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An Ex-Post Envy-Free and Efficient Allocation Mechanism:

Imperfect Information Without Common Priors

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1. Introduction

Consider a situation in which an indivisible object must be allocated to one of a number of agents under imperfect information regarding the value each agent places on it. We seek a mechanism that is guaranteed to both assign the object to an appropriate agent and determine the compensation he must pay the others, so that the resulting outcome is ex-post envy-free and efficient.

There are a variety of settings in which such a mechanism is of value. For example, (i) Suppose that a department must decide on a new chairman. Who should be chosen and how much should the others compensate him? (ii) Who should keep the family residence in a divorce or inheritance settlement and what compensation should s/he pay the others? (iii) When a partnership is dissolved, who should buy out the associates and at what price? There are, of course, many other similar situations.

For the case of two agents under perfect information about preferences, there is a well known solution, the so-called *cut and choose* method. Luce and Raiffa [1957] show that the *cut and choose* mechanism results in an efficient and envy-free outcome via iterative elimination of (weakly) dominated strategies (see section 3 below). However, their result relies heavily on the assumption of perfect information.

Under imperfect information, and common priors, Cramton, Gibbons, and Klemperer [1987]; van Damme [1992]; and McAfee [1992], each study a version of a game in which the agents simultaneously bid for the object. Based on the bids, the object is allocated and side payments are determined. All three employ the *Bayesian Nash Equilibrium* solution concept. However, only in van Damme is an efficient and envy-free outcome guaranteed. Significantly, van Damme's mechanism depends upon the agents' (common) prior. Thus, not only must the common prior be commonly known among the *agents*, the *planner too* must know the prior in order to design the right mechanism.

The assumption that players will coordinate on a Bayesian Nash Equilibrium often embodies numerous subsidiary and rather strong assumptions (i.e. common priors and common knowledge of equilibrium strategies). With this in mind, we have weakened the informational assumptions as far as we are able. In particular, we assume only that it is common knowledge among the agents which one of them values the object most, and that neither agent places probability zero on the other agents' true type.[1] We refer to this latter assumption as *truth inclusivity*. (It is significantly weaker than mutual absolute continuity of beliefs and much weaker still than the common prior assumption.) With these two assumptions we implement an efficient and envy-free allocation in *iteratively (weakly) undominated strategies*. Agents do not have to know the others' beliefs nor the others' chosen strategies in order that the desired outcome is obtained.[2]

Assuming that agents know who assigns the object the highest value is by no means an innocuous assumption, and if we could, we would relax it. Even so, it would appear that it may well be satisfied in many situations, especially when the agents know each other well, as is likely the case when dissolving a business partnership or a marriage.

In summary, the present work is an attempt to extend the cut and choose mechanism to the case of imperfect information while maintaining the spirit of the original work. We wish to emphasize that relaxing the informational assumptions comes at the cost

[1] In the sequel, an agent's type includes not only the value he places on the object, but also his beliefs over the other agents' types. This is standard. See Mertens and Zamir [1985] or Aumann and Brandenburger [1991].

[2] Thus, this line of research continues that of Perry and Reny [1995], where the emphasis is also on simplicity and practical success.

of complicating the implementing mechanism. We assign a high priority to further simplifying the mechanism proposed here.

2. *The Environment*

The formal model of knowledge below is based upon Aumann and Brandenburger [1991]. For simplicity, we consider only the case in which there are two agents.[3]

Let T denote the finite set of possible pairs of types (t_1,t_2) of the two agents, where t_i denotes the type of agent i. Let $T_i(t_{-i}) = \{t_i : (t_i,t_{-i}) \in T\}$, and let $T_i = \{t_i : (t_i,t_{-i}) \in T$, for some $t_{-i}\}$. Each of i's possible types, $t_i \in T_i$, is a pair (π_i,v_i) where π_i is a probability distribution over $T_{-i}(t_i)$ denoting i's beliefs over -i's type, and $v_i > 0$ denotes i's value.[4] We maintain the following assumption throughout the paper.

<u>*Assumption 1:*</u> For all $t \in T$, with $t_i = (\pi_i,v_i)$ $i = 1,2$

(a) $\pi_i(t_{-i}) > 0$

(b) $v_1 \neq v_2$

(c) if $v_i > v_{-i}$, $\pi_i(t_{-i}') > 0$, and $t_{-i}' = (\pi_{-i}', v_{-i}')$, then $v_i > v_{-i}'$.

