A Model of Random Matching and Price Formation

Kultti, K.K.

Publication date:
1997

Citation for published version (APA):
Kultti, K. K. (1997). *A Model of Random Matching and Price Formation*. (CentER Discussion Paper; Vol. 1997-32). CentER, Center for Economic Research.

CentER for Economic Research

Discussion paper

[redacted]

Tilburg University

Center
for
Economic Research

No. 9732

A MODEL OF RANDOM MATCHING AND PRICE FORMATION

By Klaus Kultti

March 1997

JEL C78
JEL D40

ISSN 0924-7815

A Model of Random Matching and Price Formation

by

Klaus Kultti

Center for Economic Research, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, The Netherlands (Tel.: +31-13-4663077, Fax.: +31-13-4663066, e-mail: kultti@kub.nl)

March 1997

ABSTRACT

We study a model where agents willing to trade meet randomly. There are two types of agents of whom some have an indivisible object and some a perfectly divisible object for sale. The reservation value of the former type is zero, and the latter type value the indivisible object at unity. The meeting probabilities are endogenous and are derived from the basics of the model. The agents can decide to either search or wait. The searchers are distributed randomly on the waiters. Prices are determined by bargaining if exactly two agents are matched. If more than one agent of one type are matched with an agent of another type an auction ensues. There exist potentially three equilibria whose stability is studied in evolutionary dynamics. We also use the model to identify frictions that are responsible for non-Walrasian outcomes. *Journal of Economic Literature* Classification Numbers: C78, D40, D44, D83. Keywords: Random matching, bargaining, auctions, evolutionary stability.

1. INTRODUCTION

The Walrasian paradigm of complete markets is unable to explain many economic phenomena of interest. For instance, the price formation story by an auctioneer is rather barren in explanatory terms. Walrasian theory is of no help when more detailed behaviour of agents like the actual process of trading is of interest. One popular way to deviate from the assumption of complete markets originates from the work of Diamond (1971) where he models agents searching for trading opportunities. The model can be seen as a step towards modelling the microstructure of markets. Diamond's particular insight was that trading is not frictionless but it takes time to find a trading partner. This idea has found a multitude of applications (see Mortensen, 1986; McMillan and Rothschild, 1994). In these models search usually takes place in continuous time. The corresponding discrete time models typically have agents randomly matched.

Both search models and random matching models enable quite detailed study of market microstructure, and the effect of frictions to market equilibrium, as the behaviour of a single agent can be studied, and his optimal behaviour determined. Advances in bargaining theory (Rubinstein, 1982) made it possible to study price determination in markets. Rubinstein and Wolinsky (1985) (RW in the sequel) study markets where buyers and sellers are randomly matched, and agree on the conditions of trade, or price, in pairwise bargaining. In RW the equilibrium price differs from the Walrasian price of complete markets. RW also study the what happens if the frictions vanish, and find out that depending on the way they vanish the price approaches different limits none of which is the Walrasian price. RW regard this as remarkable as they think of their model "...a reasonable mechanism of price formation in the context of a market whose basic characteristics are very similar to those of a competitive market" (p. 1148).

Any surprising results are, of course, embedded in the assumptions, and even though search and random matching models of markets take a step towards specifying the market microstructure they are still silent about the precise environment in which the agents live or operate. An exception is a model by Lu and McAfee (1996). They study the relation of two trading mechanisms, auctions and bargaining. The economy proceeds in discrete time and buyers and sellers are randomly matched. The sellers are in fixed positions in either auction markets or bargaining markets. The buyers are randomly distributed amongst the sellers. In the auction markets the price is determined in an auction, and in the bargaining markets by pairwise bargaining. Lu and McAfee study which markets will survive in an evolutionary environment where agents can choose which markets to enter. This is a slightly contrived question to ask in that particular model for two reasons. First the agents have to commit to a particular mechanism which means that a seller who meets many buyers forgoes playing buyers against each other if he has committed to bargaining. Secondly, the buyers could as well be called sellers and vice versa as both parties want what the other possesses , but the agents are not treated symmetrically in the model.

The aim of this article is to develop a well specified and easily applicable model of random matching and price formation. Market structure is determined endogenously as the agents are allowed to choose the markets they go to. The agents are not constrained to any particular trading mechanism. If just two agents meet they bargain, and if there are more agents of one type an auction is held. This method of price formation brings a competitive element to the markets that is missing in models of pairwise bargaining. Bargaining takes time during which parties are locked to each other. This decreases any competitive pressures from excess demand or supply that drive prices to a certain direction in competitive markets. We also shortly elaborate on the frictions of random matching models. It turns our that the discount factor or rate of time preference is not the only friction compared to complete Walrasian markets. Another friction is that not everybody meets everybody else. The rest of the article is organised as follows. In section 2 we present the model, in section 3 we analyse the stability of equilibria, in section 4 we discuss the connection between random matching models and Walrasian markets, and in section 5 we present conclusions.

2. THE MODEL

Consider a situation with B buyers and S sellers who desire to engage into exchange for an indivisible good. Strictly speaking the names 'buyer' and 'seller' are somewhat arbitrary and not very descriptive; both types of agents want to exchange at least some of the good they possess for the good the other type possesses. To keep in line with previous literature the agents with an indivisible good are called sellers and the agents with a divisible good are called buyers. We assume that both buyers and sellers are divided into two groups. A proportion y of the buyers are thought to wait for sellers, and a proportion x of the sellers are thought to wait for buyers. We can think of them as not moving or searching but staying in fixed positions. Correspondingly, a proportion 1-y of the buyers and a proportion 1-x of the sellers search for partners.

We take search to mean the following: The searchers are distributed randomly on the waiters of the opposite type. One can think that each searcher has a random device that assigns him to one of the fixed positions each position being equally likely. Consider, for instance, a waiting buyer. The number of sellers that he meets is distributed according to the binomial distribution with parameters $(1\text{-}x)S$ and $1/yB$. Similarly, the number of buyers a particular seller expects to meet follows the binomial distribution with parameters $(1\text{-}y)B$ and $1/xS$. When the probabilities are small the binomials can be approximated with Poisson-distributions with parameters $\alpha = \dfrac{1-x}{y\theta}$ and $\beta = \dfrac{1-y}{x}\theta$, respectively, where $\theta = \dfrac{B}{S}$. From now on we take it for granted that this approximation is justified.

Although it is not necessary we can think that the economy is divided into two separate markets; in a buyers' market the buyers wait, and in the sellers' market the sellers wait. Consider the buyers' market, and a particular buyer. The probabilities of him meeting no seller, exactly one seller, and two or more sellers are $e^{-\alpha}$, $\alpha e^{-\alpha}$, and $1-e^{-\alpha}-\alpha e^{-\alpha}$, respectively. A seller in this market is certain to be matched with a buyer but there may be other sellers matched to the same buyer, too. We denote the probability that a particular seller is the only one matched to the buyer by $a = e^{-\alpha}$. This is the same as the probability that a waiting buyer does not meet any sellers.

Analogously, in the sellers' market we denote the probabilities of meeting no buyer, exactly one buyer, and two or more buyers by , $e^{-\beta}$, $\beta e^{-\beta}$, and $1 - e^{-\beta} - \beta e^{-\beta}$, respectively. A buyer in this market is the only buyer to meet the seller with probability $b = e^{-\beta}$.

The crux of this model is the trading procedure. Needless to emphasise we regard it as more natural than the ones in the previous literature. There are two widely accepted micro-level mechanisms of price determination, namely pairwise bargaining and auctions, and we utilise both. When there is exactly one buyer and seller matched they bargain about the price of the object. When there are more than one buyer (or seller depending on the market) there is an auction in which the buyers engage in a Bertrand competition like situation. Competition pushes the price to a level on which those that do not trade are as well off as those who trade. The price is such that the buyers are pushed to their reservation utility level; they are as well off as would have been a buyer who has just entered the market and has not been matched with anyone. When exactly one seller meets exactly one buyer they conduct a Rubinstein-type alternating offers bargaining game in which one of the players is selected to be the proposer with probability one half. If his proposal is not accepted bargaining extends to the next period when the searching party can leave his partner but whatever he does he runs a risk of ending up in an auction as other buyers may emerge. In this set-up the bargaining partners are not bound to each other.

