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A DECISION THEORETIC ANALYSIS OF THE UNIT ROOT HYPOTHESIS USING MIXTURES OF ELLIPTICAL MODELS

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ABSTRACT: This paper develops a formal decision theoretic approach to testing for a unit root in economic time series. The approach is empirically implemented by specifying a loss function based on predictive variances; models are chosen so as to minimize expected loss. In addition, the paper broadens the class of likelihood functions traditionally considered in the Bayesian unit root literature by i) allowing for departures from normality via the specification of a likelihood based on general elliptical densities; ii) allowing for structural breaks to occur; iii) allowing for moving average errors; and iv) using mixtures of various submodels to create a very flexible overall likelihood.

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The economic literature devoted to the issue of unit roots in economic time series has grown immensely since the seminal papers of Dickey and Fuller (1979) and Nelson and Plosser (1982). Although the majority of the literature assumes a classical econometric perspective, a growing Bayesian unit root literature has emerged (see DeJong and Whiteman (1991a,b), Phillips (1991), Sims (1988), Koop (1991a,b), Schotman and van Dijk (1991a,b), Wago and Tsurumi (1990), Zivot and Phillips (1991)). In many cases, Bayesian results differ substantially from their classical counterparts.

This paper makes a contribution to this growing body of Bayesian unit root literature. It considers more general classes of models and methods of drawing inferences than presently exist. The paper uses models that are mixtures over various submodels with general elliptical distributions and differ in both their covariance structure and their treatment of structural breaks. The resulting mixed model is very flexible and encompasses a wide variety of dynamic structures. In addition, the paper uses a formal decision theoretic framework based on predictive variances and the conservative notion that it is worse to underestimate than to overestimate predictive variances. This approach accords naturally with a Bayesian paradigm and provides an explicit forum for choosing between stationary, unit root, and explosive models.

Section 1 of the paper introduces our hypothesis of interest and the methodology we use to test it. Section 2 discusses the sampling model, Section 3 the prior density, and Section 4 the posterior density. Section 5 treats the decision problem while Section 6 applies the methods to the extended Nelson-Plosser data set. Section 7 concludes.

Section 1: What Are We Testing?

Our aim is to determine whether a unit root is present, ie. to test an exact restriction. One obvious way is to calculate posterior odds comparing the model with a unit root imposed against the unrestricted model. This method requires that an informative (proper) prior be placed over ρ, the coefficient which equals one under a unit root. Koop (1991a,b) calculates posterior odds using informative natural conjugate priors. Schotman and van Dijk (1991a) use proper priors that require less subjective prior input but at the cost that their priors are data-based.

Rather than test explicitly for a unit root, an alternative methodology (DeJong and Whiteman (1991a,b) and Phillips (1991)) is to calculate the posterior probability that ρ is in some region near one. This method has the advantage that proper priors are no longer necessary and thus the analysis may be made more "objective". The disadvantage is that the definition of ρ as "close to one" is highly subjective. By way of example, consider Phillips (1991) who calculates the probability that $|\rho| \geq .975$ and $|\rho| \geq 1$. The former is highly subjective whereas the latter is suitable for testing for nonstationarities (ie. unit root or explosive behavior) but not for the presence of a unit root *per se*. In this paper, we use proper priors on ρ which allow us to compute posterior odds for the exact unit root null.

Furthermore we use a decision theoretic framework to carry out the unit root tests. The loss function used in the decision analysis is based on predictive behavior which can differ crucially for stationary (H_1: $|\rho|<1$), unit root (H_2: $\rho=1$), and explosive (H_3: $|\rho|>1$) models. Hence our decision problem is set up in terms of these three regions for ρ.

Section 2: The Likelihood Function

Bayesian methods require the specification of a likelihood function. Phillips (1991), for example, bases his likelihood function on the following specification:

$$y_t = \mu+\beta t+\rho y_{t-1}+\sum_{i=1}^{k-1}\phi \Delta y_{t-i}+\varepsilon_t \tag{1}$$

with ε_t i.i.d. $N(0,\tau^{-2})$,

while DeJong and Whiteman (1991a) use a different parameterization. Since analytical results cannot always be obtained for their parameterization and extensive Monte Carlo integration is required, Phillips' parameterization is preferred (see Phillips (1991)).[1]

We expand the class of considered likelihoods in three important directions: a) By relaxing the normality assumption; b) By relaxing the i.i.d. assumption; and c) By allowing for structural breaks (Perron (1989), Banerjee, Lumsdaine and Stock (1990) and Zivot and Phillips (1990)). We let y (where $y=(y_1,...,y_T)'$) have any density within the class of multivariate elliptical densities, thereby covering such densities as the multivariate normal, multivariate-t and Pearson type-II. Moreover, we allow the covariance matrix to take the form $\tau^{-2}V(\eta)$, where $V(\eta)$ is any positive definite symmetric matrix parameterized by a finite vector η. Techniques for handling extensions a) and b) are described in Osiewalski and Steel (1990) and Chib *et al.* (1990).

In this paper, no single model need be selected for final analysis. Several different structural breaks and structures for $V(\eta)$ can be chosen, and a supermodel, which is a finite mixture of the various submodels, used. We allow $V(\eta)$ to have various structures in the ARMA class. The motivation behind the inclusion of a moving average component is discussed in Schwert (1987).

Formalizing the ideas described in the preceding paragraphs, we begin with the model with no structural breaks (M_N). We mix over m different correlation structures so that each individual model is labelled M_{Ni} (i=1,...,m). For each model M_{Ni} we take:

$$\begin{aligned} P(y \mid \theta_N,\tau^2,\eta,y_{(0)},M_{Ni}) = &\mid \tau^{-2}V_{Ni}(\eta)\mid^{-1/2} \\ &g_{Ni}[(y-h_{Ni}(\theta_N))'\tau^2V_{Ni}^{-1}(\eta)(y-h_{Ni}(\theta_N))] \end{aligned} \tag{2}$$

[1] In the empirical section we follow DeJong and Whiteman (1991a,b) and set k=3.

where $g_{Nl}(.)$ is a nonnegative function which satisfies (for all l and T),

$$\int_0^\infty u^{\frac{T}{2}-1} g_{Nl}(u)du = \Gamma(T/2)\pi^{-T/2}. \tag{3}$$

In other words, we assume y has a T-variate elliptical density. Note that (3) is a necessary and sufficient condition for (2) to be a proper density, $y_{(0)}$ is the vector of initial observations $(y_{1-k}, \ldots, y_0)'$, and $h_{Nl}(\theta_N)$ is a vector of length T with property:

$$[h_{Nl}(\theta_N)]_t = \mu + \beta t + \rho y_{t-1} + \sum_{j=1}^{k-1} \phi \Delta y_{t-j}.$$

Since we assume this function to be identical for all covariance structures, we drop the l subscript and write:

$$h_N(\theta_N) = \rho y_{-1} + X_N \alpha_N \tag{4}$$

where

$$\begin{aligned} y_{-1} &= (y_0, y_1, \ldots, y_{T-1})', \\ X_N &= (X_1^N, \ldots, X_T^N)', \\ X_t^N &= (1, t, \Delta y_{t-1}, \ldots, \Delta y_{t-k+1})', \\ \alpha_N &= (\mu, \beta, \phi_1, \ldots, \phi_{k-1})' = (\mu, \beta, \phi')', \end{aligned}$$

and hence $\theta_N = (\rho, \alpha_N')'$.

For future reference we define:

$$d_{Nl}(\theta_N, \eta) = (y - h_N(\theta_N))' V_{Nl}^{-1}(\eta)(y - h_N(\theta_N)) \tag{5}$$

The model without structural breaks (M_N) is then given by the mixture of the probabilities in (2) over the m covariance structures:

$$\begin{aligned} &P(y \mid \theta_N, \tau^2, \eta, \delta, y_{(0)}, M_N) \\ &= \sum_{l=1}^{m} \delta_l \, P(y \mid \theta_N, \tau^2, \eta, y_{(0)}, M_{Nl}), \end{aligned} \tag{6}$$

where $\delta = (\delta_1, \ldots, \delta_m)'$ is a vector of mixing parameters with $\delta_l \geq 0$ and $\Sigma\delta_l = 1$.

