_1, are computed.

The asymptotic properties of the MSM estimator are similar to those of the GMM estimator. The distance between sample moments and the simulated population moments is

$$(21) \qquad d(\vartheta) = \frac{1}{T}\sum_{t=1}^{T} h(e_t) - \frac{1}{N}\sum_{i=1}^{N} h(\tilde{e}_i(\vartheta))$$

If the model is correctly specified, the distance vector has expected value zero in the true parameter value ϑ_0. A well known result on Generalised Method of Moments estimation is that the metric that gives the smallest asymptotic variance of the estimator is the inverse of the covariance matrix of the distance vector. Lee and Ingram (1991) show that if the simulated exchange rates are independent of the observed series the asymptotic variance-covariance matrix of the distance vector is $(1+\frac{1}{n})$ times the variance of the moments $h(e;\vartheta_0)$, where n is the limit of N/T. The precision of the MSM estimator thus increases with the number of simulated values. We use n=50 our applications, so that MSM standard errors are only slightly bigger than would be the case if the exact population moments were available. In our applications, the variance of the moments is estimated by the method of Newey and West (1987) from the simulated values because that estimator is not contaminated by possible irregularities in the data. The asymptotic distribution of the MSM estimator with the optimal weight matrix Σ is

$$\text{(22)} \qquad \sqrt{T}(\hat{\vartheta}-\vartheta_0) \Rightarrow N\left(0,\ (D'\Sigma^{-1}D)^{-1}\right)$$

where D is the matrix of derivatives from the distance vector to the parameters.

5.3 The choice of moments for estimating and testing.

In actual applications the number of moments used for estimation is limited for computational reasons. Therefore, given a number of moments, we want to use moments that are most informative about the parameters to be estimated. In order to achieve this it is desirable that the derivative of the moments with respect to the parameters is big relative to the variance of the moments. In other words, we are looking for moments that are especially sensitive to changes in the value of the parametervector ϑ. It is also desirable that D is well-conditioned in the metric defined by Σ^{-1}, preferably orthogonal, so that all parameters are well-identified.

Although the exchange rate is a stationary process in the target zone model the moments of the *level* of the exchange rate are not very informative about the parameters, the reason being that the observations on the levels are very dependent if the time interval between two observations is short relative to the variance of the process. Moreover, not all parameters of the target zone model are identified by the moments of the marginal distribution of the exchange rate alone. Only the ratio μ/σ^2 and the product $\alpha\mu$ are identified by the function G( ) and the marginal distribution of the fundamental. This argument implies that histograms of the 'marginal' distribution of the exchange rate (e.g. used by Bertola and Caballero (1989)) alone do not provide information about all the properties of the model. It also indicates why the estimators in the papers of Smith and Spencer (1990) and Lindberg and Söderlind (1991) do not seem to work very well: especially estimating α appears to be difficult because the distance function is almost flat in that parameter. In these papers three moments, the mean and variance of the exchange rate returns and the variance of the level, are used to estimate three parameters, so that in principle a unique estimator could be found. However, of the moments used, the variance of the exchange rate level is not very informative about the parameters. This is probably the cause of their difficulties in finding an optimum for the criterion function and obtaining good estimates for the parameters of the target zone model.

The question therefore is which moments are most informative about the parameters. The expectation and variance of Δe are largely determined by the drift, μ, and the variance, σ^2, of the fundamental, so these are obvious candidates. Identification of α from data on the exchange rate alone is more complicated. One way in which α influences the distribution of Δe is via the boundaries of the fundamentals, that are determined by the smooth pasting conditions. The regulation of the fundamentals process at the boundaries induces some negative serial correlation in the process, so the autocorrelations of Δe contain valuable information. If the fundamentals process has a non-zero drift the regulation also induces skewness in the returns distribution, so the third moment is also an informative moment. In principle a lot of other moments could be included, but given computational limitations we confine ourselves to the first three central moments and the first two autocovariances of the returns.

6. Empirical Results

In this section we present the results of an empirical application of the target zone model to exchange rates of six EMS currencies against the Deutsche Mark. In subsection 6.1 the estimates and the test for non-linearities are presented and in subsection 6.2 the specification of the model is tested. In particular, it is tested whether the model can explain certain stylised facts of exchange rate data.

6.1 Data, estimates and a test for non-linearities.

In the empirical application of the target zone model, we use samples of EMS exchange rates from the period after the last realignment in January 1987. The absence of realignments is important because the Krugman target zone model assumes that the target zone is fully credible. Given the stability of the EMS in recent years we feel confident to assume that the current target zone is credible.

The time series we study are weekly observations of the logarithm of the exchange rates[4] of six major EMS currencies against the Deutsche Mark. The sample period is 14 January 1987 until 3 October 1990. The currencies used are the Belgian Franc (BFR), the Dutch Guilder (DFL), the Danish Kroner (DKR), the French Franc (FFR), the Irish Punt (IP), and the Italian Lira (LIRA). The upper and lower limit on the exchange rate in the model are put equal to the official bound of the target zone, i.e. a deviation of +2.25% upward and -2.25% downward from the central parity is allowed (+6% and -6% for the LIRA). We use weekly data to avoid problems with missing observations due to weekends and holidays. It must be stressed that the estimators developed in section 4 and 5 are perfectly suited for dealing with missing observations or any other type of nonequally spaced observations, because the conditional distributions and the simulation schemes can be adjusted for any time interval between two observations. This is certainly an area for future research.

