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ABSTRACT

N identical sellers, selling an homogeneous product, each choose a price and an advertising policy. Buyers have usual demand functions and search optimally among sellers. A Symmetric Nash Equilibrium (SNE) is derived and characterized. If the marginal cost of advertising exceeds a critical level, then the monopoly price with zero advertising is the unique SNE; otherwise, there is a mixed-strategy SNE which can be expressed as a price distribution and an advertising policy conditional on price. Remarkably, as the number of sellers increases without limit, the SNE converges to the monopoly price and advertising per seller vanishes. On the other hand, the mean price at which sales occur is less than the monopoly price. Still for some advertising technologies, total social welfare is declining in the number of sellers.

1. INTRODUCTION.

Economists have long been interested in understanding how markets work with imperfect information. Rothschild (1975) provided a critical survey, pointing out that most of the literature is concerned with only one side of the market - such as buyer search from a known distribution of seller prices. A complete analysis would include optimal buyer and seller behavior simultaneously determining a market equilibrium. A bothersome result is that if sellers do not advertise and search is costly, then the unique equilibrium is monopolistic [Diamond, 1971]. Even with a mass of zero-search-cost buyers, the equilibrium distribution of prices converges to the monopoly price as the number of sellers increases [Stahl, 1989a]. But surely, it seems, sellers would have an incentive to advertise lower prices, thereby undermining the monopolistic equilibrium.

In a seminal article, Butters (1977) studied how sellers can influence buyer information by advertising. He demonstrated the existence of equilibrium price dispersion and actually presented an analytic solution and comparative statics results. He presented only a limit model with infinitely many buyers and sellers, so the influence of the number of buyers and sellers could not be explored. He also considered only the case of unit demands for buyers, and a specific advertising technology. On the other hand, Butters did consider both the case with no buyer search, and the case with a fixed buyer reservation price search rule.

The first case with no buyer search (buyers not receiving advertisements purchase nothing) was extended in Stahl (1989b) to encompass any finite number of sellers and buyers, a general buyer demand function, and a general advertising technology. A unique Symmetric Nash Equilibrium (SNE) was derived in which the level of advertising was less than the socially optimal level except when buyers have unit demand functions (in which case, it is efficient). With no buyer search, sellers do not have the alternative of simply charging the monopoly price to the random share of buyers who stop at their store first, foregoing sales to other buyers. Thus, one force in the direction of monopoly pricing is absent. Hence, the SNE price distribution is non-degenerate with no tendency (as the number of sellers increases) to converge to monopoly pricing; on the other hand, it does not converge to marginal cost pricing either. Remarkably, for some advertising technologies, total welfare can decrease as the number of sellers increases.

The purpose of this paper is to extend the second case of Butters with buyer search to encompass any finite number of sellers and buyers, a general buyer demand function, optimal sequential buyer search, and a general advertising technology. We develop a complete game-theoretic solution (in mixed-strategies), and conduct comparative statics in terms of the advertising technology and the number of buyers and sellers. An explicit solution is presented for the special case of linear advertising costs.

Our search model, however, differs substantially from Butters'. He assumed that buyers who do not receive advertisements distribute themselves among sellers in the same proportion as the buyers who do receive advertisements. Butters defends this assumption on two grounds. First, he dismisses the model we will analyze as "unpalatably complicated". Second, he suggests that "word-of-mouth" among buyers would result in the uninformed following the pattern of the informed. In contrast, the search model of this paper adheres to the traditional optimal search literature despite its complications. Since the fundamental purpose of our inquiry is to study the effects of information transmission and acquisition technologies on the strategic outcomes, we shy from bold assumptions about leakage of information via word-of-mouth. Rather, we suggest that such phenomena are captured in our model with an appropriate interpretation of the parameter that represents advertising effectiveness.

Our model has a finite number of sellers and buyers. Sellers offer a homogeneous product, and have identical technologies. Each seller simultaneously chooses a price and an advertising level. Buyers have identical demand functions and positive search costs. Conditional on the advertisements received and the SNE price distribution, a buyer decides whether simply to buy from the seller with the lowest advertised price, or to search further. The optimal buyer search rule is characterized by a reservation price.

A SNE can be characterized by a distribution of prices and an advertising policy conditional on the realization of a seller's own price. The optimal advertising effort is a decreasing function of the realized price. The upper bound of the SNE price distribution is the lessor of the monopoly price and the buyer's reservation price. The SNE price distribution is atomless except possibly at the upper bound, in which case the advertising effort must be precisely zero at the upper bound.

If the marginal advertising costs exceed a critical level, the unique SNE is the pure-strategy SNE at the monopoly price with no advertising. However, for lower marginal advertising costs, there is no pure-strategy SNE, but there is a mixed-strategy SNE; i.e. there is price dispersion.

As the number of sellers increases, the advertising effort per seller vanishes at all prices, and the SNE price distribution converges to the monopoly price. Nonetheless, the total industry level of advertising remains strictly positive, and the distribution of the prices at which sales actually occur converges to a non-degenerate distribution bounded above marginal costs with an atom at the monopoly price. As in the model without buyer search [Stahl, 1989b], for some advertising technologies, total welfare can be decreasing in the number of sellers. In the special case of unit buyer demand, there is over-advertising. More generally there can be over- or under-advertising at the SNE.

As the cost of advertising shrinks, the SNE converges towards marginal-cost pricing, but as the search cost shrinks, the SNE stays bounded above marginal-cost pricing. Thus, the two channels of information transmission have different impacts in that as the comparative advantage shifts towards buyer search, sellers curtail advertising; consequently buyers face higher prices than when search costs are higher but advertising costs are very low.

The paper is organized as follows. Section 2 presents the basic model and derives optimal buyer and seller behavior. Section 3 derives the SNE, and Section 4 presents explicit solutions for linear and quadratic advertising costs. Section 5 presents results on the asymptotic behavior of the SNE. Section 6 considers the welfare implications. Finally, Section 7 summarizes the results and compares them with related work. All proofs are relegated to an Appendix.

2. THE MODEL.

There are N identical sellers all offering the same homogeneous good. With identical linear production technologies, there is no loss of generality in taking marginal costs to be zero. Let p_j denote the price set by seller j. Alternatively, interpret p_j as net of marginal costs.

Advertising efforts generate a distribution of informed and uninformed buyers (before any buyer search). Let α_j denote the fraction of buyers who become informed about j's price. Sellers are assumed to have identical advertising technologies, which can be represented by a cost function $\gamma(\alpha,M)$ that gives the cost per buyer of informing a fraction α of M buyers. For example, given the classic urn technology assumed by Butters, we would have $\gamma(\alpha,M) = b\ln(1-\alpha)/\ln(1 - 1/M)$, where b is the cost per advertisement (hereafter abbreviated "ad").[1] Unless explicitly considering the effects of M, we will suppress this argument and write $\gamma(\alpha)$. We assume that $\gamma(\alpha)$ is strictly convex and twice continuously differentiable with $\gamma'(0) > 0 = \gamma(0)$.

There are M identical buyers with demand functions D(p). Alternatively, interpret D(p) as the average demand function with idiosyncratic characteristics distributed independently of all other variables. Define the revenue function $R(p) \equiv pD(p)$. We assume that R(p) is continuously differentiable with a unique maximum at $\hat{p}$, and is strictly increasing for all $p < \hat{p}$. Further, we assume that $0 \leq D(p) < \infty$ for all $p \in [0,\hat{p}]$.

2.1 **Buyer Search.**

Buyers receive information about seller prices through seller ads and individual search efforts. Each search costs a fixed amount $c > 0$.[2] Let

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[1]In this case, α is precisely the probability of receiving an ad. However, in the general case, there may be word-of-mouth leakage to the buyers who do not receive ads, in which case α, which is the total fraction of buyers who become informed (directly and indirectly), could be greater than the probability of receiving an ad. With this interpretation, buyers who are "uninformed" are truly uninformed both directly and indirectly prior to search, so it will be reasonable to assume that they distribute themselves uniformly among the sellers, in contrast to Butters' assumption.

