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Bester, H.

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by Helmut Bester

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Helmut Bester*

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**Abstract**

*This paper investigates the role of price advertising in a market where consumers are imperfectly informed about prices. We consider a monopolist whose demand depends on price and advertising expenditure. This demand function is derived from optimizing behaviour of consumers. Uninformed consumers may pay a cost to visit the seller and obtain price information. Advertising enables the monopolist to increase the number of informed consumers. In equilibrium the uninformed consumers form rational price expectations and the seller necessarily adopts a random pricing and advertising strategy. We show that the feasibility of costly advertising generates a Pareto welfare improvement even though the equilibrium advertising level is less than socially efficient. Finally, we derive some comparative statics results.*

*Keywords:* Advertising, Price Dispersion, Sales. *JEL Classification No.:* D42, D83.

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*CentER, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, The Netherlands. I wish to thank Eric van Damme and Dale O. Stahl for helpful comments.

# 1 Introduction

The goal of this paper is to investigate the role of price advertising in markets where buyers have imperfect information about prices. We consider a monopolistic seller whose demand is a function of price and advertising. This demand function is derived from optimizing behaviour on the part of consumers. Some consumers cannot directly observe the seller's price quotation. In principle, there are two ways by which these consumers may obtain price information. First, consumers may find out the price by visiting the seller. Second, the seller may increase the fraction of informed consumers through price advertising. In either case there is a cost to improving information. Going to the store is costly for the buyer, and the seller has to spend resources on advertising. We study the monopolist's pricing and advertising strategy in such a market when the uninformed consumer's have rational expectations about prices.

In their seminal paper on advertising, Dorfman and Steiner (1954) postulate a demand function that depends on price and advertising expenditures. The optimal price-advertising combination is then obtained from the first-order conditions for profit maximization. This approach does not explicitly take account of rational consumer decisions. When consumers do not enjoy advertising per se, it remains unclear why demand is affected by advertising. In particular, this feature makes their model inappropriate for welfare analysis. The assumption of imperfect price information allows us to overcome these difficulties by explicitly modelling the impact of advertising on demand. It turns out that this reformulation of the Dorfman-Steiner model generates some features that are not present in the 'reduced-form' approach. For instance, the seller's optimal decisions cannot simply be deduced from the first-order conditions for profit maximization. The reason is that profits are not jointly concave in price and advertising expenditure. This fact underlies an important property of the market outcome, namely that the seller's strategy must involve randomization. In equilibrium the monopolist stochastically chooses between high and low prices. Advertising expenditures are higher, the lower the selected price is. The role of this random behaviour is to prevent the uninformed consumers from precisely predicting the actual price in a rational expectations equilibrium. Indeed, the seller would have no incentive to advertise if the uninformed consumers were able to exactly forecast the equilibrium price.

Our equilibrium resembles Varian's (1980) model of sales in that prices are chosen stochastically. This may be interpreted as the seller having randomly chosen sales. As in our model, imperfect price information of consumers is important to generate Varian's sales equilibria. Yet, in his model the strategic interaction between oligopolistic sellers is the source of price randomization. In contrast, in our model of a monopolistic market the feasibility of advertising is ultimately responsible for price dispersion. In the absence of an advertising technology the monopolist would always charge a fixed price.

The literature distinguishes between 'persuasive' and 'informative' advertising. Advertising is said to be persuasive when it directly influences consumer preferences. Dixit and Norman (1987) study a model of this sort and conclude that from a welfare viewpoint it leads to excessive investment in advertising. Informative advertising conveys information about existence of products, prices, location of stores, and so on. Butters (1977), Grossman and Shapiro (1984), and our model fall into this category. Butters (1977) and Grossman and Shapiro (1984) assume that advertising provides information about the price of a product *and* its existence. Consumers cannot purchase a good without receiving an ad. In contrast, we assume that all consumers are well informed about the existence and the characteristics of the product; only information about its price is imperfect. This seems to be relevant for many markets as a high proportion of newspaper advertising is directed at informing potential customers about sales of food, clothing, and appliances. In our model the uninformed consumer's decision to visit the seller depends on his price expectation. The interaction between expectation formation and the seller's strategy plays a central role in the analysis of equilibrium.

Oligopolistic price advertising in a sequential search model is studied by Robert and Stahl (1991). Their paper is closely related to ours in that it focuses on pure price advertising. Firms adopt a random pricing and advertising strategy and the consumer either purchases the good after observing an ad or he follows a search strategy that is characterized by some reservation price. As in Varian (1980), oligopolistic competition generates a discontinuity in the firm's profit function so that the equilibrium is in mixed strategies. In the case of a single seller their model would not exhibit price dispersion. Indeed, all consumers have identical search costs in the Robert-Stahl model and by advertising each firm seeks to undercut the price offers of its rivals. In contrast, in our model consumers differ in their visiting cost and the goal of monopolistic advertising is to induce the high cost consumers to visit the store. The observation that differences in consumer information costs may lead to monopolistic price dispersion is similar to Salop (1977). In his model the seller operates a number of retail outlets in the market and charges different prices at different locations. In this way the market is split up into submarkets and less efficient searchers end up paying a higher purchase price. We assume that such a device for sorting consumers is not feasible since the seller offers the good at a single location.