The results of estimating the target zone model with Maximum Likelihood[5], reported in table 1, reveal significant non-linearities for three currencies, the Belgian Franc, the Danish Kroner and the French Franc. The EMS exchange rate system exerts a significant effect on future expectations of these currencies. Graphical inspection of the

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4 For ease of interpretation of the parameter estimates the logaritms of the exchange rate were taken in deviation from the central parity and multiplied by 100.

5 We only use estimators that condition on the first observation, because just after a realingment it is not very realistic to assume that the first observation is drawn from the marginal distribution.

data series shows that these exchange rates come close to the margin, but always revert to the central parity. Also, the volatility of these exchange rates appears to be smaller close to the margin. The target zone model picks up these effects, resulting in a significantly positive estimate of α and a rejection of a linear process for the exchange rate. It must be noted that the large estimates of σ^2 and α for the DKR series and their large standard errors are caused by a high correlation between the estimators of these parameters; the estimated asymptotic correlation between $\hat{\sigma}^2$ and $\hat{\alpha}$ is nearly one, and the likelihood function for the reported values does not differ much from the likelihood evaluated in much smaller values of α and σ^2. This indicates that the model is misspecified; the estimates of α are certainly not consistent with the interpretation as the semi-elasticity of money demand in the monatary model.

The estimates for the Dutch Guilder, the Irish Punt and the Italian Lira show that the target zone model is essentially linear for these currencies; the point estimate of α is very close to 0, and the hypothesis that $\alpha=0$ cannot be rejected by the Likelihood Ratio test at any usual level of significance[6]. The failure of the target zone model to detect any nonlinearities in the Guilder and Lira rates is probably not too surprising, as the Guilder is always very close to its central parity in the sample, and also the Lira doesn't get close to the margins of its relatively wide target zone, probably due to the

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[6] The Wald test (t-test) is not very reliable when α is close to zero, because the different estimates of the asymptotic covariance matrix (outer product of gradient and hessian) are very different. As an aside, all the standard errors reported were computed from the White (1980) estimator of the asymptotic covariance matrix.

possibility of infra-marginal interventions.

One might argue that the significance of α for some currencies is not the result of the presence of a target zone, but just picks up some non-linear effects that might be present in any exchange rate series. To test this hypothesis, the target zone model was estimated on a 'control group' of exchange rate series for which there exists no target zone. The exchange rates of the six currencies already mentioned plus the Deutsche Mark against the US dollar over the same sample period as the EMS data were used for this purpose. The specification of the target zone model requires values for the upper and lower limit of the exchange rate. We imposed a very narrow band on the log exchange rate: the highest and lowest value of the exchange rate in the sample, rounded upward and downward, respectively, to the closest one-hundreth, were used as upper- and lower limit. If there is any credible implicit target zone for these exchange rates, this imposed target zone will be contained in it. This gives a test for non-linearities as powerful as possible within the framework of this model. The results are that the ML estimate of α for all US dollar series is zero, so there are no target zone-type non-linearities in the US dollar exchange rates. This strenghtens our conclusion that the significant non-linearities in EMS exchange rates are caused by the effect of the credible target zone on future expectations.

The Method of Simulated Moments estimates reported in table 2 are usually far from the Maximum Likelihood estimates. The standard errors are large, except when α is estimated very close to zero and the model is essentially linear. In the latter case, the drift and variance of the fundamental (μ and σ^2) are well identified by the first two moments of the exchange rate returns. If α is greater than zero identification of the parameters by the five moments used is weak, but a minimum of the distance function is always found. Thus, our choice of moments improves upon the methods of Smith and Spencer (1990) and Lindberg and Söderlind (1991), although there remain difficulties in obtaining precise parameter estimates for the target zone model by the method of moments. This result is worrying, because the Method of Simulated Moments seems to be the only feasible method of estimating more complex target zone models.

6.2 Testing the target zone model.

Having estimated the model, we want to test whether the model is correctly specified. One way to do this is to check whether the model can explain certain stylised facts about exchange rate returns, in particular ARCH effects and non-normality of the returns distribution.

Most common specification test statistics are defined on the residuals of the model, and test whether the residuals satisfy certain distributional assumptions. In the target zone model residuals are not directly available (we could compute $e_t-E_{t-1}(e_t)$, but there is no analytic expression for the conditional expectation, so that requires numerical integration) and more seriously, the residuals do not come from the same distribution, which makes it hard to define a test statistic on them.

An alternative specification test procedure is the M-test developed by Newey (1985). The principle of the test is to check whether some moments of the exchange rate distribution generated by the theoretical model[7] are significantly different from their sample counterparts. Note that in the method of moments we used the distance between a set of population and sample moments to *estimate* the parameters of the model. We now use the distance between moments to *test* the model.

