Tilburg University

How Homogeneous Should a Team Be?

Prat, A.

Publication date:
1998

Link to publication in Tilburg University Research Portal

Citation for published version (APA):
Prat, A. (1998). *How Homogeneous Should a Team Be?* (CentER Discussion Paper; Vol. 1998-45). CentER.

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Take down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Download date: 06. Oct. 2022

CBM
R
8414
1998
nr.45

ntER
for
omic Research

Discussion paper

R 8414
1998
45

[redacted]

R 36

t organizational theory

D21
L20
M12

* C I N 0 1 7 1 7 *

Tilburg University

Center
for
Economic Research

No. 9845

HOW HOMOGENEOUS SHOULD A TEAM BE?

By Andrea Prat

April 1998

ISSN 0924-7815

How Homogeneous Should a Team Be?

Andrea Prat*
Department of Econometrics and Center for Economic Research
Tilburg University†

April 28, 1998

Abstract

Should an organization hire people with similar backgrounds or with different backgrounds? We formulate this question within the framework of team theory. The team manager must fill n jobs and can choose the type of each of the agents she hires. The type of an agent determines his information structure and his market wage. We show that if the team's payoff function is supermodular, then the manager finds it optimal to hire n agents of the same type. On the other hand, if the payoff function is submodular, and if two additional assumptions hold, the manager hires agents of at least two different types. These results do not rely on restrictions on the way uncertainty is modeled or on the feasible set of agent types.

Keywords: information structures, lattice theory, team theory, workforce homogeneity.

JEL Classification: D21, L20, M12.

*This is a revised version of a chapter of my Ph.D. dissertation. I have greatly benefited from discussions with Masahiko Aoki, Kenneth Arrow, and Dimitri Vayanos. I thank Jacques Crémer, Mathias Dewatripont, Joshua Gans, Thomas Hellmann, Marcos Lisboa, Paul Milgrom, Dolf Talman, Stef Tijs and audiences at Bocconi, IAE (Barcelona), Pompeu Fabra, Rochester, Stanford, Stockholm School of Economics, Texas A&M, Tilburg, and Toulouse (ESEM 1997) for their helpful comments.

†Address: B923, Department of Econometrics, Tilburg University, Postbus 90153, 5000 LE Tilburg, The Netherlands; Email: a.prat@kub.nl.

1 Introduction

The backgrounds of the people who work in an organization are not exogenously given. The organization (be it a firm, a government agency, a nonprofit, etc.) chooses whom to hire. This paper is concerned with one dimension of the hiring policy: the degree of variety in the backgrounds of the people who are hired.

On this dimension, different organizations can adopt strikingly different policies. Some organizations pursue a policy of hiring people with homogeneous backgrounds, while other organizations actively seek a degree of background diversity in their workforce. Perhaps, the most extreme example of homogeneous organization is the army. In most countries, officers are trained in a small number of schools under the direct control of the army. After undergoing a long, common instruction period, they are expected to share an extensive body of knowledge and a clear code of behavior.[1]

On the other hand, a familiar example of an organization that favors workforce heterogeneity is a university department. It would be surprising to encounter a department in which all faculty have been educated in the same institution, the way it happens in the army. Most departments hire from a number of schools and have an explicit policy not to hire their own Ph.D. graduates altogether. The general feeling in academe is that too much homogeneity is bad for research. While the army and the department are two extreme cases, a variety of practices can be observed across firms as well. Some firms hire people with quite similar profiles, while other firms strive to achieve a high degree of diversity.

What characteristics of an organization determines its optimal degree of background homogeneity? Clearly, this is a complex question and one could try to approach it from several angles. Motivational factors, incentive issues, the need for secrecy all play a role in this choice. This paper, however, will restrict its attention to one factor: informational efficiency. Even when examined in isolation, informational efficiency constitutes a highly complex problem. To formalize it, we use the concepts of team theory (developed by Marschak and Radner [14]), which can be regarded as a theory of decision making with multiple decision makers and endogenous information structures.

The problem is modeled as follows. An organization is made of n jobs (n is given exogenously). The organization manager must hire n agents to fill the jobs. There is a large, competitive labor supply which comprises agents of several types. The type of an agent

[1] For a discussion on the value of homogeneity in military organization, see the famous treaty by Von Clausewitz [6, Book 2, Chapter "Methods and Routines"].

determines his information structure – the most important concept of this paper. The information structure is the grid through which the agent observes the world. Mathematically, it is a mapping from the state of the world (a random variable not directly observed) into a signal available to the agent. For instance, an agent with the type "doctor" has an information structure which, when confronted with a patient, provides him with a signal on the patient's health. The agent's type of an agent also determines the cost of hiring that agent. If the manager wants to hire an agent with the type "doctor," she has to pay him the market wage for doctors. For each job, the manager can hire any agent available on the market.

Once agents are hired, the manager instructs them on how to respond to each signal they may receive.[2] In other words, the manager endows each agent with a decision function. Of course, the decision function can vary from agent to agent. When the state of the world is realized, agents observe their signals through their information structures and choose their actions through their decision functions. The gross payoff to the organization is a function of the state of the world and of the actions taken by the agents. The ultimate goal of the team manager is to choose a type and a decision function for each of the n jobs in order to maximize the expected value of the gross payoff minus the sum of wages paid to agents. Figure 1 depicts the problem for an organization in which $n = 2$.

We first assume that the payoff function is anonymous in the agents' actions. This means that the total payoff does not depend on the job labels the agents carry but only on the actions they choose. With a sport analogy, this assumption implies that the number on a player's shirt is immaterial in determining the outcome of the player's actions (true in basketball; false in soccer because of the goal-keeper's special status).

With this assumption, we prove the two central results of the paper. First, if the agents' actions are complements in the payoff function, then the set of optimal solutions contains a solution in which all agents have the same type. In that case, the organization designer can restrict her attention to configurations in which she hires only one type of agents. Second, if two additional assumptions hold (concavity and nonuniqueness of the single-type optima), we prove that, if the agents' actions are substitutes in the payoff function, then the set of

[2] In contrast to most of the recent economic literature on organizations, this paper is not directly concerned with incentive issues. We assume that there is no moral hazard on the part of agents: if an agent is given a decision function, he will follow it. However, the present model can be seen as a reduced form of a model with moral hazard in which non-incentive-compatible decision functions have already been deleted from the set of feasible decision functions and the relative costs have been incorporated into the payoff function.

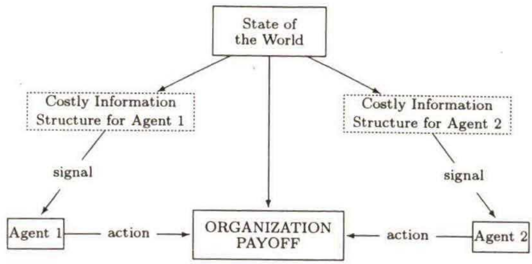

Figure 1: An Organization with Two Agents

optimal solution contains at least one solution in which at least two types of agents are hired. Thus, the crucial notion is that of complementarity, which is represented by the lattice theory concept of supermodular function.[3]

The intuition behind the two results above is provided by another proposition. Consider a function the argument of which is a vector of random variables. We show that if the function is supermodular, then the expected value of the function is higher if the random variables are perfectly correlated rather than stochastically independent. On the other hand, if the function is submodular, the expected value is higher if the random variables are stochastically independent. In the problem at hand, agents do not in general have perfect information. Thus, they are bound to deviate from the full-information solution. If their actions are complements, it is optimal for them to deviate in a coordinated manner, which occurs if their information structures are identical. Thus, hiring agents of the same type is optimal. An analogous line of reasoning can be followed when the agents' actions are substitutes.

As the intuition is general, one would expect the results to hold in a very general setting. Indeed they do. In this paper, no particular functional form is assumed for the payoff function. Moreover, the set of possible states of the world and the probability distribution on it are defined in a general way. Finally, no assumption is made on the set of agents' type, on information structures, or on decision functions.

As an extension, we discuss what happens if the payoff function is non-anonymous.

[3]See for instance Milgrom and Shannon [17]

Clearly, ex ante differences between jobs drive the optimal solution towards heterogeneity. If two positions are intrinsically different, the team will not hire the same type of worker for both positions. However, in a specialized model, it is possible to recover a restricted notion of workforce homogeneity and to find a parallel with the results above. This more limited model is of applied interest because it captures some crucial characteristics of multinational organizations. We show that a multinational with positive complementarity tends to staff offices across the world with workers of the same type, even though different offices will be affected by independent environments.

In conclusion, based on our results, one can predict that organizations in which agents actions are complements will have a homogeneous workforce, while organizations in which actions are substitutes will have a heterogeneous workforce. This prediction appears to be fulfilled in the extreme examples of the army and the university.

In the case of the army, the success of an operation depends on how well the troops coordinate. If soldiers behave in contradictory ways there can be disastrous consequences. As Von Clausewitz [6, p. 153] notes, "A battalion is made up of individuals, the least important of whom may chance to delay things or somehow make them go wrong." The actions of agents seem to be complementary. Indeed, as our results predict, the army employs agents with similar backgrounds. Soldiers of the same army receive a highly homogeneous instruction, so that in combat they will interpret contingencies uniformly and will respond in a harmonious manner. The value of teaching troops common rules for the interpretation of contingencies is exalted by Von Clausewitz [6, p. 152]:

> Cooking in the enemy camp at unusual times suggests that he is about to move. The intentional exposure of troops in combat indicates a feint. This manner of inferring the truth may be called a rule because one deduces the enemy's intentions from a single visible fact connected with them.
>
> If the rule enjoins that one should resume attacking the enemy as soon as he starts to withdraw his artillery, then a whole course of action is determined by a single phenomenon which has revealed his entire condition: the fact that he is ready to give up the fight. While he is doing so, he cannot offer serious resistance or even avoid action as he could once he is fully on the move.
>
> To the extent that *regulations* and *methods* have been drilled into troops as active principles, theoretical preparation for war is part of its active conduct. All standing instructions on formations, drill, and field-service are regulations and methods. Drill instructions are mainly regulations; field manuals, mainly

> methods. The actual conduct of war is based on these things; they are accepted as given procedures and as such must have their place in the theory of the conduct of war.