[2]The act of obtaining a price quotation is assumed to be a costly effort relative to the cost of the ultimate purchase transaction which is assumed to be zero. Thus, if a buyer receives an ad for a price she wants to accept, there is no additional transaction cost.

$F_0(p_j)$ denote the posterior probability distribution of the price charged by seller j conditional on not having received an ad from seller j. If p* is the lowest price observed so far, then the net benefit of an additional search is

$$H(p^*,c,F_0) = \int_0^{p^*} D(p)F_0(p)dp - c \quad . \tag{1}$$

We assume perfect recall. It is well-known [e.g. Kohn and Shavel, 1974] that the optimal search rule is characterized by a stationary reservation price, r, implicitly defined by

$$H(r,c,F_0) = 0, \tag{2}$$

if a root exists; otherwise, we define the reservation price to be $+\infty$. Note that $c > 0$ implies $r > 0$. For later use, note also that if F_0 is concentrated at one price, say $\tilde{p}$, then $c > 0$ implies $r > \tilde{p}$. Similarly, if $F_{0,n}$ is a sequence converging weakly to a distribution concentrated at $\tilde{p}$, then there is an n' such that for all $n > n'$, $r > \tilde{p}$.

Buyers search if and only if the lowest price they are aware of (either through ads or prior search) is more than r. After search is terminated, a buyer purchases an amount $D(p_j)$ from the lowest priced seller (say j) she has found. A buyer who has received no ads and sampled no sellers is assumed to select a seller at random (say k) and purchase an amount $D(p_k)$, provided $p_k \leq r$.[3]

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[3]We are implicitly assuming that the expected value of a transaction with some seller is always at least c. For instance, the buyer may need to purchase a number of other commodities known to be available at competitive prices from all

2.2 Seller Advertising and Pricing Decisions.

A *SNE* is a triplet $\{F(\cdot),\alpha(\cdot),r^*\}$ such that (i) when any subset of N-1 sellers choose prices from the probability distribution $F(\cdot)$ and advertising effort $\alpha(p)$, then the remaining seller has no incentive to deviate, and (ii) r^* is the optimal buyer reservation price given $\{F(\cdot),\alpha(\cdot)\}$.

In this section, we take the buyer reservation price r as given, and consider seller pricing decisions. To be explicit, we denote the price distribution by $F(p;r)$. Let P denote the maximum price in the support of $F(p)$, and let B denote the minimum price in the support.[4] Ultimately, we will derive a *consistent reservation price* r^* that is optimal for $F(p;r^*)$.

Take a random buyer, and let $\varphi(p_j)$ denote the probability that p_j is the lowest price known by this buyer conditional on knowing that j is charging p_j. Let β denote the probability that a random buyer is uninformed about the other N-1 sellers. Consider seller j's decision about advertising effort α_j at price $p_j \leq r$. The expected profits per buyer (i.e. total expected profits divided by M) can be expressed as

$$E\pi(p_j,\alpha_j) = R(p_j)\{\alpha_j\varphi(p_j) + (1-\alpha_j)\beta/N\} - \gamma(\alpha_j) \quad . \tag{3}$$

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the sellers, so the expected value of a shopping trip is substantially larger than c independent of the separate expected surplus of the advertised item. Stahl (1989b) considers a model in which uninformed buyers do not enter the market.

[4]The support is the smallest closed set with probability one.

$\partial E\pi(p_j,\alpha_j)/\partial\alpha_j = R(p_j)(\varphi(p_j) - \beta/N) - \partial\gamma/\partial\alpha_j$, and $\partial^2 E\pi(p_j,\alpha_j)/(\partial\alpha_j)^2 = -\partial^2\gamma/(\partial\alpha_j)^2 < 0$. Thus, $E\pi(p_j,\alpha_j)$ is strictly concave in α_j, which implies that there exists a unique function $\alpha(p_j)$ that maximizes eq(3). The parameters (such as r) that affect $\alpha()$ will be made explicit later.

Therefore, the strategy space for sellers can be reduced to a price distribution $F(p;r)$ and a function $\alpha(p)$, with the interpretation that sellers randomly choose a price p_j and then choose $\alpha(p_j)$.

Let $A(p_j;r)$ denote the probability that a random buyer is informed that seller j has a price $p \le p_j$. Given $F(p;r)$ and $\alpha(p)$,

$$A(p_j;r) = \int_B^{p_j} \alpha(p)dF(p;r) \quad . \tag{4}$$

It is notationally convenient to define $\bar{\alpha} = A(P;r)$, the expected value of α. Note that $\bar{\alpha}$ is also the probability that a random buyer is informed about (say) seller j. Hence, in eq(3), $\beta = (1 - \bar{\alpha})^{N-1}$: the probability that a random buyer is uninformed about the N-1 sellers playing $F(p;r)$ and $\alpha(p;r)$. Further, if $F(p;r)$ contains no atoms at or below p_j, then in eq(3), $\varphi(p_j) = [1 - A(p_j;r)]^{N-1}$.

We will now state a result that is very useful because it leads to an expression for payoffs that is continuous function.

<u>Lemma 1</u>. If a SNE $F(p;r)$ contains an atom at $\tilde{p} > 0$, then $\alpha(\tilde{p}) = 0$.

In other words, if $F(p;r)$ contains an atom at any price, then that price will not be advertised. Intuitively, if F contains an atom at $\tilde{p} > 0$, then a seller can increase profits by undercutting $\tilde{p}$ slightly and advertising the lower price.

Next, observe that there can be no atom at $\tilde{p} = 0$, because then expected profits would be precisely zero, but since $r > 0$, a seller would make a positive profit at positive prices up to r. Coupled with this fact, an immediate corollary of Lemma 1 is that $A(\cdot;r)$ is continuous over the relevant domain. Hence, for $p_j \leq r$, we can rewrite eq(3) as

$$E\pi(p_j,\alpha_j) = R(p_j)\{\alpha_j[1-A(p_j;r)]^{N-1} + (1-\alpha_j)(1-\bar{\alpha})^{N-1}/N\} - \gamma(\alpha_j) \quad . \qquad (5)$$

Two very useful and important result follows. First, SNE expected profits, $E\pi^*$, are strictly positive, because a store charging $r > 0$ will always capture a share of the completely uninformed buyers, and the latter is always positive because setting $\alpha = 1$ would generate negative profits. Second, the maximum price in the support of the SNE price distribution is the lessor of the monopoly price $\hat{p}$ and the buyers' reservation price r. Intuitively, prices higher than r are suboptimal because buyers will always search further. Prices above the monopoly price can never be optimal because revenues are declining. If every seller prices below $\min(r,\hat{p})$, then since expected profits, eq(5), are continuous, a seller can raise prices slightly without losing any customers, thereby increasing sales.

<u>Lemma 2</u>. (a) $E\pi^* > 0$, and (b) $P = \min\{r,\hat{p}\}$.

One of the implications of Lemma 2 is that buyers will not engage in any real search. If they receive any information, then with probability one the known prices are at or below their reservation price, so they will simply go to the lowest priced seller and purchase. If they are completely uninformed, then they will pick a seller at random and purchase since the price they find will be at or below their reservation price with probability one.[5] In section 3, we will completely characterize the SNE $F(p;r)$ and $\alpha(p)$.

3. DERIVATION OF SNE.

We first state a number of properties of a SNE. Initially, we will take the buyer reservation price r as exogenously fixed.

Lemma 3. The support of $F(p;r)$ is a connected interval $[B,P]$.

Lemma 4. There exists at most one price p' in the support of $F(p;r)$ such that $\alpha(p') = 0$.

Lemma 5. If there is an atom in $F(p;r)$, then it must be at P.

In other words, the price distribution must be atomless below P with connected support; there cannot be multiple price in the support of $F(p;r)$ that are not advertised; and if there is an atom at P, then that price will not be advertised. We next consider the existence of pure-strategy SNE.

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[5]This "no-real-search" implication is an artifact of the assumption of homogeneous search costs. Given one reservation price for all buyers, sellers will never choose higher prices, and hence, buyers never find it optimal to search further. Rob (1985) and Stahl (1988) consider models with heterogeneous search costs and find real search for low-search-cost buyers, but no search for a positive mass of high-search-cost buyers. Extending the present model to heterogeneous search costs is left for future study.