We obtain the model with structural breaks (M_S) by mixing over various covariance structures (j=1,...,m) and breakpoints (q=1,...,T-1). Note that we use the same covariance structures as in the previous model. Although not necessary, doing so simplifies the notation such that $V_{Nl} = V_{Sj} = V_l$ for $l=j$. Moreover, we conceptually allow for the structural breaks to occur at any point in our sample. Two types of structural breaks, level breaks and trend breaks, can occur at any time q=1,...,T-1. Perron (1989) argues for the presence of a level break in 1929 and a trend break in 1973 for most U.S. macroeconomic time series. To reduce the burden of computation only the latter two breaks are included in the empirical analysis although the general notation is retained throughout this section.

M_S is a mixture over models with different structural breakpoints and covariance structures (M_{Sjq}). Note that each of these submodels has the likelihood function:

$$P(y \mid \theta_S, \tau^2, \eta, y_{(0)}, M_{Sjq}) = \mid \tau^{-2} V_j(\eta) \mid^{-\frac{1}{2}} \\ g_{Sjq}[(y-h_{Sjq}(\theta_S))' \tau^2 V_j^{-1}(\eta)(y-h_{Sjq}(\theta_S))] \tag{7}$$

where $g_{Sjq}(.)$ satisfies (3) for all j, q and T, and

$$h_{Sjq}(\theta_S) = h_{Sq}(\theta_S) = \rho y_{-1} + X_N \alpha_N + X_{Dq} \alpha_D = \rho y_{-1} + X_{Sq} \alpha_S.$$

We define $X_{Dq} = (X_1^{Dq}, \ldots, X_T^{Dq})'$

and $X_t^{Dq} = (DU(q)_t, DT(q)_t)'$,

where $DU(q)_t = 1$ if $t>q$ and 0 otherwise (level break)

and $DT(q)_t = t-q$ if $t>q$ and 0 otherwise (trend break)

Furthermore $\alpha_D = (d_\mu, d_\beta)'$ and hence

$$X_{Sq} = (X_N \; X_{Dq}) \quad \text{and} \quad \alpha_S = (\alpha_N \; \alpha_D)'.$$

In this setup $\theta_s = (\rho \; \alpha_S')' = (\rho \; \alpha_N' \; \alpha_D')' = (\theta_N' \; \alpha_D')'$ and the structural break models have two parameters more than those lacking structural breaks. In our empirical setup we restrict d_μ to zero for 1973 and take d_β to be zero for 1929, leaving just one parameter in α_D for each of the structural break models. For future reference we define

$$d_{Sjq}(\theta_S, \eta) = (y-h_{Sq}(\theta_S))' \; V_j^{-1}(\eta)(y-h_{Sq}(\theta_S)). \tag{8}$$

The overall model (M_S), mixed over structural breaks and covariance structures, is:

$$P(y \mid \theta_S, \tau^2, \eta, \gamma, \kappa, y_{(0)}, M_S) \\ = \sum_{j=1}^{m} \gamma_j \sum_{q=1}^{T-1} \kappa_q \; P(y \mid \theta_S, \tau^2, \eta, y_{(0)}, M_{Sjq}) \tag{9}$$

where $\gamma = (\gamma_1, \ldots, \gamma_m)'$ and $\kappa = (\kappa_1, \ldots, \kappa_{T-1})'$ are mixing parameters with $\gamma_j, \kappa_q \geq 0 \; \forall$ j, q and $\Sigma \gamma_j = \Sigma \kappa_q = 1$.

Finally, we mix over the no-structural-break and structural-break models to obtain the sampling model

$$P(y \mid \theta_S, \tau^2, \eta, \lambda, \delta, \gamma, \kappa, y_{(0)}) = \lambda \sum_{j=1}^{m} \delta_j \; P(y \mid \theta_N, \tau^2, \eta, y_{(0)}, M_{Nj}) \\ + (1-\lambda) \sum_{j=1}^{m} \gamma_j \sum_{q=1}^{T-1} \kappa_q \; P(y \mid \theta_S, \tau^2, \eta, y_{(0)}, M_{Sjq}) \tag{10}$$

with $0 \leq \lambda \leq 1$.

To summarize: (10) is the overall sampling model to be used in this paper. It mixes over two models, one with and one without structural breaks. We weight the model with no structural breaks over covariance structures (see (6)) and the model with structural breaks over covariance structures and structural breaks (see (9)). Each of the mT submodels in (10) can have a different type of elliptical density. Not only do our likelihoods allow for normal, Cauchy and Student-t densities, but for densities with truncated tails (eg. Pearson type-II densities) as well.

It remains to specify the choices for $V_j(\eta)$. Since most, if not all the residual autocorrelation will be removed by including the lagged Δy_ts in the model, V_j is restricted to two choices: $V_1 = I_T$ and $V_2 = (1 + \eta^2)I_T - \eta A$, where $\eta \epsilon (-1,1)$ and A is a tridiagonal matrix with 2's on the diagonal and -1's on the off-diagonal. In other words, we allow the errors to be uncorrelated (which, only for the normal distribution, implies independence) under V_1 and to exhibit MA(1) behavior under V_2. Choi (1990) argues that ignoring the MA(1) component of the errors results in a bias in classical estimates of ρ equal to $\eta(1-\rho)/(1+\eta)$ for infinite k which tends to drive results towards the unit root for $\eta > 0$.

Section 3: The Prior Density

A controversy surrounding the use of Bayesian methods is the role of prior information. Many researchers use priors that are noninformative or objective in order to avoid the issue (see DeJong and Whiteman (1991a,b), Koop (1991a) and Phillips (1991)). Koop and Steel (1991) discuss the hazards involved in the use of such "objective" priors. Moreover, improper noninformative priors make it impossible to calculate posterior odds required to test for unit roots (see Section 1). For the reasons noted, noninformative priors for ρ are not used in this paper.

An alternative, following Schotman and van Dijk (1991a) and Koop (1991b), is to introduce explicit prior information into the analysis. Schotman and van Dijk minimize the amount of subjective prior information by allowing the prior to depend on the data, an approach which violates the likelihood principle and thus is avoided here. Koop (1991b) uses natural conjugate priors centered over the unit root restriction and performs a sensitivity analysis with respect to the prior covariance matrix of the regression parameters. In this paper a prior is used which is uniform in the regression parameters other than ρ and in $\log(\tau^{-2})$. As well as being improper, the prior is noninformative in certain dimensions in that the posterior is proportional to the likelihood function. However, before posterior odds can be calculated, the prior must be made proper in the remaining dimensions by bounding it. A sensitivity analysis can easily be performed over the choice of bounding region. We formalize these steps in the remainder of this section.

The prior density for the parameters of the sampling model can be written as:

$$P(\theta_S,\tau^2,\eta,\lambda,\delta,\gamma,\kappa) - P(\theta_S,\tau^2,\eta)P(\lambda)P(\delta)P(\gamma)P(\kappa) \tag{11}$$

That is, we *a priori* assume the mixing parameters to be independent of each other and of the parameters in each submodel. Since the mixing parameters are of no interest to us, we need only specify prior means whose existence is assumed (see Chib *et al.* (1990)). In order to be as noninformative as possible, all models receive equal prior weight. Specifically, we set

$$E(\lambda)=1/3;\ E(\delta_1)=E(\delta_2)=E(\gamma_1)=E(\gamma_2)=1/2 \text{ and } E(\kappa_q)=1/2 \text{ for } q=1,2.$$

Full robustness with respect to the choices for $g_{S|q}(.)$ and $g_{N|l}(.)$ is achieved by assuming (see Osiewalski and Steel (1990)):

$$P(\theta_S,\tau^2,\eta)=c_1\tau^{-2}P(\theta_S,\eta).$$

This assumption implies a uniform prior for $\log(\tau^{-2})$. Note that c_1 is a constant which cancels out of the posterior odds ratio and hence is irrelevant for our analysis. All that remains is to specify $P(\theta_S,\eta)$:

$$P(\theta_S,\eta)=P(\theta_N,\alpha_D,\eta)=P(\theta_N \mid \alpha_D,\eta)P(\alpha_D,\eta).$$

Since the parameters α_D and η are not present in all models, we must ensure that $P(\alpha_D,\eta)$ is proper. For the sake of convenience we assume that $P(\alpha_D,\eta)=P(\alpha_D)P(\eta)=P(d_\mu)P(d_\beta)P(\eta)$ and specify:

$$\begin{aligned} P(d_\mu) &= 1/(A_2-A_1) && \text{on } [A_1,A_2] \text{ and } 0 \text{ elsewhere} \\ P(d_\beta) &= 1/(B_2-B_1) && \text{on } [B_1,B_2] \text{ and } 0 \text{ elsewhere} \\ P(\eta) &= 1/2 && \text{on } (-1,1) \text{ and } 0 \text{ elsewhere}. \end{aligned} \qquad (12)$$

In practice, $[A_1,A_2]$ and $[B_1,B_2]$ are chosen so as to cover the area where the likelihood function is *a priori* assumed to be appreciable (see Prior Appendix). Finally, it remains to specify

$$P(\theta_N \mid \alpha_D,\eta) = P(\alpha_N \mid \rho,\alpha_D,\eta)P(\rho \mid \alpha_D,\eta).$$

Since the parameters α_N are present in all models we allow them to have an unbounded uniform prior.