To start with we study a case similar to that of Rubinstein and Wolinsky (1985), and Lu and McAfee (1996). The buyers value the sellers' good at unity, and the sellers' reservation value of the good is zero. In other words the buyers and sellers have a unit surplus to divide, and after trade takes place the traders exit the market, and are replaced with identical traders. Thus, we study a situation in which the relative numbers of buyers and sellers remain the same over time. In the beginning of a period the agents are matched, and their instantaneous utility is determined by bargaining or auction depending on in which situation they end up. The stayers may also end up with no partner in which case they just wait till next period to be matched afresh. The agents' utility functions are linear $u(x) = x$, and they discount utility with a common factor δ, $0 < \delta < 1$. Denote the value functions, or expected life time utilities with equilibrium strategies, of the buyers who wait, buyers who search, sellers who wait, and sellers who search by W_b, S_b, W_s, and S_s. We used S to denote the number of sellers but this should not cause any confusion. Next we study one particular market, and once we understand how it functions we shall determine the overall equilibrium with potentially two active markets.

Buyers' market

Consider first the buyers' market after which the corresponding results for the sellers' market follow by analogy. When a buyer and seller bargain one of them is randomly selected to propose a division of the surplus. Denote the seller's proposal by (t,1-t), and the buyer's proposal by (v, 1-v), where the first co-ordinate is the buyer's share. We assume that each one is equally likely to be the proposer the asymmetric case being an easy exercise. In case there are more than one seller matched with a buyer an auction is conducted, and we denote the buyers share by z. Bargaining takes the form of alternating offers procedure. If the proposal is not accepted it takes one period before another proposal can be made, and meanwhile more sellers may appear. Our concept of market equilibrium is the same as in RW, and consists of a pair of semi-stationary strategies that prescribe the same tactics in all bargaining situations and auction situations the agent is involved in. The existence of equilibrium can be proved exactly as in RW. We evaluate the sellers' and buyers' value functions or expected life time utilities at the end of a period. They are determined by the following equations.

$$S_s = \delta\left\{a\left[\frac{1}{2}(1-v)+\frac{1}{2}(1-t)\right]+(1\text{-a})\mathrm{S_s}\right\} \tag{1}$$

$$W_b = \delta\left\{aW_b + \alpha a\left[\frac{1}{2}v+\frac{1}{2}t\right]+(1-a-\alpha a)z\right\} \tag{2}$$

In (1) the RHS is the seller's expected utility in the next period. With probability a there are no other sellers matched with the buyer and bargaining ensues. With probability one half either of them makes an offer that is accepted in equilibrium. With probability $1\text{-}a$ other sellers appear and there will be an auction. Then the sellers are driven to their reservation utility regardless of who gets the object. The reservation utility is S_s, the expected utility of leaving the current partner. Equation (2) depicts the corresponding situation of the buyer. Notice that with probability a the buyer does not meet anybody.

The following two equations determine the subgame perfect proposals of the buyer and the seller.

$$1 - v = S_s = \delta \left\{ a \left[\frac{1}{2}(1-v) + \frac{1}{2}(1-t) \right] + (1-a)S_s \right\} \tag{3}$$

$$t = W_b = \delta \left\{ aW_b + \alpha a \left[\frac{1}{2}v + \frac{1}{2}t \right] + (1 - a - \alpha a)z \right\} \tag{4}$$

In equilibrium the proposer makes an offer that leaves the responder just indifferent between accepting and rejecting. This means that the proposal gives the responder his reservation utility. Notice in (4) that as the sellers are always matched with somebody they leave their current partner if he does not accept their proposition. Leaving him or staying does not affect the probability of being the only seller matched to a buyer but decreases the buyer's outside option as he runs the risk of ending up with no partner at all.[1]

To determine z it is enough to notice that a seller who strikes a deal receives $1\text{-}z$ which has to equal the seller's reservation utility. Thus, the final condition is

$$1 - z = S_s \tag{5}$$

From (1) to (5) it is a straightforward task to solve for the values of interest.

[1] We could also postulate that the sellers stay with their partner. Then the buyer would be certain that he has a trading partner next period, too. It is a matter of taste which way to model the situation.

$$z = v = \frac{2 - 2\delta a - \delta\alpha a}{2 - \delta a - \delta\alpha a} \tag{6}$$

$$t = W_b = \frac{\delta(2 - 2a - \alpha a)}{2 - \delta a - \delta\alpha a} \tag{7}$$

$$S_s = \frac{\delta a}{2 - \delta a - \delta\alpha a} \tag{8}$$

We have to analyse the sellers' market, too, since the sellers and buyers have the choice of entering either market. In equilibrium they should be indifferent between the markets if both of them are active. Still, already at this stage we can readily compare the situation to that of RW. They claim that the competitive equilibrium price is unity if there are more buyers than sellers and zero if there are more sellers than buyers. Within a period these prices would equate demand and supply there were a centralised Walrasian market. As Gale (1986a, b, 1987) has pointed out this is not necessarily a correct interpretation of a competitive market since there is infinite demand and supply when taking into account new entrants. Let us, however, for a moment take the same stand as in RW for the sake of comparison. In RW the prices are far from competitive even when the discount factor approaches unity, i.e. when frictions connected to search vanish. The prices do not converge to the competitive equilibrium in this framework either which should not be particularly surprising as the whole point of search or matching models is to get rid of the perfect market paradigm. Compared to perfect markets discounting is not the only friction; one friction is that not everybody meets everybody else, and consequently one should not expect competitive prices even if the discount factor approaches unity.

In RW the matching process is left unmodelled; it is assumed that meetings happen with exogenous probabilities, and frictions are reduced in two ways. First, the discount factor approaches unity. Secondly, the time period shrinks. In this case also the flow of new agents per period reduces, and as a result in RW the prices in limit differ depending on the way of reducing frictions. In this model there is only one way to decrease frictions associated to discounting. One may regard this as a desirable or undesirable property but the reason is that the matching probabilities are determined from the data of the model, and they cannot be just stipulated. When the discount factor approaches unity the price or buyers' share is the same in auction and bargaining regardless of who makes the first offer. The remaining deviation from Walrasian price is due to the frictions of the meeting technology. Like in RW when the number of, say, buyers increases θ increases and both v and w approach unity if δ also approaches unity, otherwise w approaches δ. Thus, excess demand (and supply) moves prices to the right direction and in the limit the prices approach Walrasian prices (with the qualification of the discount factor).

Sellers' market

In the sellers' market the seller is formally in the same position as a buyer in the buyers' market and vice versa. Denote the seller's proposal by $(\bar{t}, 1-\bar{t})$, the buyer's proposal in bargaining by $(\bar{v}, 1-\bar{v})$, and the buyer's share in an auction by $\bar{z}$.