3.1 Pure-Strategy SNE.

Recall that if there is a pure-strategy at price P, then by eq(1) and (2), $r > P$. It then follows from Lemma 2 that the only possible consistent pure-strategy price is the monopoly price $\hat{p}$. By Lemma 1, it must be optimal not to advertise this price, otherwise undercutting would be profitable. Thus, we need to find conditions under which it is not optimal to advertise given every seller chooses the monopoly price. Intuitively, for sufficiently high advertising costs, it will not be optimal to advertise. Then, costly search implies monopoly prices [Diamond, 1971]. More precisely:

Theorem 1. If $\gamma'(0) \geq [(N-1)/N]R(\hat{p})$, then $F(\hat{p}) = 1$ and $\alpha(\hat{p}) = 0$ is the unique consistent SNE; otherwise, there does not exist a pure-strategy SNE.

A pure-strategy SNE exists if and only if the marginal advertising cost is sufficiently high relative to monopoly revenues; otherwise, there is no pure-strategy SNE. In particular, note that if $\gamma'(0) \geq R(\hat{p})$, then $\hat{p}$ is a pure-strategy SNE for all $N \geq 2$. If $\gamma'(0) < R(\hat{p})/2$, then there does not exist a pure-strategy SNE for any $N \geq 2$. In the interim range, there exists an N' such that for all $N \leq N'$, there is a unique (pure-strategy) SNE at $\hat{p}$, and for all $N > N'$ there is no pure-strategy SNE. Thus, entry of sellers appears to reduce the likelihood of pure monopoly pricing. [We will re-examine the effects of seller entry in Sections 5 and 6.]

3.2 Derivation of Mixed-Strategy SNE.

In the remainder of the paper, unless otherwise stated, we assume $\gamma'(0) < [(N-1)/N]R(\hat{p})$, so there does not exist a pure-strategy SNE. The first step is to derive an expression for $[1-A(p;r)]^{N-1}$. It is notationally convenient to define $\alpha_P = \alpha(P)$. A necessary condition of a NE is that $E\pi(p,\alpha(p))$ be equal to a constant for all $p \in [B,P]$. Using eq(5), $E\pi(P,\alpha_P) = R(P)\{\alpha_P + (1-\alpha_P)/N\}(1-\bar{\alpha})^{N-1} - \gamma(\alpha_P)$, and hence,

$$[1-A(p;r)]^{N-1} = \left[\left(\frac{R(P)}{R(p)}\right)\left(\alpha_P + \frac{(1-\alpha_P)}{N}\right) - \frac{(1-\alpha(p))}{N}\right]\left(\frac{(1-\bar{\alpha})^{N-1}}{\alpha(p)}\right)$$

$$+ \frac{\gamma[\alpha(p)] - \gamma(\alpha_P)}{R(p)\alpha(p)} . \qquad (6)$$

Given a solution for $\alpha(p)$, eq(6) can be used to calculate $A(p;r)$. But before that, we will use eq(6) to derive a parametric solution for $\alpha(p)$.

Since $E\pi(p,\alpha_j)$, eq(5), is strictly concave in α_j, the Kuhn-Tucker conditions are necessary and sufficient to characterize the optimal α_j:

$$\partial E\pi(p,\alpha_j)/\partial\alpha_j = R(p)\{[1-A(p;r)]^{N-1} - (1-\bar{\alpha})^{N-1}/N\} - \gamma'(\alpha_j) = 0 , \qquad (7)$$

for interior optima; otherwise, $\partial E\pi(p,0)/\partial\alpha_j \le 0$ implies $\alpha_j = 0$, and $\partial E\pi(p,1)/\partial\alpha_j \ge 0$ implies $\alpha_j = 1$.

Substituting eq(6) into eq(7) yields

$$\alpha_j\gamma'(\alpha_j) - \gamma(\alpha_j) = |R(P)|\alpha_P + (1-\alpha_P)/N| - R(p)/N|\cdot(1-\bar{\alpha})^{N-1} - \gamma(\alpha_P), \qquad (8)$$

for interior optima. The "corner" conditions translate to: (i) if the right-hand-side (r.h.s.) of eq(8) is no less than $\gamma'(1) - \gamma(1)$, then $\alpha_j = 1$ is the optimal solution; (ii) if the r.h.s. is non-positive, then $\alpha_j = 0$ is the optimal solution. The relevant parameters of eq(8) are P, $\bar{\alpha}$, and α_P. Using Lemma 2, $(r,\bar{\alpha},\alpha_P)$ is a sufficient set of parameters so eq(8) and the corner conditions define a unique solution for α_j which we denote $a(p;r,\bar{\alpha},\alpha_P)$. We will see momentarily that there is a one-to-one relationship between $\bar{\alpha}$ and α_P, so (r,α_P) will be a necessary and sufficient set of parameters.

Note that the left-hand side (l.h.s.) of eq(8) as a function of α_j begins at the origin and increases with slope $\alpha_j\gamma''$, while the r.h.s. is strictly decreasing in p up to $\hat{p}$. Hence, for interior solutions, $a(\cdot;r,\bar{\alpha},\alpha_P)$ is a strictly decreasing function. This result is rather intuitive. Sellers want to advertise low prices more than high prices because the chance of being the winner (and hence the payoff to advertising) is greater when they set low prices.

We will now develop a characterization of the optimal $\alpha(p)$ that will permit us to eliminate $\bar{\alpha}$ from the parameter list.

<u>Lemma 6</u>. Eq(7) must hold as a strict equality at $p = P$:

$$\gamma'(\alpha_P) = \left(\frac{N-1}{N}\right)R(P)(1-\bar{\alpha})^{N-1} . \qquad (9)$$

Using eq(9) to determine $\bar{\alpha}$ in terms of α_P, denoted $\bar{\alpha}(\alpha_P)$, we can now express the optimal advertising level as $\alpha(p;r,\alpha_P) = a[p;r,\bar{\alpha}(\alpha_P),\alpha_P]$.

The requirement that eq(9) have a positive solution puts a lower bound on P, as follows.

<u>Lemma 7</u>. Given $\gamma'(0) < [(N-1)/N]R(\hat{p})$, a SNE must have $P > R^{-1}[\gamma'(0)N/(N-1)]$; hence, $r > \hat{r} = R^{-1}[\gamma'(0)N/(N-1)]$.

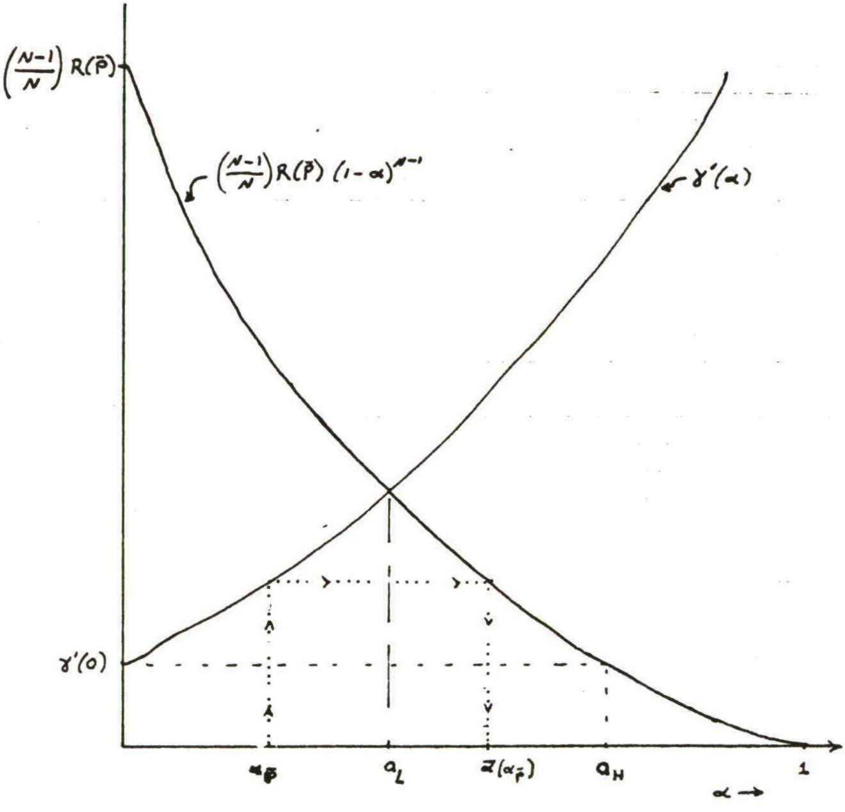
$\left(\frac{N-1}{N}\right) R(\bar{p})$
$\left(\frac{N-1}{N}\right) R(\bar{p})(1-\alpha)^{N-1}$
$\gamma'(\alpha)$
$\gamma'(0)$
$\alpha_{\bar{p}}$
a_L
$\bar{a}(\alpha_{\bar{p}})$
a_H
1
$\alpha \rightarrow$

Figure 1.

