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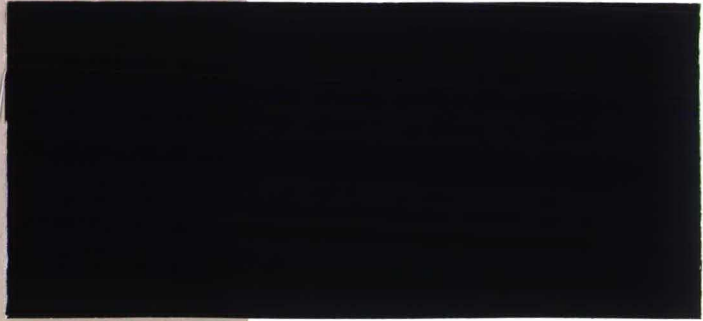
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**THE MC-VALUE FOR
MONOTONIC NTU-GAMES**

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Abstract

The MC-value is introduced as a new single-valued solution concept for monotonic NTU-games. The MC-value is based on marginal vectors, which are extensions of the well-known marginal vectors for TU-games and hyperplane games. As a result of the definition it follows that the MC-value coincides with the Shapley value for TU-games and with the consistent Shapley value for hyperplane games. It is shown that on the class of bargaining games the MC-value coincides with the Raiffa-Kalai-Smorodinsky solution. Furthermore, two characterizations of the MC-value are provided on subclasses of NTU-games which need not be convex valued. This allows for a comparison between the MC-value and the egalitarian solution introduced by Kalai and Samet (1985).

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1 Introduction

Since the introduction of NTU-games by Aumann and Peleg in 1960, many solution concepts have been proposed for this general class of games which extends both the class of TU-games and the class of bargaining games. Most of these solution concepts are based on well-known solutions for TU-games. For example, the Shapley NTU-value (Shapley (1969)), assigns to each NTU-game a set of outcomes based on so-called λ-transfer TU-games associated to this NTU-game. It is well-known that the Shapley NTU-value is an extension of the Shapley value and the Nash bargaining solution to NTU-games. Based on this fact Aumann (1985a) developed a characterization of the Shapley NTU-value using the characterizations of the Shapley value (Shapley (1953)) and the Nash solution (Nash (1950)). For the Harsanyi value (Harsanyi (1963)) a similar reasoning can be followed.

An alternative way to extend the Shapley value to NTU-games was introduced in Maschler and Owen (1989) and (1992). Their consistent Shapley value is a single-valued solution concept based on the following idea: First, the notion of the Shapley value is extended in a straightforward way to so-called hyperplane games, and, based on this extension, a value for general NTU-games is defined by associating hyperplane games to a general NTU-game.

Another solution concept for NTU-games is the compromise value introduced by Borm, Keiding, McLean, Oortwijn, and Tijs (1992). This solution concept for NTU-games is based on ideas underlying the τ-value for TU-games introduced by Tijs (1981). The compromise value is a single-valued solution concept that assigns to each NTU-game a payoff vector which is a compromise between an upper and a lower bound for the core. In Borm et al. (1992), and Otten, Borm, and Tijs (1994) it is shown that the compromise value can be considered as an extension of the τ-value for TU-games and the Raiffa-Kalai-Smorodinsky (RKS) solution for bargaining games (Raiffa (1953), Kalai and Smorodinsky (1975)).

In this paper we introduce a new single-valued solution concept for monotonic 0-normalised NTU-games, the marginal based compromise value, or shortly, the MC-value. This solution concept assigns to each game an efficient outcome lying on the line through 0 and an upper value, which is based on marginal contributions of players

to coalitions. These marginal vectors are extensions of the marginals for TU-games, which can be used to describe the Shapley value. We show that the MC-value by definition extends the Shapley value and the RKS-solution to monotonic NTU-games. Moreover, we provide two characterizations of the MC-value. The second one illustrates that also axiomatically the MC-value can be considered as an extension of the RKS-solution.