The conditions in this assumption express, respectively, that: (a) neither agent rules out the truth; (b) the agents' values are distinct; and (c) each agent knows whose value is larger. We often refer to the agents' valuations as v_h and v_l where $v_h > v_l$, and to the agents as H (high valuation) and L (low valuation) respectively. Agent i's payoff of obtaining the object at price P when its value to that agent is v_i, is v_i-P, while the payoff associated with receiving P and not receiving the object is P.

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[3] The extension to any finite number of agents is relatively straightforward.

[4] Since the agents' status quo payoffs are zero, each of them then strictly values the object. The analysis goes through even if one or both of them place a negative value on the object (which may for instance be an undesirable task to perform). The current formulation is employed for ease of exposition.

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An outcome (or allocation) consists of an agent, i, (to whom the object is given) and a payment, P, (from the chosen agent to the other). In order that the outcome (i,P) be *ex-post efficient*, it must be the case that i=H; and in order that it be *ex-post envy-free,* the payment from H to L must satisfy $v_h - P \geq P \geq v_l - P$.

Solution Concept

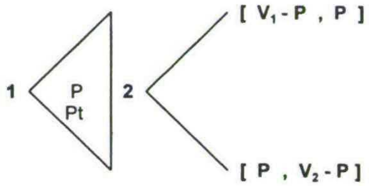
In Sections 4 and 5 below, we introduce a game form designed to implement an efficient and envy-free outcome in *iteratively (weakly) undominated* strategies. A pure (mixed) strategy for an agent is a function mapping each of his types into a vector of actions (probability distributions over actions), one for each of his information sets. Given two strategies σ_1 and σ_2 for agents 1 and 2 respectively, let $u_1(\sigma_1,\sigma_2|t_1)$ denote 1's (expected) payoff in the game when his type is t_1.[5] Let σ'_1 be a pure strategy for 1 and let Σ_2 be a subset of pure strategies for 2. We say that σ_1 (possibly mixed) *weakly dominates* σ'_1 for agent 1 against Σ_2, if $u_1(\sigma_1,\sigma_2|t_1) \geq u_1(\sigma'_1,\sigma_2|t_1)$ for all $t_1 \in T_1$ and all $\sigma_2 \in \Sigma_2$, with at least one such pair (t_1,σ_2) yielding a strict inequality. Weak dominance is similarly defined for agent 2.

3. Perfect Information

We begin with a brief review of the perfect information case. As is well known, the *cut and choose* mechanism, first analyzed more than forty years ago, nicely implements the desired outcome in two rounds of elimination of weakly dominated strategies. In this simple mechanism, player 1 (say) is chosen to move first by proposing a price P for the object. Agent 2 must then decide whether to buy or to sell the object at that price. If he chooses to buy, he receives the object and pays agent 1

[5] This is an expected payoff even if the two strategies are pure since 1 is uncertain of 2's type.

the announced price P. If he chooses to sell, the object is given to agent 1 who, in return, pays P to agent 2. (See Figure 1.)



CUT AND CHOOSE

Figure 1

Because the solution is in iteratively undominated strategies (as opposed to, say, subgame perfection), the following *tie breaking* assumption, which will be maintained throughout the paper, is needed.[6]

Assumption 2: (Tie Breaking Assumption): If agent L (H) can buy or sell at price P, and v_l-P=P (v_h-P=P), then L (H) strictly prefers to sell (buy).

The Cut and Choose Solution

Round 1: Eliminate for agent 2 all strategies in which he choose to sell when $P<v_2/2$ ($P\leq v_2/2$ if agent 2 is H) and those in which he chooses to buy when $P>v_2/2$ ($P\geq v_2/2$ if 2 is L).

Round 2: Eliminate all proposals P of agent 1 such that $P\neq v_2/2$.

Thus, the only outcomes that survive the two rounds of elimination are those in which agent H receives the object and compensates agent L by either $v_h/2$ (when agent L moves first) or $v_l/2$ (when agent H moves first).

[6] On the other hand, in the case of a fixed, discrete unit of account this assumption is vacuously satisfied for almost all choices of the agents' finite number of valuations, since such ties virtually never occur.

Remarks: *(i)* One drawback of the cut and choose mechanism is that it provides a significant first-mover advantage. The agent who moves first gets his best outcome among those that are efficient and envy free. The following *two-step cut and choose* mechanism resolves this asymmetry.

Step I: Allocate the right to be the proposer by a cut and choose mechanism.

Step II: Given the proposer determined in Step I, allocate the object by a cut and choose mechanism.

(ii) It is easy to see that the cut and choose mechanism does not work when valuations are private information. This is simply because agent 1 may have an incentive to take a risk and make a bid which is too high or too low.