On the other hand, one of the two main activities of universities is research. In search problems with multiple agents the agents' actions are typically substitutes. This is because the expected benefit of an additional agent searching in a certain direction is decreasing in the number of agents who are already searching in the same direction.[4] Our results predict that an organization with negative complementarities will hire agents with differnt backgrounds. This may explain the emphasis departments put on diversity and their avoidance of internal or highly homogeneous hiring practices.

The plan of the paper is as follows. Section 2 introduces the model. Section 3 reports the main results. Section 4 provides intuition for the main results based on the idea of error correlation. Section 5 discusses an extension to non-enonymous payoff functions. Section 6 concludes.

Related Literature As stated above, we adopt the team-theoretical framework. Team theory was developed in the Sixties by Marschak and Radner [14]. In the Seventies it gave rise to a literature on the possibility of decentralizing decision-making, such as Groves and Radner [12] and Arrow and Radner [4]. After more than a decade of limited use – which coincided with the development of principal-agent theory – team theory has been experiencing a renewed interest. Several authors have applied it to problems in organization theory that do not seem to find a satisfactory answer within the principal-agent framework. Examples are Aoki [1, 2], Crémer [7, 8, 9], Geanakoplos and Milgrom [11], Li [13], Ponssard, Steinmetz, and Tanguy [18], and Qian, Roland, and Xu [19].

Crémer [8, 9] applies team theory to the problem of workforce homogeneity, which he labels "shared knowledge." He considers a team with a quadratic objective function. The coefficient of the linear term is unknown and represents the state of the world. The state of the world is a normally distributed random variable. Each agent observes the state of the world plus a normally distributed disturbance. Crémer considers two cases: (1) the disturbances are identical across agents (shared knowledge) and (2) the disturbances are uncorrelated across agents (diversified knowledge).

[4] This may not apply to fields in which research activity displays strong economies of scale, such as medicine.

The original contribution of the present paper is to extend Crémer's problem beyond a particular formulation. This generalization is of interest in itself, but is especially valuable because it allows us to identify the sign of complementarities as the main driving force in the choice between a homogeneous and a heterogenous workforce. This finding appears to be new in economics and management science.

The assumptions made by Crémer – quadratic payoff function and normally distributed signals – are common to all the recent team-theoretical literature. Although those assumptions are quite restrictive, they are made because they allow for a closed-form solution. An incidental contribution of the present paper paper is to show that those assumptions are not needed. We formulate an organizational problem in a general way. Finding a closed-form solution is impossible and is not attempted. Instead, we apply lattice theory concepts and we study the set of optimal solution. This is sufficient to answer the question we are interested in and to generate testable implications. Hopefully, this methodology can be used to study other questions in organization theory that are still open.[5]

2 The Model of a Team

Consider a team designer who knows what the activity of the team will be but has not yet hired agents to carry out the activity. For instance, the team designer could be the manager of a soccer team at the beginning of the season. The rules of the game and the team payoff function are given. The number of team members (disregarding substitutes) is fixed. The only thing the manager needs to do is to hire eleven players from the market for soccer players and give them a game strategy.

The building blocks of the model are as follows.

Uncertainty The stochastic aspect of the model is captured by the *state of the world* $x \in X$. The state of the world is not observed directly. The team designer has prior distribution $\phi : X \rightarrow \Re$.

[5]This paper differs from the industrial organization literature on information sharing in oligopoly (See, among others, Gal-Or [10] and Vives [21]). Those works examine the incentives of oligopolists to communicate to each other the private signals they have received. The choice between sharing or not sharing information is dictated by strategic consideration. In contrast, there are no strategic considerations in the present paper. As agents do not have conflicting interests, they *always* have an incentive to share information with each other.

Payoff A team is composed of n slots and its payoff depends on the decisions taken by the agents who occupy the slots and on the state of the world. The *payoff function* is given by

$$\omega(a_1, \ldots, a_n, x) \tag{1}$$

where $a_i \in A$ represents the action taken by the agent who occupies slot i (whom we will call Agent i whenever doing so will not generate confusion between jobs and agents). A is an ordered set. No assumption is made on the form of ω except the following:

Assumption 1 (Anonymity) *For all $x \in X$,*

$$\omega(a_1, \ldots, a_n, x) = \cdots = \omega(a_n, \ldots, a_1, x) \tag{2}$$

for any rearrangement of the action indices.

Assumption 1 is an anonymity condition. In our sport analogy, the assumption has a simple meaning. Before players are hired and the game strategy is decided, the team consists of n slots. Each slot is just a t-shirt with a number on it. Then, n players with different characteristics are hired, and each is given a t-shirt. The anonymity condition says that when a player takes an action, the number on his t-shirt is immaterial. This is true in basketball (the rules are independent of the players' numbers) but false in soccer (if a player touches the ball with his hands, it matters whether he wears number 1 – the goalkeeper's number – or another number). Thus, Assumption 1 excludes that there are exogenously specified roles for team members. Of course, specialization can occur ex-post if the team players have different background or if they are given different decision functions. For instance, in basketball it is common to hire people with different abilities and to assign them different roles.[6] Of course, in the case of soccer, if we assume that the characteristics of the goalkeeper are given, we could restrict our attention to the other ten players and restore the anonymity assumption.

Types and Information Structures So far the team has been described as a payoff function with n slots to fill. Let us now model how the team designer can fill these slots. There exists a pool of agents available for hire. Agents differ according to their type $\theta \in \Theta$.

[6] In a business environment, a clear violation of Assumption 1 occurs when workers are physically separated. An example is represented by a decentralized sales force. If the action of the agent in the "Amsterdam" slot is switched with the action of the agent in the "Shanghai" slot, the team's payoff will in general change. This case is studied, in a more limited framework, in Section 5.

For each θ there are a large number of agents available for hire. Each agent of type θ has an information structure $\eta_\theta : A \to Y$. The assumption that Y does not depend on θ is without loss of generality.[7] The set of possible information structures is denoted by $\mathcal{H}$. Thus there exists a one-to-one correspondence between Θ and $\mathcal{H}$.

If the state of the world is x, an agent of type θ receives a signal

$$y = \eta_\theta(x)$$

The function η_θ induces a partition $\mathcal{P}_\theta$ on the set X. This corresponds to the standard definition of information structure (see Marschak and Radner [14, p. 48-49]).[8]

Let us introduce the following notation: θ_i represents the type of the agent that is hired to fill slot i. Let y_i denote the signal of the agent who fills slot i. Then

$$\begin{aligned} y_1 &= \eta_{\theta_1}(x) \\ \dots & \quad \dots \\ y_n &= \eta_{\theta_n}(x) \end{aligned}$$

for all $x \in X$.

Cost To hire an agent of type θ, the team must pay a wage c_θ. Without loss of generality we can disregard any cost which is unrelated to the agent types. Hence, the team's total cost is

$$C(\theta_1, \dots, \theta_n) = \sum_{i=1}^{n} c_{\theta_i}$$

Decision Functions The team tells each agent how to behave given the signal he has received. The agent in slot i is instructed to follow *decision function* $\alpha_i : Y \to A$. The decision function must be taken from a set of feasible decision functions denoted with $\mathcal{A}$.

[7] Suppose that Y_θ denotes the set of possible signals received by an agent of type θ. We just need to let $Y = \bigcup_{\theta \in \Theta} Y_\theta$.

[8] Notice that X can be augmented to accomodate noisy information structures. For instance, suppose that $X = \Re^{m+1}$ and $x = (A, \epsilon_1, \dots, \epsilon_m)$. Assume that the random variables $\epsilon_1, \dots, \epsilon_m$ do not enter the payoff function directly. $Y = \Re$. $\Theta = \{1, \dots, m\}$. Let

$$\eta_\theta = A + \epsilon_\theta$$

Then, the set of feasible information structures is the same as in Crémer [8, 9] and other team-theoretical models. The signal of an agent is the true state plus a disturbance: this disturbance is assumed to be identical across agents of the same type and uncorrelated otherwise.

It is a list of instructions such as "if you receive signal y, then you should take action a." Of course, the manager chooses the decision function for slot i after she has chosen the type of the agent that fills slot i. Thus, the choice of the decision function can depend on the agent type.

Therefore, slot i is filled with an agent of type θ_i who is instructed to behave according to α_i. Thus, the actions of the n agents are determined as follows:

$$\begin{aligned} a_1 &= \alpha_1[\eta_{\theta_1}(x)] \\ \vdots & \quad \vdots \\ a_n &= \alpha_n[\eta_{\theta_n}(x)] \end{aligned}$$

Two remarks are in order. First, we have assumed that the act of choosing an action has no cost. However, suppose that some actions are more costly than others. Such a situation could be accomodated by incorporating those action costs into the payoff function ω. Therefore, there is no loss of generality in assuming that all actions have the same cost and that that cost is zero. Second, we have not considered the problem of moral hazard. Suppose that agents find some actions more costly than others, but that neither the cost incurred nor the action taken can be contracted upon. According to the nature of the information asymmetry between the designer and the agents, some actions will not be feasible and other actions are feasible only if the designer leaves a rent to the agents. These two problems can be taken care of by deleting the unfeasible elements from the set of feasible decision functions and by incorporating the informational rent into ω or c_i. Thus, the present model can be seen as a reduced form of a more general model which includes moral hazard. However, as our conclusion only depend on informational considerations, we choose to focus on the reduced form.