The following Section describes the model and defines the rational expectations equilibrium. The existence of equilibrium and its basic properties are investigated in Section 3. Section 4 studies the welfare implications of advertising. Here we show that the existence of an advertising technology leads to a Pareto improvement; yet, from a second-best viewpoint the equilibrium advertising level is less than socially efficient. Comparative statics results are contained in Section 5. Section 6 concludes.

## 2 The Model

We consider a monopolistic seller who produces a single homogeneous good. The cost of producing one unit of the good is constant and normalized to zero. There is a continuum of consumers each of whom will buy at most one unit of the good. Without loss of generality we assume that the measure of consumers is normalized to unity. All consumers have an identical valuation $r > 0$ for the good. Visiting the seller is costly; before making a purchase consumer $d \in [0,1]$ has to pay a cost $dt$. It is assumed that $d$ is uniformly distributed on $[0,1]$ across the population of consumers. This assumption generates a simple demand structure and allows us to explicitly compute the impact of advertising on demand. One possible interpretation is that the monopolist is located at the center of some geographical market area and that $dt$ represents the transportation cost of a consumer who has to travel the distance $d$ to buy the good. Varying $t$ enables us to study how a change in the consumers' visiting cost affects the seller's pricing and advertising behavior.

There are two groups of consumers, informed and uninformed. The informed consumer observes the price $p$ quoted by the monopolist. Hence, the informed consumer $d$ will go to the store if $r - p \geq dt$. The uninformed consumer bases his decision on his price expectation $p_e$. He visits the seller if $r - p_e \geq dt$ and learns the actual price $p$ upon entering the store. At this stage the cost $dt$ is sunk so that he will buy the good as long as $p \leq r$.

A priori a fraction $0 < \gamma < 1$ of consumers is always informed about the price charged by the monopolist. Independently of his type $d$, each consumer is equally likely to be perfectly informed. The seller can increase the number of informed consumers through advertising activities. Suppose the price advertisement reaches each individual consumer with the same probability $\lambda$. Then the fraction of uninformed consumers is reduced to $(1 - \lambda)(1 - \gamma)$ and the fraction $\gamma(1 - \lambda) + \lambda$ becomes informed. We assume that the price advertisement is legally binding. The monopolist cannot induce consumers to pay the visiting cost $dt$ by advertising a low price and then upon their arrival demand a high price.

We consider only parameter constellations such that $r \leq 2t$. This rules out that the monopolist will serve the entire market. Abstracting from such boundary cases helps to simplify the analysis. For any price $p \in [0, r]$, the monopolist's demand depends on the advertising intensity $\lambda$ and the uniformed consumers' expectations $p_e \in [0, r]$ and is given by $[(\gamma(1 - \lambda) + \lambda)(r - p) + (1 - \gamma)(1 - \lambda)(r - p_e)]/t$. Accordingly, his sales revenue equals

$$p[r - (\gamma(1 - \lambda) + \lambda)p - (1 - \gamma)(1 - \lambda)p_e]/t. \tag{1}$$

The seller takes the expectation $p_e$ as fixed. Of course, in equilibrium we will require expectations to be rational which means that $p_e$ has to be consistent with the pricing and advertising strategy actually chosen by the seller. It is important to notice that the revenue function is concave in $p$ as well as in $\lambda$ but not jointly concave in $(p, \lambda)$.

Therefore, even when the costs of advertising are represented by some convex and increasing function of $\lambda$, the monopolist's overall problem is not concave and might have multiple solutions. In fact, it will turn out that randomization by the seller constitutes an important characteristic of the market equilibrium.

We proceed by first deriving the optimal pricing decision for a given advertising strategy. Maximizing (1) with respect to $p \in [0, r]$ leads to the solution

$$p^*(\lambda, p_e) = \min[\frac{r - (1-\gamma)(1-\lambda)p_e}{2(\gamma(1-\lambda)+\lambda)},\ r]. \tag{2}$$

For $p_e < r$ and $(1-\gamma)(1-\lambda)$ sufficiently large it becomes optimal for the seller to exploit the uninformed consumers by charging the reservation price $r$. This strategy is no longer optimal when the fraction of informed consumers is large enough. Moreover, in this case $p^*(\lambda, p_e)$ is strictly decreasing in $\lambda$. Thus choosing a high advertising intensity $\lambda$ can be interpreted as the monopolist having a sale.