4. Imperfect Information, Known Identities

In this section we present a mechanism that obtains the desired implementation when the identities of the agents are known to the planner. Thus in the present section, the planner knows which of them is the high valuation agent and which is the low valuation agent. He does not, however, know their valuations. This mechanism, of independent interest, will then be supplemented in the following section to yield the full solution.

Since the identities of the agents are known, it is clear which of them must get the object (agent H, by efficiency). The only thing left to be determined is the appropriate (envy-free) price that H must pay to L. In order to effect this, the planner must receive some information regarding the range of the values of v_h and v_l in order to ensure that P, the price paid by H satisfies the envy-free condition, $v_h/2 \leq P \leq v_l/2$.

Fix $t_h = (\pi_h, v_h)$ and $t_l = (\pi_l, v_l)$, the types of agent H and L, respectively, such that $(t_h, t_l) \in T$ and let $\underline{h}(t_l) = \min\{v : (\pi, v) \in \text{supp}\pi_l\}$. Thus, $\underline{h}(t_l)$ denotes the lowest possible valuation of agent H as is perceived by type t_l of agent L. By Assumption 1 ((a) and (c)), and the finiteness of the type space, $v_l < \underline{h}(t_l) \leq v_h$. Consequently, when L's type is t_l, the outcome will be envy-free if H compensates L by $\underline{h}(t_l)/2$. The mechanism below is designed to produce precisely this result. We remind the reader that $\underline{h}(t_l)$ is agent L's private information.

The Mechanism

The mechanism, depicted as a game tree in Figure 2, is defined in stages. At each stage, agents are perfectly informed of the choices made in all previous stages.

Stage 1: Agent L announces a pair of numbers, (P,F), such that $0 < P \leq F$. It is helpful to interpret P as L's proposal for the price of the object (to be paid by H to L), and F as a potential fine.

Stage 2: Agent H chooses to either:

(i) Accept, after which he pays P to L in return for the object, and then the game ends, or

(ii) Give the object to L for free; and then the game ends, or

(ii) Challenge P by announcing some $\varepsilon>0$. In this case agent L pays a fine of F as determined in Stage 1, and the game continues to Stage 3.

Stage 3: Agent L can choose to stick with his original price P>0, or reduce it to zero. Let P' denote the new price.

If P'=0, then L pays a fine of ε as determined in stage 2.

If P'=P, then H pays a fine of F as determined in stage 1.

Stage 4: Agent H chooses between buying or selling at P'.

If H chooses to sell at P' then agent L receives the object and pays agent H the amount P'.

If, agent H chooses to buy at P', then he receives the object and in return he pays P' *to the planner*, who then pays P to agent L.

(Note that by the time the game reaches this stage, the planner has collected more than enough in fines to be able to pay P to L.)

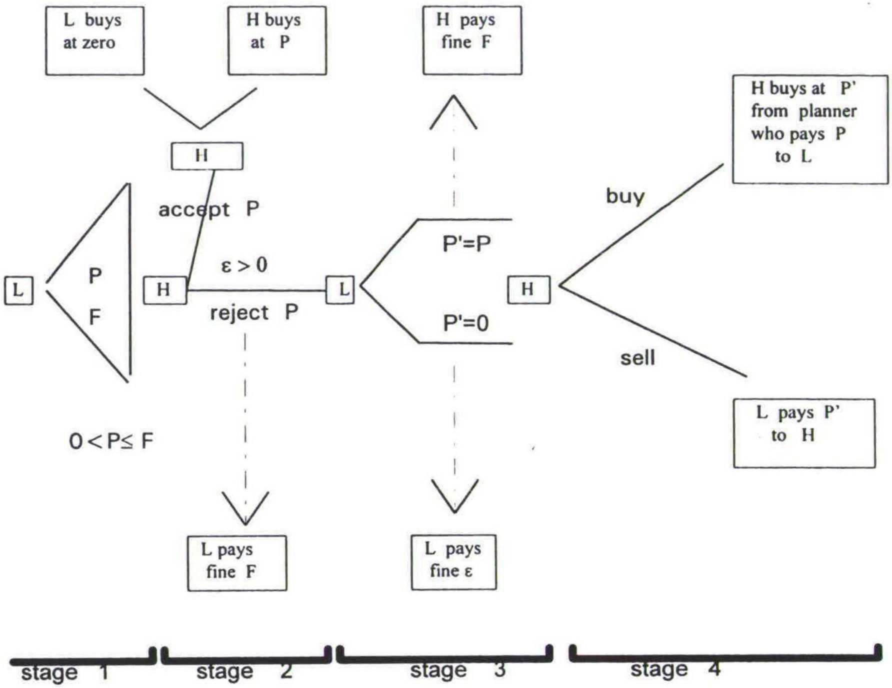


Figure 2

<u>The Solution</u>

The formal proof that the game form above implements an efficient and envy-free outcome in iteratively undominated strategies can be found in the appendix. What follows is a brief description of the main ideas behind the proof.