In the target zone model we are especially concerned about the moments of the exchange rate returns. From the literature it is well-known that the distribution of most actual exchange rate returns has 'fat tails' and that there is intertemporal dependence in the second moment, the well known ARCH effect. We wonder whether the Krugman model is capable of generating distributions for the exchange rate that have sufficient kurtosis and ARCH. The distributional assumption made in deriving the Krugman model, as in nearly all other target zone models, is that the innovations in the fundamental are normally distributed. Although the S-shaped transformation from fundamental to exchange rate may render the exchange rate distribution non-normal, we suspect that the observed first differences still show more non-normality than is implied by the model. In the target zone model we expect a positive correlation in the second moment of Δe, because due to the S-shaped mapping from fundamental to exchange rate the conditional variance is relatively large when the exchange rate is in the middle of the band, but relatively small close to the bounds. So, if the variance of the fundamentals process is not too large,

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[7] The theoretical moments of course depend on the value of the model's parameters. The M-test requires the moments to be evaluated in a consistent estimate of the parametervector. This estimate can be obtained by any consistent estimator of the parametervector, such as the Maximum Likelihood estimator or a method of moments estimator.

succesive observations on the exchange rate will not be too far apart and the conditional variances of succesive Δe's will not be very different. Hence, there is an endogenous ARCH effect in the target zone model. The question is whether this endogenous ARCH effect in the model is strong enough to explain the observed serial correlation in the second moments of the returns.

In order to assess these points we computed two M-tests. The first test, labelled M-norm, is based on the unconditional third and fourth moment of the exchange rate returns. The second test, labelled M-arch, is based on the autocovariances of the squared exchange rate returns, $(\Delta e)^2$. The moments are computed by simulation, as described in section 5. Full details on the computation of the test statistics are in an appendix that is available from the author on request. The distribution of both test statistics is $\chi^2(2)$ under the null that the model is correctly specified, so the 5% critical value of the tests is 5.99.

The tests in table 1 reveal that for all series except the BFR the model is misspecified. The M-arch test indicates that the ARCH-type effect in the data on DFL, FFR and IP rates is not fully explained by the model. However, for the other currencies there is no significant difference in observed and predicted ARCH. The M-norm test on the skewness and kurtosis of the exchange rate returns is highly significant for all series but the BFR. Clearly, the target zone model is not capable of explaining one of the most prominent stylised facts of exchange rates, namely, the fat-tailed marginal distribution of the first differences. Our conclusion is that the distributional assumptions made in deriving the target zone model are too restrictive to get a theoretical distribution that conforms to the sample distribution.

7. Conclusions.

This paper uses the basic target zone model of Krugman (1991) to test for nonlinearities in EMS exchange rates. In the Krugman model, non-linearities arise if there is a credible target zone for the exchange rate and if there is forward-looking behaviour on the foreign exchange market. Without a credible target zone, a linear solution arises. The tests reveal significant nonlinearities for three out of six EMS currencies. Our conclusion is that these non-linearities are due to the presence of a credible target zone that has a significant effect on the stochastic behaviour of the exchange rate within the band. The failure of the model to detect non-linearities in the other three currencies may be explained by the presence of a narrower implicit target zone and the threat of intra-marginal interventions.

The Krugman model is not capable of explaining the full ARCH effects and non-normality of the observed exchange rate returns. Probably the assumption of the Brownian motion with constant drift and variance for the economic 'fundamental' and the assumption of only marginal itervention are too restrictive.

It appears to be difficult to estimate the model by the Method of Simulated Moments. This is bad news for people who want to use more sophisticated target zone models because the Method of Simulated Moments seems to be the only way in which to assess the properties of such models.

There are several lines for future research. The first is to use higher frequency data, e.g. daily. Another extension is to include observed economic fundamentals in the analysis, but this requires a longer period of observations and realignments cannot be avoided for

EMS data, so we need a model that includes realignment risk. Another extension that might be useful is a model that includes intra-marginal interventions, possibly along the line of Lewis (1990).

Appendix A. Conditional distribution of the regulated Brownian motion

Svensson (1990a, Appendix A3) shows that the density of a regulated Brownian motion $f(t)$ with drift μ, variance σ^2 and support $[\underline{f},\bar{f}]$, conditional on $f(0) = f_0$ can be written as

$$p(f|f_0) = \frac{\tau e^{\tau(f-\underline{f})}}{e^{\tau(\bar{f}-\underline{f})}-1} + \frac{\exp[\tau(f-f_0)/2]}{4(\bar{f}-\underline{f})} \sum_{n=1}^{\infty} \frac{y_n(f)\cdot y_n(f_0)}{\lambda_n a^2/\sigma^2}\cdot\exp(-\lambda_n t)$$

$$\tau \equiv 2\mu/\sigma^2, \quad a \equiv (\bar{f}-\underline{f})/\pi, \quad y_n(f) \equiv 2n\cdot\cos(n(f-\underline{f})/a) + \tau a\cdot\sin(n(f-\underline{f})/a)$$

$$\lambda_n \equiv \sigma^2\left[n^2/a^2 + \tau^2/4\right]/2,$$

The speed of convergence of the infinite summation is mainly determined by the $\exp(-\lambda_n t)$ factor, where λ_n depends on the drift and variance of the fundamental in the unit time interval and is of order n^2, so that the infinite sum converges. However, if the time interval between two consecutive observations is very short, say with daily observations, convergence of the infinite summation is very slow and it takes a lot of time to compute the density function. This makes computation of the exact likelihood function considerably more computer time consuming than computation of the approximate likelihood function.