The most natural candidate for a comparison with the MC-value is the egalitarian solution introduced by Kalai and Samet (1985). Not only because both solution concepts are defined for NTU-games which are not necessarily convex valued, but mainly because for both solution concepts the outcome is determined by a starting point and a vector which indicates the direction to move in order to obtain an efficient outcome. The main difference between the MC-value and the egalitarian solution is that this direction is fixed for the egalitarian solution and for the MC-value the direction depends on the game, which seems far more natural.

The paper is organised as follows.

In section 2 we start with notations and some basic definitions. Marginal vectors for monotonic NTU-games are defined as an extension of the marginals for TU- and hyperplane games.

In section 3 the MC-value is introduced, and it is shown that the MC-value extends the Shapley value for TU-games, the consistent Shapley value for hyperplane games, and the RKS-solution for bargaining games to the general class of NTU-games.

Section 4 discusses several properties of the MC-value and yields two characterizations of the MC-value on large subclasses of NTU-games. Also a comparison between the MC-value and the egalitarian solution is provided.

Finally, we conclude this paper with some remarks and open problems in section 5.

2 NTU-games and marginal vectors

We start with some notations. Let N be a finite set. A *coalition* is a subset of N. By $\mathbf{R}^N$ we denote the set of all functions from N to $\mathbf{R}$. The elements of $\mathbf{R}^N$ will be

identified with $|N|$-dimensional vectors whose coordinates are indexed by the members of N ($|N|$ denotes the cardinality of N). If $x \in \mathbf{R}^N$ and $i \in N$, we will write x_i in stead of $x(i)$. Further, if $x \in \mathbf{R}^N$, and $\emptyset \neq S \subset N$ is a coalition, we write x_S for the restriction of x to S, i.e., $x_S := (x_i)_{i \in S} \in \mathbf{R}^S$, and $e^S \in \mathbf{R}^N$ denotes the vector with $e_i^S = 1$ if $i \in S$, and $e_i^S = 0$, otherwise.

For $x, y \in \mathbf{R}^N$, we write $x \geq y$ if $x_i \geq y_i$ for all $i \in N$, and $x > y$ if $x_i > y_i$ for all $i \in N$. Further, for $x, \lambda \in \mathbf{R}^N$, we define $\lambda * x \in \mathbf{R}^N$ by $(\lambda * x)_i := \lambda_i x_i$ for all $i \in N$. Let $\mathbf{R}_+^N := \{x \in \mathbf{R}^N \mid x \geq 0\}$, and $\mathbf{R}_{++}^N := \{x \in \mathbf{R}^N \mid x > 0\}$.

Let $A \subset \mathbf{R}^N$ and $\lambda \in \mathbf{R}^N$. Define $\lambda * A := \{\lambda * a \mid a \in A\}$. Further, the boundary of A is denoted by ∂A, $int(A)$ denotes the relative interior of A, and the convex hull of A is denoted by $conv(A)$. A is called *comprehensive* if $x \in A$ and $y \leq x$ imply $y \in A$. The *comprehensive hull* of A is the set $comp(A) := \{x \in \mathbf{R}^N \mid x \leq y \text{ for some } y \in A\}$.

Finally, the set of all permutations of N is denoted by $\Pi(N)$. For $x \in \mathbf{R}^N$ and $\sigma \in \Pi(N)$, we define $\sigma(x) \in \mathbf{R}^N$ by $\sigma(x)_i := x_{\sigma(i)}$ for all $i \in N$. $A \subset \mathbf{R}^N$ is called *symmetric* if for all $x \in A$, and all $\sigma \in \Pi(N)$, we have $\sigma(x) \in A$.

A *non-transferable utility game* or *NTU-game* is a pair (N, V), where $N = \{1, \ldots, n\}$ is a finite set of players, and V is a map assigning to each coalition $S \in 2^N \setminus \{\emptyset\}$ a subset $V(S)$ of $\mathbf{R}^S$ of *attainable payoff vectors* such that

(i) $V(\{i\}) = \{x \in \mathbf{R}^i \mid x \leq 0\}$ for all $i \in N$,

(ii) $V(S)$ is non-empty, closed and comprehensive for each $S \in 2^N \setminus \{\emptyset\}$,

(iii) The set $V_0(S) := \{x \in V(S) \mid x \geq 0\}$ is bounded for each $S \in 2^N \setminus \{\emptyset\}$.