It is useful to examine a graphical method of determining $\bar{\alpha}(\cdot)$ as presented in Figure 1. The upward sloping function is $\gamma'(\alpha)$, and the downward sloping function is $[(N-1)/N]\cdot R(P)(1-\alpha)^{N-1}$, which are the left and right hand sides of eq(9) respectively. The functions have been drawn under the assumption that $\gamma'(0) < [(N-1)/N]R(P)$ so there is a unique interior intersection point. For any α_P, start on the horizontal axis at $\alpha = \alpha_P$ and move vertically to the $\gamma'()$ function; then move horizontally to the other function. The final horizontal position is $\bar{\alpha}(\alpha_P)$. It is easy to see that $\bar{\alpha}(\cdot)$ is strictly decreasing with a maximum value of $a_H \equiv \bar{\alpha}(0)$. The unique fixed point of $\bar{\alpha}(\cdot)$ is denoted a_L. Observe that $0 < a_L < a_H$.

It should be apparent that since $\alpha(\cdot;r,\alpha_P)$ is non-increasing, it must be that $\alpha_P \le \bar{\alpha}(\alpha_P)$. Figure 1 illustrates how this requirement circumscribes the feasible range for α_P. For any $\alpha_P \in [0,a_L]$, as indicated by the dotted lines in Figure 1, $\bar{\alpha}(\alpha_P)$ lies in the interval $[a_L,a_H]$, which is compatible with a SNE solution. On the other hand, for any $\alpha_P > a_L$, $\bar{\alpha}(\alpha_P) > \alpha_P$, and hence is not compatible.

Having a parametric solution $\alpha(p;r,\alpha_P)$, we can use eq(6) to define a parametric solution $A(p;r,\alpha_P)$. Next, to determine the lower bound B, note that we must have $A(B;r,\alpha_P) = 0$, and then solve eq(6) for B in terms of $\alpha(\cdot;r,\alpha_P)$. Let $\alpha_B \equiv \alpha(B;r,\alpha_P)$, and define

$$L(B) \equiv R(B) - r(B)/\alpha_B = 0, \text{ where}$$

$$r(B) \equiv \left[R(P)\left(\alpha_P + \frac{1-\alpha_P}{N}\right) - R(B)\frac{1-\alpha_B}{N}\right]\left(\frac{N\gamma'(\alpha_P)}{(N-1)R(P)}\right) + \gamma(\alpha_B) - \gamma(\alpha_P) \;, \tag{10}$$

so $L(B) = 0$ iff $A(B;r,\alpha_P) = 0$. Hence, eq(10) defines has a unique solution $B(r,\alpha_P) \in (0,P)$.[6]

We will now derive a parametric solution for the price distribution which we will denote $F(p;r,\alpha_P)$. Differentiating eq(4) with respect to p_j, we see that $\partial A(p_j;r,\alpha_P)/\partial p_j = \alpha(p_j;r,\alpha_P)F'(p_j;r,\alpha_P)$, where F' denotes the probability density function. The easiest way to derive an expression for $\partial A/\partial p$ is to differentiate eq(5) with respect to p_j, using the Envelope Theorem to get

$$\frac{\partial A(p_j;r,\alpha_P)}{\partial p_j} = \frac{R'(p_j)}{R(p_j)} \cdot \left[\frac{\alpha[1 - A(p_j;r,\alpha_P)]^{N-1} + (1-\alpha)\gamma'(\alpha_P)/[(N-1)R(P)]}{(N-1)\alpha[1 - A(p_j;r,\alpha_P)]^{N-2}}\right], \quad (11)$$

where $\alpha = \alpha(p_j;r,\alpha_P)$. We can see that $\partial A(p;r,\alpha_P)/\partial p \geq 0$, and hence, $F'(p_j;r,\alpha_P) = [\partial A(p_j;r,\alpha_P)/\partial p_j]/\alpha(p_j;r,\alpha_P) \geq 0$. Integrating:

$$F(p_j;r,\alpha_P) = \int_B^{p_j} \frac{\partial A(p;r,\alpha_P)/\partial p_j}{\alpha(p;r,\alpha_P)} dp . \quad (12)$$

Note that by construction, $F(B;r,\alpha_P) = 0$ and the density is non-negative. The only remaining requirement is the endpoint condition that $F(p;r,\alpha_P) \leq 1$.

<u>Theorem 2</u>. Given $r \geq \hat{r}$, there exists an α_P such that $F(p;r,\alpha_P)$ and $\alpha(p;r,\alpha_P)$ defined by eqs(6 - 12) are SNE strategies conditional on r.

Thus, we have a method of computing SNE strategies (conditional on buyer reservation price $r \geq \hat{r}$). Taking α_P as a parameter, we first compute a

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[6] To see this, observe from eq(7), assuming $\alpha_B < 1$ that $\gamma'(\alpha_B) = R(B)(1-y)$, where $y = \gamma'(\alpha_P)/[(N-1)R(P)]$. Then, $r'(B) = R(B)\alpha'(B) - R'(B)(1-\alpha_B)y$, so $L'(B) = R'(B)[1 + y(1-\alpha_B)/\alpha_B] - L(B)\alpha'(B)/\alpha_B$, which is strictly positive for all B satisfying $L(B) = 0$. But since $L'(B) > 0$ for all roots of L(), there can be only one root. If $\alpha_B = 1$ and eq(7) does not hold as an equality, then $L'(B) = R'(B) > 0$, so again B is unique.

parametric advertising solution $\alpha(p;r,\alpha_P)$. Then, we use $\alpha(p;r,\alpha_P)$ to compute a parametric price distribution. Next, we adjust α_P until $F(P;r,\alpha_P) \leq 1$ and $\alpha_P[1 - F(P;r,\alpha_P)] = 0$.

We turn now to the question of uniqueness. First observe that there cannot be two SNE both with atoms at P, because an atom at P requires $\alpha_P = 0$, which in turn defines a unique solution to eq(8), and hence a unique price distribution as well. Whether or not there can be multiple SNE (not more than one with an atom at P) is an open question. To shed some light on this question, it is useful to know how $\alpha(p;r,\alpha_P)$ depends on α_P. Substituting eq(9) into eq(8) and differentiating with respect to α_P reveals that $\alpha(p;r,\alpha_P)$ is strictly increasing in α_P for all $\alpha_P \in (0,a_L)$. Thus, recalling eq(12), the direct effect of increasing α_P is to decrease $F(p;r,\alpha_P)$. Unfortunately, I have not been able to prove that the total effect is always negative. For uniqueness, it would suffice to show that $\partial F(p;r,\alpha_P)/\partial\alpha_P < 0$ when evaluated at any α_P such that $F(p;r,\alpha_P) = 1$. I suspect that for some revenue functions and advertising cost functions, there are multiple SNE. However, there will also be a wide class of revenue functions for which there will be a unique SNE.