The buyers' and sellers' value functions are

$$W_s = \delta\left\{bW_s + \beta b\left[\frac{1}{2}(1-\bar{v}) + \frac{1}{2}(1-\bar{t})\right] + (1 - \mathrm{b} - \beta\mathrm{b})(1 - \bar{\mathrm{z}})\right\} \tag{9}$$

$$S_b = \delta\left\{b\left[\frac{1}{2}\bar{v} + \frac{1}{2}\bar{t}\right] + (1-b)S_b\right\} \tag{10}$$

The equilibrium offers are determined by

$$1-\bar{v} = \delta\left\{bW_s + \beta b\left[\frac{1}{2}(1-\bar{v}) + \frac{1}{2}(1-\bar{t})\right] + (1-b-\beta b)(1-\bar{z})\right\} \tag{11}$$

$$\bar{t} = \delta\left\{b\left[\frac{1}{2}\bar{v} + \frac{1}{2}\bar{t}\right] + (1-b)S_b\right\} \tag{12}$$

and the final equilibrium condition in these markets is

$$\bar{z} = S_b \tag{13}$$

Formulae (9)-(13) are identical to (1)-(5) when we make the following transformations $\bar{v} \mapsto 1-t$, $1-\bar{v} \mapsto t$, $\bar{t} \mapsto 1-v$, $1-\bar{t} \mapsto v$, $\bar{z} \mapsto 1-z$, $1-\bar{z} \mapsto z$, $a \mapsto b$, $\alpha \mapsto \beta$, $W_s \mapsto W_b$ and $S_b \mapsto S_s$. Now we can utilise (6)-(8) and the terms of interest in these markets turn out

$$\bar{z} = \bar{t} = S_b = \frac{\delta b}{2 - \delta b - \delta\beta b} \tag{14}$$

$$W_s = \frac{\delta(2 - 2b - \beta b)}{2 - \delta b - \delta\beta b} \tag{15}$$

$$\bar{v} = \frac{2 - 2\delta + \delta b}{2 - \delta b - \delta\beta b} \tag{16}$$

Equilibrium

Overall equilibrium in the economy requires that both sellers and buyers find entering either market equally attractive if both markets are active, i.e. $W_b = S_b$ and $W_s = S_s$. It is clear that if all buyers and all sellers are in one market, say buyers' market, then it does not pay for anyone to go to the redundant sellers' market since he will be there alone. The equilibrium conditions for two active markets produce two equations

$$[2 - a(2+\alpha)][2 - \delta b(1+\beta)] - b[2 - \delta a(1+\alpha)] = 0 \tag{17}$$

$$[2 - b(2+\beta)][2 - \delta a(1+\alpha)] - a[2 - \delta b(1+\beta)] = 0 \tag{18}$$

From these we can solve for a and b

$$e^{-\alpha} = \frac{2(1+\beta)}{3 + 2\alpha + 2\beta + \alpha\beta} \tag{19}$$

$$e^{-\beta} = \frac{2(1+\alpha)}{3+2\alpha+2\beta+\alpha\beta} \tag{20}$$

From (19) and (20) we get $e^{\beta-\alpha} = \dfrac{1+\beta}{1+\alpha}$ which holds only if $\alpha = \beta$. This is equivalent to

$$\frac{1-x}{y\theta} = \frac{1-y}{x}\theta \tag{21}$$

From the equality of (7) and (14) with $\alpha = \beta$ we get

$$2 - 3e^{-\alpha} - \alpha e^{-\alpha} = 0 \tag{22}$$

The expression on the LHS of (22) is increasing and at $\alpha = 1$ it is positive, and thus the solution to (22) is less than unity. Let us denote the solution to (22) by θ_0 since the magnitude of the relative number of buyers to sellers θ compared to this number is going to play a major role in the analysis. In equilibrium $\alpha = \beta = \theta_0$ which is equivalent to $\dfrac{1-x}{y\theta} = \dfrac{1-y}{x}\theta = \theta_0$. This yields two equations from which x and y can be solved.

$$x = \frac{1-\theta_0\theta}{1-\theta_0^2} \tag{23}$$

$$y = \frac{\theta-\theta_0}{\theta(1-\theta_0^2)} \tag{24}$$

We see that buyers' and sellers' markets can co-exist only if the relative difference between buyers and sellers in not too large. In particular, it has to be true that $\dfrac{1}{\theta_0} > \theta > \theta_0$. Let us note for completeness that θ_0 is between 0.58 and 0.59.

Proposition 1. There always exist an equilibrium in which only the buyers' market is active, and an equilibrium in which only the sellers' market is active. When $\frac{1}{\theta_0} > \theta > \theta_0$ there exists a third equilibrium in which both markets are active. In this equilibrium the proportion of buyers in the buyers' market is $y = \frac{\theta - \theta_0}{\theta(1 - \theta_0^2)}$, the proportion of sellers in the sellers' market is $x = \frac{1 - \theta_0\theta}{1 - \theta_0^2}$, and the expected utility of all agents is the same.

The equilibrium with two active markets exhibits an interesting feature. Even if there are more, say, buyers than sellers the buyers' expected utility is the same as that of the sellers. This happens since if there is only one market both parties find it preferable to wait rather than search. Thus, the excess demand is counterbalanced by relatively more sellers in the sellers' market than in the buyers' market. The agents of both types compete for being a waiter, and this equates their expected utilities even though their numbers need not equal.

The test for Nash-equilibrium in this model is not very strong since it is concerned only with unilateral deviations. We view markets as a co-ordinating device, and it is plausible that more demanding considerations may be of significance. Consequently, we consider some kind of multilateral deviations. To this end we adopt the slow evolutionary dynamics that Lu and McAfee (1996) use to study the stability of equilibria.

3. STABILITY

First we note that all market structures are not only Nash-equilibria but they are stable against multilateral deviations. No group of agents can put up a market of their own in such a way that all of them are better-off than previously. This means that the equilibria satisfy a strong requirement that resembles that of coalition proof Nash-equilibrium. To see this assume that there is only, say, the buyers' market. Consider a group of agents establishing a sellers' market where the proportion of buyers to sellers is r. This is profitable to the agents if the following conditions hold

$$S_b > W_b \Leftrightarrow e^{-r}(2-\delta e^{-1/\theta}-\delta\frac{1}{\theta}e^{-1/\theta}) > (2-2e^{-1/\theta}-\frac{1}{\theta}e^{-1/\theta})(2-\delta e^{-r}-\delta re^{-r}) \tag{25}$$

$$W_s > S_s \Leftrightarrow (2-2e^{-r}-re^{-r})(2-\delta e^{-1/\theta}-\delta\frac{1}{\theta}e^{-1/\theta}) > e^{-1/\theta}(2-\delta e^{-r}-\delta re^{-r}) \tag{26}$$

These imply that $e^{-r}(1+r) < e^{-1/\theta}(1+1/\theta)$ which holds if $r > 1/\theta$, and routine calculations show that then (25) and (26) cannot hold simultaneously. The result also indicates that if there is to be only one market the agents would rather wait than search. The solution to equation (22) reflects the same fact. As θ_0 is less than unity in both markets there are more agents waiting than searching. The reason is that by waiting the agents run no risk of being engaged in a price competition, or an auction. The trade-off is the risk of ending up with no partner for the period but this involves only a delay in trading while being in the demand side of an auction involves a loss of all surplus from trade. The same argument carries to the case of two active markets.

To get more insight to which equilibrium is likely to emerge let us study the situation from an evolutionary perspective. The buyers and sellers are indifferent between markets if $W_b = S_b$ and $W_s = S_s$ which is equivalent to

$$[2-a(2+\alpha)][2-\delta b(1+\beta)]-b[2-\delta a(1+\alpha)]=0 \tag{27}$$

$$[2-b(2+\beta)][2-\delta a(1+\alpha)]-a[2-\delta b(1+\beta)]=0 \tag{28}$$

We call (27) and (28) the buyers' equilibrium curve and the sellers' equilibrium curve, BE and SE, respectively. Notice that the dependence of α, β, a and b on the proportion of sellers in the sellers' market y, and the proportion of buyers in the buyers' market x has been surpressed. As in Lu and McAfee (1996) we think of the populations adjusting slowly. As trades are completed the traders exit and are repleced by new entrants so that θ remains constant. The new entrants are myopic and expect the pay-offs to stay the same as when they enter the markets. They go to the market where their expected pay-off is highest. Thus the number of agents increases in the favourable markets and decreases in the unfavourable markets. This is the replicator dynamics of evolutionary theory (Nachbar, 1990). It is thought that the number of entrants is small so as to circumvent problems that arise from discrete adjustment steps which can lead to cycles or non-convergence. As a result of the dynamics x and y change in time but we do not make the time dependence explicit in order to keep the notation simple. To proceed we first establish the slopes of the equilibrium curves BE and SE.

Lemma 1. For both buyers and sellers x and y are negatively related on the equilibrium curves, i.e. $\frac{\partial y}{\partial x} < 0$.