Conditions (ii) and (iii) are standard. Condition (i) is a 0-normalisation which is not very restrictive either. It is imposed only for the sake of convenience. Note that we do not require the sets $V(S)$ to be convex. So this allows for utility functions which are not necessarily of the von Neumann-Morgenstern type (cf. Kalai and Samet (1985)). The set of all NTU-games with player set N is denoted by Γ^N. Often we identify an NTU-game (N, V) with V. NTU-games, introduced by Aumann and Peleg (1960), form a rather large class of games which comprises the well-known class of transferable utility games (von Neumann and Morgenstern (1944)) and the class of cooperative

(pure) bargaining games (Nash (1950)).

A *transferable utility game (TU-game)* is a pair (N, v), where N is the set of players and $v : 2^N \to \mathbf{R}$ is a function which assigns to each coalition a real number such that $v(\emptyset) = 0$. Here, we also require that $v(\{i\}) = 0$ for all $i \in N$ (0-normalisation). The NTU-game (N, V) corresponding to the TU-game (N, v) is given by

$$V(S) := \{x \in \mathbf{R}^S \mid x(S) \leq v(S)\}.$$

for each $S \in 2^N \setminus \{\emptyset\}$.

A *(pure) bargaining game* (with disagreement outcome 0) is an NTU-game (N, V), with $V_0(N) \neq \emptyset$ and

$$V(S) = \{x \in \mathbf{R}^S \mid x \leq 0\} \text{ for all } S \in 2^N \setminus \{\emptyset, N\}.$$

Let V be an NTU-game. The *core* of V, denoted $C(V)$, consists of all payoff vectors attainable for the grand coalition N which are not dominated by any coalition S, i.e.,

$$C(V) := \{x \in V(N) \mid x_S \notin int(V(S)) \text{ for all } S \in 2^N \setminus \{\emptyset\}\}.$$

In the sequel we restrict attention to the class of monotonic NTU-games. An NTU-game V is called *monotonic* if for all $S, T \in 2^N$, with $\emptyset \neq S \subset T$, and all $x \in V(S)$, there exists a $y \in V(T)$ with $y_S \geq x$, or equivalently, if the projection of $V(T)$ on $\mathbf{R}^S$ contains the set $V(S)$. The class of monotonic NTU-games $V \in \Gamma^N$ is denoted by Γ_m^N.

Definition 2.1 Let $V \in \Gamma_m^N$ be a monotonic NTU-game and let $\sigma \in \Pi(N)$. The *marginal vector* $m^\sigma(V)$ is defined by

$$m_{\sigma(i)}^\sigma(V) := \max\{t \in \mathbf{R} \mid (m_{\sigma(1)}^\sigma, \ldots, m_{\sigma(i-1)}^\sigma, t) \in V(\sigma(1), \ldots, \sigma(i))\}$$

for all $i \in N$. If there is no confusion about the game V we write m^σ instead of $m^\sigma(V)$.

Note that the marginal vectors are well-defined, because of the definition of NTU-games and monotonicity. It is also clear that $m^\sigma \in \partial V(N)$ and $m^\sigma \geq 0$ for all $\sigma \in \Pi(N)$.

The interpretation of the vector m^σ is as follows: If $\sigma(1), \ldots, \sigma(n)$ is a certain order on the players, then m^σ assigns to player $\sigma(1)$ the maximum he can obtain in $V(\{\sigma(1)\})$. $m^\sigma_{\sigma(2)}$ is the maximum player $\sigma(2)$ can get in $V(\{\sigma(1), \sigma(2)\})$ given that he should guarantee player $\sigma(1)$ a payoff of $m^\sigma_{\sigma(1)}$, etc. So the marginal vector m^σ assigns to each player the maximum he can get if he should guarantee his predecessors the payoffs already given to them.