3.3 Existence of a Consistent SNE.

In section 3.2, we took the buyer reservation price r as exogenously fixed. Now we want to close the model by finding a consistent r* such that the seller price distribution F(p;r*) induces r* as the buyer reservation price. Conditional on not receiving an ad from a seller, the posterior probability distribution on that seller's price is

$$F_0(p;r) \equiv \frac{\int_0^p [1-\alpha(x;r)]dF(x;r)}{1-\bar{\alpha}} = \frac{F(p;r) - A(p;r)}{1-\bar{\alpha}} \qquad (13)$$

Let $B_0 = \inf\{p \mid \alpha(p) < 1\}$.[7] Then F_0 has support $[B_0,P]$ and is skewed towards P relative to F(p).

First, observe that for $r \geq \hat{p}$, $P = \hat{p}$, so $F(p;\cdot)$ is independent of $r \geq \hat{p}$; hence, $H[\hat{p},c,F_0(\cdot;r)]$ is constant for $r \geq \hat{p}$. Consequently, if $H[\hat{p},c,F_0(\cdot.\hat{p})] \leq 0$, then there exists a consistent buyer reservation price $r^* \geq \hat{p}$. This will occur, for example, if c is sufficiently high. On the other hand, if $H[\hat{p},c,F_0(\cdot.\hat{p})] > 0$, then a consistent reservation price (if one exists) must lie in the open interval $(0,\hat{p})$.

Second, observe that the buyer reservation price enters the seller SNE strategy choices only via the term $R[\min\{r,\hat{p}\}]$. Recalling that R() is continuously differentiable, if there were a unique conditional SNE for every r, then the existence of a consistent r^* would be almost trivial. However, we must take account of the possibility that the SNE asserted by Theorem 2 may depend discontinuously on r.

<u>Theorem 3</u>. There exists a fully consistent SNE. That is, there exists a parameter α_p^* and a buyer reservation price r^* that is consistent with $F(p;r^*,\alpha_p^*)$ of Theorem 2.

The proof uses a fixed point argument to find a P^* and α_p^* simultaneously such that $H[P^*,c,F_0(\cdot;P^*,\alpha_p^*)] \leq 0$ and $F_0(P^*;P^*,\alpha_p^*) \leq 1$, with strict inequalities

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[7]Given strictly convex advertising costs, it is clear from Figure 1 that $B_0 < P$, since $\alpha_p < \bar{\alpha} < 1$.

implying that $P^* = \hat{p}$ and $\alpha_P^* = 0$ respectively. Hence, there exists a consistent reservation price $r^* \geq P^*$.

4. EXPLICIT SOLUTIONS FOR SPECIAL CASES.

4.1 Linear Advertising Costs.

While strict convexity of advertising costs $\gamma(\alpha)$ played a crucial role in the above derivation of the mixed-strategy SNE, we can derive a unique SNE for the case when $\gamma(\alpha)$ is linear. Accordingly, in this subsection we assume that $\gamma(\alpha) = \gamma'\alpha$, for some $0 < \gamma' < [(N-1)/N]R(\hat{p})$. Eq(7) now produces a knife-edge: there is some price p^* such that if $p_j < p^*$, then $\alpha_j = 1$, and if $p_j > p^*$, then $\alpha_j = 0$. But since by Lemma 5, P is the only price such that $\alpha(P;r) = 0$, we must have $\alpha(p;r) = 1$ for all $p < P$. Then, from eq(13), $F_0(p) = 0$ for all $p < P$, so by eq(1) and Lemma 2, we must have $P = \hat{p}$, the monopoly price.

If there were no atom at $\hat{p}$, then $\bar{\alpha} = 1$, which would be inconsistent with eq(9). Hence, there must be an atom at $\hat{p}$, and eq(9) determines $\bar{\alpha} = 1 - \{N\gamma'/[(N-1)R(\hat{p})]\}^{1/(N-1)}$. Since $\alpha(p;r) = 1$ for all $p < \hat{p}$, $(1-\bar{\alpha})$ is also the mass of the atom at $\hat{p}$. Then, expected profits are $E\pi(\hat{p},0) = R(\hat{p})(1-\bar{\alpha})^{N-1}/N$. Since $\alpha = 1$ for $p < \hat{p}$, $A(p;r) = F(p;r)$; hence, $E\pi(p,1) = R(p)[1 - F(p;r)]^{N-1} - \gamma'$. Setting $E\pi(p,1) = E\pi(\hat{p},0)$ and solving for $F(p;r)$ yields

$$F(p;r) = 1 - \left(\frac{N\gamma'}{(N-1)R(p)}\right)^{1/(N-1)} \qquad (14)$$

Observe that $\lim_{p\to\hat{p}} F(p;r) = \bar{\alpha}$, as required. The lower bound B is determined by setting $F(B;r) = 0$, which yields $B = R^{-1}[N\gamma'/(N-1)]$.

Sellers choose the monopoly price, $\hat{p}$, with positive probability and do not advertise that price (because the net marginal benefits to advertising are zero). In the event that all sellers choose $\hat{p}$, all the buyers are uninformed and simply distribute themselves uninformly over the sellers. In all other events, some seller chooses a lower price and all buyers are fully informed about these lower prices, so only the lowest-priced seller has customers. A seller must trade-off the likelihood of being the lowest-priced seller and getting all the business with the likelihood of all sellers choosing $\hat{p}$ and sharing the business. The price distribution, eq(14), equates the expected profits of these alternatives.

Note that as γ' increases toward $[(N-1)/N]R(\hat{p})$, the lower bound B converges to $\hat{p}$, and the mass of the atom at $\hat{p}$ converges to 1. Thus, the price distribution converges pointwise to the degenerate distribution at the monopoly price, which is the unique solution for all $\gamma' \geq [(N-1)/N]R(\hat{p})$. Conversely, as $\gamma' \to 0$, the lower bound B converges to 0, the mass of the atom at $\hat{p}$ converges to 0, and F(p) converges to 1 for all $p > B$. Thus, the price distribution converges weakly to the degenerate distribution at 0 [i.e. Bertrand pricing].

4.2 Quadratic Advertising Costs.

Suppose $\gamma(\alpha) = \gamma'\alpha + \gamma''\alpha^2/2$, for positive constants γ' and γ''. We can derive a closed form expression for the optimal advertising policy:

$$\alpha(p;\alpha_p) = \left\{\left[R(P)\left(\alpha_p + \frac{1-\alpha_p}{N}\right) - \frac{R(p)}{N}\right]\left(\frac{2N(\gamma'+\gamma''\alpha_p)}{(N-1)R(P)\gamma''}\right) - \frac{2\gamma'\alpha_p}{\gamma''} - (\alpha_p)^2\right\}^{\frac{1}{2}}, \tag{15}$$

subject to $\alpha(p;\alpha_p) \in [0,1]$. We then seek a α_p such that $F(P;r,\alpha_p) \leq 1$ and $\alpha_p \cdot [1-F(P;r,\alpha_p)] = 0$.