No Communication We have made the implicit assumption that agents cannot communicate with each other between the time they receive their signals and the time they choose their actions. If they could communicate, then the decision function of agent i would not depend only on y_i but also on the messages he receives from the other agents. As complete communication is in general not feasible, this model can be interpreted as a reduced form of a model in which everything that could have been communicated has already been communicated.

To summarize, the givens of the *team designer problem* are: a set of states of the world (X), the prior distribution (ϕ), the payoff function (ω), a set of actions (A), a set of information structures ($\mathcal{H}$), a set of agent types (Θ), a set of decision functions ($\mathcal{A}$), and the wage function (c). The team must select, for each slot i, an agent type θ_i and his decision function α_i. This makes a total of $2n$ choices (which we will refer to as the team's *configuration*). The goal of the team designer is to maximize the expected payoff less the wages paid:

$$\max_{\{\theta_i \in \Theta, \alpha_i \in \mathcal{A}\}_{i=1,\ldots,n}} E\{\omega[\alpha_1(\eta_{\theta_1}(x)), \ldots, \alpha_n(\eta_{\theta_n}(x)), x]\} - \sum_{i=1}^{n} c_{\theta_i}$$

In the remainder of the paper, we assume that the team problem has at least one solution.

3 General Results

3.1 Defining Complementarities

To represent complementarities, we adapt the general definition of supermodular and submodular functions to the problem at hand (See for instance Milgrom and Shannon [17]):

Definition 1 *The payoff function ω is supermodular in the agents' actions if, for any two vectors $(\hat{a}_1, \ldots, \hat{a}_n) \in A^n$ and $(\check{a}_1, \ldots, \check{a}_n) \in A^n$ and for all $x \in X$, the following holds*

$$\begin{aligned} &\omega(\hat{a}_1, \ldots, \hat{a}_n, x) + \omega(\check{a}_1, \ldots, \check{a}_n, x) \\ &\leq \omega[\min(\hat{a}_1, \check{a}_1), \ldots, \min(\hat{a}_n, \check{a}_n), x] + \omega[\max(\hat{a}_1, \check{a}_1), \ldots, \max(\hat{a}_n, \check{a}_n), x]. \end{aligned}$$

Conversely, ω is submodular in the agents' actions if, given any two vectors $(\hat{a}_1, \ldots, \hat{a}_n) \in A^n$ and $(\check{a}_1, \ldots, \check{a}_n) \in A^n$, for all $x \in X$, the following holds

$$\begin{aligned} &\omega(\hat{a}_1, \ldots, \hat{a}_n, x) + \omega(\check{a}_1, \ldots, \check{a}_n, x) \\ &\geq \omega[\min(\hat{a}_1, \check{a}_1), \ldots, \min(\hat{a}_n, \check{a}_n), x] + \omega[\max(\hat{a}_1, \check{a}_1), \ldots, \max(\hat{a}_n, \check{a}_n), x]. \end{aligned}$$

How does supermodularity relate to the notion of complementarity based on cross-derivatives? The latter definition is applicable only if the function is twice-differentiable. Topkis [20, Th. 3.2] shows that, if a function is twice-differentiable, then the function is supermodular if and only if the second-order cross derivatives are all nonnegative (in the present case: $\partial^2\omega/\partial a_i \partial a_j \geq 0$ for $i \neq j$), while the function is submodular if and only if the cross derivatives are all nonpositive. Thus, supermodularity is a generalization of the traditional notion of complementarity. Its use derives from the fact that in many problems the second-order cross derivative is not well-defined (for instance, because the agent's action is a discrete variable).

The following result (proven in the appendix) will be used repeatedly in the paper.

Lemma 1 *Given an ordered set A and a vector $a = (a_1, \ldots, a_n) \in A^n$, define $P(a)$ as the set of all vectors obtained by permuting elements of a. Consider $f : A^n \to \Re$. If f is supermodular (submodular), then*

$$\frac{1}{n!} \sum_{(p_1,\ldots,p_n)\in P(a)} f(p_1, \ldots, p_n) \leq (\geq) \frac{1}{n} \sum_{i=1,\ldots,n} f(a_i, \ldots, a_i)$$

Consider a function, the arguments of which are all defined on the same ordered set. Take a particular vector of arguments. If the function is supermodular, then the average value of the function for all possible permutations of the vector is smaller or equal to the average value of the function of vectors in which all elements are equal to one of the arguments of the initial vector. The converse holds if the function is submodular.

The following is immediate:[9]

Corollary 1 *Suppose f is such that is symmetric, that is, $f(a_1, \ldots, a_n) = \cdots = f(a_n, \ldots, a_1)$ for any rearrangement of the vector a. If f is supermodular (submodular), then*

$$f(p_1, \ldots, p_n) \leq (\geq) \frac{1}{n} \sum_{i=1,\ldots,n} f(a_i, \ldots, a_i)$$

3.2 Sufficient Condition for the Optimality of Workforce Homogeneity

It is now possible to state the main result of this paper. Provided that the team's problem has at least one optimal solution and provided that Assumption 1 holds, we have the following:

Proposition 1 *If ω is supermodular in the agents' actions, then the set of solutions to the team designer problem contains at least one configuration in which $\theta_1 = \ldots = \theta_n$.*

[9]Corollary 1 has been proven in a direct way by Meyer and Mookherjee [15, Proposition 1]. To the best of my knowledge, Lemma 1 – which is of independent interest – is new. The Corollary is sufficient to prove the results in Sections 3.2 and 3.3. The Lemma is needed for Sections 4 and 5.

Proof Suppose ω is supermodular in the agents' actions. We will prove that, for any configuration in which not all agents have the same type, we can find a configuration which gives a greater or equal expected payoff and in which all agents have the same type.

Consider a feasible choice of types, denoted with $\theta_1, \ldots, \theta_n$, and a feasible choice of decision functions, denoted with $\alpha_1(\cdot), \ldots, \alpha_n(\cdot)$. Notice that

$$\omega[\alpha_1(\eta_{\theta_1}(x)), \ldots, \alpha_n(\eta_{\theta_n}(x)), x]$$

is supermodular in $\alpha_1(\eta_{\theta_1}(x)), \ldots, \alpha_n(\eta_{\theta_n}(x))$ for all $x \in X$. By Corollary 1 and Assumption 1, we have that, for all $x \in X$,

$$\begin{aligned}&\frac{1}{n}\{\omega[\alpha_1(\eta_{\theta_1}(x)), \ldots, \alpha_1(\eta_{\theta_1}(x)), x]\\&+\ldots+\omega[\alpha_n(\eta_{\theta_n}(x)), \ldots, \alpha_n(\eta_{\theta_n}(x)), x]\}\\&\geq \omega[\alpha_1(\eta_{\theta_1}(x)), \ldots, \alpha_n(\eta_{\theta_n}(x)), x]\end{aligned}$$

Take expectations over the set of possible states and add the total cost of wages on both sides

$$\begin{aligned}&\frac{1}{n}(E\{\omega[\alpha_1(\eta_{\theta_1}(x)), \ldots, \alpha_1(\eta_{\theta_1}(x)), x]\} + nc_{\theta_1}\\&+\ldots+E\{\omega[\alpha_n(\eta_{\theta_n}(x)), \ldots, \alpha_n(\eta_{\theta_n}(x)), x]\} + nc_{\theta_n})\\&\geq E\{\omega[\alpha_1(\eta_{\theta_1}(x)), \ldots, \alpha_n(\eta_{\theta_n}(x)), x]\} + \sum_{i=1}^{n} c_{\theta_i}\end{aligned}$$

Then for at least one of the types, say θ_k,

$$\begin{aligned}&E\{\omega[\alpha_k(\eta_{\theta_k}(x)), \ldots, \alpha_k(\eta_{\theta_k}(x)), x]\} + nc_{\theta_k}\\&\geq E\{\omega[\alpha_1(\eta_{\theta_1}(x)), \ldots, \alpha_n(\eta_{\theta_n}(x)), x]\} + \sum_{i=1}^{n} c_{\theta_i}\end{aligned}$$

Thus, for any feasible choice of types and of decision functions, there exists a feasible choice of types and decision functions in which $\theta_1 = \ldots = \theta_n$ who does at least as well. Then, if the set of solutions to the team designer problem is not empty, as we have assumed, it must contain a solution in which $\theta_1 = \ldots = \theta_n$.

The intuition behind Proposition 1 is that if the payoff function is supermodular then the team is better off if agents commit correlated errors rather than uncorrelated errors. We will explore this theme in Section 4.[10]

[10] Suppose instead that the payoff function is *strictly* supermodular, that is, that '$<$' replaces '$\leq$' in the

Proposition 1 says that, among all the possible solutions to the team's problem, there is at least one in which all agents have the same type. It does not compute which, among all the solutions of that kind, is the optimal one. However, a proposition like 1 greatly simplifies the task of designing an optimal organizational structure. The organization designer can, without loss of generality, focus on solutions in which all agents have the same type. For example, if there are 20 agents and 4 information structures, the number of possible configurations – counting symmetric structures only once – is 8855. However, by applying Proposition 1, the organization designer knows that the number of configurations she needs to check is just 4.