Substituting (2) into (1) gives the revenue function for given values of $\lambda$ and $p_e$ :

$$\Pi(\lambda, p_e) = \frac{[r-(1-\lambda)(1-\gamma)p_e]^2}{4(\gamma(1-\lambda)+\lambda)t} \qquad \text{if } (1-\gamma)(1-\lambda) \leq r/(2r-p_e), \tag{3}$$

$$\Pi(\lambda, p_e) = [r(1-\gamma)(1-\lambda)(r-p_e)]/t \quad \text{if } (1-\gamma)(1-\lambda) \geq r/(2r-p_e).$$

Determining the optimal advertising intensity $\lambda$ is complicated by the fact that $\Pi(\lambda, p_e)$ is not concave in $\lambda$. To cope with this problem we assume that there is some finite number $n$ of advertising intensities among which the seller can choose. They are denoted by $\lambda_i, i = 0, ..., n-1$, with $\lambda_0 < \lambda_1 < ...\lambda_{n-1} < 1$. The seller must pay the cost $k_i$ when he selects advertising intensity $\lambda_i$. We denote by $k = (k_0, k_1, ..., k_{n-1})$ the vector of advertising costs and assume $k_0 < k_1 < ...k_{n-1}$. Also, we set $\lambda_0 = k_0 \equiv 0$ which gives the monopolist the freedom not to advertise his price. The assumption of a finite set of feasible advertising levels enables us to apply a standard fixed point argument to prove existence of equilibrium; the case of a continuous variable $\lambda$ may be approximated by the limit $n \to \infty$. We allow the seller to adopt a random advertising strategy represented by $q \in Q \equiv \{q \in \mathbf{R}^n | \sum_i q_i = 1\}$ such that $q_i$ is the probability of selecting $\lambda_i$. Now we are ready to define a market equilibrium with rational consumer expectations.

**Definition:** $(q^*, p_e^*)$ is an *equilibrium* if
(i) $q^* \in \text{argmax}_{q \in Q} \Sigma_i q_i[\Pi(\lambda_i, p_e^*) - k_i]$; and
(ii) $p_e^* = [\Sigma_i q_i^*(1-\lambda_i)p^*(\lambda_i, p_e^*)]/[\Sigma_i q_i^*(1-\lambda_i)]$.

Condition (i) states that the seller's choice of advertising intensities is profit maximizing. Indeed, by (i) one has $q_i^* > 0$ only if $\Pi(\lambda_i, p_e^*) - k_i \geq \Pi(\lambda_j, p_e^*) - k_j$ for all $j = 0, ..., n-1$. Condition (ii) requires the uninformed consumers' expectations to be consistent with Bayes' rule. The consumer who has not been reached by an advertisement will use this observation to update his beliefs. Conditional upon not receiving an advertisement, he concludes that the monopolist charges $p^*(\lambda_i, p_e^*)$ with probability $q_i^*(1-\lambda_i)/[\Sigma_j q_j^*(1-\lambda_j)]$.

# 3 Equilibrium Advertising

Before studying the equilibrium defined in the previous section, we disregard for a moment the monopolist's option to advertise. This will give us a reference point for our further analysis. If there is no advertising, the seller will always charge the same price $p^*(\lambda_0, p_e)$ so that rational expectations require $p^*(\lambda_0, p_e^*) = p_e^*$. Solving for $p_e^*$ then yields the equilibrium price

$$\hat{p} = r/(1+\gamma) \tag{4}$$

The higher the fraction of a priori informed consumers, the lower is $\hat{p}$. The intuition is simply that the uninformed consumers' demand is completely inelastic so that demand becomes more elastic when there are more informed consumers. When $\gamma = 0$, one has $\hat{p} = r$ and the market effectively collapses. This is so because the seller would optimally ask $p = r$ from any consumer who arrives at his store. Quoting a lower price cannot increase his demand as all consumers observe $p$ only after entering the store.

With rational expectations the consumers anticipate this pricing behavior and refrain from visiting the seller. Indeed, this type of market break-down is well known from the search literature (see, e.g. Stiglitz (1979)). To avoid the autarky equilibrium some consumers have to be well informed about prices, either exogenously or endogenously through advertising.

We now return to the analysis of the advertising equilibrium by first investigating its existence.

**Proposition 1:** *There is an equilibrium* $(q^*, p_e^*)$. *Moreover, for* $||k||$ *sufficiently small it satisfies* $q_0^* < 1$, *i.e. the seller advertises his price with positive probability.*

**Proof:** Define

$$\varphi_1(p_e) \equiv \text{argmax}_{q \in Q}\ \Sigma_i q_i[\Pi(\lambda_i, p_e) - k_i]; \tag{5}$$

$$\varphi_2(q, p_e) \equiv [\Sigma_i q_i(1-\lambda_i)p^*(\lambda_i, p_e)]/[\Sigma_i q_i(1-\lambda_i)].$$

Clearly, $\varphi_1(.)$ is convex-valued and, by the Maximum Theorem, an u.h.c. correspondence from $P = \{p_e | 0 \leq p_e \leq r\}$ into $Q$. Similarly, $\varphi_2(.)$ is a continuous mapping from $Q \times P$ into $P$ because by (2), $0 \leq p^*(\lambda_i, p_e) \leq r$ for all $p_e \in P$. As a result, $\varphi = \varphi_1 \times \varphi_2$ is a convex-valued and u.h.c. correspondence from $Q \times P$ into itself. Thus, Kakutani's fixed point theorem guarantees the existence of a fixed point $(q^*, p_e^*)$. By definition of $\varphi$ it is easily seen that $(q^*, p_e^*)$ constitutes an equilibrium.

To prove the second part of the Proposition, we will show that $q_0^* = 1$ together with part (ii) of the definition of equilibrium implies $\Pi(\lambda_0, p_e^*) < \Pi(\lambda_i, p_e^*)$ for $i > 0$. Clearly, for $||k||$ sufficiently small this contradicts optimality of $q^* = (1, 0, ..., 0)$ as required by part (i) of the definition.