Appendix B. Simulating the target zone model

The estimation methods discussed in this paper often require simulation of the stochastic process of the target zone model. In this section we describe how numerical simulation of this and more general models can be done. The method is based on the work of Duffie and Singleton (1988) who developed a method for computing the moments of a Brownian motion with possibly changing drift and variance

$$\text{(B.1)} \qquad dy(t) = \mu(y(t),t)dt + \sigma(y(t),t)dW(t).$$

where $W(t)$ is a Wiener process and μ and σ are continuous functions of $y(t)$ and t. Integrating this stochastic difference equation from t_0 to t_1 gives

$$\text{(B.2)} \qquad y(t_1) = y(t_0) + \int_{t_0}^{t_1}\mu(y(\tau),\tau)d\tau + \int_{t_0}^{t_1}\sigma(y(\tau),\tau)dW(\tau).$$

One can approximate $y(t_1)$ by replacing the integrals with the sum of a finite number of function evaluations. Using an equidistant partition of the interval $[t_0,t_1]$ one obtains

$$\text{(B.3)} \qquad y(t_1) \approx y(t_0) + \Delta t \sum_{i=0}^{N-1} \mu(y(t_0+i\Delta t),t_0+i\Delta t)$$

$$+ \Delta t \sum_{i=0}^{N-1} \sigma(y(t_0+i\Delta t),t_0+i\Delta t)\cdot\varepsilon(t_0+i\Delta t)/\sqrt{\Delta t}$$

where $\Delta t = (t_1-t_0)/N$ and the $\varepsilon(t)$'s are independent standard normal random variables. This approximation can be used to simulate the stochastic process *recursively*, starting from $y(t_0)$, by running the following simulation scheme

(B.4) $$y(t+\Delta t) = y(t) + \mu(y(t),t)\Delta t + \sigma(y(t),t)\sqrt{\Delta t}\cdot\hat{\varepsilon}(t)$$

where $\hat{\varepsilon}(t)$ is a drawing from the standard normal distribution. This simulation procedure is called the **Euler scheme**. If μ and σ are constant over time it is clear that $E[y(t+\Delta t)|y(t)] = \mu\Delta t$ and $Var[y(t+\Delta t)|y(t)] = \sigma^2\Delta t$. Using the Euler scheme in that case gives no approximation errors for any Δt. If μ and σ do depend on t or on y(t) the approximation error is minimized by chosing Δt as small as possible. Simulation of a sample of size T with observation interval of lenght 1 can be done by choosing a starting value y(0), going $T/\Delta t$ times through the scheme and recording only the values y(t) for which t is an integer. The smaller Δt, the better the distribution of the simulated series y(t) will approximate the distribution of the continuous time process.

The difficulty of a discrete time computer simulation of the continuous time regulated Brownian motion is caused by the assumption that there is only infenitessimal intervention at the margins, so that the process does not jump, but also has a zero probability of being exactly at the margin. In a discrete time approximation, interventions are strictly positive, and we have to make some assumption on where the process goes after an intervention. The scheme used by Smith and Spencer (1990) and Beetsma (1991) is

$$f^*(t+\Delta t) = f(t) + \mu\Delta t + \sigma\sqrt{\Delta t}\cdot\hat{\varepsilon}(t)$$

(B.5)

$$f(t+\Delta t) = \begin{cases} \bar{f} \text{ if } f^*(t+\Delta t)>\bar{f} \\ \underline{f} \text{ if } f^*(t+\Delta t)<\underline{f} \\ f^*(t+\Delta t) \text{ otherwise} \end{cases}$$

It is clear that for $\Delta t>0$ there will be a point mass at $\bar{f}$ and $\underline{f}$ in the distribution of $f(t+\Delta t)$, whereas the mass at those points in the continuous time model is zero. This point mass can be large if $f(t)$ is close to $\bar{f}$ or $\underline{f}$ relative to the magnitude of Δt. A way to improve the accuracy of the simulation is to choose Δt very small if $f(t)$ is close to $\bar{f}$ or $\underline{f}$.