First, we shall make use of a result due to Marx and Swinkels (1993) which allows us to concentrate on a particular order of eliminating weakly dominated strategies. A consequence of their result is that if a unique outcome survives the particular order of elimination that we consider, then every order of elimination results in this same outcome. Section 6 contains a more detailed discussion of this point. The order of elimination that we shall consider begins by applying backward induction for a number of rounds. Hence in these early rounds the game tree is "pruned" from the end.

Consider then the last stage of the game where agent H has the choice of buying or selling at P'. It is dominant for H to sell if the current price exceeds $v_h/2$, and it is dominant to buy otherwise. Prune the tree accordingly.

Now move back through the tree to Stage 3 where one of the two main features of the game comes into play. At this stage, agent L must choose between paying $\varepsilon>0$ and reducing P to 0, or saving the ε and sticking to his original proposal P. Recall that $\underline{h}(t_l)$ is the lowest possible value of v_h according to L when L's type is t_l. Assume first that $P>\underline{h}(t_l)/2$ and note that if L reduces his proposal to 0, agent H is sure to buy next stage, which guarantees a payoff of $P-\varepsilon$ to L (since in this case the planner pays P to L). If instead L chooses to stick to his original proposal P, he receives a payoff of P *only* when H chooses to buy, and (v_l-P) when H chooses to sell. But since $P>\underline{h}(t_l)/2$, it follows that (i) L assigns positive probability to the latter possibility and (ii) $(v_l-P)<P$. Thus, for each proposal $P>\underline{h}(t_l)/2$, there is an adjustment cost $\varepsilon>0$ small enough such that reducing the proposal to 0 for a cost of ε dominates leaving it unchanged at P.

An analogous argument establishes that for every $\varepsilon>0$, when $P \leq \underline{h}(t_l)/2$, choosing to reduce the proposal to 0 at cost $\varepsilon>0$ is dominated for L by the choice of leaving it unchanged at P. Prune the tree accordingly.

Move back to Stage 2 of the game, where agent H chooses between accepting P or challenging P by announcing some $\varepsilon>0$. Since by truth inclusivity (i.e. Assumption 1(a)) v_h is in the support of L's belief about H's value (and this is common knowledge), we must have $\underline{h}(t_l) \leq v_h$. Consequenlty, if $P>v_h/2$, then H knows that $P> \underline{h}(t_l)/2$. Given the eliminations carried out above, it is then dominant for H to reject P by announcing ε sufficiently small since this will guarantee him the object for free. (This makes use of the finite type space.) Similarly, if according to H's information $P\leq \underline{h}(t_l)/2$, then challenging P is a dominated strategy for H. This is because in the unique continuation after such a challenge H is sure to buy the object at P anyway, and in addition pay a fine of F. Prune the tree accordingly.

This brings us to the first stage of the game where agent L must choose a price P and a fine F. Now, what we wish to argue is that after a few more rounds of elimination, all remaining strategies will involve agent L proposing a price $P= \underline{h}(t_l)/2$, and that agent H will choose to buy the object at that price. To see the difficulty involved in establishing this, recall that $\underline{h}(t_l)/2$ is agent L's private information, and that agent H will definintely challenge L's proposal P if he believes that $P>\underline{h}(t_l)/2$. Consequently, we must establish through iterative dominance arguments that L's unique best proposal is $P=\underline{h}(t_l)/2$. Now although this is what is formally required, we will not give the full argument here (the details are in the appendix). Instead, we shall provide only the main intuition, which is nicely captured through a familiar signaling-like story.

So with this in mind, suppose that agent L wishes to propose $P=\underline{h}(t_l)/2$. In order to be certain not to pay the fine F, he must convince agent H that the announced P is indeed equal to $\underline{h}(t_l)/2$ (recall that $\underline{h}(t_l)/2$ is L's private information). It is here where the fine F plays a role. Indeed, choosing a sufficiently high F while simultaneously proposing $P>\underline{h}(t_l)/2$ is a dominated move. This is because L's proposal is sure to be challenged by every type of H for whom $v_h/2<P$, an event which L assigns strictly positive probability. Thus, the potential gain from successfully proposing $P>\underline{h}(t_l)/2$ is (for F high enough) less than the (expected) cost of being challenged and paying F. Thus, announcing a high enough fine F is a sure signal that the simultaneously proposed P is not above $\underline{h}(t_l)/2$. Consequently, (because of our previous pruning of the tree) it is dominant for H to accept such a P. This, in turn, implies that if the proposal P is not higher than $\underline{h}(t_l)/2$, then it is dominant to also announce such a high F. Consequently, L's announced P is not above $\underline{h}(t_l)/2$ if and only if the simultaneously announced fine is sufficiently high. Such high fines therefore perfectly signal that L's announced proposal is $P=\underline{h}(t_l)/2$ (since it is in L's interest to raise P to the point where H is just unwilling to challenge).