By (ii), $q_0^* = 1$ and $q_i^* = 0$ for $i > 0$ implies $p_e^* = p^*(\lambda_0, p_e^*)$. Therefore one has $p_e^* = \hat{p}$, as given by (4). By (3),

$$\Pi(\lambda,\hat{p}) = \frac{r^2(2\gamma - \lambda\gamma + \lambda)^2}{4(1+\gamma)^2(\gamma(1-\lambda)+\lambda)t}. \tag{6}$$

As $\partial\Pi(\lambda,\hat{p})/\partial\lambda > 0$, this proves $\Pi(\lambda_0,\hat{p}) < \Pi(\lambda_i,\hat{p})$ for $i > 0$. Q.E.D.

Proposition 1 shows that the monopolist has an incentive to advertise if the resulting cost is not too high. The intuition is that without advertising all consumers would expect the seller to charge $\hat{p}$. Given these expectations the seller can increase his sales revenue by reducing the fraction of uninformed consumers and advertising a lower price. This increases his demand because those consumers who observe the advertisement realize that their expectations have been too pessimistic. Of course, with rational expectations this cannot always be the case, which explains the following result.

**Proposition 2:** *Any equilibrium $(q^*, p_e^*)$ satisfies $q_0^* > 0$, i.e. with positive probability the seller does not advertise his price.*

**Proof:** As $\gamma > 0$ the seller can certainly make positive profits. This implies $p_e^* < r$ in any equilibrium. Define $\lambda^*(p_e^*)$ by $p^*(\lambda^*, p_e^*) = p_e^*$. Then $p^*(\lambda, p_e^*) < p_e^*$ if and only if $\lambda > \lambda^*(p_e^*)$ because $p^*(\lambda, p_e^*)$ is strictly decreasing in $\lambda$ whenever $p^*(\lambda, p_e^*) < r$. Using (1) and the Envelope Theorem one has $\partial\Pi(\lambda, p_e^*)/\partial\lambda > 0$ if and only if $p^*(\lambda, p_e^*) < p_e^*$. This means $\Pi(\lambda, p_e^*)$ is decreasing in $\lambda$ for $\lambda < \lambda^*(p_e^*)$ and strictly increasing only if $\lambda > \lambda^*(p_e^*)$. Accordingly $\Pi(\lambda_i, p_e^*) > \Pi(\lambda_0, p_e^*)$ implies $\lambda_i > \lambda^*(p_e^*)$ and, therefore, $p^*(\lambda_i, p_e^*) < p_e^*$.

Suppose there is an equilibrium with $q_0^* = 0$ so that $\Sigma_{i>0}\, q_i^* = 1$. Because $k_i > 0$, equilibrium condition (i) implies $\Pi(\lambda_i, p_e^*) > \Pi(\lambda_0, p_e^*)$ for all $i$ such that $q_i^* > 0$. By the above argument this implies $p^*(\lambda_i, p_e^*) < p_e^*$ for all $i$ such that $q_i^* > 0$. As $q_0^* = 0$, we obtain $[\Sigma_i q_i^*(1-\lambda_i)p^*(\lambda_i, p_e^*)]/[\Sigma_i q_i^*(1-\lambda_i)] < p_e^*$, a contradiction to equilibrium condition (ii). This proves $q_0^* > 0$ in any equilibrium. Q.E.D.

The monopolist engages in advertising following a random strategy. Occasionally he announces a 'special offer', but with some chance he provides no price information and charges a high price. By randomly having a sale he keeps the uninformed consumers uncertain about his price. It is easy to see why this must be the case in a rational expectations equilibrium. In the absence of price randomness the uninformed consumers would precisely forecast the equilibrium price. But then spending money on advertising can no longer be profitable for the seller. This argument is similar to the observation of Grossman and Stiglitz (1980) that the equilibrium price in a capital market with rational expectations cannot fully reveal the informed traders' private knowledge. Indeed, if this were the case then no agent would have an incentive to spend resources on gathering information. Therefore, there must be some exogenous source of noise which prevents the equilibrium from being fully revealing. In our model the monopolist endogenously generates such noise to maintain a role for advertising.

**Proposition 3:** *Let* $(q^*, p_e^*)$ *be an equilibrium with* $q_0^* < 1$. *Then* $p^*(\lambda_0, p_e^*) > p_e^* > p^*(\lambda_i, p_e^*)$ *for all* $i > 0$ *such that* $q_i^* > 0$. *Moreover,* $p^*(\lambda_0, p_e^*) > \hat{p} > p_e^*$.

**Proof:** The first part of the proof of Proposition 2 shows that $q_i^* > 0$ implies $p_e^* > p^*(\lambda_i, p_e^*)$ for all $i > 0$. As $\lambda_i < 1$ for all $i$, equilibrium condition (ii) therefore necessitates $p^*(\lambda_0, p_e^*) > p_e^*$.