We assume that the parameter of interest, ρ, is independent of α_D and η. Under the hypothesis that a unit root is present (H_2) we set $\rho=1$. Under the hypothesis that a unit root is not present we try two priors for ρ. Our first choice is a bounded uniform prior which, for the stationary region (H_1), takes the form:

$$\begin{aligned} P(\rho) &= 2\tfrac{2}{9} && \text{if } \rho \in [.55,1.00) \\ &= 0 && \text{otherwise}, \end{aligned}$$

and for the explosive region (H_3):

$$\begin{aligned} P(\rho) &= 10 && \text{if } \rho \in (1.00,\ 1.10] \\ &= 0 && \text{otherwise}. \end{aligned}$$

This type of bounded uniform prior leads to a truncated Student-t posterior for ρ under V_1 and for $\rho|\eta$ under V_2. Alternatively, an independent Student-t prior for ρ with identical first two moments[2] can be used to yield a 2-0 poly-t posterior density for ρ (or $\rho|\eta$) (Dreze (1977)). Note that pseudo-random drawings to be used in the Monte Carlo integration can easily be made from all these densities (Richard and Tompa (1980)).

Although the first two posterior moments of ρ may not be crucially affected by the difference between priors, Koop *et al.* (1991) show that results for n-step ahead prediction can differ dramatically.[3] That is, predictive means and variances will exist for any horizon (n) in the case of a bounded uniform prior; however the Student prior for ρ allows only for finite predictive means (given η) for n up to approximately T, and for finite predictive variances if n is less than approximately T/2. In Section 5 we introduce a loss function based on predictive variances whose behavior is expected to differ across priors if n is close to T/2. Of course, the fact that moments may not exist will not necessarily show up clearly here given the inevitable limitations of a Monte Carlo analysis which uses a finite number of replications.

This concludes our development of a prior for the parameters of our sampling model. It is worth emphasizing that, with four exceptions, ρ, η, d_μ and d_β, the priors for all our parameters are noninformative. We believe that the priors we specify for these exceptions will not be considered unreasonable by other researchers.

Section 4: The Posterior Density

Combining our results from the two previous sections yields our Bayesian model:

$$\begin{aligned} P(y,\theta_S,\tau^2,\eta,\lambda,\delta,\gamma,\kappa \mid y_{(0)}) = c_1\tau^{-2}P(\rho)P(\alpha_D)P(\eta)P(\lambda)P(\delta)P(\gamma)P(\kappa) \\ \{\lambda\sum_{l=1}^{2}\delta_l P(y \mid \theta_N,\tau^2,\eta,y_{(0)},M_{Nl}) \\ +(1-\lambda)\sum_{l=1}^{2}\gamma_l\sum_{q=1}^{2}\kappa_q P(y \mid \theta_S,\tau^2,\eta,y_{(0)},M_{Slq})\} \end{aligned} \tag{13}$$

Using results from Osiewalski and Steel (1990), we integrate out τ^2 and the mixing parameters, yielding:

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[2] We use truncated (at $\rho=1$) Student priors for both H_1 and H_3 which are constructed in such a way that their untruncated counterparts mimic the moments of the relevant uniform prior mirrored around $\rho=1$. This yields half-Students with a mode at $\rho=1$. Finally, the degrees of freedom parameter is chosen to be 3 so that this alternative prior has fat tails yet still allows the first two moments to exist.

[3] The n-step ahead prediction involves moments of ρ of order n for predictive means and of order 2n for predictive variances.

$$P(y,\theta_S,\eta \mid y_{(0)}) = c_2 P(\rho)P(\alpha_D)P(\eta)\{\frac{1}{3}\ |V_i(\eta)|^{-1/2}[d_{Ni}(\theta_N,\eta)]^{-T/2} + \frac{2}{3}\sum_{j=1}^{2}\frac{1}{2}\sum_{q=1}^{2}\frac{1}{2}\ |V_j(\eta)|^{-1/2}[d_{Sjq}(\theta_S,\eta)]^{-T/2}\} \tag{14}$$

where $c_2 = c_1\Gamma(T/2)\pi^{-T/2}$ and definitions (5) and (8) are used.

For the individual models we obtain:

$$P(\theta_N,\eta \mid y,y_{(0)},M_{Ni}) = C_{Ni}^{-1}\ P(\rho)P(\eta)|V_i(\eta)|^{-\frac{1}{2}}[d_{Ni}(\theta_N,\eta)]^{-T/2} \tag{15}$$

and

$$P(\theta_S,\eta \mid y,y_{(0)},M_{Sjq}) = C_{Sjq}^{-1}\ P(\rho)P(\eta)|V_j(\eta)|^{-\frac{1}{2}}[d_{Sjq}(\theta_S,\eta)]^{-T/2}\ , \tag{16}$$

where C_{Ni} and C_{Sjq} are the integrating constants needed to construct posterior odds (ie. $C_{Ni} = P(y \mid y_{(0)},M_{Ni})$ and $C_{Sjq} = P(y \mid y_{(0)},M_{Sjq})$). Although the integrating constants may be calculated directly, it should be noted that α_N may be integrated out of (15) and (16) analytically using the properties of multivariate Student distributions. Once α_N is integrated out, the C_{Ni}'s and C_{Sjq}'s may be calculated using Monte Carlo integration. One-dimensional integration is required for calculation of C_{N1}; two-dimensional integration for C_{N2} and C_{S1q}; and three-dimensional integration for C_{S2q}. Formally, the posterior density for α_D, given ρ and η, is a truncated Student-t over the region given in (12). If this region covers most of the parameter space where the likelihood function is appreciable, the truncation will not matter. In this case we can integrate out the full α_S vector as a joint Student density, leaving only one and two dimensional integrals for C_{S1q} and C_{S2q} which we calculate using Monte Carlo integration. A check on this approximation is to perform the integration with respect to α_D numerically by direct simulation with rejection.

The integrating constant for the sampling model, $C = P(y \mid y_{(0)})$, is given by:

$$C = \frac{1}{6}(\sum_{i=1}^{2} C_{Ni} + \sum_{j=1}^{2}\sum_{q=1}^{2} C_{Sjq}). \tag{17}$$

These integrating constants can be used to calculate the posterior probabilities of the various submodels.

$$P(M_{Ni} \mid y,y_{(0)}) = C_{Ni}/6C$$

$$P(M_N \mid y,y_{(0)}) = (C_{N1}+C_{N2})/6C$$

$$P(M_{Sjq} \mid y,y_{(0)}) = C_{Sjq}\ /6C$$

$$P(M_{Sj} \mid y,y_{(0)}) = (C_{Sj1}+C_{Sj2})/6C$$

$$P(M_{Sq} \mid y,y_{(0)}) = (C_{S1q}+C_{S2q})/6C.$$

The posterior model probabilities may indicate, among other things, whether structural breaks are present or if errors exhibit MA(1) behavior. Although not given here, inference on the parameters could be obtained from weighted averages of (15) and (16) (with α_N possibly integrated out), where the weights are the relevant model probabilities.

Under all hypotheses, we use the same general mixture of submodels for the sampling density. Note, however, that in all cases, the relative posterior weights given to the submodels depend on the data.