5. Imperfect Information, Unknown Identities

We now drop the assumption that the identities of the agents are known to the planner. In order to accommodate this change, we precede the previous section's mechanism with one whose purpose is to identify which of the agent's is L. Thus, the main task is to show that indeed this "identity game" performs the desired task. Thereafter, we may rely on the analysis of section 4.

In what follows we sometimes call the agents 1 and 2, while at other times L and H. This is because the perspectives of the agents differ from that of the planner. From the planner's point of view, the agent's identities are unknown, while their identities are common knowledge between the agents themselves. Consequently, the labels 1 and 2 are used when we take the planner's perspective, and L and H are used when we take the agents' perspective. Finally, to simplify the analysis to follow, we assume that agent H strictly prefers the outcome in which he gets the object for a price equal to half of his value, to the lottery giving him a 50-50 chance of getting the object or nothing. This assumption can be dropped at the cost of complicating the mechanism slightly.

The Identity Game

The game proceeds in stages such that at each stage there is perfect information regarding the choices in all previous stages.

Stage 1: Agent 1 announces H or L, and the game proceeds to stage 2.

Stage 2: Agent 2 announces H or L.

(i) If the resulting history is (H,H) then they play the King Solomon game of Perry and Reny [1994]. It proceeds as follows.

> The two agents submit bids for the object in a second-price sealed-bid all-pay auction *with an option*. The option is that after the bids are revealed to the agents, the winner (highest bidder) can either choose to stick with his bid (in which case he receives the object and both bidders pay the second-highest bid), or he can choose to *quit* and give the object to the other agent in which case no payments are made by either agent. If the two bids are identical, then the object is sold to one of them (determined by the toss of a fair coin) at a price equal to the common bid. In this case the other agent pays nothing.

Subsequently, the game ends. (In particular, the mechanism of Section 4 is not played.)

(ii) If the resulting history is (H,L) then they play the mechanism described in Section 4 where each agent plays the role of the player matching his announced identity (i.e. agent 1 is H and agent 2 is L). After the Section 4 mechanism is played, the game ends.

(iii) If the resulting history is (L,L) then the object is allocated between the agents by the toss of a fair coin. End of game.

(iv) If the resulting history is (L,H) then the game proceeds to stage 3.

Stage 3: Agent 1 once again chooses either L or H.

(i) If agent 1 chooses H, they then play the King Solomon game as described above, after which the game ends.

(ii) If agent 1 chooses L, then they play the mechanism described in section 4 where agent 1 is L and agent 2 is H, after which the game ends.

The full proof that this extended game form implements the desired outcome in iteratively undominated strategies is left to the appendix. Before providing the ideas behind the proof, we provide two central observations.

(i) In the unique outcome that survives iterative elimination of weakly dominated strategies in the King Solomon game, agent H receives the object at no cost to him while agent L receives a payoff of zero.

(ii) In the game form of Section 4 wherein the agents' identities are assumed known, there is, for each joint type t, a unique outcome (which is also efficient and envy-free) surviving iterative elimination of weakly dominated strategies. In this outcome (which may depend on the joint type), agent H receives the object and pays agent L a strictly positive amount which is no more than half of the value that H places on the object. All of this holds regardless of the agents' types.

Owing to (ii) above, it suffices to show that the Identity Game induces truthful revelation of the agents' identities (i.e. H or L).

So consider the identity game when agent L happens to move first. If L falsely identifies himself as H, agent H will follow by also identifying himself as H, thereby triggering the play of the King Solomon game (in which H receives the object for free).[7] Thus agent L, when moving first, will identify himself as L, ensuring himself a positive payoff in the continuation. But once L truthfully identifies himself, H will follow by truthfully identifying himself also. This is because doing so guarantees that H can purchase the object for a price no more than half of his value, and this he prefers (by assumption) to the 50-50 lottery. Thus if L hapens to move first, truthful revelation occurs.

Next, consider the case in which H happens to move first. Suppose that he falsely identifies himself as L. Clearly, agent L will then also identify himself as L since by doing so he induces the 50-50 lottery over the object, while doing otherwise he gives H the opportunity to trigger the play of the King Solomon game in which H receives the object for free (so H will sieze this opportunity) and L receives nothing. On the other hand, if H truthfully identifies himself, it is clearly best for L to truthfully identify himself also since this triggers the game form of Section 4 and guarantees L a strictly positive payoff. Therefore by (ii), H is better off identifying himself truthfully, and so in this case too both agents truthfully reveal their identities.