As $p^*(\lambda_0, p_e^*) > p_e^*$, (2) and (4) imply $p_e^* < r/(1+\gamma) = \hat{p}$. Because $p^*(\lambda_0, p_e)$ is decreasing in $p_e$, one obtains $p^*(\lambda_0, p_e^*) > p^*(\lambda_0, \hat{p}) = \hat{p}$. Q.E.D.

The seller advertises his price only if he wants to inform consumers that they have to pay less than they expect. This also means that the uninformed consumer who has not observed the advertisement will be positively surprised when he arrives at the store. The opposite is true if the seller happens to charge $p^*(\lambda_0, p_e^*)$. Indeed, in this case some of the uninformed consumers will ex post regret visiting the seller. Those consumers for whom $r - p_e^* - dt > 0 > r - p^*(\lambda_0, p_0^*) - dt$ will find that they would have been better off by not going to the seller. Yet, they realize this only after the cost $dt$ is sunk.

As $p_e^* < \hat{p}$ the uninformed consumers' price expectation is more optimistic than in the reference equilibrium without advertising. Thus even when the seller selects advertising intensity $\lambda_0 = 0$, he faces a higher demand than in the equilibrium $\hat{p}$. Therefore, he optimally quotes a price $p^*(\lambda_0, p_e^*) > \hat{p}$. Altogether Proposition 3 shows that advertising has a dual impact on the seller's pricing behavior. Sometimes he will post a higher price and sometimes a lower price than $\hat{p}$.

# 4 Welfare

This Section investigates the welfare implications of advertising. We compare the equilibrium outcome with the hypothetical situation where the monopolist chooses the price $\hat{p}$ because he is unable to communicate price information. It will turn out that advertising generates a Pareto improvement; all market participants are better off when the seller follows the random strategy described in the foregoing Section. We begin by considering the seller. As our comparison is concerned with equilibrium payoffs, one cannot conclude that the seller must gain simply because he has the option not to invest in advertising. This argument would neglect the equilibrium interaction between the seller's strategy and the consumers' expectation formation.

**Proposition 4:** *Let* $(q^*, p_e^*)$ *be an equilibrium with* $q_0^* < 1$. *Then* $\Sigma_i q_i^*[\Pi(\lambda_i, p_e^*) - k_i] > \Pi(\lambda_0, \hat{p})$, *i.e. the seller earns higher profits with than without advertising.*

**Proof:** By Proposition 2 one has $q_0^* > 0$ so that $\Sigma_i q_i^*[\Pi(\lambda_i, p_e^*) - k_i] = \Pi(\lambda_0, p_e^*)$. By Proposition 3, $p_e^* < \hat{p}$ whenever $q_0^* < 1$. Therefore, (3) implies $\Pi(\lambda_0, p_e^*) > \Pi(\lambda_0, \hat{p})$.
Q.E.D.

Since $p_e^* < \hat{p}$, the seller attracts more uninformed consumers and so he can guarantee himself higher profits. To investigate consumer welfare we define

$$V_d(p) \equiv \max[r - p - dt,\ 0]. \tag{7}$$

The function $V_d(p)$ represents buyer $d$'s utility when he either observes or expects that the monopolist demands $p$. It will be important that $V_d(.)$ is convex, which means that consumers prefer price riskiness. This is a typical property of indirect utility functions (see, e.g. Waugh (1944)). The expected utility of a consumer $d$ who belongs to the fraction $\gamma$ of perfectly informed consumers equals $\Sigma_i q_i^* V_d(p^*(\lambda_i, p_e^*))$. A buyer in the group of initially uninformed consumers observes the seller's price advertisement $p^*(\lambda_i, p_e^*)$ with probability $\lambda_i$; with probability $(1-\lambda_i)$ he remains uninformed and expects $p_e^*$. Therefore, his expected equilibrium utility is $\Sigma_i q_i^*[(1-\lambda_i)V_d(p_e^*) + \lambda_i V_d(p^*(\lambda_i, p_e^*))]$.

**Proposition 5:** *Let $(q^*, p_e^*)$ be an equilibrium with $q_0^* < 1$. Then, $\Sigma_i q_i^* V_d(p^*(\lambda_i, p_e^*)) \geq V_d(\hat{p})$ and $\Sigma_i q_i^*[(1-\lambda_i)V_d(p_e^*) + \lambda_i V_d(p^*(\lambda_i, p_e^*))] \geq V_d(\hat{p})$ for all $d$ and the inequality holds in both cases for $d$ sufficiently small. That is, the expected utility of all consumers is higher with than without advertising.*

**Proof:** First we will show that $p_e^* \geq \Sigma_i q_i^* p^*(\lambda_i, p_e^*)$. Define the probability vectors $z$ and $z'$ by

$$z_i = q_{n-1-i}^*,\ z_i' = q_{n-1-i}^*(1-\lambda_{n-1-i})/\Sigma_j q_j^*(1-\lambda_j), \tag{8}$$

where $i = 0, ..., n-1$. We show that $z'$ first-order dominates $z$. We proceed by induction. Clearly $z_0 = q_{n-1}^* \geq q_{n-1}^*(1-\lambda_{n-1})/\Sigma_i q_i^*(1-\lambda_i) = z_0'$ because $1-\lambda_i \geq 1-\lambda_{n-1}$ for all $i$. It remains to show that $\Sigma_i^{k-1} z_i \geq \Sigma_i^{k-1} z_i'$ implies $\Sigma_i^k z_i \geq \Sigma_i^k z_i'$ for all $k < n-1$. Suppose, for some $k$, $\Sigma_i^k z_i < \Sigma_i^k z_i'$, i.e.