The concept of marginal vectors is not a new idea. In the context of TU-games marginal vectors can be used to describe the Shapley value (Shapley (1953)). Also in the field of NTU-games marginal vectors are known: Maschler and Owen (1989) used marginal vectors to define the consistent Shapley value, which is an extension of the Shapley value to the class of hyperplane games. Our definition of marginal vectors for monotonic NTU-games is a straightforward extension of the previous notions of marginal vectors.

3 The MC-value

In this section we will introduce a new single-valued solution concept for monotonic NTU-games based on the marginal vectors, the MC-value.

Definition 3.1 Let $G^N \subset \Gamma^N$. A *solution concept* on G^N is a map F which assigns to each game $V \in G^N$ a (possibly empty) subset $F(V)$ of $\mathbf{R}^N$. F is called a *value* on G^N if it assigns to each $V \in G^N$ a single point in $\mathbf{R}^N$.

Many solution concepts have been proposed for NTU-games. The ones which received most attention in the literature are the Shapley NTU-value (Shapley (1969)) and the Harsanyi value (Harsanyi (1963)). These solution concepts are based on the Shapley value for TU-games. Other solution concepts for NTU-games based on different principles and for various classes are the egalitarian solutions by Kalai and Samet (1985), the consistent Shapley value (Maschler and Owen (1989), (1992)) and the compromise value (Borm et al. (1992)).

Before we introduce the MC-value, we first remark that for TU-games the Shapley value can be viewed as the unique efficient convex combination of 0 and the sum

of the marginal vectors. [1] Therefore, the Shapley value can be regarded as a compromise value, i.e., a value which assigns to each game an outcome which is an efficient compromise between an upper value and a lower value for the game. This observation leads us to the following definition.

Definition 3.2 Let $V \in \Gamma_m^N$ be a monotonic NTU-game. Denote

$$b(V) := \sum_{\sigma \in \Pi(N)} m^\sigma.$$

The *marginal based compromise value* of V, or shortly, the *MC-value* of V is the largest convex combination of 0 and $b(V)$ which is an element of $\partial V(N)$. Formally,

$$MC(V) := \max\{\alpha b(V) \mid \alpha \in \mathbf{R}_+, \alpha b(V) \in V(N)\}.$$

The MC-value is well-defined since $V_0(N)$ is nonempty and compact and the vector $b(V)$ is nonnegative (note that $b(V) = 0$ if and only if $V_0(S) = \{0\}$ for all $S \subset N$, $S \neq \emptyset$). The vector $b(V)$ can be regarded as an upper value for V. [2]

Clearly, the MC-value is a generalisation of the Shapley value to the class of monotonic, 0-normalised NTU-games. The following theorem shows that the MC-value not only by definition extends the Shapley value, but also the consistent Shapley value for hyperplane games and the Raiffa-Kalai-Smorodinsky solution for bargaining games (Raiffa (1953), Kalai and Smorodinsky (1975)).

Theorem 3.3

(i) On the class of monotonic, 0-normalised TU-games the MC-value coincides with the Shapley value.

(ii) On the class of monotonic, 0-normalised hyperplane games the MC-value coincides with the consistent Shapley value.

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[1] This was pointed out by Carles Rafels.

[2] The 0-normalisation we imposed is not very restrictive: the MC-value can be extended to a covariant value on the class of all monotonic games in the following way: Translate an arbitrary monotonic game into a 0-normalised game, compute the MC-value for this game, and then translate it back to obtain a solution for the original game.

(iii) On the class of bargaining games the MC-value coincides with the Raiffa-Kalai-Smorodinsky solution.

The proof is straightforward and therefore it is left to the reader.

We conclude this section with a modified version of an example of Shafer (1980) which has led to an interesting debate on the interpretation of the Shapley NTU-value (cf. Aumann (1985b, 1986), Roth (1986)).