The following example illustrates the use of Proposition 1:

Example 1 (A Product Made of Two Components): Consider a firm made of two divisions. Agent 1 is the manager of Division 1 and Agent 2 is the manager of Division 2. The final product of the firm is obtained by assembling a component produced by Division 1 and a component produced by Division 2. Agent 1 decides a_1, the quantity produced by Division 1, and Agent 2 decides a_2, the quantity produced by Division 2. Because each product needs both components, the number of items produced is $\min(a_1, a_2)$. The firm faces an inelastic demand curve. It can sell up to x products at a unit price p. If it produces more than x products, the excess will be unsold. Therefore, the number of products sold is the minimum between the number of products produced and the number of products demanded: $\min(a_1, a_2, x)$. Demand depends on the state of the world represented by the real random variable x with a given probability distribution $p(x)$. The unit cost is the same for both components: k (let us assume that $k < 0.5p$). The payoff function of the firm is

$$\omega(a_1, a_2, x) = p \min(a_1, a_2, x) - k(a_1 + a_2) \tag{3}$$

The structure of the problem suggests that the agents' actions are complements. Indeed, it can be verified that the payoff function (3) is supermodular in a_1 and a_2 for all x (See the Appendix for a formal verification). Therefore, we can apply Proposition 1: the set of optimal solutions contains at least one solution in which the type of the agent in slot 1 is the same as the type of the agent in slot 2. This result is independent of the distribution of x and of the feasible set of types.

definition of supermodular function given in Section 4. Then, it is easy to see that, for any solution in which $\theta_1 = \ldots = \theta_n$ does not hold, it is almost always possible to find at least one solution in which $\theta_1 = \ldots = \theta_n$ and which yields a strictly higher expected payoff. Then, the set of optimal solutions will generically contain *only* configurations in which $\theta_1 = \ldots = \theta_n$.

3.3 Sufficient Condition for the Optimality of Workforce Heterogeneity

This subsection presents a partial parallel of Proposition 1 for workforce heterogeneity. Two additional assumptions are necessary. First,

Assumption 2 (Concavity of the Payoff Function) *The set A is convex and the payoff function ω is concave in $a_1, a_2, \ldots, a_n$.*

Assumption 2 does two important things. First, by assuming that the action space is convex, it avoids the possibility that agents have to coordinate on an asymmetric solution because the symmetric solution is not feasible. Second, by assuming the concavity of the payoff function, it guarantees that the team is risk-averse. A risk-loving manager may want to hire homogeneous agents in order to coordinate on riskier actions.

The second additional assumption excludes situations in which one type of worker is superior to all other types:

Definition 2 *A one-type optimum is a solution to*

$$\max_{\theta^*, \{\alpha_i^*\}_{i=1,\ldots,n}} E\{\omega[\alpha_1^*(\eta_{\theta^*}(x)), \ldots, \alpha_n^*(\eta_{\theta^*}(x)), x]\} - n\theta^*$$

Assumption 3 (Nonuniqueness of One-Type Optima) *There exist two distinct values θ^* and θ^{**} and two sets of decision functions $\{\alpha_i^*\}_{i=1,\ldots,n}$ and $\{\alpha_i^{**}\}_{i=1,\ldots,n}$ such that $(\theta^*, \{\alpha_i^*\}_{i=1,\ldots,n})$ and $\theta^{**}, \{\alpha_i^{**}\}_{i=1,\ldots,n})$ are both one-type optima.*

Assumption 3 considers a restricted problem. Suppose the team can only hire agents of one type: which type of agents would it hire? The assumption requires that there are at least two optimal types. Without this assumption, it could be the case that a type of agent is strictly 'better' than the others. This would imply that workforce homogeneity is optimal in a trivial way. On the contrary, in reality, the labor supply is heterogeneous and, for any given profile, it comprises several types of workers, none of which clearly dominates the other.[11]

Of course, Assumptions 2 and 3 do not imply that heterogeneity is the optimal solution. In particular, if the payoff function is strictly supermodular, then all optimal configurations still require full homogeneity, as predicted by Proposition 1.

With Assumptions 1 through 3, the following holds:

[11] For instance, if a department wants to hire a faculty, it can choose from graduates of various graduate schools. There will be several schools with similar rankings. However, within the set of schools in the same ranking, there may be large differences in terms of focus or style.

Proposition 2 *If ω is submodular, then the set of optimal solutions contains at least one solution in which it is not true that $\theta_1 = \ldots = \theta_2$.*

To illustrate the scope of Proposition 2,, consider the following:

Example 2 (A Search Problem): Consider a team of two researchers: 1 and 2. There are two possible fields of research: Left and Right. Only one of the fields is promising, but the researchers do not know which. Let z be a random variable which can assume the values 0 and 1 with equal probability. If $z = 0$, Left is promising. If $z = 1$, Right is promising. Each agent works for a unitary amount of time. The agent in slot i chooses $a_i \in [0, 1]$, which represents the percentage of his work time that he devotes to searching Left. The percentage of time that he devotes to searching Right is given by $1 - a_i$.

Within a research field, there are decreasing returns to scale. For instance, assume that the probability of success is proportional to the square root of the total research time spent in that field. If the research is successful, the team receives a payoff of Q. The team's payoff can be written as:

$$\omega(a_1, a_2, y) = Qz\frac{\sqrt{2 - a_1 - a_2}}{\sqrt{2}} + Q(1 - z)\frac{\sqrt{a_1 + a_2}}{\sqrt{2}} \tag{4}$$

where the denominator $\sqrt{2}$ is used to normalize the probabilities.

Suppose that there are two types of reseachers: those educated in university A (denoted with θ_A) and those educated in university B (denoted with θ_A). Both types observe y with some error, but the errors are uncorrelated across types. Let the state of the world be $x = (y, \epsilon_A, \epsilon_B)$, where ϵ_A and ϵ_B are independently uniformly distributed on $[0, 1]$. For $\theta = A, B$,

$$\eta_\theta(z) = \begin{cases} z & \text{if } \epsilon_\theta \geq p \\ 1 - z & \text{if } \epsilon_\theta \geq p \end{cases}$$

where $p \in (0.5, 1]$ denotes the precision of the signal.

It is easy to check that the function in (4) is submodular in a_1 and a_2. Moreover, Assumptions 2 and 3 hold as well. Therefore, by Proposition 2, the search problem described here always has an optimal solution in which one agent is of type A and the other is of type B.

Search problems typically entail submodular payoff functions. The more one agent searches in a direction, the more the other agents should search in other directions. Of course, the team can always order the agents to spread equally on all possible directions.

However, this decision function is clearly not optimal because it does not take into account the agents' signals. The best thing the team designer can do is to hire agents who receive uncorrelated signals, so that agents can spread on different directions without renouncing their signals.[12]

4 Error Coordination

This section does not directly refer to the central theme of the paper. However, it illustrates a property of supermodular and submodular functions that is useful in interpreting the results of this paper. Consider a function of random variables. Suppose the random variables can be either perfectly codependent or mutually independent. This section proves that, if the function is supermodular, the expected value of the function is higher when the variables are perfectly codependent, while, if the function is submodular, it is higher when the variables are mutually independent. In the light of this result, we can provide some intuition on the results presented in Section 3.

Consider two random vectors:

$$y^I = (y_1, y_2, \cdots, y_n)$$
$$y^D = (y_0, y_0, \cdots, y_0)$$

where $y_0, y_1, y_2, \cdots, y_n$ are identically distributed, mutually independent random variables. Consider a function $f : \Re^n \to \Re$. The following can be proven.[13]

Proposition 3 *If f is supermodular, then $E[f(y^D)] \geq E[f(y^I)]$, while if f is submodular, then $E[f(y^D)] \leq E[f(y^I)]$.*

If a function is supermodular, then the expected value is higher in the case of correlated errors than in the case of uncorrelated errors. The opposite holds if the function is submodular. Although Proposition 3 is not used to prove Propositions 1 and 2, it provides intuition for those results.

Suppose the agents of an organization are bound to commit errors. The y's can be interpreted as the actions of the agents. The actions are random because the agent's signal

[12] For a discussion of the role of uncorrelated information in search problems, see also Bassan and Scarsini [5]. They consider a class of multi-agent search problems and demonstrate the value of heterogeneity based on the idea of experimentation externalities.

[13] Proposition 3 was conjectured by Milgrom and Roberts [16]. The proof is in the Appendix.

has a random disturbance. Assume that the organization designer cannot reduce the entity of errors, but can choose whether the errors are perfectly codependent or mutually independent. If the team payoff function is supermodular, the organization designer will want the errors to be perfectly codependent. Workforce homogeneity is a device to make the errors perfectly codependent. On the other hand, if the team payoff function is submodular, the organization designer will want the errors to be mutually independent and heterogeneity is a device to make errors mutually independent.

5 Non-anonymous Payoff Functions

In the main part of this paper we assumed that the team payoff function is anonymous, that is, it does not matter which agent takes which action, but only which actions are taken. Anonymity actually implies two restrictions: (i) the payoff function is symmetric in the agents' actions; (ii) the interaction between the state of the world and the action is symmetric across agents.

Restriction (i) is essential to our results. If the payoff function is asymmetric in the agents' actions, it means that slots are a priori differentiated. Then, it is clear that the team will want to hire people with differentiated backgrounds. Thus, relaxing (i) will push the team toward diversity. This is, however, hardly surprising. If jobs are heterogeneous from the start, a homogeneous solution is unlikely to be optimal.