Section 5: Decision Theory

In the previous sections we have described how the posterior probabilities of various hypotheses can be calculated using Bayesian methods. However, econometricians must frequently make decisions. For instance, in a pre-testing exercise a decision must frequently be made as to whether a unit root is present in a series. If present, the series may have to be differenced in a larger VAR model. The Bayesian paradigm provides a formal framework for making such decisions. To make a decision the researcher specifies a loss function and chooses the action which minimizes expected loss (see Zellner (1971)). By focussing on posterior probabilities, previous Bayesian researchers have implicitly used a very simple loss function where all losses attached to incorrect decisions are equal. (That is, the loss associated with the choice of a unit root when the series is stationary is equal to that associated with the assumption of stationarity when a unit root is present). Classical researchers use a loss function where losses are asymmetric, viz. where the choice of a level of significance implicitly defines the loss function. Lacking a measure over the parameter space, classical researchers are forced to look for, say, dominating strategies (which are rare) or minimax solutions. It is this lack of formal development and justification of a loss function which is, in our opinion, a serious weakness of previous Bayesian and classical unit root studies. This section proposes a loss function which we use to make decisions on whether to accept or reject the unit root hypothesis.

Our criterion for the evaluation of losses associated with incorrect decisions is prediction. This criterion is important because the macroeconomic time series in this study are frequently used for prediction (eg. to forecast from VAR models or to calculate impulse responses). The cost of assuming stationarity with such models when the series are really nonstationary may be drastically different from the converse. Since differences between nonstationary and stationary models are more pronounced for predictive variances than for predictive means, we base a loss function on predictive variances. Given that the precision of forecasts is often a crucial issue we believe this approach to be a sensible one.

For the simple AR(1) model, with intercept and trend[4], the predictive variance for forecasting n periods ahead is given by Koop *et al.* (1991):

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[4] Formally speaking, using this model corresponds to conditioning on ϕ and α_D in our more general model and assuming uncorrelated errors.

$$Var(y_{T+n} \mid y, y_{(0)}, \rho, \mu, \beta, \tau^2) = \tau^{-2} \sum_{i=0}^{n-1} \rho^{2i} \quad , \tag{18}$$

when we condition on all the parameters; and by the more complicated formula (for T>4):

$$Var(y_{T+n} \mid y, y_{(0)}, \rho) = \frac{SSE_\rho}{T-4} \Big(\sum_{i=0}^{n-1} \rho^{2i} + \frac{2}{T(T^2-1)} \sum_{i=1}^{n} \sum_{j=1}^{n} r(i,j) \rho^{2n-i-j} \Big) \tag{19}$$

when we integrate out μ, β and τ^2 using the noninformative priors given in Section 3. In (19) we use $r(i,j) \equiv 6ij + 3(T-1)(i+j) + 2T^2 - 3T + 1$ and $SSE_\rho = (y - \rho y_{-1})'M(y - \rho y_{-1})$ where M is the identity matrix minus the usual projection matrix on the intercept and trend. To ensure computational tractability, we do not fully marginalize the variance with respect to ρ. Rather, we replace the powers of ρ in (18) and (19) by their expected values (ie. we replace ρ^i with $E(\rho^i)$ which we calculate using Monte Carlo integration).[5]

To develop the loss function we first define:

$$g^j_{n,1}(\rho) = \sum_{i=0}^{n-1} \rho^{2i} \mid H_j \tag{20}$$

and

$$g^j_{n,2}(\rho) = g^j_{n,1}(\rho) + \frac{2}{T(T^2-1)} \sum_{i=1}^{n} \sum_{j=1}^{n} r(i,j) \rho^{2n-i-j} \mid H_j \tag{21}$$

where H_1: $\rho < 1$ (stationary model)

H_2: $\rho = 1$ (unit root model)

H_3: $\rho > 1$ (explosive model).

For each H_j, we can use the marginal posterior density of ρ to calculate

$$Eg^j_{n,1}(\rho) = E\Big(\sum_{i=0}^{n-1} \rho^{2i} \mid H_j, y, y_{(0)} \Big),$$

where we have already mixed over the different models in the likelihood function using the relevant posterior probabilities. $Eg^j_{n,2}(\rho)$ is calculated in the same fashion. Our loss function takes the form:

$$l^{n,j}_{d,s} = \max(1, Eg^d_{n,j}(\rho)/Eg^s_{n,j}(\rho)) + \delta \ \max(1, Eg^s_{n,j}(\rho)/Eg^d_{n,j}(\rho)) - (1+\delta),$$

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[5] If we had fully marginalized with respect to ρ, an additional term would have been added to the predictive variances under H_1 and H_3. Therefore, predictive variances for the trend-stationary and explosive models are slightly underestimated relative to the unit root model.

where $i=1$ or 2; H_d is the hypothesis chosen; H_s is the "correct" hypothesis; and δ, which is greater than or equal to 1, reflects our aversion to underestimating the predictive variance.[6] For each decision, d, we compute the expected loss:

$$l_d^{n,i} = \sum l_{d,s}^{n,i}\, p(H_s \mid y, y_{(0)}).$$

and choose d for which the loss is minimal for a given forecast horizon, n.

The expression in (20) refers to that part of the variance due to sampling uncertainty that differs crucially across the three regions for ρ.[7] Note that this quantity is bounded as n grows for H_1, is linear in n for H_2, and grows exponentially for H_3. Thus as n becomes moderately large, it displays very different characteristics for these three regions. The loss function based on $g^i_{n,1}$ differs from that based on $g^i_{n,2}$ in its treatment of parameter uncertainty about μ, β and τ^2. For both loss functions the random nature of ρ is only partially taken into account. We know that by not marginalizing fully with respect to ρ, we favor H_2 if $\delta > 1$, since predictive variances under H_1 and H_3 are underestimated.

Note that it is crucial to consider multi-period predictions since they bring out the differences in predictive behavior between stationary, unit root, and explosive models (see Chow (1974) for some specific problems when $n > 1$).

The parameter δ plays an important role in our loss function. If $\delta = 1$, the loss function is symmetric in the sense that underestimating and overestimating the predictive variance are equally costly. For values of δ greater than one underestimating the predictive variance (and giving a researcher excessive confidence in her forecasts) is more costly than overestimating the predictive variance. The loss function is normalized such that losses are zero for correct decisions but are: i) equal to the variance ratio (which is bigger than one) if the chosen model has a bigger variance than the "correct" model (ie. if we overestimate the predictive variance); and ii) equal to δ times the inverse of the variance ratio if the chosen model has a smaller variance than the "correct" model (ie. the predictive variance is underestimated).

At short horizons the losses do not differ much across models (unless δ is very large) and the model is chosen largely on the basis of its posterior probability. Indeed when $n=1$ all losses are zero by definition. At long forecast horizons, the differences in predictive variances between stationary and nonstationary models grow large; and assuming $\delta > 1$, nonstationary models grow concomitantly more attractive. So if there is any chance that the correct model is nonstationary, our loss function will choose it at some forecast horizon (ie. the cost of incorrectly choosing the stationary model and seriously underestimating the predictive variance will eventually dominate at some forecast horizon). Under both loss functions, H_2 will be chosen if n goes to infinity (holding δ constant), while H_3 will be chosen if δ goes to infinity (holding n constant). Since the decision taken depends crucially on the choice of n and

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[6] For computational ease, we assume that SSE_ρ approximately cancels out in our loss function.

[7] In classical analyses, the MSE of a forecast will also have the same analytical form if parameter uncertainty is not taken into account.

δ, we do a sensitivity analysis over these two parameters.

The decision theory approach is based on the assumption that researchers are interested in choosing a particular region for ρ since they may wish, for instance, to difference the data. However, in cases where such a pretest strategy is not required, we suggest basing predictions on a mixture over regions for ρ weighted with the relevant posterior probabilities.

Section 6: Empirical Results

This section presents evidence on the existence of a unit root in the Nelson-Plosser series. The data used are extended to cover the period until 1988 (see Data Appendix). Tables 1 and 2 present posterior means and standard deviations for ρ and η under H_1 and H_3, while Table 3 presents evidence on the presence of structural breaks and moving average errors. Table 4 contains the posterior probabilities of H_1, H_2 and H_3, and Tables 5 and 6 summarize the results of the decision analysis. Posterior odds are calculated for testing the various hypotheses with respect to ρ by using the sampling model weighted over all the submodels. Although our primary focus is on the unit root hypothesis, two subsidiary questions are simultaneously addressed: (1) Is there evidence of one or more structural breaks in our economic time series? (2) Is there evidence of MA(1) behavior in the error terms?