6. Final Remarks

We have demonstrated that there is a particular order of elimination of weakly dominated strategies yielding the desired efficient and envy-free outcome. This leaves open the possibility that other orders of elimination might yeild a different outcome. But in fact this is not possible. Any iterative weak dominance algorithm applied to our game that ends in finitely many rounds must result in precisely the same outcome as that identified in the analysis above. This fact follows from a result due to Marks and Swinkels [199~~4~~3]. They introduce the notion of a "nice" weak dominance

[7] One must of course verify that agent H can do no better by falsely identifying himself as L and thereby triggering the game of Section 4 but with the roles of the agents reversed. This is taken up in the appendix.

algorithm. A corollary of their result is that if a unique outcome results through iterative nice weak dominance, then the order of elimination of weakly dominated strategies (in the usual sense) does not matter. A strategy for player i is nicely weakly dominated by another strategy, if whenever some strategy of the others renders i's payoff the same for the two of his strategies, all other players' payoffs are the same as well. It is not hard to check that all of the eliminations carried out in the particular order that we chose are nice.

Finally, we wish to comment on the role of large fines. There are a number of criticisms which are levied against the use of large fines, or so-called holocaust outcomes. The most common is that the imposition of such outcomes is often not credible. However, in many economic contexts (including the present one) this difficulty is easily avoided by simply insisting that the fine be paid to an independent third party. In this way, issues of rengotiation-proofness do not arise. A second criticism is the absolute magnitude of the fine itself. In the present setting, the size of the fine required to convince the high agent that the low agent is truthfully revealing his beliefs depends upon the severity of the informational asymmetries between the agents. The fine must be large when these asymmetries are severe. Of course, Abreu and Matsushima [1992] have shown that one can make do with arbitrarily small fines. However, their mechanism requires a potentially very large number of rounds of elimination of dominated strategies, which, regarding the practical success of the mechanism, seems to raise legitimate concerns (see Glazer and Rosenthal [1992]). But the issue has never been about large fines per se. After all, a very practical and convincing way for someone to convince another that he knows that a gun is not loaded is to point it at himself and pull the trigger. What is crucial to this example however, is that although the fine for lying (i.e. saying you know the gun is not loaded when in fact you don't know this) is very large (you are killed with positive probability), when you are telling the truth, there is no chance whatever that you will be shot. This is in contrast to our mechanism, where the low-value agent must rely on the high-value agent not to challenge his proposal (thereby triggering the fine); even when the low-value agent is being truthful. Whether or not one can rid the mechanism of this feature while still keeping the number of rounds of elimination bounded (and reasonably small) is an important question to which we, at least, do not have an answer.

Appendix

Solving the Section 4 Game of Known Identities

Round 1: Consider any point in the mechanism where agent H makes the last move. If at this point, H has the choice to buy or sell, eliminate for H all strategies in which at such a point he chooses to buy when the current price exceeds $v_h/2$, or to sell when it is less than or equal to $v_h/2$.[8] If at such a point he has the choice to buy or give away the object, eliminate all strategies in which he buys when the price is above his value, or gives the object away when the price is below his value.

Round 2: Proceed backward to Stage 3 of the mechanism. At this stage agent L must choose between paying ε and adjusting P downward to zero, or saving the ε and sticking to his original proposal. Recall that $\underline{h}(t_l)$ is the lowest possible value of v_h according to L if his type is t_l. Eliminate all strategies for L which specify at this point in the game that he stick to his original proposal when his type is t_l, $P>\underline{h}(t_l)/2$, and ε is small enough so that

$$(*) \quad P-\varepsilon>\alpha P + (1-\alpha)(v_l-P),$$

where α denotes the probability, according to $\pi_l(t_l)$, that $v_h/2\geq P$. (Such an ε exists since $v_l-P<P$ which in turn follows from $P>\underline{h}(t_l)/2$.) Each of these strategies is dominated against those of H that remain, by a strategy which is identical except that in this situation it specifies that L adjust his proposal to zero. To see why, note that from the first round of elimination L is sure to receive P from the planner by adjusting downward (and paying ε). On the other hand, by sticking with his proposal, he receives the expected payoff on the right-hand side of (*). By similar reasoning, we may also eliminate all strategies for L specifying at this point in the game that he revise downward when his type is t_l and $P \leq \underline{h}(t_l)/2$.