On the other hand, restriction (ii) is not essential to our results. This section relaxes (ii) and considers local states of the world. Suppose that the state of the world is $x = (z_1, z_2, \epsilon_A, \epsilon_B)$ with $z_i \in \Re$ and $\epsilon_j \in \Re$. However, ϵ_A and ϵ_B do not enter the payoff function directly, so that

$$\omega(a_1, a_2, x) \equiv w(a_1, a_2, z_1, z_2)$$

For $i = 1, 2$, z_i represents the *local* state of the world for Agent i. We assume that w is twice continuously differentiable and concave. w has the following symmetry property:

$$w(a_1, a_2, z_1, z_2) = w(a_2, a_1, z_2, z_1) \tag{5}$$

Condition (5) implies restriction (i). If agents swapped both their local states and their actions, the payoff would not change. However (5) does not satisfy (ii). If the agents swapped their actions only the team payoff would in general change. Thus, anonymity is violated.

The local state of the world of one agent interacts with the action taken by that agent but not with the action taken by the other agent. This fact is represented by the following assumption

$$\frac{\partial^2 w}{\partial a_i \partial z_j} \begin{cases} > 0 & \text{if } i = j \\ = 0 & \text{if } i \neq j \end{cases} \tag{6}$$

Moreover, it is assumed that z_1 and z_2 are identically and independently distributed.

The set of possible agent types is $\Theta = \{A, B\}$. If the agent in slot i has type θ, he observes

$$y_i = \eta_\theta(x) = z_i + \epsilon_\theta$$

Agents of the same type have the same disturbance (This, of course, does not mean they receive the same signal, because they face two different local states of the world). Agents of different types have different disturbances which are assumed to be uncorrelated. We assume that ϵ_A and ϵ_B are identically and independently distributed.

Thus, the team designer has two options: $\theta_1 = \theta_2$ or $\theta_1 \neq \theta_2$. Moreover, she must give a decision function $\alpha_1(\cdot)$ to Agent 1 and a decision function $\alpha_2(\cdot)$ to Agent 2.

Proposition 4 *If w is supermodular in a_1 and a_2, then the set of optimal solutions contains a configuration in which $\theta_1 = \theta_2$. If w is submodular in a_1 and a_2, then the set of optimal solutions contains a configuration in which $\theta_1 \neq \theta_2$.*

Proof of Proposition 4: Given the concavity and differentiablity of w, and given that agents have information structures η_1 and η_2, the necessary and sufficient condition for the optimality of $\alpha_i(y_i)$ is person-by-person optimality (see Marschak and Radner [14, p. 157]:

$$\begin{cases} \frac{\partial}{\partial a_1} E_{z_1,z_2,y_2}[w(\alpha_1(y_1), \alpha_2[\eta_2(y_2)], z_1, z_2 | y_1] = 0 & \forall y_1 \in \Re \\ \frac{\partial}{\partial a_2} E_{z_1,z_2,y_1}[w(\alpha_1[\eta_1(y_1)], \alpha_2(y_2), z_1, z_2 | y_2] = 0 & \forall y_2 \in \Re \end{cases}$$

Because of the strict concavity of w, $\alpha_1(\cdot)$ and $\alpha_2(\cdot)$ are unique. Thus, the symmetry of the problem implies that, for all $y \in \Re$, $\alpha_1(y) = \alpha_2(y) \equiv \alpha(y)$.

Claim: If $\theta_1 \neq \theta_2$, then $\alpha(\cdot)$ is nondecreasing.

Proof of the Claim: Without loss of generality, let $\theta_1 = A$. Let us restrict our attention on the first of the two person-by-person optimality conditions, which can be rewritten as

$$\frac{\partial}{\partial a_1} E_{\epsilon_A, z_2, \epsilon_B}[w(\alpha_1(y_1), \alpha_2[\eta_2(z_2 + \epsilon_B)], y_1 - \epsilon_A, z_2 | y_1] = 0$$

for all $y_1 \in \Re$. Because ϵ_A, z_2, and ϵ_B are stochastically independent, and by switching operators, the person-by-person optimality condition becomes

$$\frac{\partial}{\partial a_1} E_{\epsilon_A} E_{z_2} E_{\epsilon_B}[w(\alpha_1(y_1), \alpha_2[\eta_2(z_2 + \epsilon_B)], y_1 - \epsilon_A, z_2|y_1] = 0 \tag{7}$$

The fact that w is supermodular in a_1 and z_1 implies that $\frac{\partial}{\partial a_1} w(\alpha_1(y_1), \alpha_2[\eta_2(z_2+\epsilon_B)], y_1 - \epsilon_A, z_2|y_1$ is nondecreasing in y_1. Therefore, the left-hand side of (7) is nondecreasing in y_1 and $\alpha_1(y_1)$ must be nondecreasing in y_1. The proof goes through in a similar way for the second person-by-person optimality condition.

We will prove that, for any configuration with $\theta_1 \neq \theta_2$, there exists a configuration with $\theta_1 = \theta_2$, which yields a greater or equal expected payoff. Thus, suppose, without loss of generality, that $\theta_1 = A$ and $\theta_2 = B$. Let us define

$$\tilde{w}(\epsilon_A, \epsilon_B|\alpha(\cdot), z_1, z_2) \equiv \omega(\alpha(z_1 + \epsilon_A), \alpha(z_2 + \epsilon_B, z_1, z_2)$$

Given any $\alpha(\cdot)$, z_1, and z_2, by the fact that $\alpha(\cdot)$ is nondecreasing, the supermodularity of ω implies that $\tilde{w}$ is supermodular in ϵ_A and ϵ_B (notice however that $\tilde{w}$ need not be symmetric in ϵ_A and ϵ_B – a departure from the main part of this paper).

By Proposition 3

$$E_{\epsilon_A,\epsilon_B}[\tilde{w}(\epsilon_A, \epsilon_B|\alpha(\cdot), z_1, z_2)] \leq E_{\epsilon_A,\epsilon_B}[\tilde{w}(\epsilon_A, \epsilon_A|\alpha(\cdot), z_1, z_2)]$$

implying

$$E_{\epsilon_A,\epsilon_B}[w(\alpha(z_1 + \epsilon_A), \alpha(z_2 + \epsilon_B), z_1, z_2)] \leq E_{\epsilon_A}[w(\alpha(z_1 + \epsilon_A), \alpha(z_2 + \epsilon_A), z_1, z_2)]$$

Take expectations on z_1 and z_2 on both sides

$$E_x[w(\alpha(z_1 + \epsilon_A), \alpha(z_2 + \epsilon_B), z_1, z_2)] \leq E_x[w(\alpha(z_1 + \epsilon_A), \alpha(z_2 + \epsilon_A), z_1, z_2)]$$

which proves that a configuration with $\theta_1 = \theta_2 = A$ dominates a configuration with $\theta_1 = A$ and $\theta_1 = B$.

If ω is submodular, the proof goes through in a similar fashion.

Proposition 4 is an application of Proposition 3. If the payoff function is supermodular, the team wants agents to commit correlated errors, while, if the payoff function is submodular, it is better if agents commit uncorrelated errors.

The model developed in this section captures some stylized features of multinational organizations. Offices in different countries are subjected to different business environments (local states). In the extreme case, local states are uncorrelated across countries. The local managers can be of the same type or of different types. For instance, the type could be the educational background of managers. The educational background is imperfect in the sense that it allows only for imperfect observations of reality. In particular, it may induces systematic deviations from the true value (e.g. engineers are more conservative than MBA holders). Then, the question is whether the firm wants these deviations to be correlated or uncorrelated across countries. As we have shown, if the firm's objective is supermodular in the action taken by local manager, then the firm benefits from having managers with the same background.

6 Conclusion

We have considered the problem of a team designer. The designer has n vacant slots and must decide if she wants to fill them with agents of the same type or with heterogeneous agents. The type of an agent determines the agent's information structure, which provides him with a signal on the state of the world. The designer can instruct the agent on what action to take conditional on the signal he has received. This paper has established a general connection between complementarities across agents and the opportunity of hiring agents with similar characteristics. If the payoff function is supermodular, agents should belong to the same type. If the payoff function is submodular, agents should be of different types.

While the analysis presented here has been purely theoretical, its main ideas can be applied to important organizational issues. This paper predicts that the workforce homogeneity of a company is determined by the type of interaction between its agents. Therefore, activities for which good fit between various units is the first concern will have a homogeneous workforce in order to maximize coordination. On the other hand, activities that revolve around exploitation of new opportunities will have a more heterogeneous workforce in order to maximize the chance of developing successful innovations.

Perhaps the most limiting assumption of this paper is that agents cannot communicate with each other between the time they observe their signals and the time they take their actions. Of course, if there is an exogenous level of communication, the present model can easily be extended to apply to the that part of information which has not been communicated. However, the real challenge is to let communication be endogenous. Arrow [3, p. 56-59] noted that each organization develops its *code* – a set of channels of intra-organizational communication. How organizations develop their codes is a problem which is central to real organizations but has not yet been studies in economic theory. Future research might use a model similar to the present one to study coding.