The behavior of the SNE $\alpha(p)$ is illustrated in Figure 2. The example

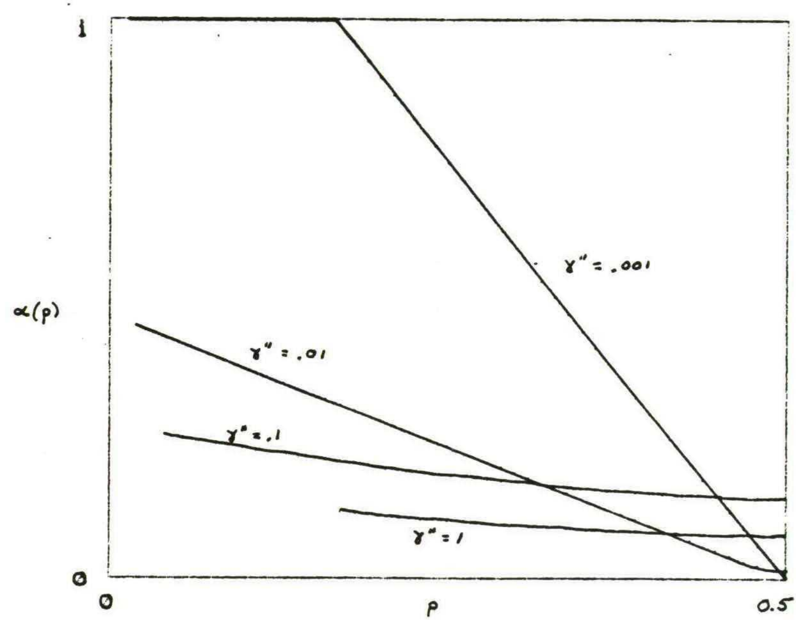

Figure 2

assumes search cost sufficiently high so $P = \hat{p}$, linear demand $D(p) = 1-p$, $N = 10$, and $\gamma' = 0.01$. For sufficiently small γ'', the SNE has an atom at P and $\alpha_P = 0$. The SNE advertising level has $\alpha(p) = 1$ for low prices and sharply declines towards zero for prices near P. As $\gamma'' \to 0$, the range of fully advertised prices ($\alpha = 1$) increases to $[B,P)$; i.e. the advertising solution converges to that for linear advertising costs. As γ'' increases, advertising is curtailed on more and more prices; i.e. $\alpha(p)$ rotates downward and counterclockwise. For sufficiently high γ'', $\alpha_P > 0$, and as γ'' increases further, $\alpha(p)$ declines towards zero.

Figure 3 displays several SNE price density functions for this example. As $\gamma'' \to 0$, the lowest price B declines towards 0.0112, and eventually an atom forms at $\hat{p}$ which increases in mass towards 0.2925. The limit as $\gamma'' \to 0$ is the SNE for the linear advertising case presented above. Notice that for $\gamma'' \leq 0.01$, the density is "U-shaped"; thus, a seller is more likely to charge a high price or a low "sale" price, when advertising costs do not increase too sharply.

5. ASYMPTOTIC BEHAVIOR.

We are interested in how the qualitative behavior of a consistent SNE is affected by the number of buyers and sellers, advertising costs and search costs.

Assuming that $\gamma(\alpha,M)$ and $\partial\gamma(\alpha,M)/\partial\alpha$ increase monotonically to limits $\gamma(\alpha,\infty)$ and $\gamma'(\alpha,\infty)$, the limit set of SNE is completely characterized by the limit marginal advertising costs $\gamma'(\alpha,\infty)$. From Figure 1, it is apparent that both a_L and a_H decrease, which suggests that α_P and hence $\alpha(p;r)$ decrease. Unfortunately, due to the potential non-uniqueness of the SNE, we cannot derive any general comparative statics about the price distribution, other than that a

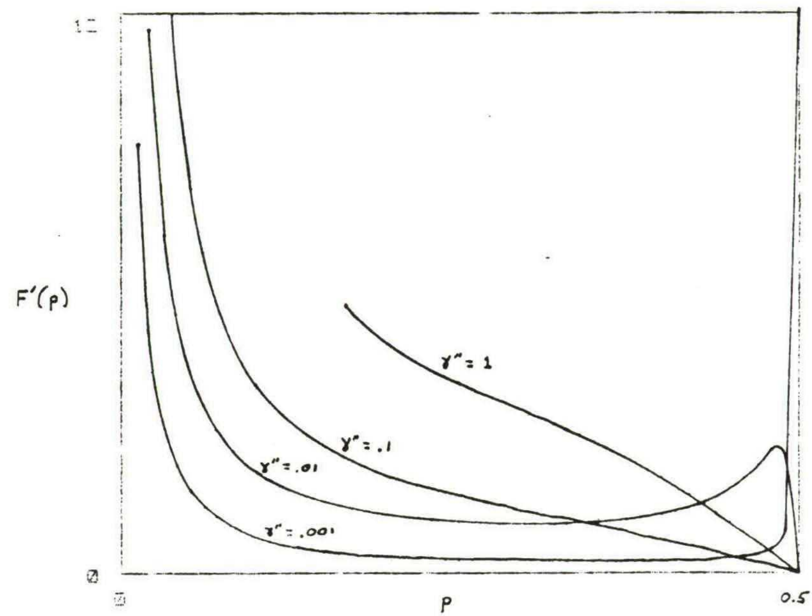

Figure 3.

well-defined limit distribution exists.

5.3 The Effect of the Number of Sellers.

We will prove the remarkable result that the price distribution converges to the monopoly price as $N \to \infty$. It is easiest to see this result for the case of linear advertising costs. Following this, we will derive this result for the general case. Recalling from Theorem 1, that pure monopoly pricing is the unique SNE when $\gamma'(0) \geq R(\hat{p})$, we will focus exclusively on the remaining case when $\gamma'(0) < R(\hat{p})$.

From eq(14), given linear advertising costs $\gamma' > 0$,[8] as $N \to \infty$, $F(p;r) \to 0$ for all $p < P$, and the mass at P approaches unity. While B declines monotonically to $R^{-1}[\gamma'/R(P)]$, the SNE price distribution converges to the degenerate distribution with unit mass at P. But then recalling the remarks of Section 2.1, there exists an N' such that for all $N > N'$, $r > P$, so by Lemma 2, $P = \hat{p}$. In other words, the upper bound of the price distribution hits the monopoly price for sufficiently many (finite) sellers, and becomes more concentrated at the monopoly price as the number of sellers increases without bound.[9]

To develop this result for the case of strictly convex advertising costs, we proceed with a number of lemmas. Note that while there may be multiple SNE for various values of N, the results apply to any SNE, so we will be showing not just that some SNE converges to monopoly pricing, but that *all* SNE converge to

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[8]That $\gamma'(0)$ is strictly positive is crucial to our asymptotic results. It can be easily seen from eq(14) why $\gamma' = 0$ is a singular case.

[9]A curious non-monotonic behavior is exhibited when $R(\hat{p})/2 < \gamma' < R(\hat{p})$. For example, suppose $\gamma = 0.75 \cdot R(\hat{p})$. By Theorem 1, pure monopoly pricing is the unique SNE for N = 2,3 and 4. Then for all $N \geq 5$, there is no pure-strategy SNE, meaning that the mean price is less than $\hat{p}$. However, as $N \to \infty$, the mixed-strategy SNE converges back to the monopoly price.

monopoly pricing.

Lemma 8. As $N \to \infty$, $\bar{\alpha}$ and α_P converge to 0, $(1-\bar{\alpha})^{N-1} \to \gamma'(0)/R(P)$, and $N\bar{\alpha} \to \ln[R(P)/\gamma'(0)]$.

From Lemma 8, we see that while the average effort of advertising per seller $\bar{\alpha}$ vanishes as $N \to \infty$, the aggregate average effort $N\bar{\alpha}$ does not. Moreover, the probability that a typical buyer is totally uninformed, $(1-\bar{\alpha})^{N-1}$, remains bounded below 1, given $\gamma'(0)/R(P) < (N-1)/N$.

Lemma 9. As $N \to \infty$, $\alpha(p) \to 0$, but $N\alpha(p) \to \infty$ for all $p < P$.

In other words, the advertising effort per seller shrinks to zero for all prices, but slowly enough so the aggregate advertising effort increases without bound for all $p < P$. Note that this result does not imply that expected advertising expenditures increase without bound, because expected advertising expenditures per seller are $E\gamma = \int\gamma[\alpha(p)]dF(p)$, which clearly depend not only on $\alpha(p)$ but also on $F(p)$. Indeed, given Lemma 9, to keep $E\gamma$ bounded, it must be that $F(p)$ has less and less probability mass at $p < P$. This conclusion is formalized in the Theorem 5 below, whose proof uses a different insight. But first it is useful to present one last Lemma concerning the distribution of received prices.

Lemma 10. As $N \to \infty$, $[1 - A(p)]^{N-1} \to \gamma'(0)/R(p)$ and $A(p) \to 0$ for all p.

In other words, the probability that a typical buyer will be informed that seller j has a price $p_j \leq p$, $A(p)$, converges to zero as $N \to \infty$. On the other hand, because the aggregate advertising effort does not vanish, the probability that p_j is the lowest price of the other N-1 sellers about whom a typical buyer is informed, $[1 - A(p_j)]^{N-1}$, is bounded above zero.