This can be established by totally differentiating the BE and SE curves. The proofs of this and other results are relegated to the appendix. Lemma 1 says that when the number of sellers in the sellers' market goes up so does the number of buyers, and consequently their number in the buyers' market goes down. More sellers in the sellers' market make the buyers there better-off, and if the buyers are to be equally well-off in both markets some of them have to exit the buyers' market and enter the sellers' market.

Let θ_1 be the solution to $2(1-\delta) = e^{-y}\left[4 - 3\delta + 2y(1-\delta)\right]$. It depends on the discount factor but it is easy to establish that θ_1 is always greater than unity, and thus greater than θ_0.

Lemma 2. i) The BE-curve always contains point $(x, y) = (1,0)$, and the SE-curve always contains point $(x, y) = (0,1)$.

ii) If $\theta < \frac{1}{\theta_1}$ there exists $x_\theta \in (0,1)$ such that the BE-curve contains $(x, y) = (x_\theta, 1)$.

iii) If $\theta > \theta_1$ there exists $y_\theta \in (0,1)$ such that the SE-curve contains $(x,y) = (1, y_\theta)$.

Lemma 2 is a step towards establishing the position of the equilibrium curves which depends on the relative amounts of buyers and sellers. Case i) says that buyers' equilibrium curve always contains a point with the sellers' market only in existence, and the sellers' equilibrium curve always contains a point with the buyers' market only in existence. For instance, in the latter case if there were any buyers in the sellers' market any seller would find it profitable to go to sellers' market since there he would be matched with several buyers who would engage in an auction. But then sellers would not be indifferent between which markets to participate in. Analogous reasoning applies to buyers' equilibrium curve. The heuristics of ii) and iii) are analogous. Consider, for instance, a case with very many buyers, i.e. case iii). Even if there are some buyers in the buyers' market none of the sellers wants to go there since in the sellers' market ($x = 1$) they are almost certain to meet buyers who are engaged in an auction. This means that the sellers reap all the surplus from trade, and the risk of remaining without a partner is so small that it is not worthwhile to go to the buyers' market for a match which leads to bargaining. If in cases ii) and iii) BE and SE do not intersect we immediately see that in case ii) BE is above SE and in case iii) SE is above BE.

We still have to study several cases. Let us first establish the position of BE and SE when they do not intersect.

Lemma 3. When $\theta < \theta_0$ SE is below BE.

Lemma 4. When $\frac{1}{\theta_0} < \theta$ BE is below SE.

Lemmata 3 and 4 with Lemma 2 determine the position of BE and SE as a function of the relative number of buyers to sellers completely. We still have to establish their position when they intersect. To this end we calculate their derivatives at the point of intersection.

Lemma 5. Assume that $\frac{1}{\theta_0} > \theta > \theta_0$. At the point of intersection BE cuts SE from above.

Note that depending on δ it is possible that $\frac{1}{\theta_1} > \theta_0$ as well as $\frac{1}{\theta_1} < \theta_0$. Very low values of δ result in the first case.

The evolutionary or stability analysis of the markets is based on the next lemma. It tells which markets the agents go to when they are not on their equilibrium curves. In evolutionary terms it determines to which direction the proportions of agents in various markets change as a result of new entrants. The idea is that only the entrants are free to choose which markets to participate in, and will, of course, go where the expected utility is highest. As time goes these decisions change the proportion of agents in different markets, and as far as this process converges we attain equilibrium selection. As Lu and McAfee (1996) note the analysis is appropriate only if we ignore the discrete time, and assume that the number of entrants is small since discrete dynamics with many entrants may lead to cycles. There are several ways to motivate this (see Lu and McAfee, 1996) but one can simply assume that only a small proportion of agents actually trade this proportion being the same as that when everybody who is capable trades.

Lemma 6. In *x-y*-space above BE buyers prefer the sellers' market, and below BE the buyers prefer the buyers' market. Above SE sellers prefer the buyers' market, and below SE sellers prefer the sellers' market.

In evolutionary terms Lemma 6 says that above BE y decreases and below it y increases. Similarly above SE x decreases and below it x increases. In figure 1-3 three of the seven possible configurations of BE and SE are depicted. The arrows show the directions to which x and y change, and we immediately see which market structure the dynamics selects. Figure 1 depicts the case in which there are three equilibria. When the numbers of buyers and sellers do not differ too much the dynamics may select anyone of the three equilibria depending from where it starts. However, the market structure with two active markets is not stable since any neighbourhood of this point contains points that are between BE and SE from where the dynamics converges either to the buyers' market or sellers' market. The higher the proportion of buyers to sellers is the closer to point $(0,1)$ the curves intersect. In this sense the more likely it is that sellers' market emerges as equilibrium.

In figure 2 BE is above SE, the curves do not intersect and BE does not contain point $(0,1)$. The dynamics selects the buyers' market, i.e. the market where buyers wait and sellers are distributed on them. There are relatively few buyers and many sellers, and the dynamics selects the equilibrium that is desirable from the buyers point of view. This is a sound result as one should regard it as strange if buyers being in short supply did not profit from their situation. Heuristically, if there we only the sellers' market the buyers would meet sellers for certain but run a small risk of ending up in an auction. The sellers in turn run a large risk of ending up with no partner at all. Any buyer that moves to the buyers market attracts at least one seller, and both are better-off.

In figure 3 SE is above BE and the curves do not intersect. Analogously to the previous case the dynamics selects the sellers' market, i.e. the market where sellers wait and buyers are distributed on them. It should be noted that in no picture the form of the curves is accurate as the only important matter is their position.

The remaining four cases differ from the above three only so far as the curves intersect each other and contain the points $(0,1)$ and $(1,0)$. We summarise these findings in the following proposition whose proof is omitted as it does not add anything to the pictures.

Proposition 2. i) If $\frac{1}{\theta_0} < \theta < \theta_0$ there are two stable equilibria in which only one market is active, and one unstable equilibrium in which both buyers' and sellers' markets are active.
ii) If $\theta \leq \theta_0$ there is one stable equilibrium that consists of the buyers' market.
iii) If $\frac{1}{\theta_0} \leq \theta$ there is one stable equilibrium that consists of the sellers' market.

When BE and SE intersect it is not clear which of the equilibria is most plausible. However, it is seen that if the dynamics starts from a random point it is more likely to select the buyers' market if there are more sellers and vice versa. One should not disregard the equilibrium with two active markets. Even though it is unstable in this dynamics the dynamics is derived from myopic or very stupid behaviour. It is not at all clear how seriously replicator-like dynamics should be taken in market settings with rational agents. In this specific case the dynamics can be defended by the fact that agents exit the markets after trading, and thus only short run considerations are important to them. But the model can be altered to depict repeated interaction, and then criticism of the replicator dynamics is valid. Further, if one has to select a market structure on a priori basis the equilibrium with two active markets has the advantage of treating both types of agents symmetrically. Thus, the only conclusion we are willing to draw from this exercise is that when there are significantly more agents of one type one should model the markets with random matching in such a way that the agents on the short side wait and the agents on the long side search.

4. WALRASIAN MARKETS AND MATCHING

In this section we shortly investigate the relationship between Walrasian or perfectly competitive markets and markets where agents are randomly matched. To do this we have to specify what Walrasian markets in this context mean. Usually it is thought that in Walrasian markets prices emerge so that they equate demand and supply. In the random matching model which we have dealt with the demand and supply are infinite as agents who trade exit and are replaced by new agents. Thus demand and supply are not well defined. Focusing on active agents only demand and supply are well defined in any period the former being the same as the number of buyers B and the latter the same as the number of sellers S. We study this one period economy like RW do while acknowledging that there are problems with this choice. The Walrasian price in the one period economy is unity if demand is greater than supply, and zero if demand is less than supply.

The model of the previous sections demonstrates that equilibrium prices are different from Walrasian prices in a matching model. This holds good even if the agents are perfectly patient, and even when competitive pressures are introduced by allowing auctions as well as bargaining. The reason behind this feature of the model is that not everybody meets everybody. The easiest way to see this is to consider a matching model slightly different from that of the previous sections.