A scheme that gives no point mass at the bounds is found by reflecting the stochastic process in the upper or lower bound if $f^*(t+\Delta t)$ exceeds that bound:

$$f^*(t+\Delta t) = f(t) + \mu\Delta t + \sigma\sqrt{\Delta t}\cdot\hat{\varepsilon}(t)$$

(B.6)

$$f(t+\Delta t) = \begin{cases} 2\bar{f}-f^*(t+\Delta t)-f(t) & \text{if } f^*(t+\Delta t)>\bar{f} \\ 2\underline{f}-f^*(t+\Delta t)-f(t) & \text{if } f^*(t+\Delta t)<\underline{f} \\ f^*(t+\Delta t) \text{ otherwise} \end{cases}$$

We prefer this scheme because it generates no observations exactly on the bounds. Such observations cause problems in computing the likelihood function for simulated data because, as a result of the smooth pasting conditions, $G'(f)$ for such observations is zero, so that the Jacobian of the transformation from fundamental to exchange rate and hence the likelihood function are infinite.

The conditional distribution function of $f(t+\Delta t)$ generated by this scheme with initial value $f(t)=f_t$ is

$$\text{(B.7)} \qquad P(f|f_t) = \Phi\left(\frac{f-f_t-\mu\Delta t}{\sigma\sqrt{\Delta t}}\right) - \Phi\left(\frac{2\underline{f}-f-f_t-\mu\Delta t}{\sigma\sqrt{\Delta t}}\right) + (1-\Phi\left(\frac{2\bar{f}-f-f_t-\mu\Delta t}{\sigma\sqrt{\Delta t}}\right))$$

The first part is the usual normal distribution function, the second part represents the probability that f(t) has been refelected in the lower bound, and the third part is the probability mass reflected in the upper bound. Comparing this distribution with the approximate distribution function of the continuous time model, (14), we see that the discrete simulation overestimates the probabilities of reflection somewhat, due to the omission of interventions that possibly take place within the time interval $(t,t+\Delta t]$. So, for a good approximation to the continuous time distribution it is necessary to use a simulation interval Δt that is small compared with the lenght of the observation interval (in our applications we use 10 drawings per observation).

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Table 1. Maximum Likelihood estimates for DM exchange rates

	μ	σ^2	α	LR	M-norm	M-arch
BFR	-0.148	1.399	4.089	5.32	0.89	1.19
	(0.387)	(2.124)	(9.099)			
DFL	0.028	0.170	0.100	0.00	17.01	170.75
	(0.153)	(0.027)	(a)			
DKR	-0.028	4972.03	139.37	31.47	12.75	1.96
	(0.017)	(4021.01)	(71.61)			
FFR	1.196	8.256	4.375	7.97	316.14	19.92
	(0.710)	(2.889)	(1.362)			
IP	0.181	1.625	0.008	0.00	8.93	26.47
	(0.710)	(0.282)	(0.002)			
LIRA	-1.296	5.723	0.240	0.10	105.18	3.71
	(1.286)	(1.515)	(0.443)			

Notes: weekly data from 14/01/1987 to 3/10/1990
Standard errors in parentheses
LR: likelihood ratio test for $\alpha=0 \simeq \chi^2(1)$
M-norm: M test on third and fourth moment of $\Delta e \simeq \chi^2(2)$
M-arch: M test on first and second autocorrelation of $\Delta e^2 \simeq \chi^2(2)$
(a) not identified for this series

Table 2. Simulated moments estimates for DM exchange rates

	μ	σ^2	α	dist	M-norm	M-arch
BFR	-0.432 (11.57)	1.505 (77.39)	5.109 (383.66)	2.05	11.99	0.18
DFL	0.541 (0.248)	0.172 (0.023)	0.000 (0.031)	12.67	15.28	16.23
DKR	1.600 (31.40)	9.507 (385.68)	0.867 (44.00)	4.32	4.68	0.19
FFR	1.094 (14.58)	6.853 (182.84)	0.737 (30.75)	47.44	29.30	0.54
IP	0.859 (0.744)	1.719 (0.180)	0.000 (0.003)	1.40	52.73	20.99
LIRA	1.933 (5.371)	6.278 (5.286)	0.001 (0.082)	84.46	155.76	1.27

Notes: see table 1
dist is the minimum of the distance function $\approx \chi^2(2)$
moments used for estimation: mean, variance, skewness, first and second autocorrelation of exchange rate return (Δe)

Legend to the figures

Figure 1.

S-shaped mapping from fundamental to exchange rate $(\alpha,\mu,\sigma^2) = (0,2,1)$.

Figure 2.

Solid lines: empirical distribution functions of $\ln(\hat{\alpha})$ (figure 2a), $\hat{\mu}$ (figure 2b) and $\ln(\hat{\sigma}^2)$ (figure 2c). Dotted lines: normal distribution function with zero mean and same variance as the empirical distribution. The average overestimation (with t-value) is

$\ln(\hat{\alpha})$	0.0639 (1.630)
$\hat{\mu}$	-0.0398 (0.866)
$\ln(\hat{\sigma}^2)$	0.0485 (6.070)

Figure 3.

Solid line: empirical distribution of Likelihood Ratio test statistic for α=0. Dotted line: $\frac{1}{2}\chi_0^2 + \frac{1}{2}\chi_1^2$ distribution function.