7 Appendix: Proofs

Proof of Lemma 1 Suppose that S is an ordered set and that $q : S^m \to \Re$ is supermodular. Then, it is immediate from the definition of supermodularity that, for any $t \in S$ and $w \in S$,

$$q(t, \overbrace{w, \ldots, w}^{m-1 \text{ times}}) + q(w, \overbrace{t, \ldots, t}^{m-1 \text{ times}}) \leq q(\overbrace{w, \ldots, w}^{m \text{ times}}) + q(\overbrace{t, \ldots, t}^{m \text{ times}})$$

Consider now $\tilde{q} : S^{l+m} \to \Re$ and assume that $\tilde{q}$ is supermodular in all its arguments. Then, for any vector $(z_1, \ldots, z_l) \in S^l$, any $t \in S$, and any $w \in S$,

$$\begin{aligned} & \tilde{q}(z_1, \ldots, z_l, t, \overbrace{w, \ldots, w}^{m-1 \text{ times}}) + \tilde{q}(z_1, \ldots, z_l, w, \overbrace{t, \ldots, t}^{m-1 \text{ times}}) \\ \leq \; & \tilde{q}(z_1, \ldots, z_l, \overbrace{t, \ldots, t}^{m \text{ times}}) + \tilde{q}(z_1, \ldots, z_l, \overbrace{w, \ldots, w}^{m \text{ times}}) \end{aligned} \tag{8}$$

If we apply (8) to the problem at hand, we have that for any $k = 2, 3, \ldots, n$ and for any $(p_1, \ldots, p_k) \in A^k$,

$$\begin{aligned} & f(p_1, \ldots, p_{k-2}, p_{k-1}, \overbrace{p_k, \ldots, p_k}^{n-k+1 \text{ times}}) + f(p_1, \ldots, p_{k-2}, p_k, \overbrace{p_{k-1}, \ldots, p_{k-1}}^{n-k+1 \text{ times}}) \\ \leq \; & f(p_1, \ldots, p_{k-2}, \overbrace{p_k, \ldots, p_k}^{n-k+2 \text{ times}}) + f(p_1, \ldots, p_{k-2}, \overbrace{p_{k-1}, \ldots, p_{k-1}}^{n-k+2 \text{ times}}) \end{aligned} \tag{9}$$

Let $P_k(a) = \{(p_1, \ldots, p_k) | (p_1, \ldots, p_n) \in P(a)\}$. Let $\bar{p}_k = (p_1, \ldots, p_k)$. By (9),

$$\begin{aligned} & \sum_{\bar{p}_k \in P_k(a)} [f(p_1, \ldots, p_{k-2}, p_{k-1}, \overbrace{p_k, \ldots, p_k}^{n-k+1 \text{ times}}) + f(p_1, \ldots, p_{k-2}, p_k, \overbrace{p_{k-1}, \ldots, p_{k-1}}^{n-k+1 \text{ times}})] \\ \leq \; & \sum_{\bar{p}_k \in P_k(a)} [f(p_1, \ldots, p_{k-2}, \overbrace{p_k, \ldots, p_k}^{n-k+2 \text{ times}}) + f(p_1, \ldots, p_{k-2}, \overbrace{p_{k-1}, \ldots, p_{k-1}}^{n-k+2 \text{ times}})] \end{aligned} \tag{10}$$

However, it is easy to see that

$$\begin{aligned}
& \sum_{\bar{p}_k \in P_k(a)} f(p_1, \ldots, p_{k-2}, p_{k-1}, \overbrace{p_k, \ldots, p_k}^{n-k+1 \text{ times}}) \\
= & \sum_{\bar{p}_k \in P_k(a)} f(p_1, \ldots, p_{k-2}, p_k, \overbrace{p_{k-1}, \ldots, p_{k-1}}^{n-k+1 \text{ times}})
\end{aligned}$$

and

$$\begin{aligned}
& \sum_{\bar{p}_k \in P_k(a)} f(p_1, \ldots, p_{k-2}, \overbrace{p_k, \ldots, p_k}^{n-k+2 \text{ times}}) \\
= & \sum_{\bar{p}_k \in P_k(a)} f(p_1, \ldots, p_{k-2}, \overbrace{p_{k-1}, \ldots, p_{k-1}}^{n-k+2 \text{ times}})
\end{aligned}$$

so that (10) becomes:

$$\begin{aligned}
& \sum_{\bar{p}_k \in P_k(a)} f(p_1, \ldots, p_{k-1}, \overbrace{p_k, \ldots, p_k}^{n-k+1 \text{ times}}) \\
\leq & \sum_{\bar{p}_k \in P_k(a)} f(p_1, \ldots, p_{k-2}, \overbrace{p_{k-1}, \ldots, p_{k-1}}^{n-k+2 \text{ times}}) \qquad (11)
\end{aligned}$$

Notice, however, that now p_k does not appear in the right-hand side of (11). Thus, for any $(p_1, \ldots, p_{k-1}) \in P_{k-1}(a)$, the summation contains $n-k+1$ identical elements corresponding to the possible values of p_k. Hence, (11) becomes

$$\begin{aligned}
& \sum_{\bar{p}_k \in P_k(a)} f(p_1, \ldots, p_{k-1}, \overbrace{p_k, \ldots, p_k}^{n-k+1 \text{ times}}) \\
\leq & \ (n-k+1) \sum_{\bar{p}_{k-1} \in P_{k-1}(a)} f(p_1, \ldots, p_{k-2}, \overbrace{p_{k-1}, \ldots, p_{k-1}}^{n-k+2 \text{ times}}) \qquad (12)
\end{aligned}$$

By applying (12) recursively, we have

$$\begin{aligned}
& \sum_{\bar{p}_n \in P_n(a)} f(p_1, \ldots, p_{n-2}, p_{n-1}, p_n) \\
\leq & \ 1 \cdot \sum_{\bar{p}_{n-1} \in P_{n-1}(a)} f(p_1, \ldots, p_{n-2}, p_{n-1}, p_{n-1}) \\
\vdots & \qquad \qquad \vdots \\
\leq & \ 1 \cdot 2 \cdot \cdots \cdot (n-k) \sum_{\bar{p}_k \in P_k(a)} f(p_1, \ldots, p_{k-1}, \overbrace{p_k, \ldots, p_k}^{n-k+1 \text{ times}})
\end{aligned}$$

$$
\begin{aligned}
&\vdots \qquad\qquad \vdots \\
&\leq \ (n-1)! \sum_{\bar{p}_1 \in P_1(a)} f(\overbrace{p_1, \ldots, p_1}^{n \text{ times}})
\end{aligned}
\tag{13}
$$

Notice that the left-hand side of (13) is equal to

$$
\sum_{(p_1,\ldots,p_n)\in P(a)} f(p_1, \ldots, p_n)
$$

and that

$$
\sum_{i=1,\ldots,n} f(a_i, \ldots, a_i) = \sum_{\bar{p}_1 \in P_1(a)} f(\overbrace{p_1, \ldots, p_1}^{n \text{ times}})
$$

Hence, (13) becomes

$$
\sum_{(p_1,\ldots,p_n)\in P(a)} f(p_1, \ldots, p_n) \leq (n-1)! \sum_{i=1,\ldots,n} f(a_i, \ldots, a_i)
$$

By dividing both sides of (7) by $n!$, the proposition is proven for f supermodular. If f is submodular, the proof goes through in the same fashion with switched inequality signs.

Verification That the Payoff Function in Eq. (3) Is Supermodular Consider any two vectors of strategies (a_1', a_2') and (a_1'', a_2''). By Definition 1, we have to prove that, for all x,

$$
\begin{aligned}
&\omega[\min(a_1', a_1''), \min(a_2', a_2''), x] + \omega[\max(a_1', a_1''), \max(a_2', a_2''), x] \\
&\geq \omega(a_1', a_2', x) + \omega(a_1'', a_2'', x)
\end{aligned}
\tag{14}
$$

Assume without loss of generality that $a_1'' \geq a_1'$. If also $a_2'' > a_2'$, then (14) holds as an equality. Hence, suppose that $a_2'' \leq a_2'$. Then, (14) becomes

$$
\min(a_1'', a_2', x) + \min(a_1', a_2'', x) \geq \min(a_1', a_2', x) + \min(a_1'', a_2'', x) \tag{15}
$$

Given that $a_1'' \geq a_1'$ and $a_2'' \leq a_2'$, without loss of generality, we can assume that $a_1'' \geq a_2'$ (if it happens that $a_1'' < a_2'$, we can switch the suffixes $'$ and $''$ and also switch the indexes 1 and 2). There are three possible cases:

$$
a_1'' \geq a_2' \geq a_2'' \geq a_1'
$$

$$
a_1'' \geq a_2' \geq a_1' \geq a_2''
$$

$$
a_1'' \geq a_1' \geq a_2' \geq a_2''
$$

For all three cases it is easy to verify that (15) holds. Therefore, the payoff function in Eq. (3) is supermodular.

To prove Proposition 2 the following is useful

Lemma 2 *Under Assumption 2, the set of one-type optima includes a solution in which*

$$\alpha_1(y) = \ldots = \alpha_1(y)$$

Lemma 2 refers to the problem in which agents are artificially restricted to be of the same type. In that case, convexity of the action space and concavity of the payoff function are together a sufficient condition to have an optimal solution in which agents all agents have the same decision function.

Proof of Lemma 2 Suppose all agents have the same type θ and therefore are endowed with the same information structure $\eta(\cdot)$. For all $y \in Y$, the team chooses a rule of action which maximizes

$$g[\alpha_1(y), \ldots, \alpha_n(y)] = E\{\omega[a_1, \ldots, a_n, x]|\eta(x) = y\} \tag{16}$$

Suppose ω is symmetric and concave in a. Because the expectation is a linear operator, also g is symmetric and concave in a.