Since parameter estimates are only slightly relevant to the issues we address in this paper, we discuss results only briefly. Note first that Tables 1 and 2 support the conclusions of Choi (1990): Omitting the MA(1) component of the error term does indeed tend to drive estimates of ρ towards one in a manner consistent with the asymptotic bias derived by Choi. Table 3 contains the probability that an MA(1) error term is present as well as the AR(3) component already allowed for in our specification. For many series this probability is very high and for no series is it small enough to be ignored. Thus Choi's results are more than just theoretically interesting. The inclusion of a moving average error term would appear to be an important part of any specification. A second point worth noting about Tables 1 and 2 is that posterior means and standard deviations alone should not be used to infer the probability of a hypothesis. For example, Table 4 indicates that a high probability exists that the real wage series contains a unit root but the nominal wage does not. This cannot be ascertained simply by examining the posterior means and standard deviations in Tables 1 and 2, a point which exemplifies the hazards of using highest posterior density intervals for testing purposes.

With respect to structural breaks in Table 3, note that, although our results are consistent with Perron's contention that a level break occurred in 1929 in many macroeconomic time series, we find virtually no evidence for the presence of a trend break in 1973 for any of the series.[8] As Perron (1989) notes, models with structural breaks tend to yield less evidence of a unit root.

We do not discuss Table 4 in detail but we do use the results to calculate the expected losses

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[8] Perron indicates that models with a 1973 trend break are more relevant for post-war quarterly data sets than the long annual data sets used here.

required for our decision analysis. For our purposes it is sufficient to note that results show that trend-stationarity (H_1) is the most probable hypothesis for most of the series (notable exceptions are the CPI and velocity); however, without a formal loss function it would be rash to rule out the unit root model at this time.

It is worth emphasizing that our loss function has two key properties. First, as long as δ is greater than one, it is better to overestimate than to underestimate predictive variances. This property tends to favor H_2 over H_1 and H_3 over H_2 and H_1. Indeed as δ goes to infinity (holding n constant) H_3 will always be chosen. Second, there is a tendency in our loss function to favor H_2. H_2 lies between H_1 and H_3 such that a researcher will, loosely speaking, never go too far wrong in choosing H_2. (Potential losses would be very large if, say, H_1 were chosen when H_3 was the "correct" model). In fact, as n goes to infinity (holding δ constant) H_2 will always be chosen.[9] These two properties account for most of the findings in Tables 5 and 6, which present the model chosen for different values of n and δ.[10] With the exception of the CPI and velocity series and, to a lesser extent, the GNP deflator and real wage series, H_1 is the model chosen (so long as δ or n is not large). However, clear scope exists for choosing nonstationarity if underestimating predictive variances is felt to be a serious problem. If $\delta = 100$ a researcher would almost never select the trend-stationary model. There appears to be less sensitivity of our loss function with respect to n. If we restrict attention to short- or medium-term forecasts (eg. $n < 10$), only a few cases exist where different values of n yield different conclusions. A typical example is real GNP, where, unless the researcher is interested in forecasting four or more decades into the future, the trend-stationary model is chosen for $\delta = 1$ or 10. Only if $\delta = 100$ (a strong penalty for underestimating predictive variances) is the unit root model selected. Overall, we conclude that there is strong evidence in favor of trend-stationarity for virtually all the series analyzed in this paper (especially as the conditional results given in this paper are biased in favor of H_2); however, as we show, researchers with different loss functions may make different inferences.

It is interesting to note that our results for $\delta = 10$ correspond closely to those given in Phillips (1991, reply) who uses the Phillips-Ploberger posterior odds test on the same data. The chief difference is that Phillips finds the nominal wage series to contain a unit root, whereas we only match this finding if n is very large or $\delta = 100$. Note, however, that Phillips'results are obtained by using an improper Jeffreys' prior for ρ, whereas we use a formal decision theoretic approach based on a strong aversion to underestimating predictive variances. Researchers who do not wish to include such an aversion in their analysis will tend to choose trend-stationarity more often.

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[9] It is worth emphasizing that our failure to fully marginalize with respect to ρ favors the unit root hypothesis.

[10] Table 5 and 6 correspond to our two loss functions. Because their results are very similar their different treatment of parameter uncertainty in the predictive variance may not be too important for the purposes of our analysis for finite n. As n goes to infinity these differences may become important (see Koop *et al.* (1991)).

A final issue worth discussing is the sensitivity of our results to various priors. As described in Section 4, we use two different priors for ρ: a half-Student and a bounded uniform prior. The first and second moments of the half-Student prior are chosen so as to match the uniform prior (see footnote 2). The differences between the two priors occur in third and higher moments. Tables 1 and 2 indicate that posterior first and second moments do not differ much across the two priors. The remaining tables, however, indicate somewhat larger differences. This is especially true of Tables 5 and 6, where in some cases, the two very similar priors yield different conclusions (eg. Nominal GNP for $\delta=10$ or the GNP deflator for $\delta=1$ or 10).[11] Our decision analysis depends upon high order moments of ρ and our priors differ in these high moments. Recall that, while all moments exist for our bounded uniform prior, none beyond 2 exist for our half-Student prior. Although Bayesians who use informative priors typically do not worry about third or higher prior moments, our analysis suggests that care should be taken in eliciting such prior moments when a decision analysis which involves high order moments is carried out. The effect of prior moments on the existence of predictive variances for multi-period forecasting is formally analyzed in Koop *et al.* (1991).

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[11] As described in Section 3, predictive variances exist only for n less than approximately T/2, a fact which is ignored in Tables 5 and 6 where results are occasionally reported for n>T/2.

Table 1: Posterior Means for ρ and η under H_1 (Standard deviations in parentheses)

			Uniform Prior ρ			Student Prior ρ	
		No MA ρ	MA ρ	MA η	No MA ρ	MA ρ	MA η
Real GNP	nb	0.8134	0.7462	0.4416	0.8291	0.7836	0.3483
		(.0570)	(.0889)	(.3377)	(.0594)	(.0894)	(.3705)
	lb	0.7409	0.6941	0.3815	0.7669	0.7242	0.3484
		(.0681)	(.0829)	(.2880)	(.0689)	(.0999)	(.3127)
	tb	0.8127	0.7338	0.5178	0.8288	0.7732	0.4372
		(.0562)	(.0862)	(.2943)	(.0547)	(.0877)	(.3365)
Nominal GNP	nb	0.9411	0.9031	0.6737	0.9434	0.9025	0.7512
		(.0296)	(.0448)	(.1683)	(.0287)	(.0485)	(.1290)
	lb	0.7777	0.7555	0.3228	0.7991	0.7862	0.3168
		(.0630)	(.0763)	(.2410)	(.0634)	(.0760)	(.2612)
	tb	0.9209	0.8514	0.7762	0.9251	0.8728	0.7744
		(.0371)	(.0659)	(.1206)	(.0355)	(.0625)	(.1182)
Real per cap. GNP	nb	0.8032	0.7363	0.4321	0.8201	0.7782	0.3303
		(.0579)	(.0889)	(.3407)	(.0577)	(.0914)	(.3838)
	lb	0.7564	0.7022	0.4263	0.7813	0.7345	0.3753
		(.0671)	(.0845)	(.2970)	(.0688)	(.0984)	(.3260)
	tb	0.8032	0.7256	0.5152	0.8205	0.7636	0.4365
		(.0583)	(.0866)	(.3004)	(.0579)	(.0918)	(.3383)
Ind. Prod.	nb	0.8256	0.7626	0.3843	0.8392	0.7985	0.3003
		(.0523)	(.0859)	(.3072)	(.0515)	(.0832)	(.3356)
	lb	0.7498	0.6952	0.3530	0.7743	0.7244	0.3181
		(.0678)	(.0811)	(.2401)	(.0666)	(.0984)	(.2620)
	tb	0.8149	0.7386	0.4430	0.8296	0.7731	0.3819
		(.0536)	(.0847)	(.2833)	(.0538)	(.0849)	(.2976)
Employ-ment	nb	0.8637	0.8024	0.4442	0.8734	0.8273	0.4160
		(.0473)	(.0747)	(.2357)	(.0458)	(.0694)	(.2253)
	lb	0.7982	0.7300	0.4209	0.8150	0.7599	0.3953
		(.0563)	(.0767)	(.1916)	(.0555)	(.0773)	(.1954)
	tb	0.8578	0.7866	0.4873	0.8679	0.8148	0.4525
		(.0484)	(.0774)	(.2190)	(.0471)	(.0739)	(.2260)
Unempl. Rate	nb	0.7454	0.6586	0.5935	0.7747	0.6644	0.6001
		(.0736)	(.0748)	(.1303)	(.0750)	(.1117)	(.1242)
	lb	0.7144	0.6523	0.5866	0.7459	0.6412	0.5912
		(.0764)	(.0740)	(.1244)	(.0824)	(.1170)	(.1278)
	tb	0.7378	0.6587	0.5922	0.7682	0.6542	0.6055
		(.0758)	(.0739)	(.1362)	(.0770)	(.1140)	(.1275)
GNP De-flator	nb	0.9634	0.9474	0.4973	0.9640	0.9468	0.5646
		(.0189)	(.0294)	(.3127)	(.0188)	(.0294)	(.2417)
	lb	0.9166	0.8843	0.5462	0.9194	0.8909	0.5313
		(.0289)	(.0423)	(.2314)	(.0285)	(.0396)	(.2400)
	tb	0.9321	0.8942	0.6154	0.9347	0.9095	0.4408
		(.0300)	(.0477)	(.2196)	(.0295)	(.0427)	(.3704)