Figure 1

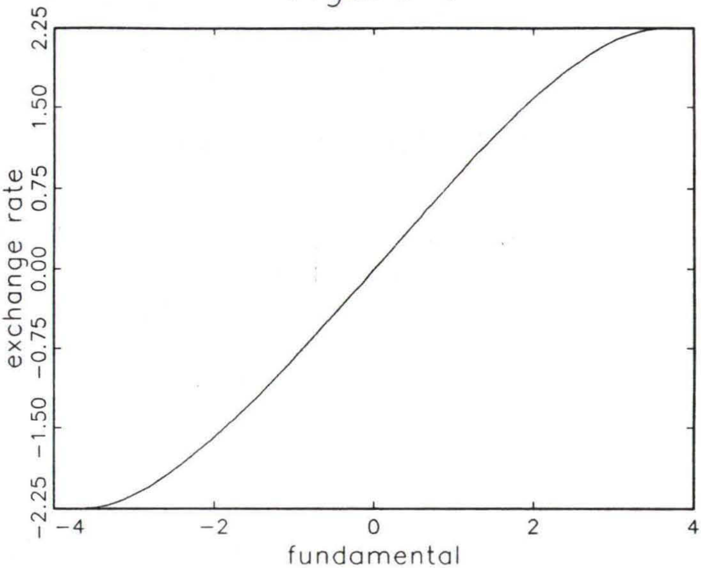


Figure 2a

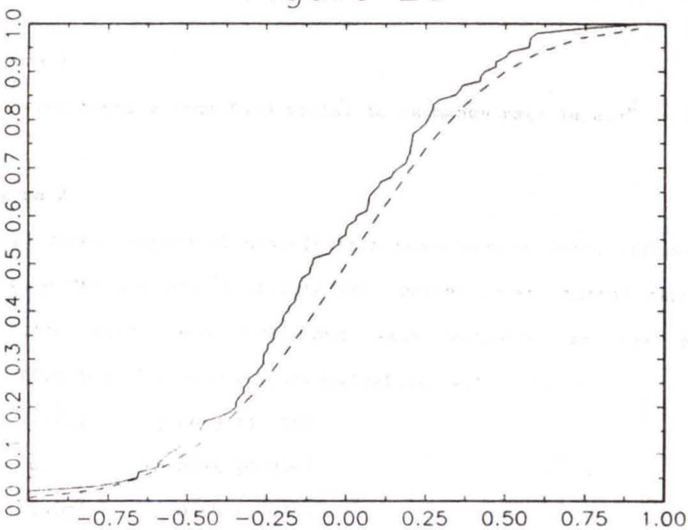


Figure 2b

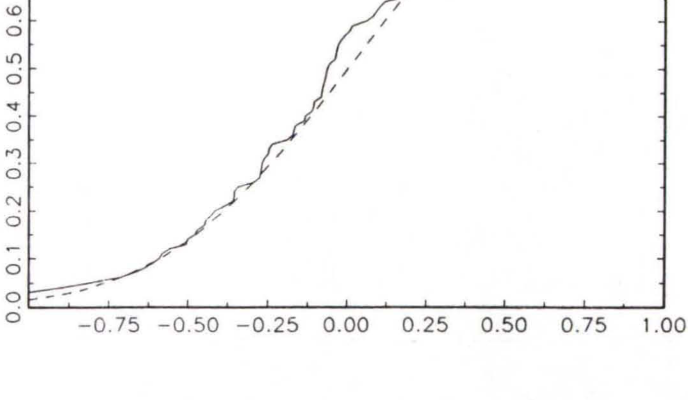


Figure 2c