[8] That is, eliminate every strategy σ_h for H such that for at least one of his types $t_h = (\pi_h, v_h)$, $\sigma_h(t_h)$ specifies that at this point in the game, he sell at a price less than or equal to, or buy at a price above, $v_h/2$.

Round 3: Move back to Stage 2, where agent H chooses between accepting P or challenging P by announcing some $\varepsilon>0$. Given the previous 2 rounds of elimination, we eliminate at this stage additional strategies for H. Suppose that H's type is $t_h = (\pi_h, v_h)$. If for all $t_l \in \operatorname{supp}\pi_h(t_h)$, $P> \underline{h}(t_l)/2$, which in particular is the case when $P>v_h/2$ (by Assumption 1(a)), then all strategies for H in which he chooses (when his type is t_h) to accept and then buy at P are dominated. Indeed, each of these strategies is dominated by one which is identical to it except that instead, P is rejected with an ε small enough to ensure that L will adjust P downward to zero in the continuation. The existence of such an ε follows from previous rounds of elimination and the assumed finite type space. Similarly, if for all $t_l \in \operatorname{supp}\pi_h(t_h)$, $P \leq \underline{h}(t_l)/2$, then all strategies for H in which (when his type is t_h) he chooses to challenge P are each dominated by a strategy which is identical except that it instead specifies that he accept the proposal and then buy at P. Consequently, we may eliminate, for every possible type t_h of H, all such strategies of H described above.

Round 4: We are now ready to analyze the first stage of the game. So far we have eliminated weakly dominated strategies by using the familiar backward induction reasoning. We shall now continue our elimination procedure, but switch to a forward induction type of reasoning. Consider the choice of (P,F) by agent L on his first move. Note that given the strategies that have survived so far, a proposal of P will be challenged by H if $v_h/2<P$ (see round three). Hence, if $P > \underline{h}(t_l)/2$ then agent t_l assigns a strictly positive (and bounded away from zero because there are finitely many types) probability to the event that P will be challenged by agent H, and that he (agent L) will pay the fine F. It follows that for every type t_l, and $P > \underline{h}(t_l)/2$ there exists a fine $F(P,t_l)$ large enough such that every strategy for L in which he announces $(P,F) > (\underline{h}(t_l)/2,\ F(P,t_l))$ when type t_l, is dominated by (say) a strategy in which t_l announces $P = F = \underline{h}(t_l)/2$. Thus, eliminate for agent L, all strategies in which for some type t_l, the strategy specifies $P > \underline{h}(t_l)/2$ and $F > F(P,t_l)$. For future reference, let $F(P)$ be the largest value of $F(P,t_l)$ over all of L's types t_l.

Round 5: Recall that in round 3 we concluded that it is dominated for H to challenge a proposal of L when he is sure that $P \leq \underline{h}(t_l)/2$. Consequently, we may eliminate for agent H all strategies in which he challenges a proposal (P,F) with F>F(P), since according to the previous round's eliminations, type t_l announces F > F(P) only if $P \leq \underline{h}(t_l)/2$.

Round 6: Consider a proposal (P,F) by agent L of type t_l in which $P \leq \underline{h}(t_l)/2$. It follows from round 5 that if F>F(P) then this proposal is sure to be accepted by H. Moreover, if F≤F(P), there are strategies of H remaining in which H challenges. Now note that for every proposal P made by agent L, his payoff is always higher when P is accepted than when P is challenged. (Since when it is challenged, L must pay a fine at least as large as P and given the previous eliminations H always buys in the continuation and L then receives P. Thus L's overall payoff is non positive.) Thus we may eliminate for L all strategies in which some type t_l announces (P,F) such that $P \leq \underline{h}(t_l)/2$ and F ≤ F(P).

Round 7: Given the strategies just eliminated for L, it follows that any proposal (P,F) with F≤F(P) is a sure indication that L's type t_l satisfies $P > \underline{h}(t_l)/2$. By the reasoning of round 3, we eliminate all strategies for H in which such a proposal is not challenged.

Round 8: Given the strategies remaining for H, an announcement of $P = \underline{h}(t_l)/2$ and F > F(P) by type t_l, is sure to be accepted by H with H then buying the object. Refer to this as type t_l's *sure thing* announcement. Now, recall that whenever a proposal by agent L is challenged, his payoff is ultimately non positive. Given the strategies remaining for H, among the (F,P) announcements still remaining for t_l, if $P > \underline{h}(t_l)/2$, then P is challenged, while the most favorable outcome from announcing $P < \underline{h}(t_l)/2$ is that L obtain P. Thus, all such strategies for L are dominated by identical ones in which t_l makes his sure thing announcement and therefore can be eliminated.