Table 1 (continued): Posterior Means for ρ and η under H_1 (Standard deviations in parentheses)

			Uniform Prior ρ			Student Prior ρ	
		No MA ρ	MA ρ	MA η	No MA ρ	MA ρ	MA η
CPI	nb	0.9886	0.9804	0.6531	0.9887	0.9808	0.6286
		(.0077)	(.0134)	(.1406)	(.0078)	(.0129)	(.1665)
	lb	0.9888	0.9804	0.6539	0.9888	0.9838	0.4881
		(.0077)	(.0134)	(.1474)	(.0077)	(.0099)	(.3427)
	tb	0.9820	0.9679	0.6582	0.9820	0.9694	0.6412
		(.0114)	(.0120)	(.1456)	(.0115)	(.0198)	(.1718)
Wages	nb	0.9373	0.9053	0.5068	0.9393	0.9032	0.5165
		(.0279)	(.0459)	(.3128)	(.0273)	(.0487)	(.3155)
	lb	0.7999	0.7818	0.2137	0.8120	0.7822	0.2225
		(.0471)	(.0593)	(.2292)	(.0472)	(.0596)	(.2338)
	tb	0.9212	0.8725	0.6076	0.9247	0.8723	0.5960
		(.0345)	(.0596)	(.2479)	(.0332)	(.0591)	(.2659)
Real Wages	nb	0.9280	0.8818	0.6506	0.9322	0.9038	0.5466
		(.0395)	(.0659)	(.2152)	(.0377)	(.0560)	(.2737)
	lb	0.9276	0.8751	0.7391	0.9324	0.8867	0.7954
		(.0397)	(.0672)	(.2361)	(.0377)	(.0614)	(.1909)
	tb	0.8112	0.7159	0.6047	0.8316	0.7668	0.4624
		(.0574)	(.0807)	(.2278)	(.0502)	(.0886)	(.3292)
Money Stock	nb	0.9402	0.9070	0.5721	0.9415	0.9123	0.5534
		(.0233)	(.0380)	(.1882)	(.0229)	(.0357)	(.2060)
	lb	0.8807	0.8454	0.4773	0.8848	0.8550	0.4623
		(.0318)	(.0446)	(.2041)	(.0316)	(.0432)	(.2158)
	tb	0.9187	0.8726	0.5924	0.9210	0.8811	0.5700
		(.0270)	(.0440)	(.1789)	(.0269)	(.0424)	(.2008)
Velocity	nb	0.9629	0.9395	0.5648	0.9635	0.9437	0.5481
		(.0212)	(.0356)	(.3094)	(.0207)	(.0341)	(.3220)
	lb	0.9635	0.9383	0.6035	0.9642	0.9418	0.5976
		(.0209)	(.0360)	(.2607)	(.0206)	(.0342)	(.2668)
	tb	0.9580	0.9289	0.6083	0.9594	0.9329	0.6248
		(.0253)	(.0431)	(.2823)	(.0246)	(.0412)	(.2509)
Bond Yield	nb	0.9466	0.9195	0.4860	0.9488	0.9277	0.4560
		(.0299)	(.0466)	(.1987)	(.0289)	(.0427)	(.2314)
	lb	0.8931	0.8386	0.5516	0.9003	0.8583	0.5355
		(.0441)	(.0674)	(.2024)	(.0430)	(.0638)	(.2088)
	tb	0.9449	0.9152	0.4917	0.9501	0.9283	0.4897
		(.0410)	(.0647)	(.2198)	(.0380)	(.0538)	(.2005)
Stock Prices	nb	0.9297	0.8991	0.3569	0.9329	0.9080	0.3339
		(.0333)	(.0527)	(.3018)	(.0320)	(.0493)	(.3032)
	lb	0.9135	0.8829	0.3322	0.9175	0.8932	0.3152
		(.0351)	(.0512)	(.2367)	(.0346)	(.0480)	(.2333)
	tb	0.9069	0.8581	0.4342	0.9120	0.8751	0.3934
		(.0378)	(.0619)	(.2663)	(.0362)	(.0579)	(.2811)

* nb = no break, lb = level break, tb = trend break.

Table 2: Posterior Means for ρ and η under H_3 (Standard deviations in parentheses)

			Uniform Prior ρ			Student Prior ρ	
		No MA ρ	MA ρ	MA η	No MA ρ	MA ρ	MA η
Real GNP	nb	1.0167	1.0285	0.4292	1.0134	1.0155	0.3609
		(.0155)	(.0292)	(.4019)	(.0126)	(.0143)	(.4043)
	lb	1.0189	1.0266	0.6654	1.0153	1.0184	0.4962
		(.0175)	(.0227)	(.2331)	(.0144)	(.0168)	(.4123)
	tb	1.0172	1.0219	0.4941	1.0136	1.0172	0.4811
		(.0159)	(.0191)	(.3357)	(.0125)	(.0162)	(.3491)
Nominal GNP	nb	1.0138	1.0196	0.5850	1.0117	1.0155	0.5143
		(.0124)	(.0177)	(.2637)	(.0105)	(.0138)	(.3284)
	lb	1.0186	1.0260	0.6703	1.0144	1.0180	0.5999
		(.0174)	(.0221)	(.2414)	(.0136)	(.0163)	(.3356)
	tb	1.0158	1.0225	0.6064	1.0129	1.0181	0.6181
		(.0142)	(.0200)	(.2720)	(.0177)	(.0153)	(.2546)
Real per cap. GNP	nb	1.0174	1.0230	0.4118	1.0135	1.0159	0.3805
		(.0163)	(.0217)	(.3828)	(.0126)	(.0151)	(.4010)
	lb	1.0193	1.0241	0.5183	1.0152	1.0184	0.5182
		(.0181)	(.0201)	(.4057)	(.0138)	(.0163)	(.3998)
	tb	1.0177	1.0230	0.5088	1.0138	1.0169	0.4824
		(.0166)	(.0202)	(.3344)	(.0128)	(.0154)	(.3488)
Ind. Prod.	nb	1.0150	1.0184	0.2981	1.0125	1.0140	0.2737
		(.0142)	(.0178)	(.3381)	(.0115)	(.0132)	(.3477)
	lb	1.0179	1.0215	0.3533	1.0141	1.0159	0.3517
		(.0169)	(.0195)	(.4593)	(.0131)	(.0139)	(.4285)
	tb	1.0153	1.0193	0.3767	1.0124	1.0143	0.3615
		(.0144)	(.0183)	(.3064)	(.0117)	(.0127)	(.3066)
Employment	nb	1.0150	1.0192	0.3709	1.0128	1.0151	0.3713
		(.0141)	(.0176)	(.2335)	(.0115)	(.0147)	(.2287)
	lb	1.0156	1.0199	0.4118	1.0127	1.0150	0.4007
		(.0152)	(.0187)	(.2173)	(.0117)	(.0135)	(.2260)
	tb	1.0154	1.0193	0.4164	1.0123	1.0153	0.4163
		(.0144)	(.0180)	(.2198)	(.0115)	(.0134)	(.2069)
Unempl. Rate	nb	1.0211	1.0272	0.5908	1.0161	1.0192	0.5823
		(.0189)	(.0228)	(.1231)	(.0149)	(.0172)	(.1394)
	lb	1.0226	1.0287	0.6006	1.0166	1.0203	0.6018
		(.0205)	(.0241)	(.1287)	(.0153)	(.0198)	(.1285)
	tb	1.0218	1.0256	0.5998	1.0165	1.0188	0.5942
		(.0198)	(.0213)	(.1306)	(.0156)	(.0164)	(.1286)
GNP Deflator	nb	1.0091	1.0138	0.5207	1.0083	1.0116	0.5074
		(.0082)	(.0136)	(.3016)	(.0075)	(.0108)	(.3097)
	lb	1.0096	1.0131	0.5662	1.0087	1.0110	0.5730
		(.0089)	(.0125)	(.2805)	(.0079)	(.0103)	(.2737)
	tb	1.0120	1.0175	0.5726	1.0105	1.0137	0.5647
		(.0110)	(.0153)	(.2737)	(.0095)	(.0120)	(.2793)