$$1/\Sigma_j q_j^*(1-\lambda_j) > \Sigma_i^k z_i / \Sigma_i^k q_{n-1-i}^*(1-\lambda_{n-1-i}). \tag{9}$$

As $\Sigma_i^{k-1} z_i \geq \Sigma_i^{k-1} z_i'$, (9) implies

$$(\Sigma_i^{k-1} z_i)(\Sigma_i^k q_{n-1-i}^*(1 - \lambda_{n-1-i})) > (\Sigma_i^k z_i)(\Sigma_i^{k-1} q_{n-1-i}^*(1 - \lambda_{n-1-i})). \tag{10}$$

Simplifying (10) yields

$$\Sigma_i^{k-1} q_{n-1-i}^*(1 - \lambda_{n-1-k}) > \Sigma_i^{k-1} q_{n-1-i}^*(1 - \lambda_{n-1-i}),$$

a contradiction to $(1-\lambda_{n-1-k}) < (1-\lambda_{n-1-i})$ for all $i < k$. Now, by first order dominance and equilibrium condition (ii) one obtains

$$p_e^* = \Sigma_i z_i' p^*(\lambda_{n-1-i}, p_e^*) \geq \Sigma_i z_i p^*(\lambda_{n-1-i}, p_e^*) = \Sigma_i q_i^* p^*(\lambda_i, p_e^*), \tag{11}$$

because $p^*(\lambda_{n-1-i}, p_e^*) \leq p^*(\lambda_{n-1-j}, p_e^*)$ for $j \geq i$.

By convexity of $V_d(.)$ and our above result we get

$$\Sigma_i q_i^* V_d(p^*(\lambda_i, p_e^*)) \geq V_d(\Sigma_i q_i^* p^*(\lambda_i, p_e^*)) \geq V_d(p_e^*). \tag{12}$$

As $p_e^* < \hat{p}$ by Proposition 3, one has $V_d(p_e^*) \geq V_d(\hat{p})$ for all $d$ with strict inequality for $d$ small enough. This together with (12) proves the first claim of Proposition 5. The second claim follows directly from Proposition 3 since $\lambda_0 = 0$ and $p^*(\lambda_i, p_e^*) < p_e^* < \hat{p}$ for all $i > 0$ such that $q_i^* > 0$. Q.E.D.

Propositions 4 and 5 establish that the feasibility of price communication makes all agents better off. The source of this welfare gain is the surplus generated by the increase in the monopolist's production. Nonetheless, equilibrium output is still below the social optimum, which would be attained by setting $p = 0$ and not spending resources on advertising. Of course, this first-best outcome is incompatible with monopolistic pricing. But even when the pricing rule (2) is taken as fixed, the equilibrium advertising level fails to yield a second-best or constrained welfare optimum. The monopolist selects $\lambda$ to maximize his profit ignoring the impact on consumer welfare. Since $p^*(\lambda, p_e^*)$ decreases with $\lambda$, consumer surplus is the higher, the higher the advertising intensity chosen by the seller. Yet, in any equilibrium with advertising the seller is indifferent between choosing some $\lambda_i > 0$ and $\lambda_0 = 0$ and he sets $q_i^* < 1$ and $q_0^* > 0$. The divergence between the private and social gains leads to underinvestment in advertising. Without directly interfering in the seller's pricing decisions, welfare may be increased by a subsidy on advertising expenditures.

# 5 Comparative Statics

This Section provides some comparative statics insights into the nature of the equilibrium. We consider exogenous changes in the seller's advertising cost $k$, the visiting cost parameter $t$, and the number of fully informed consumers $\gamma$.

**Proposition 6:** *Let $(q^*, p_e^*)$ be an equilibrium corresponding to $k = k'$ and let $(q^{**}, p_e^{**})$ be an equilibrium corresponding $k = k''$. If $k_i'' > k_i'$ for all $i > 0$ and $q_0^* < 1$, then $p_e^{**} > p_e^*$, i.e. an increase in advertising costs increases the uninformed consumers' price expectation.*

**Proof:** If $q_0^{**} = 1$, then $p_e^{**} = \hat{p}$ so that $p_e^* < \hat{p} = p_e^{**}$ by Proposition 3. If $q_0^{**} < 1$, there is a $j > 0$ such that $q_j^{**} > 0$. Also, Proposition 2 shows that $q_0^{**} > 0$ and $q_0^* > 0$. Therefore, equilibrium condition (i) implies that for some $j > 0$

$$\Pi(\lambda_j, p_e^*) - \Pi(\lambda_0, p_e^*) \leq k_j', \quad \Pi(\lambda_j, p_e^{**}) - \Pi(\lambda_0, p_e^{**}) = k_j''. \tag{13}$$