Assume the rule of action $a^* = (a_1^*, a_2^*, \cdots, a_n^*)$ is a maximum of $g(a, y)$. By symmetry, all reorderings of a^* are maxima, too. If we define

$$\bar{a}^* = \left(\frac{\sum_{i=1}^n a_i^*}{n}, \frac{\sum_{i=1}^n a_i^*}{n}, \ldots, \frac{\sum_{i=1}^n a_i^*}{n} \right)$$

then, by concavity of g, we get $g(\bar{a}^*, y) \geq g(a^*, y)$. This holds for all possible signals y. Moreover, $\bar{a}^*$ is a symmetric rule of action. Then, if there exists an optimal rule of action – as has been assumed throughout this paper – then there also exists a symmetric optimal rule of action.

Proof of Proposition 2 By Assumption 3, there exist distinct types θ' and θ'' with associated information strcutures $\eta_{\theta'}(\cdot) \equiv \eta'(\cdot)$ and $\eta_{\theta''}(\cdot) \equiv \eta''(\cdot)$ such that

$$(\theta', \theta'') \in \text{argmax}_{\alpha_1, \alpha_2, \theta_1, \theta_2} \omega[\alpha_1(\eta_{\theta_1}(x)), \ldots, \alpha_n(\eta_{\theta_n}(x)), x] - c_{\theta_1} - c_{\theta_2}$$

Suppose that α' is the optimal rule of action associated to η' and α'' is the optimal rule of action associated to η''. By Lemma 2, for θ' there exists an optimal set of decision functions $\alpha'_1(\cdot) = \cdots = \alpha'_n(\cdot) = \alpha'(\cdot)$ and for θ' there exists an optimal set of decision functions $\alpha''_1(\cdot) = \cdots = \alpha''_n(\cdot) = \alpha''(\cdot)$. Suppose ω is submodular. For simplicity, let $c(\eta') \equiv c_{\theta'}$ and $c(\eta'') \equiv c_{\theta''}$. Then, by Corollary 1,

$$\begin{aligned}
&\omega\{\overbrace{\alpha'[\eta'(x)], \ldots, \alpha'[\eta'(x)]}^{k \text{ agents}}, \overbrace{\alpha''[\eta''(x)], \ldots, \alpha''[\eta''(x)]}^{n-k \text{ agents}}\} - [kc(\eta') + (n-k)c(\eta'')] \\
\geq\ & \frac{k}{n}\omega\{\overbrace{\alpha'[\eta'(x)], \ldots, \alpha'[\eta'(x)]}^{n \text{ agents}}\} - kc(\eta') \\
&+ \frac{n-k}{n}\omega\{\overbrace{\alpha''[\eta''(x)], \ldots, \alpha''[\eta''(x)]}^{n \text{ agents}}\} - (n-k)c(\eta')
\end{aligned}$$

for all $x \in X$ and for any $k = 1, 2, \ldots, n$. Take expectations and recall that η' and η'' yield the same expected net profit. Then, for $k = 1, 2, \ldots, n-1$,

$$\begin{aligned}
&E(\omega\{\overbrace{\alpha'[\eta'(x)], \ldots, \alpha'[\eta'(x)]}^{k \text{ agents}}, \overbrace{\alpha''[\eta''(x)], \ldots, \alpha''[\eta''(x)]}^{n-k \text{ agents}}\}) - [kc(\eta') + (n-k)c(\eta'')] \\
\geq\ & E\{\omega[\overbrace{\alpha'[\eta'(x)], \ldots, \alpha'[\eta'(x)]}^{n \text{ agents}}]\} - nc(\eta') \\
=\ & E\omega[\overbrace{\alpha''[\eta''(x)], \ldots, \alpha''[\eta''(x)]}^{n \text{ agents}}] - nc(\eta'')
\end{aligned}$$

It follows that diversified knowledge is optimal.

Proof of Proposition 3 Suppose f is supermodular. Because the y's are identically distributed, the expected value of f is invariant to permutations of the y's:

$$E[f(y_1, \cdots, y_n)] = \cdots = E[f(y_n, \cdots, y_1)]$$

and, obviously,

$$E[f(y_i, \cdots, y_i)] = E[f(y_0, \cdots, y_0)] \text{ for } i = 1, 2, \ldots, n$$

If f is supermodular, its expected value is supermodular as well. Consider the arithmetic average of all the possible permutation of the y's. By Lemma 1,

$$\frac{\sum_{(y_{j_1}, \ldots, y_{j_n}) \in P(y)} f(y_{j_1}, \ldots, y_{j_n})}{N[P(y)]} \leq (\geq) \frac{1}{n}[f(y_1, \ldots, y_1) + f(y_2, \ldots, y_2) + \cdots + f(y_n, \ldots, y_n)]$$

implying

$$E[f(y_1, \cdots, y_n)] \leq E[f(y_0, \cdots, y_0)]$$
$$E[f(y^I)] \leq E[f(y^D)]$$

and conversely when f is submodular.

References

[1] Masahiko Aoki. Horizontal vs. vertical information structure of the firm. *American Economic Review*, 76:971–983, 1986.

[2] Masahiko Aoki. An evolving diversity of organizational mode and its implications for transitional economies. *Journal of the Japanese and International Economies*, 9(4):330–353, 1995.

[3] Kenneth J. Arrow. *The Limits of Organizations.* Norton, New York, 1974.

[4] Kenneth J. Arrow and Roy Radner. Allocation of resources in large teams. *Econometrica*, 47:361–386, 1979.

[5] Bruno Bassan and Marco Scarsini. On the value of information in multi-agent decision theory. *Journal of Mathematical Economics*, 24:557–576, 1995.

[6] Carl von Clausewitz. *On War.* Princeton University Press, 1976.

[7] Jacques Crémer. A partial theory of the optimal organization of bureaucracy. *Bell Journal of Economics*, 11:683–93, 1980.

[8] Jacques Crémer. Common knowledge and the coordination of human activities. In Aoki et al., editor, *The Firm as a Nexus of Treaties.* Sage Publications, 1990.

[9] Jacques Crémer. Corporate culture and shared knowledge. *Industrial and Corporate Change*, 2(3):351–386, 1993.

[10] Esther Gal-Or. Information sharing in oligolopoly. *Econometrica*, 53:329–343, 1985.

[11] John Geanakoplos and Paul Milgrom. A theory of hierarchies based on limited managerial attention. *Journal of the Japanese and Interanational Economies*, 5:205–225, 1991.

[12] Theodore Groves and Roy Radner. Allocation of resources in a team. *Journal of Economic Theory*, 3:415–444, 1972.

[13] Bo Li. The optimal design of communication networks: M-form, u-form, and matrix management. mimeo, Stanford University, 1995.

[14] Jakob Marschak and Roy Radner. *Economic Theory of Teams.* Yale University Press, New Haven, Connecticut, 1972.

[15] Margaret A. Meyer and Dilip Mookherjee. Incentives, compensation, and social welfare. *Review of Economic Studies*, 54:209–226, 1987.

[16] Paul Milgrom and John Roberts. Complementarities and fit: Strategy, structure and organizational change in manufacturing. *Journal of Accounting and Economics*, 19:179–208, 1994.

[17] Paul Milgrom and Christina Shannon. Monotone comparative statics. *Econometrica*, 62(1):157–180, 1994.

[18] Jean-Pierre Ponssard, Sébastien Steinmetz, and Hervé Tanguy. Team models as a framework to analyze coordination problems within the firm. mimeo, Laboratoire D'Econométrie de l'Ecole Polytechnique, 1996.

[19] Yingyi Qian, Gérard Roland, and Chenggang Xu. Information and coordination in u-form and m-form organizations. mimeo, Stanford University, 1995.

[20] Donald Topkis. Minimizing a submodular function. *Operations Research*, 26(2):305–321, 1978.

[21] Xavier Vives. Duopoly information equilibrium: Cournot and Bertrand. *Journal of Economic Theory*, 34:71–94, 1985.

No.	Author(s)	Title
9773	H. Huizinga and S.B. Nielsen	The Taxation of Interest in Europe: A Minimum Withholding Tax?
9774	E. Charlier	Equivalence Scales for the Former West Germany
9775	M. Berliant and T. ten Raa	Increasing Returns and Perfect Competition: The Role of Land
9776	A. Kalwij, R. Alessie and P. Fontein	Household Commodity Demand and Demographics in the Netherlands: a Microeconometric Analysis
9777	P.J.J. Herings	Two Simple Proofs of the Feasibility of the Linear Tracing Procedure
9778	G. Gürkan, A.Y. Özge and S.M. Robinson	Sample-Path Solutions for Simulation Optimization Problems and Stochastic Variational Inequalities
9779	S. Smulders	Should Environmental Standards be Tighter if Technological Change is Endogenous?
9780	B.J. Heijdra and L. Meijdam	Public Investment in a Small Open Economy
9781	E.G.F. Stancanelli	Do the Rich Stay Unemployed Longer? An Empirical Study for the UK
9782	J.C. Engwerda and R.C. Douven	Local Strong *d*-Monotonicity of the Kalai-Smorodinsky and Nash Bargaining Solution
9783	J.C. Engwerda	Computational Aspects of the Open-Loop Nash Equilibrium in Linear Quadratic Games
9784	J.C. Engwerda, B. van Aarle J.E.J. Plasmans	The (In)Finite Horizon Open-Loop Nash LQ-Game: An Application to EMU
9785	J. Osiewalski, G. Koop and M.F.J. Steel	A Stochastic Frontier Analysis of Output Level and Growth in Poland and Western Economies
9786	F. de Jong	Time-Series and Cross-Section Information in Affine Term Structure Models
9787	G. Gürkan, A.Y. Özge and S.M. Robinson	Sample-Path Solution of Stochastic Variational Inequalities
9788	A.N. Banerjee	Sensitivity of Univariate AR(1) Time-Series Forecasts Near the Unit Root
9789	G. Brennan, W. Güth and H. Kliemt	Trust in the Shadow of the Courts
9790	A.N. Banerjee and J.R. Magnus	On the Sensitivity of the usual *t*- and *F*-tests to AR(1) misspecification
9791	A. Cukierman and M. Tommasi	When does it take a Nixon to go to China?