Table 2 (continued): Posterior Means for ρ and η under H_3 (Standard deviations in parentheses)

			Uniform Prior ρ			Student Prior ρ	
		No MA ρ	MA ρ	MA η	No MA ρ	MA ρ	MA η
CPI	nb	1.0067	1.0095	0.5818	1.0065	1.0075	0.3119
		(.0055)	(.0083)	(.1977)	(.0053)	(.0071)	(.4508)
	lb	1.0069	1.0103	0.6316	1.0065	1.0081	0.3985
		(.0056)	(.0085)	(.1626)	(.0054)	(.0074)	(.4384)
	tb	1.0081	1.0119	0.6430	1.0078	1.0097	0.4469
		(.0069)	(.0105)	(.1466)	(.0065)	(.0088)	(.3990)
Wages	nb	1.0111	1.0153	0.4781	1.0122	1.0126	0.4522
		(.0104)	(.0144)	(.3230)	(.0109)	(.0118)	(.3378)
	lb	1.0125	1.0184	0.5453	1.0112	1.0143	0.5508
		(.0123)	(.0169)	(.3167)	(.0103)	(.0138)	(.3138)
	tb	1.0132	1.0189	0.5277	1.0120	1.0147	0.5181
		(.0124)	(.0171)	(.3097)	(.0111)	(.0133)	(.3045)
Real Wages	nb	1.0207	1.0258	0.4960	1.0159	1.0188	0.4289
		(.0181)	(.0221)	(.3116)	(.0139)	(.0174)	(.3881)
	lb	1.0204	1.0293	0.8209	1.0161	1.0209	0.7014
		(.0178)	(.0229)	(.1540)	(.0143)	(.0188)	(.3332)
	tb	1.0171	1.0237	0.5541	1.0137	1.0173	0.5212
		(.0162)	(.0205)	(.3000)	(.0127)	(.0165)	(.3473)
Money Stock	nb	1.0082	1.0116	0.5326	1.0078	1.0102	0.5142
		(.0077)	(.0108)	(.2143)	(.0070)	(.0095)	(.2366)
	lb	1.0085	1.0123	0.5477	1.0079	1.0107	0.5431
		(.0082)	(.0120)	(.2010)	(.0074)	(.0100)	(.2059)
	tb	1.0087	1.0124	0.5525	1.0079	1.0106	0.5504
		(.0081)	(.0120)	(.2007)	(.0073)	(.0098)	(.1982)
Velocity	nb	1.0120	1.0170	0.5176	1.0106	1.0137	0.4942
		(.0104)	(.0157)	(.3398)	(.0091)	(.0120)	(.3588)
	lb	1.0122	1.0177	0.5781	1.0109	1.0144	0.5535
		(.0107)	(.0160)	(.2870)	(.0093)	(.0129)	(.3158)
	tb	1.0156	1.0230	0.5782	1.0132	1.0167	0.5696
		(.0135)	(.0209)	(.3056)	(.0113)	(.0150)	(.2953)
Bond Yield	nb	1.0163	1.0224	0.4531	1.0136	1.0222	0.4531
		(.0143)	(.0191)	(.2135)	(.0120)	(.0196)	(.2173)
	lb	1.0162	1.0209	0.4942	1.0134	1.0217	0.4966
		(.0151)	(.0187)	(.2095)	(.0120)	(.0196)	(.2062)
	tb	1.0401	1.0423	0.4346	1.0255	1.0314	0.4362
		(.0267)	(.0274)	(.1976)	(.0188)	(.0275)	(.2063)
Stock Prices	nb	1.0141	1.0164	0.2389	1.0120	1.0130	0.2203
		(.0130)	(.0155)	(.3432)	(.0108)	(.0116)	(.3461)
	lb	1.0130	1.0159	0.2940	1.0110	1.0133	0.2846
		(.0121)	(.0150)	(.2713)	(.0102)	(.0133)	(.2722)
	tb	1.0137	1.0168	0.3215	1.0115	1.0135	0.3092
		(.0126)	(.0157)	(.3071)	(.0106)	(.0122)	(.3099)

* nb = no break, lb = level break, tb = trend break.

Table 3: Posterior Probabilities of Elements in Mixtures

		Uniform Prior for ρ			Student Prior for ρ	
	Level Break	Trend Break	Moving Average	Level Break	Trend Break	Moving Average
Real GNP	0.0614	1.2E-5	0.5856	0.1556	3.6E-5	0.4977
Nominal GNP	0.6639	2.9E-5	0.4740	0.8489	4.5E-5	0.4146
Real per cap. GNP	0.1728	2.3E-5	0.5719	0.1391	2.0E-5	0.4991
Industrial Production	0.2449	0.0001	0.5211	0.1636	9.6E-5	0.4829
Employ-ment	0.4488	1.0E-5	0.7137	0.3626	1.1E-5	0.6454
Unempl. Rate	0.4447	4.9E-4	0.9930	0.4102	4.8E-4	0.9857
GNP De-flator	0.2676	1.4E-4	0.5892	0.3034	1.7E-4	0.5950
CPI	0.0438	5.7E-5	0.4718	0.0461	5.9E-5	0.5148
Wages	0.9453	2.4E-6	0.3240	0.9427	5.3E-6	0.2960
Real Wages	0.1428	0.0033	0.5554	0.1252	0.0085	0.5773
Money Stock	0.5586	1.2E-4	0.7550	0.5194	1.3E-4	0.7399
Velocity	0.0108	0.0002	0.7345	0.0108	0.0002	0.7341
Bond Yield	0.7527	0.0125	0.7763	0.9180	0.0070	0.7661
Stock Prices	0.2838	0.0018	0.4587	0.2935	0.0018	0.4485

Table 4: Posterior Probabilities of Regions for ρ

		Uniform Prior for ρ			Student Prior for ρ	
	H_1: $\rho<1$	H_2: $\rho=1$	H_3: $\rho>1$	H_1: $\rho<1$	H_2: $\rho=1$	H_3: $\rho>1$
Real GNP	0.9824	0.0147	0.0029	0.9816	0.0133	0.0051
Nominal GNP	0.8275	0.1371	0.0354	0.9232	0.0476	0.0292
Real per cap. GNP	0.9883	0.0094	0.0023	0.9852	0.0099	0.0049
Industrial Production	0.9869	0.0108	0.0023	0.9863	0.0099	0.0039
Employ-ment	0.9678	0.0263	0.0059	0.9657	0.0242	0.0101
Unempl. Rate	0.9968	0.0023	0.0009	0.9905	0.0059	0.0036
GNP De-flator	0.4940	0.4339	0.0722	0.6107	0.2896	0.0997
CPI	0.0789	0.8087	0.1124	0.1376	0.6192	0.2432
Wages	0.9794	0.0176	0.0030	0.9803	0.0162	0.0035
Real Wages	0.4355	0.4198	0.1447	0.5515	0.2720	0.1765
Money Stock	0.9097	0.0789	0.0011	0.9360	0.0495	0.0145
Velocity	0.3198	0.5650	0.1152	0.4389	0.4141	0.1470
Bond Yield	0.7057	0.2315	0.0627	0.8790	0.0746	0.0464
Stock Prices	0.6114	0.3243	0.0643	0.7131	0.2068	0.0801