As $k_j'' > k_j'$, one obtains $\psi(p_e^{**}) > \psi(p_e^*)$, where $\psi(p_e) \equiv \Pi(\lambda_j, p_e) - \Pi(\lambda_0, p_e)$. To prove the Proposition, we will show that $\psi'(p_e) > 0$. Using (1) and the Envelope Theorem, one obtains $\partial\Pi(\lambda, p_e)/\partial p_e = -(1-\gamma)(1-\lambda)p^*(\lambda, p_e)/t$. As $\lambda_0 = 0$, this implies

$$\psi'(p_e) = (1-\gamma)[p^*(\lambda_0, p_e) - (1-\lambda_j)p^*(\lambda_j, p_e)]/t > 0 \tag{14}$$

because $p^*(\lambda_0, p_e) > p^*(\lambda_j, p_e)$ and $0 < \lambda_j < 1$. Q.E.D.

With higher advertising costs the uniformed consumers' equilibrium expectations become more pessimistic. They will be convinced that the seller is less likely to hold a sale. Indeed, it is easy to see that in a probabilistic sense the equilibrium $(q^{**}, p_e^{**})$ involves less advertising than $(q^*, p_e^*)$. By (3), $p^*(\lambda_i, p_e^{**}) < p^*(\lambda_i, p_e^*)$ since $p_e^{**} > p_e^*$. That is, the higher the advertising cost the lower is the price charged by the seller for any given advertising intensity. If nonetheless the uninformed consumer expects to pay a higher price, it must be the case that the seller puts more weight on lower advertising intensities, which are associated with higher prices. In the special case $n = 2$, where the seller can choose only not to advertise or to advertise with intensity $\lambda_1$, this argument directly shows that $q_0^{**} > q_0^*$; i.e. higher information costs make the monopolist more likely to refrain from advertising.

**Proposition 7:** *Let $(q^*, p_e^*)$ be an equilibrium corresponding to $t = t'$ and let $(q^{**}, p_e^{**})$ be an equilibrium corresponding to $t = t''$. If $t'' > t'$ and $q_0^* < 1$, then $p_e^{**} > p_e^*$, i.e. an increase in the consumers' visiting cost increases the uninformed consumers' price expectation.*

**Proof:** Let $\Pi'(\lambda, p_e)$ and $\Pi''(\lambda, p_e)$ denote the seller's sales revenue corresponding to $t'$ and $t''$, respectively. As $\Pi''(\lambda, p_e) = \Pi'(\lambda, p_e)t'/t''$, the increase in $t$ changes the seller's payoff from $\Pi'(\lambda_i, p_e) - k_i$ to $\Pi'(\lambda, p_e)t'/t'' - k_i$, By an affine transformation of payoffs $q_0^{**}$ thus maximizes $\Sigma_i q_i[\Pi'(\lambda_i, p_e) - k_i t''/t']$. Accordingly, an increase in $t$ has the same effect as an increase in $k$ so that Proposition 6 applies. Q.E.D.

As the proof of Proposition 7 reveals, an increase in consumer visiting costs is equivalent to an increase in advertising costs in the following sense: If $(q^{**}, p_e^{**})$ is an equilibrium for given parameter values $(t'', k')$, then it is also an equilibrium under the parameter constellation $(t', k'')$ with $k'' = k't''/t'$. Perhaps surprisingly, the monopolist reacts to an increase in the consumers' cost of obtaining price information by advertising less frequently. But, this is so because his sales revenue is lower the higher is $t$. Accordingly, advertising yields a lower rate of return, $[\Pi(\lambda_i, p_e^*) - \Pi(\lambda_0, p_e^*)]/k_i$, when $t$ is raised.

**Proposition 8:** *Let $(q^*, p_e^*)$ be an equilibrium corresponding to $\gamma = \gamma'$ and let $(q^{**}, p_e^{**})$ be an equilibrium corresponding to $\gamma = \gamma''$. If $\gamma'' > \gamma'$ and $q_0^* < 1$, then $p_e^{**} < p_e^*$, i.e. an increase in the fraction of perfectly informed consumers lowers the uninformed consumers' price expectation.*

**Proof:** Let $\Pi'(\lambda, p_e)$ and $\Pi''(\lambda, p_e)$ denote the seller's sales revenue corresponding to $\gamma'$ and $\gamma''$, respectively. As $0 < q_0^* < 1$, there is a $j > 0$ such that $q_j^* > 0$ and

$$\Pi'(\lambda_j, p_e^*) - \Pi'(\lambda_0, p_e^*) = k_j. \tag{15}$$

By (1) and the Envelope Theorem, $\partial\Pi/\partial\gamma = (1-\lambda)p^*(\lambda, p_e)(p_e - p^*(\lambda, p_e))/t$. By Proposition 3, $p^*(\lambda_0, p_e^*) > p_e^* > p^*(\lambda_j, p_e^*)$. Therefore, $\partial[\Pi(\lambda_j, p_e^*) - \Pi(\lambda_0, p_e^*)]/\partial\gamma > 0$ so that

$$\Pi''(\lambda_j, p_e^*) - \Pi''(\lambda_0, p_e^*) > k_j. \tag{16}$$

Now suppose $p_e^{**} \geq p_e^*$. The proof of Proposition 6 shows that $\Pi(\lambda_j, p_e) - \Pi(\lambda_0, p_e)$ is increasing in $p_e$. Therefore

$$\Pi''(\lambda_j, p_e^{**}) - \Pi''(\lambda_0, p_e^{**}) > k_j, \tag{17}$$

a contradiction to equilibrium condition (i) and the fact that $q_0^{**} > 0$ by Proposition 2.