Instead of distributing searchers on waiters let us assume that there is a fixed number k of trading locations in the economy. All the agents are distributed randomly on one of the locations. If there happens to be equal numbers of buyers and sellers in a particular location they bargain over the price. If there are unequal numbers an auction is held, and those on the long side are pushed to their reservation level. If there are more buyers than sellers the Walrasian price is unity as there is excess demand. The same price emerges if there is only one trading location, i.e. $k = 1$. All the agents are in this same location and as there are more buyers the auction price is unity. Increasing k means increasing deviations from the competitive equilibrium, at least as long as k does not grow much larger than the number of agents. When k is different from unity it is even possible that in some location there will be more sellers than buyers, and the sellers end up on the long side. The price in this location is then different from the price in a location with an excess of buyers. One could parameterise the degree of competitiveness by $1/k$ where unity would mean perfectly competitive markets and smaller values would mean correspondingly larger deviations from perfect competition.

In the text book case of competitive markets a law of one price is sometimes mentioned. Regardless of its lawlikeness it is generally expected that similar good are traded at a uniform price in a competitive environment. Consider the model of the previous sections, and eg. the buyers' market. The prices (say buyers share) differ depending on who makes the proposal in bargaining, and the auction price equals the buyers' share when the buyers make the proposal. In addition to being different from Walrasian prices the prices in the model deviate from the law of one price as well. As noted earlier when the discount factor approaches unity the law of one price emerges though it is not the Walrasian price. One interpretation of this is that the deviation from the law of one price is due to the impatience of the agents, and the deviation from the Walrasian price is due to the fact that not everybody meets everybody else.

Bargaining and auction are in some sense not on equal standing in the model. While bargaining is modelled explicitly auction is not. As a result auction just takes place within a period, supposedly within a very short time, while bargaining extends explicitly through several periods (potentially infinite). From the modelling point of view the crucial aspect is that the threat of refusing an offer, i.e. the concurrent actions, take place only in the next period. One may desire to have one price in bargaining and a different price in auction. A simple alteration of the bargaining procedure results in uniform price in bargaining regardless of the proposer. Assume that one of the parties always makes an offer. If it is refused there is another round in which the proposer is determined randomly so that either party is equally likely. Assume further that the first round takes so little time within a period that discounting can be ignored. It turns out that regardless of who makes the first proposal it is accepted and the share of the buyer is $\frac{1+\delta-2\delta a-\delta\alpha a}{2-\delta a-\delta\alpha a}$ while the share of the buyer in an auction is $\frac{2-2\delta a-\delta\alpha a}{2-\delta a-\delta\alpha a}$. In case one wants to make a distinction between prices in bargaining and in auction one can easily model bargaining in the above fashion; this just alters the model slightly without any significant effect as one can readily notice by comparing the above shares to (6) and (7).

5. CONCLUSION

Market models with search or random matching do not usually specify the environment in which the agents operate. Many times this is not even necessary but when the models attempt to address the question of price formation and determine equilibrium prices this aspect may be of importance. The standard way to determine prices is to postulate that the agents meet trading partners with some probability and when matched with a partner bargain over the price. In bargaining the agents are locked to each other and competitive pressures are diminished. This may lead to strange results, and what we have done in this article consists of adding an auction as a mechanism for determining prices. Auction ensues when more than one agent meet an agent with an object for sale while if only two agents meet they bargain in a familiar fashion. In this setting we cannot freely assign the probabilities with which the agents meet other agents; the probabilities of meeting zero, one, and more than one agent have to be consistent. We attain this by carefully specifying the environment and the manner in which the agents meet each other.

The model is surprisingly easy to analyse, and there are at most three equilibria when one restricts the agents' strategies to what in RW are called semi-stationary. Depending on the relative numbers of buyers and sellers stability analysis gives in some cases clear results as to which equilibrium is the most plausible. When the numbers of buyers and sellers are not too different there still remains ambiguity about the most plausible equilibrium. Still we know what the stability features of an evolutionary dynamic in this case looks like. A random matching model necessarily exhibits some forms of market imperfection compared to the Walrasian or competitive market model. A somewhat contrived comparison of these models suggests that there are two distinct frictions in a random matching model, namely impatience and the fact that not everybody meets everybody else in the markets.

I thank Juha-Pekka Niinimaki, Tuomas Takalo, Timo Seppalainen and Frans Spinnewyn for useful comments. Financial support by Yrjo Jahnsson Foundation, and in a form of a TMR grant from the European Commission is gratefully acknowledged

REFERENCES

Diamond P. (1971), 'A model of price adjustment', *Journal of Economic Theory* **3**, 156-168.

Gale D. (1986a), 'Bargaining and competition part I: Characterization', *Econometrica* **54**, 785-806.

Gale D. (1986b), 'Bargaining and competition part II: Existence', *Econometrica* **54**, 807-818.

Gale D. (1987), 'Limit theorems for markets with sequential bargaining', *Journal of Economic Theory* **43**, 20-54.

Lu X. and R.P. McAfee (1996), 'The evolutionary stability of auctions over bargaining', *Games and Economic Behavior* **15**, 228-254.

McMillan J. and M. Rothschild (1994), 'Search' in (eds.) Aumann R. and S. Hart, Handbook of game theory, vol. 2, Amsterdam: North-Holland, 905-927.

Mortensen D. (1986), 'Job search and labor market analysis' in (eds.) Ashenfelter O. and R. Layard, Handbook of labor economics, vol. 2. Handbooks in Economics series, no. 5, Amsterdam: North-Holland, 849-919.

Nachbar J. (1990), 'Evolutionary selection dynamics in games: convergence and limit properties, *International Journal of Game Theory* **19**, 59-89.

Rubinstein A. (1982), 'Perfect equilibrium in a bargaining model', *Econometrica* **50**, 97-110.

Rubinstein A. and A. Wolinsky (1985), 'Equilibrium in market with sequential bargaining', *Econometrica* **53**, 1133-1150.

APPENDIX

Proof of Lemma 1.

Totally differentiating BE we get

$$\frac{dy}{dx} = \frac{-yx^2 A - \delta y^2(1-y)\theta^2 B - y^2(1-y)\theta^2 C + \delta yx^2 D}{x^2(1-x)A + \delta y^2 x\theta^2 B + y^2 x\theta^2 C - \delta x^2(1-x)D} \tag{A0}$$

where

$$A = (1+\frac{1-x}{y\theta})\left[2-\delta e^{-\frac{1-y}{x}\theta}(1+\frac{1-y}{x}\theta)\right]e^{-\frac{1-x}{y\theta}}$$

$$B = \frac{1-y}{x}\theta\left[2-e^{-\frac{1-x}{y\theta}}(2+\frac{1-x}{y\theta})\right]e^{-\frac{1-y}{x}\theta}$$

$$C = \left[2-\delta e^{-\frac{1-x}{y\theta}}(1+\frac{1-x}{y\theta})\right]e^{-\frac{1-y}{x}\theta}$$

$$D = \frac{1-x}{y\theta}e^{-\frac{1-y}{x}\theta}e^{-\frac{1-x}{y\theta}}$$

A, B, C, and D are all positive and as $A > \delta D$ the result follows.

Totally differentiating SE gives the following expression

$$\frac{dy}{dx} = \frac{-y^2(1-y)\theta^2 A' - \delta yx^2 B' - yx^2 C' + \delta y^2(1-y)\theta^2 D'}{y^2 x\theta^2 A' + \delta x^2(1-x)B' + x^2(1-x)C' - \delta y^2 x\theta^2 D'} \tag{A1}$$

where

$$A' = (1+\frac{1-y}{x}\theta)\left[2-\delta e^{-\frac{1-x}{y\theta}}(1+\frac{1-x}{y\theta})\right]e^{-\frac{1-y}{x}\theta}$$

$$B' = \frac{1-x}{y\theta}\left[2-e^{-\frac{1-y}{x}\theta}(2+\frac{1-y}{x}\theta)\right]e^{-\frac{1-x}{y\theta}}$$

$$C' = \left[2-\delta e^{-\frac{1-y}{x}\theta}(1+\frac{1-y}{x}\theta)\right]e^{-\frac{1-x}{y\theta}}$$

$$D' = \frac{1-y}{x}\theta e^{-\frac{1-y}{x}\theta}e^{-\frac{1-x}{y\theta}}$$

As before the result follows since $A' > \delta D'$. ■

Proof of Lemma 2.