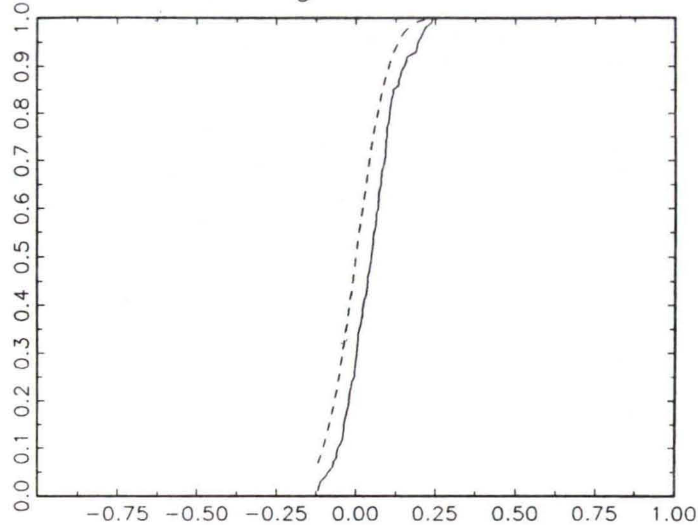
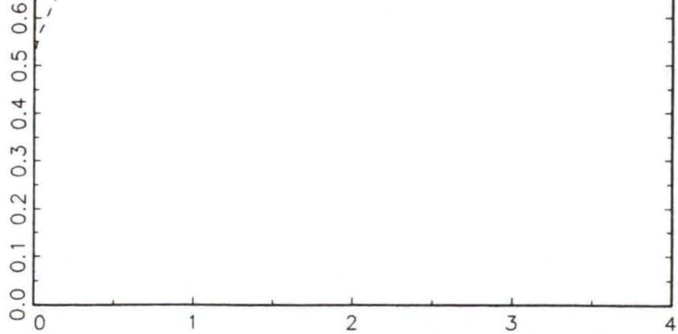
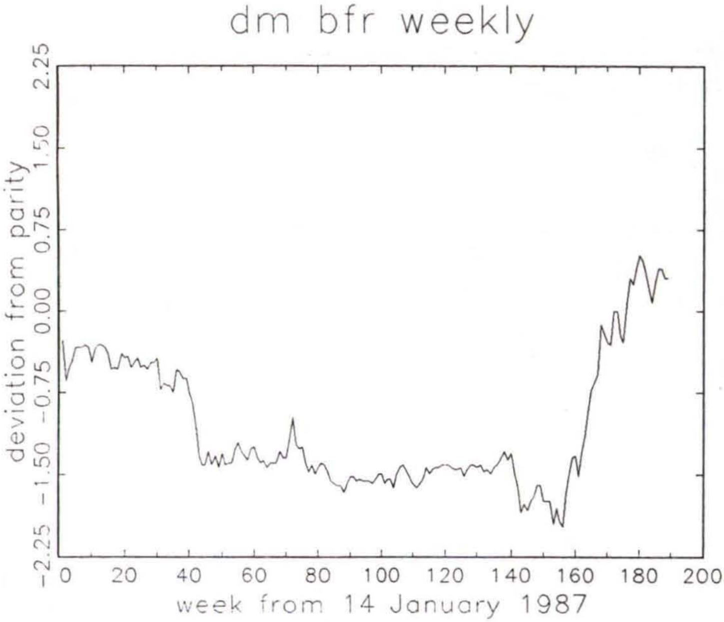
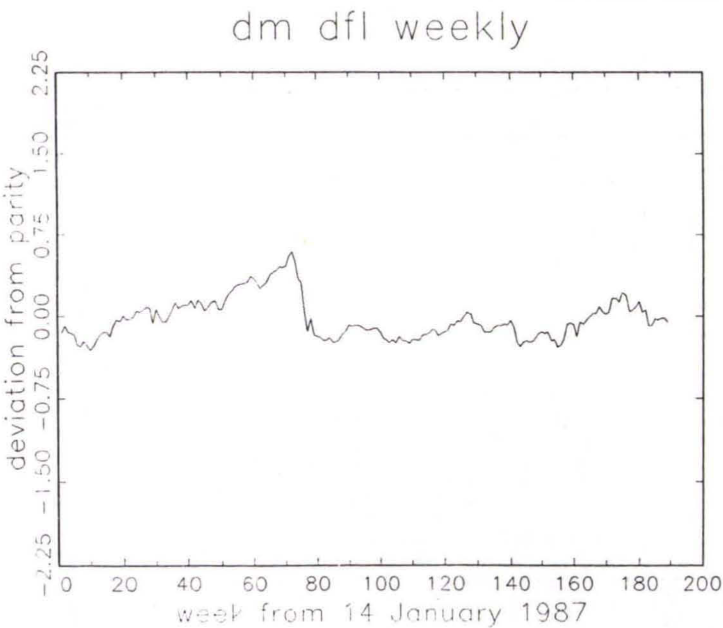
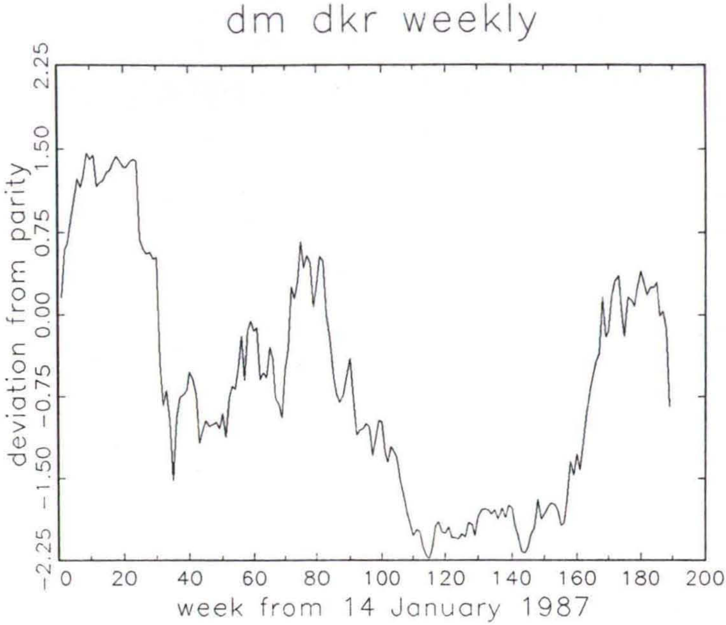