Example 3.4 Consider the following exchange market $\mathcal{E}$ with three agents and two commodities. The initial commodity bundles ω^i of the agents 1,2 and 3, and the utility functions u_i are given by

$$\omega^1 = (1 - \epsilon, 0), \ \omega^2 = (0, 1 - \epsilon), \ \omega^3 = (\epsilon, \epsilon), \text{ and for } (x_1, x_2) \in \mathbf{R}_+^2$$

$$u_1(x_1, x_2) = u_2(x_1, x_2) = \min\{x_1, x_2\}, \ u_3(x_1, x_2) = \tfrac{1}{2}(x_1 + x_2).$$

Here $0 \leq \epsilon < \tfrac{1}{5}$.

The corresponding NTU-game (N, V) is given by

$$V(\{i\}) = \{t \in \mathbf{R} \mid t \leq 0\}, \ i = 1, 2$$
$$V(\{3\}) = \{t \in \mathbf{R} \mid t \leq \epsilon\},$$
$$V(\{1,2\}) = \{(t_1, t_2) \in \mathbf{R}^{\{1,2\}} \mid t_1 + t_2 \leq 1 - \epsilon, \ t_1 \leq 1 - \epsilon, \ t_2 \leq 1 - \epsilon\},$$
$$V(\{1,3\}) = \{(t_1, t_3) \in \mathbf{R}^{\{1,3\}} \mid t_1 + t_3 \leq \tfrac{1}{2} + \tfrac{1}{2}\epsilon, \ t_1 \leq \epsilon, \ t_3 \leq \tfrac{1}{2} + \tfrac{1}{2}\epsilon\},$$
$$V(\{2,3\}) = \{(t_2, t_3) \in \mathbf{R}^{\{2,3\}} \mid t_2 + t_3 \leq \tfrac{1}{2} + \tfrac{1}{2}\epsilon, \ t_2 \leq \epsilon, \ t_3 \leq \tfrac{1}{2} + \tfrac{1}{2}\epsilon\},$$
$$V(\{1,2,3\}) = \{(t_1, t_2, t_3) \in \mathbf{R}^N \mid t_1 + t_2 + t_3 \leq 1, \ t_1 \leq 1, \ t_2 \leq 1, \ t_3 \leq 1\}.$$

Note that V is not 0-normalised if $\epsilon \neq 0$. If we compute the MC-value of this game by following the approach described in footnote 2, we obtain

$$MC(V) = (\frac{5}{12} - \frac{5}{12}\epsilon, \frac{5}{12} - \frac{5}{12}\epsilon, \frac{1}{6} + \frac{5}{6}\epsilon).$$

This outcome is also prescribed by the Shapley NTU-value. However, other solution concepts such as the compromise value and the Harsanyi solution yield different outcomes (see for example Borm et al. (1992)). There the outcome for player 3 is 0 if $\epsilon = 0$, while the MC-value and the Shapley NTU-value always give a positive payoff to player 3 of at least $\tfrac{1}{6}$. This fact has been a topic in an extensive discussion about the Shapley NTU-value between Aumann and Roth.

4 Characterizations of the MC-value

In this section we investigate several properties of the MC-value and moreover, two characterizations of the MC-value are provided. We conclude this section with a comparison between the MC-value and the egalitarian solution.

Some properties of the MC-value are summarised in proposition 4.1.

Proposition 4.1 On the class Γ_m^N of monotonic NTU-games the MC-value satisfies the following properties.

(i) *weak Pareto optimality:* $MC(V) \in \partial V(N)$ for all $V \in \Gamma_m^N$.

(ii) *scale covariance:* $MC(\lambda * V) = \lambda * MC(V)$ for all $\lambda \in \mathbf{R}_{++}^N$ and all $V \in \Gamma_m^N$. (The game $\lambda * V$ is defined by $(\lambda * V)(S) := \lambda * V(S)$ for all $S \subset N, S \neq \emptyset$.)

(iii) *symmetry:* $MC_i(V) = MC_j(V)$ for all $V \in \Gamma_m^N$ and all $i, j \in N$ which are symmetric in V. Here, players $i, j \in N$ are called *symmetric* in V if

   (1) for all $S \subset N \setminus \{i, j\}$, all $x \in V(S \cup \{i\})$ it holds that $y \in V(S \cup \{j\})$, where $y \in \mathbf{R}^{S \cup \{j\}}$ is defined by $y_j = x_i$ and $y_S = x_S$,

   (2) for all $S \subset N$, $i, j \in S$ and all $x \in V(S)$, we have $y \in V(S)$, where $y \in \mathbf{R}^S$ is defined by $y_i = x_j$, $y_j = x_i$ and $y_{S \setminus \{i,j\}} = x_{S \setminus \{i,j\}}$.