Let us see how BE behaves when one of the co-ordinates approaches its end point.

i) Let $(x,y)\to(0,y)$. Now BE becomes $2-e^{-\frac{1}{y\theta}}(2+\frac{1}{y\theta})=0$ which never holds.

ii) Let $(x,y)\to(1,y)$. Now BE becomes $(2-\delta)e^{-(1-y)\theta}=0$ which never holds.

iii) Let $(x,y)\to(x,1)$. Now BE becomes

$$2(1-\delta)=e^{-\frac{1-x}{\theta}}\left[4-3\delta+2\frac{1-x}{\theta}(1-\delta)\right] \tag{A2}$$

which is of the form $2(1-\delta)=e^{-w}\left[4-3\delta+2w(1-\delta)\right]\equiv\xi(w)$. Since $\frac{\partial}{\partial w}\xi(w)=-e^{-w}\left[2-\delta+2w(1-\delta)\right]<0$ and as w grows without limit $\xi(w)$ approaches zero, there exists a unique solution θ_1 to (A2). $\frac{1-x}{\theta}=\theta_1\Rightarrow x=1-\theta\theta_1$ and thus when $\theta<\frac{1}{\theta_1}$ BE goes through point $(x,1)$ where $x\in(0,1)$. If $\theta\geq\frac{1}{\theta_1}$ BE converges to $(0,1)$.

iv) Let $(x,y)\to(x,0)$. Now BE becomes $2-e^{-\frac{\theta}{x}}(1+\delta+\delta\frac{\theta}{x})=0$ which never holds.

From ii) and iv) we see that BE always converges to $(1,0)$.

Let us see how SE behaves when one of the co-ordinates approaches its end point.

i) Let $(x,y)\to(0,y)$. Now SE becomes $2-e^{-\frac{1}{y\theta}}(1+\delta+\delta\frac{1}{y\theta})=0$ which never holds.

ii) Let $(x,y)\to(1,y)$. Now SE becomes

$$2(1-\delta)=e^{-(1-y)\theta}\left[4-3\delta+2(1-y)\theta(1-\delta)\right] \tag{A3}$$

which is of the form $2(1-\delta)=e^{-w}\left[4-3\delta+2w(1-\delta)\right]\equiv\xi(w)$. As above $(1-y)\theta=\theta_1\Rightarrow y=\frac{\theta-\theta_1}{\theta}$ and thus when $\theta>\theta_1$ SE goes through point $(1,y)$ where $y\in(0,1)$. If $\theta\leq\theta_1$ SE converges to $(1,0)$.

iii) Let $(x,y)\to(x,1)$. Now SE becomes $(2-\delta)e^{-\frac{1-x}{\theta}}=0$ which never holds.

iv) Let $(x,y)\to(x,0)$. Now SE becomes $2-e^{-\frac{\theta}{x}}(2+\frac{\theta}{x})=0$ which never holds.

From i) and iii) we see that SE always converges to $(0,1)$.■

Proof of Lemmata 3 and 4.

Next we have to determine the position of BE and SE for various parameter values. When $\theta < \dfrac{1}{\theta_1}$ BE is above SE if they do not intersect, and when $\theta > \theta_1$ SE is above BE if they do not intersect. Let us next study the case $\theta \le \theta_0$. As BE and SE do not intersect it is sufficient to establish their position for one value of y say $y = \dfrac{1}{2}$. Now BE becomes

$$\frac{2 - e^{-\frac{2(1-x)}{\theta}}\left(2 + \frac{2(1-x)}{\theta}\right)}{2 - \delta e^{-\frac{2(1-x)}{\theta}}\left(1 + \frac{2(1-x)}{\theta}\right)} = \frac{e^{-\frac{\theta}{2x}}}{2 - \delta e^{-\frac{\theta}{2x}}\left(1 + \frac{\theta}{2x}\right)} \tag{A4}$$

and SE becomes

$$\frac{2 - e^{-\frac{\theta}{2x}}\left(1 + \frac{\theta}{2x}\right)}{2 - \delta e^{-\frac{\theta}{2x}}\left(1 + \frac{\theta}{2x}\right)} - \frac{e^{-\frac{2(1-x)}{\theta}}}{2 - \delta e^{-\frac{2(1-x)}{\theta}}\left(1 + \frac{2(1-x)}{\theta}\right)} = \frac{e^{-\frac{\theta}{2x}}}{2 - \delta e^{-\frac{\theta}{2x}}\left(1 + \frac{\theta}{2x}\right)} \tag{A5}$$

Let us write (A4) in a more compact form $f(x) = h(x)$, and (A5), too, $g(x) = h(x)$.

Routine calculations show that $\dfrac{\partial f}{\partial x} = \dfrac{-e^{-\frac{2(1-x)}{\theta}}\dfrac{2}{\theta}\left[2\dfrac{2(1-x)}{\theta}(1-\delta) + 2 - \delta e^{-\frac{2(1-x)}{\theta}}\right]}{\left[2 - \delta\delta e^{-\frac{2(1-x)}{\theta}}\left(1 + \dfrac{2(1-x)}{\theta}\right)\right]^2} < 0$ for all $x \in [0,1]$, $\dfrac{\partial g}{\partial x} = \dfrac{-e^{-\frac{\theta}{2x}}\dfrac{\theta^2}{2x^3}(1-\delta)}{\left[2 - \delta e^{-\frac{\theta}{2x}}\left(1 + \dfrac{\theta}{2x}\right)\right]^2} + \dfrac{-e^{-\frac{2(1-x)}{\theta}}\dfrac{2}{\theta}\left(2 - \delta e^{-\frac{2(1-x)}{\theta}}\right)}{\left[2 - \delta\delta e^{-\frac{2(1-x)}{\theta}}\left(1 + \dfrac{2(1-x)}{\theta}\right)\right]^2} < 0$ for all $x \in [0,1]$, and $\dfrac{\partial h}{\partial x} = \dfrac{e^{-\frac{\theta}{2x}}\left(2 - \delta e^{-\frac{\theta}{2x}}\right)}{\left[2 - \delta e^{-\frac{\theta}{2x}}\left(1 + \dfrac{\theta}{2x}\right)\right]^2} > 0$ for all $x \in [0,1]$. We also need the following results $f(0)=\dfrac{2-e^{-\frac{2}{\theta}}\left(2+\frac{2}{\theta}\right)}{2-\delta e^{-\frac{2}{\theta}}\left(1+\frac{2}{\theta}\right)}$, $g(0)=1-\dfrac{e^{-\frac{2}{\theta}}}{2-\delta e^{-\frac{2}{\theta}}\left(1+\frac{2}{\theta}\right)}$, $h(0)=0$ and $f(0)<g(0)$. It is easy to see that $h(1)>g(1)$ when $\theta\leq\theta_0$. f equals g if $\theta^2=4x(1-x)$ which has two solutions $x_1=\frac{1}{2}-\frac{\sqrt{1-\theta^2}}{2}$ and $x_2=1-x_1=\frac{1}{2}+\frac{\sqrt{1-\theta^2}}{2}$ as θ is less than unity. We show that at $x_1=\frac{1}{2}-\frac{\sqrt{1-\theta^2}}{2}$

$$h(x_1)<g(x_1)=f(x_1) \tag{A6}$$

This means that h intersects g before f provided that at $x_2=1-x_1=\frac{1}{2}+\frac{\sqrt{1-\theta^2}}{2}$ h is above both f and g. (A6) is equivalent to

$$2-3e^{-\frac{\theta}{2x_1}}-\frac{\theta}{2x_1}e^{-\frac{\theta}{2x_1}}>0 \tag{A7}$$

where we have used the fact that $\frac{\theta}{2x_1}=\frac{2(1-x_1)}{\theta}$. (A7) holds only if $\frac{\theta}{2x_1}>\theta_0\Leftrightarrow\varphi(\theta)\equiv\frac{\theta}{1-\sqrt{1-\theta^2}}>\theta_0$. The derivative of $\varphi(\theta)$ is negative for $\theta\leq\theta_0$, and thus $\varphi(\theta)$ achieves its minimum at $\theta=\theta_0$. At $\theta=\theta_0$ we have the desired result $\varphi(\theta_0)=\frac{\theta_0}{1-\sqrt{1-\theta^2{}_0}}>\theta_0$.