Finally, we come to our main results concerning the effect of seller entry on the SNE mixed-strategies.

Theorem 4. As $N \to \infty$, $F(p) \to 0$ for all $p < P$, and $B \to R^{-1}[\gamma'(0)]$.

Thus, we see that the SNE price distribution converges to the degenerate distribution with all mass at one price P. The next question concerns the effect of N on this price P and the mass at P.

Theorem 5. There is an N' such that for all N > N', there is a unique SNE, $P = \hat{p}$, and $F(P) > 0$.

Thus, Theorem 5 together with Theorem 4, establishes that the SNE converges to pure monopoly pricing as the number of sellers increases without bound. Indeed, once entry exceeds some finite level, there is an atom in the price distribution at the monopoly price, and the mass of that atom increases to 1 as $N \to \infty$.

The reason for this remarkable result is that sellers always have the option of being content with their captive share of buyers who are totally uninformed: asymptotically, a mass of $\gamma'(0)/R(\hat{p})$ buyers. On the other hand, competing for buyers requires expenditures for advertising and a cut in price, and these efforts are rewarded only if the seller has the lowest price in the information set of a typical buyer.[10]

This result seems counterintuitive if one focuses on the limit as $N \to \infty$: monopoly pricing with no advertising. Surely (you might say) advertising a price just below the monopoly price would be profitable. Indeed, the limit strategies are not equilibrium strategies for the model with $N = \infty$. It appears that [as in Stahl (1988)] there does not exist a SNE for the limit model with $N = \infty$. However, Theorems 4 and 5 are about the asymptotic behavior of the model with a large but finite number of sellers. To understand better how the limit model and the large-but-finite model differ, we can extract more insights from the foregoing Lemmas.

First, let's examine the advertising effort. From Lemma 8, we observe that the average effort of advertising per seller $\bar{\alpha}$ vanishes as $N \to \infty$. Expected advertising expenditures per seller are $E\gamma = \int\gamma[\alpha(p)]dF(p)$. Using Lemma 9, $E\gamma$ vanishes as $N \to \infty$. Thus, average advertising effort and expenditures shrink towards zero with seller entry. On the other hand, by Lemma 8, the aggregate average effort $N\bar{\alpha}$ is bounded above zero. Similarly, total industry advertising expenditures $NE\gamma \approx N\bar{\alpha}\gamma'(0)$, which, by Lemma 8, converges to $\gamma'(0)\ln[R(\hat{p})/\gamma'(0)] > 0$. Because of this aggregate advertising effort, the probability that a buyer is completely uninformed, $(1-\bar{\alpha})^N$, converges to $\gamma'(0)/R(\hat{p}) \in (0,1)$, despite the fact that $\bar{\alpha} \to 0$. Thus, there is always some positive mass of uninformed buyers, <u>and</u> a positive mass of (partially) informed buyers.

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[10]The model of Stahl (1989a) is similar under the interpretation that the proportion of zero-search-cost types is like having a group that has received ads from every seller and treating advertising as a prior decision. The result was that the SNE price distribution converged to monopoly pricing because the rewards for price competition declined with seller entry. Now with endogenous advertising which turns out to be more intense at lower prices, the forces that drove the former result are even stronger: competition is too fierce, so sellers opt for their captive market share instead.

Given that there is always a positive mass of informed buyers, it does not follow that by slightly undercutting the monopoly price a seller can gain a discrete increase in sales. The seller must still be the lowest-priced seller among those about whom the buyer is informed. The probability that p_j is the lowest price among the sellers about whom a typical buyer is informed is $[1 - A(p_j)]^{N-1}$, which, by Lemma 10 converges to $\gamma'(0)/R(p_j)$. Thus, the marginal revenue gained is approximately $\gamma'(0)$, while the direct marginal advertising cost is $\gamma'(0)$, so the net direct marginal benefit of additional advertising $p_j < \hat{p}$ is zero. But then we must also take account of the foregone profits from the mass of uninformed buyers, so the total net marginal benefit from more advertising at low prices is non-positive. Thus, we see that even though F(p) may be arbitrarily close to the degenerate distribution at $\hat{p}$, and even though the advertising effort per seller is arbitrarily small, it does not follow that selecting and advertising a price below $\hat{p}$ would be profitable. Hopefully this discussion makes Theorems 4 and 5 more understandable.

We can also deduce the effect of entry on SNE profits. Expected profits per seller are $E\pi^* = E\pi(P,\alpha_p) = R(P)\{\alpha_p + (1-\alpha_p)/N\}(1-\bar{\alpha})^{N-1} - \gamma(\alpha_p) = [\alpha_p\gamma'(\alpha_p) - \gamma(\alpha_p)] + \gamma'(\alpha_p)/(N-1)$. Then, by Theorem 5, there is an N' such that for all N > N', $\alpha_p = 0$, so $E\pi^* = \gamma'(0)/(N-1)$. Therefore, $E\pi^* \to 0$, as $N \to \infty$. On the other hand, total industry profits, $NE\pi^*$, converge to $\gamma'(0) > 0$.

5.2 Multiplicative Changes in Advertising Costs.

To examine the effects of advertising costs, we consider multiplicative changes in the advertising cost function $\gamma(\alpha,M)$. Fix some $\gamma_0(\alpha)$ and let $\gamma(\alpha) = \lambda\gamma_0(\alpha)$ for $\lambda > 0$.

Theorem 6. As $\lambda \to 0$, $\bar{\alpha} \to 1$, $B \to 0$, and $F(p;r^*) \to 1$ for all $p > 0$.

In other words, advertising saturates the market and the price distribution weakly converges to the degenerate distribution at $p = 0$ (the Bertrand outcome). Note that by eq(1), $r > 0$, so there is a positive lower bound on P. Since $\bar{\alpha} \to 1$, it follows that $\alpha_B \to 1$, but, when $\gamma_0(1) = \infty$, α_B approaches 1 at a rate slow enough so $\lambda\gamma_0(\alpha_B) \to 0$.

As marginal advertising costs increase, eventually $\gamma'(0) \geq [(N-1)/N]R(\hat{p})$, so by Theorem 1, the SNE converges to pure monopoly pricing. In general, the SNE lies between the Bertrand outcome and the monopoly outcome.

5.3 The Effect of Search Costs.

It is interesting to ask how the SNE behaves as the buyer search costs become arbitrarily small, and to compare these results with the case of very low advertising costs. Recall that without advertising [Diamond, 1971], the unique equilibrium is monopoly-pricing, so there is a sharp discontinuity at $c = 0$. With advertising, we will see that there is still a discontinuity, albeit of smaller magnitude.

Theorem 7. As $c \to 0$, (a) B and P converge to $\hat{r} = R^{-1}[\gamma'(0)(N-1)/N]$, and (b) $\alpha(p) \to 0$.

In other words, the SNE price distribution converges strongly to a single atom at $\hat{r} < \hat{p}$.[11] This follows directly from eq(1). Then, from Figure 1 we can see

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[11]Note that this limit is *not* an equilibrium of the model with $c = 0$, since clearly marginal-cost pricing is the unique equilibrium when $c = 0$. Further,

that advertising vanishes.

This result stands in sharp contrast to Theorem 4: that marginal-cost pricing prevails as advertising costs diminish. We see that as the comparative advantage in information transmission switches from sellers to buyers, sellers drastically curtail their advertising efforts.

6. WELFARE EFFECTS.

A crucial factor in determining welfare effects is the cumulative probability distribution of the price at which buyers purchase. First, the probability distribution of the lowest known price is just $1 - [1-A(p)]^N$. But there is a probability of $(1-\bar{\alpha})^N$ that a buyer will be uninformed, in which case the buyer faces $F_0(p)$. Letting $G(p)$ denote the total cumulative distribution of the price at which buyers purchase,

$$G(p) = 1 - [1-A(p)]^N + (1-\bar{\alpha})^N F_0(p) \qquad (16)$$

At the upper bound P, $G(P) = 1 - (1-\bar{\alpha})^N[1 - F_0(P)]$, which is identically 1, since $F_0(P) = 1$. But note that if F has an atom at P, there is also an atom in G() at P of mass $(1-\bar{\alpha})^N F_0(P)$.