Table 5: Results of Conditional Decision Analysis Using $I_d^{n,1}$ (n=2,...,100)

		Uniform Prior for ρ			Student Prior for ρ	
	$\delta=1$	$\delta=10$	$\delta=100$	$\delta=1$	$\delta=10$	$\delta=100$
Real GNP	n<60: H_1 else: H_2	n<45: H_1 else: H_2	H_2	n<76: H_1 else: H_2	n<60: H_1 else: H_2	H_2
Nominal GNP	n<55: H_1 else: H_2	H_2	n<15: H_3 else: H_2	n<63: H_1 else: H_2	n<14: H_1 else: H_2	n<10: H_3 else: H_2
Real per cap. GNP	n<73: H_1 else: H_2	n<56: H_1 else: H_2	H_2	n<77: H_1 else: H_2	n<60: H_1 else: H_2	H_2
Ind. Prod.	n<76: H_1 else: H_2	n<60: H_1 else: H_2	H_2	n<80: H_1 else: H_2	n<67: H_1 else: H_2	H_2
Employ-ment	n<72: H_1 else: H_2	n<52: H_1 else: H_2	H_2	n<57: H_1 else: H_2	n<44: H_1 else: H_2	H_2
Unempl. Rate	n<73: H_1 else: H_2	n<58: H_1 else: H_2	n<38: H_1 else: H_2	n<59: H_1 else: H_2	n<47: H_1 else: H_2	n<7: H_1 else: H_2
GNP De-flator	H_2	H_2	H_3	n<58: H_1 else: H_2	n<4: H_3 else: H_2	H_3
CPI	H_2	H_3	H_3	H_2	H_3	H_3
Wages	n<91: H_1 else: H_2	n<70: H_1 else: H_2	H_2	n<76: H_1 else: H_2	n<63: H_1 else: H_2	H_2
Real Wages	H_2	n<14: H_3 else: H_2	H_3	n<21: H_1 else: H_2	n<21: H_3 else: H_2	H_3
Money Stock	n<95: H_1 else: H_2	n<6: H_1 else: H_2	n<3: H_3 else: H_2	n<95: H_1 else: H_2	n<59: H_1 else: H_2	n<6: H_3 else: H_2
Velocity	H_2	n<16: H_3 else: H_2	H_3	H_2	n<67: H_3 else: H_2	H_3
Bond Yield	n<43: H_1 else: H_2	H_2	n<46: H_3 else: H_2	n<41: H_1 else: H_2	H_2	n<29: H_3 else: H_2
Stock Pri-ces	n<46: H_1 else: H_2	H_2	n<85: H_3 else: H_2	n<53: H_1 else: H_2	H_2	H_3

Table 6: Results of Decision Analysis Using $I_d^{n,2}$ (n=2,..,100)

		Uniform Prior for ρ			Student Prior for ρ	
	$\delta=1$	$\delta=10$	$\delta=100$	$\delta=1$	$\delta=10$	$\delta=100$
Real GNP	$n<67$: H_1 else: H_2	$n<49$: H_1 else: H_2	H_2	$n<83$: H_1 else: H_2	$n<67$: H_1 else: H_2	H_2
Nominal GNP	$n<62$: H_1 else: H_2	H_2	$n<13$: H_3 else: H_2	$n<70$: H_1 else: H_2	$n<25$: H_1 else: H_2	$n<10$: H_3 else: H_2
Real per cap. GNP	$n<81$: H_1 else: H_2	$n<62$: H_1 else: H_2	H_2	$n<85$: H_1 else: H_2	$n<67$: H_1 else: H_2	H_2
Ind. Prod.	$n<82$: H_1 else: H_2	$n<65$: H_1 else: H_2	H_2	$n<85$: H_1 else: H_2	$n<72$: H_1 else: H_2	H_2
Employ-ment	$n<80$: H_1 else: H_2	$n<57$: H_1 else: H_2	H_2	$n<68$: H_1 else: H_2	$n<56$: H_1 else: H_2	$n<7$: H_1 else: H_2
Unempl. Rate	$n<80$: H_1 else: H_2	$n<63$: H_1 else: H_2	$n<41$: H_1 else: H_2	$n<62$: H_1 else: H_2	$n<49$: H_1 else: H_2	H_2
GNP De-flator	H_2	H_2	H_3	$n<64$: H_1 else: H_2	$n<4$: H_3 else: H_2	H_3
CPI	H_2	H_3	H_3	H_2	H_3	H_3
Wages	H_1	$n<78$: H_1 else: H_2	H_2	$n<83$: H_1 else: H_2	$n<69$: H_1 else: H_2	H_2
Real Wages	H_2	$n<13$: H_3 else: H_2	H_3	$n<16$: H_1 else: H_2	$n<20$: H_3 else: H_2	H_3
Money Stock	H_1	$n<6$: H_1 else: H_2	$n<3$: H_3 else: H_2	H_1	$n<65$: H_1 else: H_2	$n<5$: H_3 else: H_2
Velocity	H_2	$n<15$: H_3 else: H_2	H_3	H_2	$n<26$: H_3 else: H_2	H_3
Bond Yield	$n<47$: H_1 else: H_2	H_2	$n<33$: H_3 else: H_2	$n<44$: H_1 else: H_2	H_2	$n<23$: H_3 else: H_2
Stock Pri-ces	$n<50$: H_1 else: H_2	H_2	$n<58$: H_3 else: H_2	$n<56$: H_1 else: H_2	H_2	$n<71$: H_3 else: H_2

Section 7: Conclusions

The paper develops a formal decision theoretic approach to testing for unit roots which involves the use of a loss function based on predictive variances. It also extends the class of likelihood functions in the Bayesian unit root literature by using a likelihood function which is a mixture over submodels which differ in covariance structure and in the treatment of structural breaks. Each of the individual likelihoods mixed into the overall likelihood function belongs to the class of general elliptical densities.

Our empirical results indicate that a high posterior probability of trend-stationarity exists for most of the economic time series. However, if there is a high cost to underestimating predictive variances, our decision analysis indicates that trend-stationarity is not necessarily the preferred choice.

Data Appendix

The data used in this paper are that of Nelson and Plosser (1982) updated to 1988 by Herman van Dijk. Primary data sources are listed in Schotman and van Dijk (1991b). All data are annual U.S. data. We take natural logs of all series except for the bond yield. The fourteen series are:

1) Real GNP (1909-1988).
2) Nominal GNP (1909-1988).
3) Real per capita GNP (1909-1988).
4) Industrial production (1860-1988).
5) Employment (1890-1988).
6) Unemployment rate (1890-1988).
7) GNP deflator (1889-1988).
8) Consumer Price Index (1860-1988).
9) Nominal wages (1900-1988).
10) Real wages (1900-1988).
11) Money stock (1889-1988).
12) Velocity (1869-1988).
13) Bond yield (1900-1988).
14) Common stock prices (1871-1988).

Prior Appendix

The Appendix discusses the selection of the bounded uniform priors for d_μ and d_β in (12). We use symmetric priors for all cases ($A_1 = -A_2$ and $B_1 = -B_2$) and set $A_2 = \varsigma_1 y_{q-1}$ and $B_2 = \varsigma_2 (y_T - y_0)/T + 1$. Since a level break of 10% is deemed to be highly unlikely, we set $\varsigma_1 = .10$ for all series except the bond yield and unemployment rate (for these series $\varsigma_1 = .4$). ς_2 is more difficult to elicit. Looking at $(y_T - y_0)/T + 1$, we set $\varsigma_2 = .1$ for real GNP, wages, employment, industrial production, money stock, and GNP per capita; $\varsigma_2 = .2$ for nominal GNP; $\varsigma_2 = .4$ for the Consumer Price Index and the GNP deflator; $\varsigma_2 = 1$ for real wages, velocity, unemployment and common stock prices; and $\varsigma_2 = 4$ for the bond yield. For no series is the posterior mean close to any of these boundaries.

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