Figure 3




dm bfr weekly
deviation from parity
2.25
1.50
0.75
0.00
-0.75
-1.50
-2.25
0 20 40 60 80 100 120 140 160 180 200
week from 14 January 1987


dm dfl weekly
deviation from parity
2.25
1.50
0.75
0.00
-0.75
-1.50
-2.25
0 20 40 60 80 100 120 140 160 180 200
week from 14 January 1987


dm dkr weekly
deviation from parity
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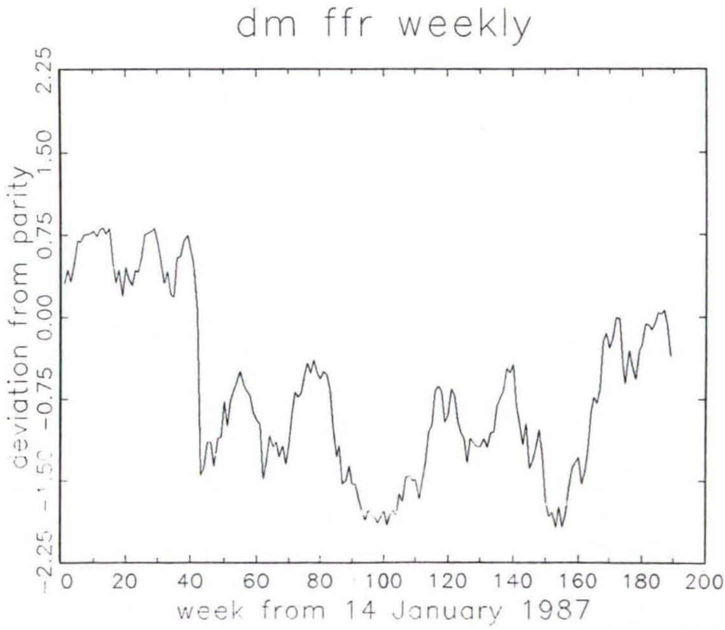
week from 14 January 1987


dm ffr weekly
deviation from parity
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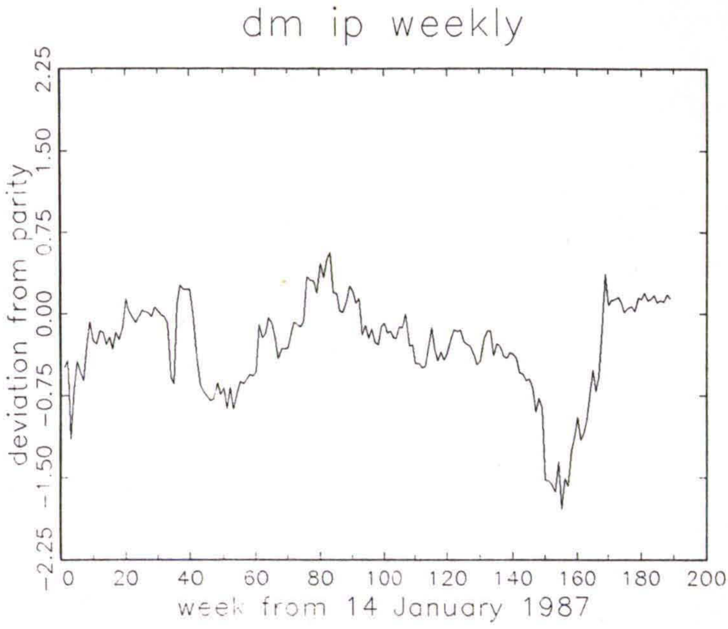
week from 14 January 1987


dm ip weekly
deviation from parity
2.25
1.50
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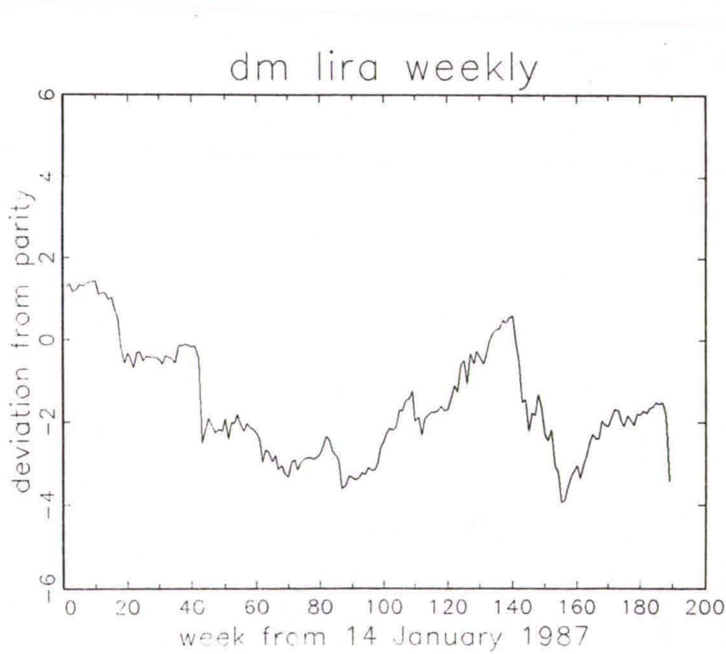
week from 14 January 1987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week from 14 January 1987

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