Like above at $x_2=1-x_1=\frac{1}{2}+\frac{\sqrt{1-\theta^2}}{2}$ h is greater than g (and f) if $\frac{\theta}{2x_2}<\theta_0\Leftrightarrow\psi(\theta)\equiv\frac{\theta}{1+\sqrt{1-\theta^2}}<\theta_0$. The derivative of $\psi(\theta)$ is positive for $\theta\leq\theta_0$ and thus $\psi(\theta)$ achieves its maximum at $\theta=\theta_0$. At $\theta=\theta_0$ we have the desired result $\psi(\theta_0)=\frac{\theta_0}{1+\sqrt{1-\theta_0{}^2}}<\theta_0$. From these calculations we conclude that SE is below BE.

Assume that $\frac{1}{\theta_0}<\theta$. As $f(0)<g(0)$ and there is no solution to $\theta^2=4x(1-x)$ we conclude that g is above f for all $x\in[0,1]$ which means that BE is below SE. ■

Proof of Lemma 5.

Assume that $\frac{1}{\theta_0} > \theta > \theta_0$. At the point of intersection of BE and SE

$\alpha = \frac{1-x}{y\theta} = \beta = \frac{1-y}{x}\theta = \theta_0$. We evaluate the derivatives of BE and SE at this point.

Note that $A = A'$, $B = B'$, $C = C'$, and $D = D'$. We claim that (A0) < (A1) which means that BE cuts SE from above. This condition simplifies to

$$[(1-x)(1-y) - xy]\left(A^2 - C^2 + \delta^2 D^2 - \delta^2 B^2\right) < 0 \tag{A8}$$

The first term in (A8) is negative, $(1-x)(1-y) - xy = -\frac{(1-\theta\theta_0)(\theta-\theta_0)}{\theta(1-\theta_0^2)} < 0$. At the intersection point $A = (1+\theta_0)\left[2 - \delta e^{-\theta_0}(1+\theta_0)\right]e^{-\theta_0} > C = \left[2 - \delta e^{-\theta_0}(1+\theta_0)\right]e^{-\theta_0}$ and $D = \theta_0 e^{-2\theta_0} = B = \theta_0\left[2 - e^{-\theta_0}(2+\theta_0)\right]e^{-\theta_0}$. Thus, (A8) holds and BE cuts SE from above.■

Proof of Lemma 6.

Since V_b, W_b, V_s, and W_s are continuous it is sufficient to study their value at one point above and below them. At point $(x,y) = (1,1,)$ $V_b = 0 < W_b = \frac{\delta}{2-\delta}$ and $V_s = 0 < W_s = \frac{\delta}{2-\delta}$. Thus above BE and SE both *y* and *x* decrease. Similarly evaluating the value functions at $(x,y) = (0,0,)$ one can conclude that below BE and SE both *y* and *x* increase.■

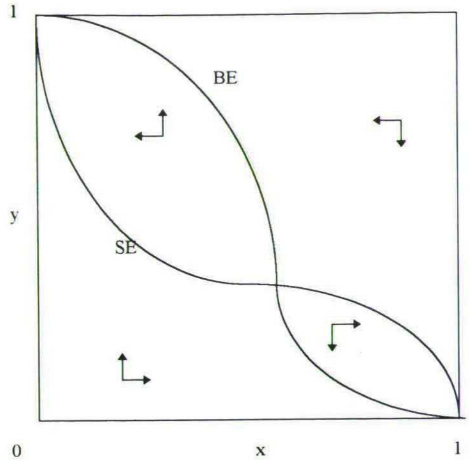

Figure 1. $\frac{1}{\theta_0} > \theta > \theta_0$

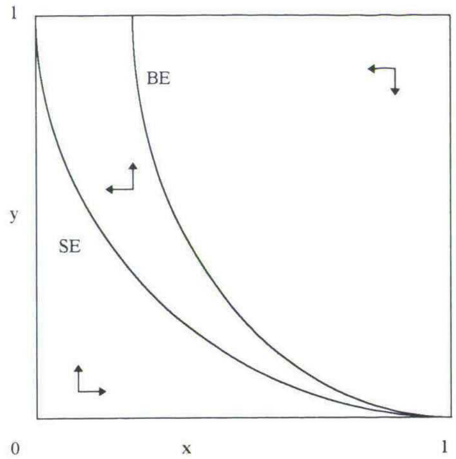

Figure 2. $\theta < \theta_0$ and $\theta < \dfrac{1}{\theta_1}$

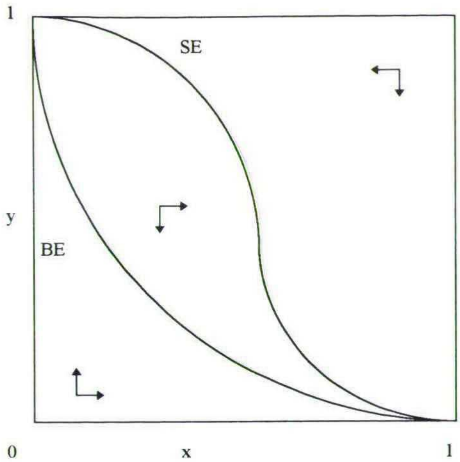

Figure 3. $\frac{1}{\theta_0} < \theta$ and $\theta < \theta_1$

No.	Author(s)	Title
9661	U. Gneezy and J. Potters	An Experiment on Risk Taking and Evaluation Periods
9662	H.J. Bierens	Nonparametric Nonlinear Co-Trending Analysis, with an Application to Interest and Inflation in the U.S.
9663	J.P.C. Blanc	Optimization of Periodic Polling Systems with Non-Preemptive, Time-Limited Service
9664	M.J. Lee	A Root-N Consistent Semiparametric Estimator for Fixed Effect Binary Response Panel Data
9665	C. Fernández, J. Osiewalski and M.F.J. Steel	Robust Bayesian Inference on Scale Parameters
9666	X. Han and H. Webers	A Comment on Shaked and Sutton's Model of Vertical Product Differentiation
9667	R. Kollmann	The Exchange Rate in a Dynamic-Optimizing Current Account Model with Nominal Rigidities: A Quantitative Investigation
9668	R.C.H. Cheng and J.P.C. Kleijnen	Improved Design of Queueing Simulation Experiments with Highly Heteroscedastic Responses
9669	E. van Heck and P.M.A. Ribbers	Economic Effects of Electronic Markets
9670	F.Y. Kumah	The Effect of Monetary Policy on Exchange Rates: How to Solve the Puzzles
9671	J. Jansen	On the First Entrance Time Distribution of the $M/D/\infty$ Queue: a Combinatorial Approach
9672	Y.H. Farzin, K.J.M. Huisman and P.M. Kort	Optimal Timing of Technology Adoption
9673	J.R. Magnus and F.J.G.M. Klaassen	Testing Some Common Tennis Hypotheses: Four Years at Wimbledon
9674	J. Fidrmuc	Political Sustainability of Economic Reforms: Dynamics and Analysis of Regional Economic Factors
9675	M. Das and A. van Soest	A Panel Data Model for Subjective Information on Household Income Growth
9676	A.M. Lejour and H.A.A. Verbon	Fiscal Policies and Endogenous Growth in Integrated Capital Markets
9677	B. van Aarle and S.-E. Hougaard Jensen	Output Stabilization in EMU: Is There a Case for an EFTS?
9678	Th.E. Nijman, F.A. de Roon and C.Veld	Pricing Term Structure Risk in Futures Markets