(iv) *b-symmetry:* $MC_i(V) = MC_j(V)$ for all $i, j \in N$ with $b_i(V) = b_j(V)$, and all $V \in \Gamma_m^N$.

(v) *conditional monotonicity:* $MC(V) \leq MC(W)$ for all $V, W \in \Gamma_m^N$ with $V(N) \subset W(N)$, $b(W) > 0$, and $\frac{b_i(V)}{b_i(W)} = \frac{b_j(V)}{b_j(W)}$ for all $i, j \in N$.

Proof. We will only prove (ii). The other properties are obvious. Let $V \in \Gamma_m^N$, $\lambda \in \mathbf{R}_{++}^N$ and $\sigma \in \Pi(N)$. The reader easily verifies that $m^\sigma(\lambda * V) = \lambda * m^\sigma(V)$, and so $b(\lambda * V) = \lambda * b(V)$. From this it immediately follows that $MC(\lambda * V) = \lambda * MC(V)$.
□

The properties (i), (ii) and (iii) are standard. Besides these three the MC-value

satisfies also other standard properties as individual rationality, unanimity, and the null player property. Property (iv) is a stronger version of symmetry. It is introduced to characterize the MC-value and states that if for two players in a game the sum of all their marginal contributions is equal, then they should get the same payoff. Property (v) is a strengthening of the (restricted) monotonicity which is used to characterize the RKS-solution for bargaining games. The interpretation is that if the set of attainable payoffs for the grand coalition becomes larger and the direction of the upper value does not change, then no player will be worse off in the new situation.

Now we will provide two characterizations of the MC-value on subclasses of monotonic NTU-games. Attention will be restricted to the class $\tilde{\Gamma}^N_m$ of monotonic NTU-games $V \in \Gamma^N_m$ satisfying $b(V) > 0$. This is a weak condition which means that every player has a positive marginal contribution to at least one coalition. Note that $b(V) > 0$ guarantees the existence of a strictly positive point in $V(N)$, so in particular, games with null players are excluded.

We have the following characterization of the MC-value on the class $\tilde{\Gamma}^N_m$.

Theorem 4.2 The MC-value is the unique value on $\tilde{\Gamma}^N_m$ which satisfies

(i) weak Pareto optimality,

(ii) scale covariance,

(iii) b-symmetry.

Proof. From proposition 4.1 it follows that the MC-value satisfies (i)-(iii). Let $F : \tilde{\Gamma}^N_m \to \mathbf{R}^N$ satisfy the three properties, and let $V \in \tilde{\Gamma}^N_m$. We show that $F(V) = MC(V)$.
Since $b(V) > 0$, the vector $\lambda \in \mathbf{R}^N_{++}$ with coordinates $\lambda_i := \frac{1}{b_i(V)}$ for all $i \in N$ is well-defined. Consider the game $\lambda * V$. Clearly, $\lambda * V \in \tilde{\Gamma}^N_m$, and $b(\lambda * V) = \lambda * b(V) = e^N$. b-symmetry of F and the MC-value implies $F_i(\lambda * V) = F_j(\lambda * V)$ for all $i, j \in N$ and $MC_i(\lambda * V) = MC_j(\lambda * V)$ for all $i, j \in N$. From weak Pareto optimality of F and the MC-value it follows that $F(\lambda * V) = MC(\lambda * V)$. Scale covariance now yields

$F(V) = MC(V).$ □

Finally, we present a characterization of the MC-value which can be considered as a generalisation of the characterization of the RKS-solution for bargaining games by Kalai and Smorodinsky (1975). For this, we have to impose an extra condition on the set $V_0(N)$. Namely,