No.	Author(s)	Title
9792	A. Cukierman, P. Rodriguez and S.B. Webb	Central Bank Autonomy and Exchange Rate Regimes - Their Effects on Monetary Accommodation and Activism
9793	B.G.C. Dellaert, M. Prodigalidad and J.J. Louvriere	Family Members' Projections of Each Other's Preference and Influence: A Two-Stage Conjoint Approach
9794	B. Dellaert, T. Arentze, M. Bierlaire, A. Borgers and H. Timmermans	Investigating Consumers' Tendency to Combine Multiple Shopping Purposes and Destinations
9795	A. Belke and D. Gros	Estimating the Costs and Benefits of EMU: The Impact of External Shocks on Labour Markets
9796	H. Daniëls, B. Kamp and W. Verkooijen	Application of Neural Networks to House Pricing and Bond Rating
9797	G. Gürkan	Simulation Optimization of Buffer Allocations in Production Lines with Unreliable Machines
9798	V. Bhaskar and E. van Damme	Moral Hazard and Private Monitoring
9799	F. Palomino	Relative Performance Equilibrium in Financial Markets
97100	G. Gürkan and A.Y. Özge	Functional Properties of Throughput in Tandem Lines with Unreliable Servers and Finite Buffers
97101	E.G.A. Gaury, J.P.C. Kleijnen and H. Pierreval	Configuring a Pull Production-Control Strategy Through a Generic Model
97102	F.A. de Roon, Th.E. Nijman and C. Veld	Analyzing Specification Errors in Models for Futures Risk Premia with Hedging Pressure
97103	M. Berg, R. Brekelmans and A. De Waegenaere	Budget Setting Strategies for the Company's Divisions
97104	C. Fernández and M.F.J. Steel	Reference Priors for Non-Normal Two-Sample Problems
97105	C. Fernández and M.F.J. Steel	Reference Priors for the General Location-Scale Model
97106	M.C.W. Janssen and E. Maasland	On the Unique D1 Equilibrium in the Stackelberg Model with asymmetric information
97107	A. Belke and M. Göcke	Multiple Equilibria in German Employment -Simultaneous Identification of Structural Breaks-
97108	D. Bergemann and U. Hege	Venture Capital Financing, Moral Hazard, and Learning
97109	U. Hege and P. Viala	Contentious Contracts
97110	P.J.-J. Herings	A Note on "Stability of Tâtonnement Processes of Short Period Equilibria with Rational Expectations"

No.	Author(s)	Title
97111	C. Fernández, E. Ley, and M.F.J. Steel	Statistical Modeling of Fishing Activities in the North Atlantic
97112	J.J.A. Moors	A Critical Evaluation of Mangat's Two-Step Procedure in Randomized Response
97113	J.J.A. Moors, B.B. van der Genugten, and L.W.G. Strijbosch	Repeated Audit Controls
97114	X. Gong and A. van Soest	Family Structure and Female Labour Supply in Mexico City
97115	A. Blume, D.V. DeJong, Y.-G. Kim and G.B. Sprinkle	Evolution of Communication with Partial Common Interest
97116	J.P.C. Kleijnen and R.G. Sargent	A Methodology for Fitting and Validating Metamodels in Simulation
97117	J. Boone	Technological Progress and Unemployment
97118	A. Prat	Campaign Advertising and Voter Welfare
9801	H. Gersbach and H. Uhlig	Debt Contracts, Collapse and Regulation as Competition Phenomena
9802	P. Peretto and S. Smulders	Specialization, Knowledge Dilution, and Scale Effects in an IO-based Growth Model
9803	K.J.M. Huisman and P.M. Kort	A Further Analysis on Strategic Timing of Adoption of New Technologies under Uncertainty
9804	P.J.-J. Herings and A. van den Elzen	Computation of the Nash Equilibrium Selected by the Tracing Procedure in *N*-Person Games
9805	P.J.-J. Herings and J.H. Drèze	Continua of Underemployment Equilibria
9806	M. Koster	Multi-Service Serial Cost Sharing: A Characterization of the Moulin-Shenker Rule
9807	F.A. de Roon, Th.E. Nijman and B.J.M. Werker	Testing for Mean-Variance Spanning with Short Sales Constraints and Transaction Costs: The Case of Emerging Markets
9808	R.M.W.J. Beetsma and P.C. Schotman	Measuring Risk Attitudes in a Natural Experiment: Data from the Television Game Show Lingo
9809	M. Bütler	The Choice between Pension Reform Options
9810	L. Bettendorf and F. Verboven	Competition on the Dutch Coffee Market
9811	E. Schaling, M. Hoeberichts and S. Eijffinger	Incentive Contracts for Central Bankers under Uncertainty: Walsh-Svensson non-Equivalence Revisited

No.	Author(s)	Title
9812	M. Slikker	Average Convexity in Communication Situations
9813	T. van de Klundert and S. Smulders	Capital Mobility and Catching Up in a Two-Country, Two-Sector Model of Endogenous Growth
9814	A.Belke and D. Gros	Evidence on the Costs of Intra-European Exchange Rate Variability
9815	J.P.C. Kleijnen and O. Pala	Maximizing the Simulation Output: a Competition
9816	C. Dustmann, N. Rajah and A. van Soest	School Quality, Exam Performance, and Career Choice
9817	H. Hamers, F. Klijn and J. Suijs	On the Balancedness of *m*-Sequencing Games
9818	S.J. Koopman and J. Durbin	Fast Filtering and Smoothing for Multivariate State Space Models
9819	E. Droste, M. Kosfeld and M. Voorneveld	Regret Equilibria in Games
9820	M. Slikker	A Note on Link Formation
9821	M. Koster, E. Molina, Y. Sprumont and S. Tijs	Core Representations of the Standard Fixed Tree Game
9822	J.P.C. Kleijnen	Validation of Simulation, With and Without Real Data
9823	M. Kosfeld	Rumours and Markets
9824	F. Karaesmen, F. van der Duyn Schouten and L.N. van Wassen-hove	Dedication versus Flexibility in Field Service Operations
9825	J. Suijs, A. De Waegenaere and P. Borm	Optimal Design of Pension Funds: A Mission Impossible
9826	U.Gneezy and W. Güth	On Competing Rewards Standards -An Experimental Study of Ultimatum Bargaining-
9827	M. Dufwenberg and U. Gneezy	Price Competition and Market Concentration: An Experimental Study
9828	A. Blume, D.V. De Jong and G.R. Neumann	Learning in Sender-Receiver Games
9829	B.G.C. Dellaert, J.D. Brazell and J.J. Louviere	Variations in Consumer Choice Consistency: The Case of Attribute-Level Driven Shifts in Consistency
9830	B.G.C. Dellaert, A.W.J. Borgers, J.J. Louviere and H.J.P. Timmermans	Consumer Choice of Modularized Products: A Conjoint choice Experiment Approach

No.	Author(s)	Title
9831	E.G.A. Gaury, H. Pierreval and J.P.C. Kleijnen	New Species of Hybrid Pull Systems
9832	S.J. Koopman and H.N. Lai	Modelling Bid-Ask Spreads in Competitive Dealership Markets
9833	F. Klijn, M. Slikker, S. Tijs and J. Zarzuelo	Characterizations of the Egalitarian Solution for Convex Games
9834	C. Fershtman, N. Gandal and S. Markovich	Estimating the Effect of Tax Reform in Differentiated Product Oligopolistic Markets
9835	M. Zeelenberg, W.W. van Dijk, J. van der Pligt, A.S.R. Manstead, P. van Empelen and D. Reinderman	Emotional Reactions to the Outcomes of Decisions: The Role of Counterfactual Thought in the Experience of Regret and Disappointment
9836	M. Zeelenberg, W.W. van Dijk and A.S.R. Manstead	Reconsidering the Relation between Regret and Responsibility
9837	M. Dufwenberg and G. Kirchsteiger	A Theory of Sequential Reciprocity
9838	A. Xepapadeas and A. de Zeeuw	Environmental Policy and Competitiveness: The Porter Hypothesis and the Composition of Capital
9839	M. Lubyova and J.C. van Ours	Unemployment Durations of Job Losers in a Labor Market in Transition
9840	P. Bolton and X. Freixas	A Dilution Cost Approach to Financial Intermediation and Securities Markets
9841	A. Rustichini	Minimizing Regret: The General Case
9842	J. Boone	Competitive Pressure, Selection and Investments in Development and Fundamental Research
9843	H.L.F. de Groot	Macroeconomic Consequences of Outsourcing. An Analysis of Growth, Welfare, and Product Variety
9844	U. Gneezy, W. Güth and F. Verboven	Presents or Investments? An Experimental Analysis
9845	A. Prat	How Homogeneous Should a Team Be?

P   RLANDS
Bibliotheek K. U. Brabant
17 000 01420372 4