Q.E.D.

Unlike the two previous results, Proposition 8 fails to have a straightforward implication regarding the change in the seller's advertising strategy $q^*$. The reason is that

**TABLE 1**

*Equilibrium for* $n = 2, r = 2, t = 1, \lambda_1 = 0.75, k_1 = 0.01$

| $\gamma$ | $q_0^*$ | $p_e^*$ | $p^*(\lambda_0, p_e^*)$ | $p^*(\lambda_1, p_e^*)$ |
|---|---|---|---|---|
| 0.1 | 0.1842 | 1.512 | 2 | 1.071 |
| 0.2 | 0.1376 | 1.433 | 2 | 1.071 |
| 0.3 | 0.1412 | 1.345 | 1.764 | 1.069 |
| 0.4 | 0.1663 | 1.276 | 1.543 | 1.064 |
| 0.5 | 0.1982 | 1.221 | 1.389 | 1.056 |
| 0.6 | 0.2539 | 1.177 | 1.274 | 1.046 |
| 0.7 | 0.4106 | 1.144 | 1.183 | 1.035 |
| 0.8 | 1 | 1.111 | 1.111 | - |

the r.h.s. of equilibrium condition (ii) depends negatively on $\gamma$. Therefore, a decrease in $p_e^*$ does not necessarily imply that advertising is intensified and the effect on $q^*$ remains unclear. Indeed, the parameter $\gamma$ has two opposing effects on the profitability of advertising. On the one hand, for a given expectation $p_e$ an increase in $\gamma$ raises the return rate $[\Pi(\lambda_i, p_e) - \Pi(\lambda_0, p_e)]/k_i$. The reason is that the seller's profit from charging a price $p^*(\lambda_i, p_e) < p_e$ is higher if more consumers are informed about this fact. The contrary is the case when he charges $p^*(\lambda_0, p_e) > p_e$. As a result, $[\Pi(\lambda_i, p_e) - \Pi(\lambda_0, p_e)]/k_i$ and $\gamma$ are positively related. On the other hand, Proposition 8 shows that the uninformed consumers' expectation $p_e$ is more optimistic the higher $\gamma$ is. This reduces the impact of advertising on demand and so $[\Pi(\lambda_i, p_e) - \Pi(\lambda_0, p_e)]/k_i$ is decreased.

To illustrate what happens, we resort to a numerical example. We consider the case $n = 2$ with $\lambda_1 = 0.75$. The other parameter values are $r = 2, t = 1$, and $k_1 = 0.01$. Table 1 reports the equilibrium outcome $(q^*, p_e^*)$ and the seller's pricing strategy as a function of $\gamma$. It turns out that in this example the seller sets $p^*(\lambda_0, p_e^*) = r$ if $\gamma < 0.23$. With probability $q_0^*$ he charges the reservation price to exploit the uninformed consumers. Interestingly, we observe that $q_0^*$ decreases with $\gamma$ as long as $p^*(\lambda_0, p_e^*) = r$, i.e. the likelihood of advertising increases when there are more informed consumers. The intuition is that exploiting the uninformed consumers becomes less profitable and so the monopolist advertises $p^*(\lambda_1, p_e^*) < r$ more often. This tendency is reversed when $\gamma$ is large enough and $p^*(\lambda_0, p_e^*) < r$. Then $\gamma$ and $q_0^*$ are positively related. This appears plausible since the provision of price information is of importance only for a fraction $1-\gamma$ of consumers. In fact, for $\gamma > 0.78$ the monopolist sets $q_0^* = 1$ and refrains from advertising.

# 6 Conclusion

This paper studies price advertising in a monopolistic market. Through advertising the monopolist informs those consumers who otherwise would not observe his price. This is profitable for the seller only if he quotes a price below the uninformed consumers' price expectation. Given rational expectations, this cannot always be the case. We demonstrate that this leads to an equilibrium in which the monopolist randomizes over prices and advertising intensities. This kind of price dispersion is a consequence of equilibrium interactions between consumer expectations and the monopolist's strategy. It differs from models of oligopolistic Bertrand competition where discontinuities in the firms' profits are well-known to generate mixed strategy equilibria in the sellers' pricing game. Indeed, one might suspect that a monopoly is less likely to be characterized by price dispersion than an oligopoly. Stigler (1961, p.223) argues that "from the manufacturer's viewpoint, uncertainty concerning his price is clearly disadvantageous, the cost of search is a cost of purchase, and consumption will be smaller the greater the dispersion of prices". In our model this is true only when the monopolist is unable to communicate price information. In this case his profit function is strictly concave in price, which makes randomizing suboptimal. This property of the profit function is destroyed by the option to advertise. As a result, profit maximization leads to stochastic pricing and advertising behaviour.

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