$V_0(N)$ is *non-level*, i.e., if $x, y \in \partial V_0(N)$ and $x \geq y$, then $x = y$

The non-levelness condition is a standard condition that is often used in characterizations of solution concepts for NTU-games (see for example, the characterization of the Shapley NTU-value by Aumann (1985a)). Let $\widehat{\Gamma}_m^N$ denote the class of all $V \in \widetilde{\Gamma}_m^N$ for which $V_0(N)$ is non-level. Then we have

Theorem 4.3 The MC-value is the unique value on $\widehat{\Gamma}_m^N$ which satisfies

(i) weak Pareto optimality,

(ii) scale covariance,

(iii) symmetry,

(iv) conditional monotonicity.

Proof. Let $F : \widehat{\Gamma}_m^N \to \mathbf{R}^N$ satisfy the properties (i)-(iv). From the proof of theorem 4.2 it follows that it is sufficient to show that $F(V) = MC(V)$ for games $V \in \widehat{\Gamma}_m^N$ with $b(V) = e^N$. Let V be such a game. Since $b(V) > 0$, it follows that $MC(V) > 0$. Moreover, b-symmetry of the MC-value implies $MC_i(V) = MC_j(V)$ for all $i, j \in N$. Consider the following bargaining game W.

$$W(S) := \begin{cases} \{x \in \mathbf{R}^S \mid x \leq 0\}, & \text{if } S \neq N, \\ \{x \in V(N) \mid \sigma(x) \in V(N) \text{ for all } \sigma \in \Pi(N)\}, & \text{if } S = N. \end{cases}$$

Note that $W(N)$ is the largest symmetric subset of $V(N)$. Since $MC_i(V) = MC_j(V)$ for all $i, j \in N$, it follows that $MC(V) \in W(N)$, and because $MC(V) \in \partial V(N)$ it also follows that $MC(V) \in \partial W(N)$. Hence, $b(W) > 0$, and since $W(N)$ is symmetric, it easily follows that $b_i(W) = b_j(W)$ for all $i, j \in N$. So the origin 0, $b(V)$ and $b(W)$

are lying on one line.

Claim: $W \in \hat{\Gamma}_m^N$.

Proof of the claim. It is sufficient to show that $W_0(N)$ is non-level. Let $x \in \partial W_0(N)$, and suppose there exists a $y \in \partial W_0(N)$ with $y \geq x, y \neq x$. Since $x, y \in W_0(N)$, we have $\sigma(x), \sigma(y) \in V_0(N)$ for all $\sigma \in \Pi(N)$. Moreover, $\sigma(y) \geq \sigma(x), \sigma(y) \neq \sigma(x)$ for all σ. Non-levelness of $V_0(N)$ implies that $\sigma(x) \in int(V_0(N))$ for all σ. Hence, for each $\sigma \in \Pi(N)$ there exists an $\epsilon_\sigma \in \mathbf{R}_{++}$ such that $\mathcal{B}(\sigma(x), \epsilon_\sigma) := \{z \in \mathbf{R}^N \mid \|z - \sigma(x)\| < \epsilon_\sigma\} \subset V_0(N)$. [3] Take $\epsilon := \min\{\epsilon_\sigma \mid \sigma \in \Pi(N)\}$. Then $\mathcal{B}(\sigma(x), \epsilon) \subset V_0(N)$ for all $\sigma \in \Pi(N)$, and since $\mathcal{B}(\sigma(x), \epsilon) = \sigma(\mathcal{B}(x, \epsilon))$, it follows that $\mathcal{B}(x, \epsilon) \subset W_0(N)$. Hence, $x \in int(W_0(N))$, which yields a contradiction. So the claim is proved.