Thus the only remaining proposal for every type t_l is $P=\underline{h}(t_l)/2$ and $F>F(P)$, which is accepted by H who then buys the object. Consequently, the unique outcome surviving the iterative elimination of weakly dominated strategies is the efficient and envy-free one in which H receives the object and pays t_l the amount $\underline{h}(t_l)/2$.

Solving the Extended Game:

As before, we consider a particular order of eliminating weakly dominated strategies. The irrelevance of the order of elimination is discussed in Section 6.

Rounds 1-8: Apply rounds 1-8 of the elimination process of the previous section to the portions of the agents' strategies in which the history in the identity game is: (a) (H,L) and agent 1's identity is H and 2's is L, or (b) (L,H,L), and agent 1's identity is L and 2's is H. In addition, during rounds 1 through 4, apply the elimination procedure from Perry and Reny [1994] to the portion of the agents strategies in which King Solomon's game is played. Lastly, in round 1, eliminate for L all strategies in which he buys the object at a price above its value to him. (In particular, he may have such an opportunity when the two agents lie about their identities triggering the mechanism of section 4, and H then makes a price offer above L's value.)

In particular then, given the strategies now remaining: (i) whenever the King Solomon game is entered, agent H gets the object for free (see Perry and Reny [1994]), (ii) whenever the agents announce their true identities so that the mechanism of section 4 is entered, agent H buys the object at the price $\underline{h}(t_l)/2$, and (iii) whenever the agents announce false identities so that the mechanism of section 4 is entered, agent L never buys the object for more than he values it.

Round 9: Eliminate for H, all strategies in which (in the identity game) he fails to announce H after L announces L. (This includes the history (H,L,H) as well as (L,H).) These are weakly dominated by announcing H at that point since by doing so H guarantees that he gets the object for free (since the King Solomon game is entered). Not doing so, however, triggers the mechanism from section 4, but where the roles of the payers are reversed. Since by (iii) above L never buys the object for more than his value, there is no outcome in which H obtains a payoff above v_h, while there are outcomes (reached by remaining strategies) in which he obtains strictly less.

Round 10: Eliminate for L all strategies in which he announces H when it is his turn to move in the identity game. These are dominated by announcing L, since by round 9, an announcement of H (whether or not agent H gets another move) results in a payoff of zero; while an announcement of L results in a payoff of $v_l/2>0$ if H previously announced or subsequently announces L, and a payoff of $\underline{h}(t_l)/2>0$ if agent H previously announced or subsequently announces H.

Round 11: Eliminate for H all strategies in which he announces L at the beginning of the identity game or after agent L has announced L (by round 10, L does not at this stage ever announce H). These are dominated by announcing H, since by round 10, announcing L in these circumstances results in a 50-50 chance of getting the object or nothing, while announcing H results in a payoff of $v_h - \underline{h}(t_l)/2$ which is strictly preferred to the lottery (where in case $v_h = \underline{h}(t_l)$, we make use of the lottery tie-breaking assumption).

Thus, the only remaining strategies have each player truthfully identify himself in the identity game, thereby triggering the mechanism of Section 4, and yield the desired outcome as before there.

References

Abreu, D. and H. Matsushima (1992): "Virtual Implementation in Iteratively Undominated Strategies: Complete Information," *Econometrica*, 60, 993-1008.

Aumann, R.J., and A. Brandenburger (1991): "Epistemic Conditions for Nash Equilibrium," Working Paper 91-042, Division of Research, Harvard Business School.

Cramton, P., R. Gibbons, and P. Klemperer (1987): "Dissolving a Partnership Efficiently," *Econometrica* **55**, 615-632.

van Damme, E. (1992): "Fair Division under Asymmetric Information," in *Rational Interaction, Essays in Honor of John C. Harsanyi,* Springer-Verlag Berlin Heidelberg New York, London, 121-144.

Glazer, J. and R.W. Rosenthal (1992): "A Note on Abreu-Matsushima Mechanisms," *Econometrica,* 60, 1435-1438.

Luce, R.D. and H. Raiffa (1957): *Games and Decisions*, John Wiley, New York.

Marks, L. and J. Swinkels (1993): "Order Independence & Iterated Weak Dominance," working paper; (forthcoming in *Games and Economic Behavior).*

McAfee, R.P. (1992): "Amicable Divorce," *Journal of Economic Theory* **56**, 266-293.

Mertens, J.F. and S. Zamir (1985): "Formulation of Bayesian Analysis for Games with Incomplete Information," *International Journal of Game Theory,* 14, 1-29.

Perry, M. and P.J. Reny (1995): "A General Solution to King Solomon's Dilemma," working paper.

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