No.	Author(s)	Title
9679	M. Dufwenberg and U. Gneezy	Efficiency, Reciprocity, and Expectations in an Experimental Game
9680	P. Bolton and E.-L. von Thadden	Blocks, Liquidity, and Corporate Control
9681	T. ten Raa and P. Mohnen	The Location of Comparative Advantages on the Basis of Fundamentals only
9682	S. Hochguertel and van Soest	The Relation between Financial and Housing Wealth of Dutch A. Households
9683	F.A. de Roon, Th.E. Nijman and B.J.M. Werker	Testing for Spanning with Futures Contracts and Nontraded Assets: A General Approach
9684	F.Y. Kumah	Common Stochastic Trends in the Current Account
9685	U.Gneezy and M. Das	Experimental Investigation of Perceived Risk in Finite Random Walk Processes
9686	B. von Stengel, A. van den Elzen and D. Talman	Tracing Equilibria in Extensive Games by Complementary Pivoting
9687	S.Tijs and M. Koster	General Aggregation of Demand and Cost Sharing Methods
9688	S.C.W. Eijffinger, H.P. Huizinga and J.J.G. Lemmen	Short-Term and Long-Term Government Debt and Nonresident Interest Withholding Taxes
9689	T. ten Raa and E.N. Wolff	Outsourcing of Services and the Productivity Recovery in U.S. Manufacturing in the 1980s
9690	J. Suijs	A Nucleolus for Stochastic Cooperative Games
9691	C. Seidl and S.Traub	Rational Choice and the Relevance of Irrelevant Alternatives
9692	C. Seidl and S.Traub	Testing Decision Rules for Multiattribute Decision Making
9693	R.M.W.J. Beetsma and H. Jensen	Inflation Targets and Contracts with Uncertain Central Banker Preferences
9694	M. Voorneveld	Equilibria and Approximate Equilibria in Infinite Potential Games
9695	F.B.S.L.P. Janssen and A.G. de Kok	A Two-Supplier Inventory Model
9696	L. Ljungqvist and H. Uhlig	Catching up with the Keynesians
9697	A. Rustichini	Dynamic Programming Solution of Incentive Constrained Problems

No.	Author(s)	Title
9698	G.Gürkan and A.Y. Özge	Sample-Path Optimization of Buffer Allocations in a Tandem Queue - Part I: Theoretical Issues
9699	H. Huizinga	The Dual Role of Money and Optimal Financial Taxes
96100	H. Huizinga	The Taxation Implicit in Two-Tiered Exchange Rate Systems
96101	H. Norde, F. Patrone and S. Tijs	Characterizing Properties of Approximate Solutions for Optimization Problems
96102	M. Berg, A. De Waegenaere and J. Wielhouwer	Optimal Tax Reduction by Depreciation: A Stochastic Model
96103	G. van der Laan, D. Talman and Z. Yang	Existence and Approximation of Robust Stationary Points on Polytopes
96104	H. Huizinga and S.B. Nielsen	The Coordination of Capital Income and Profit Taxation with Cross-Ownership of Firms
96105	H. Degryse	The Total Cost of Trading Belgian Shares: Brussels Versus London
96106	H. Huizinga and S.B. Nielsen	The Political Economy of Capital Income and Profit Taxation in a Small Open Economy
96107	T. Dieckmann	The Evolution of Conventions with Endogenous Interactions
96108	F. de Jong and M.W.M. Donders	Intraday Lead-Lag Relationships Between the Futures-, Options and Stock Market
96109	F. Verboven	Brand Rivalry, Market Segmentation, and the Pricing of Optional Engine Power on Automobiles
96110	D. Granot, H. Hamers and S. Tijs	Weakly Cyclic Graphs and Delivery Games
96111	P. Aghion, P. Bolton and S. Fries	Financial Restructuring in Transition Economics
96112	A. De Waegenaere, R. Kast and A. Lapied	Non-linear Asset Valuation on Markets with Frictions
96113	R. van den Brink and P.H.M. Ruys	The Internal Organization of the Firm and its External Environment
96114	F. Palomino	Conflicting Trading Objectives and Market Efficiency
96115	E. van Damme and S. Hurkens	Endogenous Stackelberg Leadership
96116	E. Canton	Business Cycles in a Two-Sector Model of Endogenous Growth
9701	J.P.J.F. Scheepens	Collusion and Hierarchy in Banking

No.	Author(s)	Title
9702	H.G. Bloemen and E.G.F. Stancanelli	Individual Wealth, Reservation Wages and Transitions into Employment
9703	P.J.J. Herings and V.J. Vannetelbosch	Refinements of Rationalizability for Normal-Form Games
9704	F. de Jong, F.C. Drost and B.J.M. Werker	Exchange Rate Target Zones: A New Approach
9705	C. Fernández and M.F.J. Steel	On the Dangers of Modelling Through Continuous Distributions: A Bayesian Perspective
9706	M.A. Odijk, P.J. Zwaneveld, J.S. Hooghiemstra, L.G. Kroon and M. Salomon	Decision Support Systems Help Railned to Search for 'Win-Win' Solutions in Railway Network Design
9707	G. Bekaert, R.J. Hodrick and D.A. Marshall	The Implications of First-Order Risk Aversion for Asset Market Risk Premiums
9708	C. Fernández and M.F.J. Steel	Multivariate Student-t Regression Models: Pitfalls and Inference
9709	H. Huizinga and S.B. Nielsen	Privatization, Public Investment, and Capital Income Taxation
9710	S. Eijffinger, E. Schaling and M. Hoeberichts	Central Bank Independence: a Sensitivity Analysis
9711	H. Uhlig	Capital Income Taxation and the Sustainability of Permanent Primary Deficits
9712	M. Dufwenberg and W. Güth	Indirect Evolution Versus Strategic Delegation: A Comparison of Two Approaches to Explaining Economic Institutions
9713	H. Uhlig	Long Term Debt and the Political Support for a Monetary Union
9714	E. Charlier, B. Melenberg and A. van Soest	An Analysis of Housing Expenditure Using Semiparametric Models and Panel Data
9715	E. Charlier, B. Melenberg and A. van Soest	An Analysis of Housing Expenditure Using Semiparametric Cross-Section Models
9716	J.P. Choi and S.-S. Yi	Vertical Foreclosure with the Choice of Input Specifications
9717	J.P. Choi	Patent Litigation as an Information Transmission Mechanism
9718	H.Degryse and A. Irmen	Attribute Dependence and the Provision of Quality
9719	A. Possajennikov	An Analysis of a Simple Reinforcing Dynamics: Learning to Play an "Egalitarian" Equilibrium
9720	J. Jansen	Regulating Complementary Input Supply: Cost Correlation and Limited Liability
9721	J. ter Horst and M. Verbeek	Estimating Short-Run Persistence in Mutual Fund Performance

No.	Author(s)	Title
9722	G. Bekaert and S.F. Gray	Target Zones and Exchange Rates: An Empirical Investigation
9723	M. Slikker and A. van den Nouweland	A One-Stage Model of Link Formation and Payoff Division
9724	T. ten Raa	Club Efficiency and Lindahl Equilibrium
9725	R. Euwals, B. Melenberg and A. van Soest	Testing the Predictive Value of Subjective Labour Supply Data
9726	C. Fershtman and U. Gneezy	Strategic Delegation: An Experiment
9727	J. Potters, R. Sloof and F. van Winden	Campaign Expenditures, Contributions and Direct Endorsements: The Strategic Use of Information and Money to Influence Voter Behavior
9728	F.H. Page, Jr.	Existence of Optimal Auctions in General Environments
9729	M. Berliant and F.H. Page, Jr.	Optimal Budget Balancing Income Tax Mechanisms and the Provision of Public Goods
9730	S.C.W. Eijffinger and Willem H. Verhagen	The Advantage of Hiding Both Hands: Foreign Exchange Intervention, Ambiguity and Private Information
9731	A. Ridder, E. van der Laan and M. Salomon	How Larger Demand Variability may Lead to Lower Costs in the Newsvendor Problem
9732	K. Kultti	A Model of Random Matching and Price Formation