Symmetry and weak Pareto optimality of F and the MC-value imply $F(W) = MC(W)$. [4] Since $MC(V) \in \partial W(N)$, and 0, $b(V)$ and $b(W)$ are lying on one line, it follows that $MC(W) = MC(V)$. Further, conditional monotonicity of F yields that $F(V) \geq F(W) = MC(V)$. Since $MC(V) \in \partial V_0(N)$, non-levelness of $V_0(N)$ implies $F(V) = MC(V)$. $\qquad\qquad\square$

Theorem 4.2 and 4.3 provide characterizations of the MC-value on large classes of NTU-games, where in particular, the sets $V(S)$ need not be convex as usually is required for characterizations of solution concepts for NTU-games (cf. Aumann (1985a), Borm et al. (1992)). This makes the egalitarian solution introduced by Kalai and Samet (1985) a natural candidate for comparison with the MC-value, because the egalitarian solution is also characterized on a class of NTU-games which need not be convex valued. The main difference in the domain of the characterizations of the egalitarian solution and the MC-value is that the egalitarian solution is characterized on a class of NTU-games for which no monotonicity and non-levelness condition is required.

If we compare theorem 4.3 with the characterization of the egalitarian solution given

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[3]$\|x\|$ denotes the Euclidean norm of $x \in \mathbf{R}^N$.

[4]Note that we only use an even weaker version of symmetry, which states that if all players in a game are symmetric, then they will all receive the same payoff.

in Kalai and Samet (1985), it is striking that in both characterizations a monotonicity property plays a crucial role. It should be remarked that the MC-value does not satisfy the monotonicity property which is used to characterize the egalitarian solution, and the latter does not satisfy the conditional monotonicity property introduced above.

Another aspect which justifies the comparison between the egalitarian solution and the MC-value is that both solution concepts are based on a payoff vector which can be considered as a starting point and a direction in which to move from the starting point to an efficient outcome for the grand coalition. However, for the egalitarian solution this direction is independent of the game under consideration, while for the MC-value the direction is determined by the game. As a consequence of the fixed direction the egalitarian solution sometimes yields counterintuitive outcomes, and moreover, it does not satisfy the covariance property. Therefore, the egalitarian solution depends on the utility representation of the preferences of the players.

5   Concluding remarks

Theorem 3.3 shows that the MC-value by definition generalises the Shapley value for TU-games and the RKS-solution for bargaining games. As theorem 4.3 illustrates the MC-value also axiomatically can be considered as an extension of the RKS-solution. It remains an open problem, however, whether an axiom system for the MC-value can be developed based on characterizations of the Shapley value (cf. Shapley (1953), Young (1985)).

It is well-known that for TU-games the core is contained in the convex hull of the marginal vectors (Weber (1988), cf. Derks (1992)). Hence, for TU-games this 'Weber set' is a 'core catcher'. Shapley (1971) proved that for convex TU-games the core coincides with the convex hull of the marginal vectors. It is an interesting problem to examine possible relations between marginal vectors and the core for monotonic NTU-games. Some results are listed below.

In the context of NTU-games there are at least two notions of convexity: ordinal convexity (Vilkov (1977)) and cardinal convexity (Sharkey (1982)). Both ordinal and cardinal convexity are extensions of the notion of convex TU-games to NTU-games. Although for convex TU-games all marginal vectors are core elements, this result can not be extended to the NTU- case: There are counterexamples both for ordinal and cardinal convex games. [5]

A possible extension of the Weber set towards the general class of NTU-games is the following. For a game (N, V) define the Weber set $W(V)$ of V as the set of all weak Pareto optimal points $x \in V(N)$ for which there exists a point $y \in conv(\{m^\sigma \mid \sigma \in \Pi(N)\})$ with $x \geq y$. Clearly, this definition extends the Weber set for TU-games to the NTU-case. For the very simple class of 1-corner games, i.e., games $(N, V) \in \Gamma_m^N$ where $V(S)$ is the comprehensive hull of a single point for each S, it is not difficult to show that $W(V)$ indeed is a core catcher. However, for monotonic NTU-games where $V(N)$ is not convex obviously the Weber set need not be a core catcher. We do not know whether $C(V) \subset W(V)$ for all NTU-games $V \in \Gamma_m^N$ for which $V(N)$ a convex set. Clearly, it is true for convex valued bargaining games.

Acknowledgements

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[5] We are grateful to Sjaak Hurkens for providing the counterexample for ordinal convex NTU-games.

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