Of particular interest is the behavior of $G(p)$ as $N \to \infty$. From Lemma 8 and Theorem 5, the last term of eq(16) converges to zero for all $p < P$. From eq(6) and Lemma 10, $[1-A(p)]^N \to \gamma'(0)/R(p)$, which is strictly less than 1 for all $p \in$

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this result holds if and only if advertising costs are strictly convex, since as shown in Section 4.1, with linear advertising costs, $P = \hat{p}$, independent of c.

(B,P]. Hence, G(p) converges to a distribution with an atom of mass $\gamma'(0)/R(P)$ at the upper bound. Indeed, from Theorem 6, there exists an N' such that for all N > N', F() and hence G() have atoms at P.

The appropriate welfare measures are consumer surplus, producer surplus, and total surplus. Let $CS(P) = \int_P^\infty D(p)dp$. Then, we can express expected consumer surplus as

$$ECS \quad = \quad CS(P) + \int_B^P D(p)G(p)dp \ . \qquad (17)$$

Expected producer surplus is just aggregate profits $NE\pi$. Expected total surplus can be expressed as

$$ETS \quad = \quad CS(P) + R(P) - \int_B^P pD'(p)G(p)dp - N\int_B^P \gamma[\alpha(p)]dF(p) \ . \qquad (18)$$

We can see that anything that causes G(p) to increase (in the sense of stochastic dominance) will unambiguously increase ECS. However, except for a very special case, the effects on $NE\pi$ and ETS are far from obvious.

Consider the special case of unit demand for prices less than or equal to $\hat{p}$, and zero demand otherwise. Then, since D' = 0, G(p) is irrelevant to ETS. In fact, it is immediate from eq(18) that the social optimum is a zero level of advertising. The price merely allocates the maximum total surplus between buyers and sellers. Therefore, we can conclude that there is *over-advertising* in the SNE in the special case of unit demands. It is also clear that the demands could be perturbed slightly so D'(p) < 0, and still preserve the over-advertising conclusion.

To derive more definite welfare results, we turn to the case of linear advertising costs. The solution for the finite case with linear advertising costs was derived in section 4.1. Recall that there is an atom in the SNE price distribution F() at P of mass $f = (1-\bar{\alpha})^{N-1} = \{N\gamma'/[(N-1)R(P)]\}^{1/(N-1)}$ at P.

First, consider entry of sellers. A key factor is how the mass at P in G() behaves. This mass is equal to $(1-\bar{\alpha})^N = \{N\gamma'/[(N-1)R(P)]\}^{N/(N-1)}$. We should like to know whether this atom is increasing or decreasing in mass as a function of N. It is straightforward to verify that $(1-\bar{\alpha})^N$ is increasing in N for all N ≥ 2 if $\gamma'/R(P) \leq 1/2e \simeq 0.184$. In other words, if the marginal advertising costs are small in comparison to monopoly revenues per buyer, then seller entry makes it more likely that buyers will be uninformed. Consequently, some buyers may be made worse off.

For example, if $D(p) = 1-p$, and $\gamma' \leq .0125$, then ECS, $NE\pi$ and ETS are monotonically decreasing and total industry advertising expenditures, $NE\gamma$, are increasing in N for all $N \geq 2$. For $\gamma' = .125$, $NE\pi$, ECS and ETS are decreasing while $NE\gamma$ is increasing for all $N \geq 2$. For intermediate ranges of γ', $NE\pi$, ETS and $-NE\gamma$ are decreasing, while ECS first increases and then decreases in N. In other words, while buyers can sometimes benefit from entry, advertising expenditures typically increase more than enough to offset the buyers' benefits, resulting in a decline in social welfare.

Second, consider multiplicative changes in advertising costs. It is obvious that the mass is increasing in advertising costs, γ'. Indeed, since F() collapses to the degenerate distribution at $\hat{p}$ for sufficiently large γ', G() also collapses to the monopoly price. Thus, for all $p < P$, G(p) is decreasing, so ECS is also decreasing. Total profits, $NE\pi = R(P)f^{N-1} = N\gamma'/(N-1)$, so total profits are increasing with γ'. Total advertising costs $N\int\gamma[\alpha(p)]dF = N\gamma'(1-f)$.

Using the expression for f and differentiating with respect to γ', we find that total advertising costs are increasing (decreasing) as $\gamma'/R(P) < (>) (1 - 1/N)^N < 1/e$.

To study the welfare effects of search costs, we need to subtract c from ECS. Then, we can write $d[ECS - c]/dc = \int_B^{\hat{p}} D(p)[\partial G(p)/\partial c]dp - 1$. Trivially, when advertising costs are linear, since the SNE is independent of c, buyers are unambiguously better off with lower search costs. The conclusion is not so immediate when advertising costs are strictly convex. From Theorem 4, we know that the SNE concentrates mass at $\hat{r}$ and $\alpha \to 0$. Hence for $c \approx 0$, from eq(16), $\partial G/\partial c \approx \partial F_0/\partial c$, and from eq(1), $\int_B^{\hat{p}} D(p)[\partial F_0/\partial c] - 1 < 0$. Thus, buyers benefit from reductions in search costs when search costs are already very low. On the other hand, ECS-c is not necessarily monotonic in c. Indeed, the effect of advertising cutbacks and a rising lower bound on prices (B) can result in buyers being worse off after an exogenous reduction in search costs.

7. CONCLUSIONS.

We have studied a model of seller pricing behavior in which buyers can become informed through seller advertising and direct search. Thus, the amount of information buyers have is an endogenous phenomena. Our representation of advertising is very general and can encompass word-of-mouth and other leakage effects beyond simple broadcasting of ads. We completely characterized the Symmetric Nash Equilibria.

If the marginal advertising cost is too high, then the monopoly price with zero advertising is the unique SNE; otherwise, there is a mixed-strategy SNE which can be expressed as a price distribution and an advertising policy conditional on price. The price distribution has a connected support, and is atomless below the highest price in the support. There can be an atom at the highest price in the support, in which case, it is optimal to refrain from advertising that price. The advertising effort is decreasing in price, so lower prices are advertised more intensively than higher prices.

As advertising costs become negligible, the SNE price distribution converges to the Bertrand price, and as advertising costs increase, the SNE price distribution converges to the monopoly price. On the other hand, as search costs become negligible, the SNE price distribution converges to a price higher than marginal production costs. Thus, the two information channels have different effects on the outcome.

The comparative statics on the effect of the number of sellers is most interesting. There is a finite number N' such that if there are more sellers than N', then there is an atom at the monopoly price. As the number of sellers increases without bound, advertising per seller vanishes and the SNE price distribution converges to the monopoly price.

To analyze the welfare effects, we derived the distribution of the price at which sales actually occur. This distribution is considerably skewed towards lower prices since it is a high order statistic. In particular, as the number of sellers increases without bound, this distribution converges to a non-degenerate distribution with a mean less than the monopoly price; on the other hand, the limit distribution has an atom at the monopoly price. We presented examples for a variety of demand functions and linear advertising costs in which welfare declined with entry of sellers.

The question of the efficiency of the SNE advertising effort is difficult to fully analyze. In the case of constant demand, there clearly is over-advertising. For other demand functions, there can be either over- or under-advertising.[12]

Comparing this paper with Stahl (1989a), both find that pricing becomes more monopolistic with entry, and both find that welfare can decrease with entry. Thus, *allowing sellers to advertise does not induce more competitive pricing*. As the number of sellers increases, a positive proportion of the buyers remain uninformed, so the asymptotics are similar.[13]

These results stand in sharp contrast to Cournot models [e.g. MasColell, 1982] and to Bertrand models with and without capacity constraints [e.g. Allen and Hellwig, 1986] which find that prices become more competitive as the number of firms increases. The crucial ingredient in the model of this paper is costly buyer search. We know from Diamond (1971), that costly buyer search can lead to monopoly pricing. However, in our model we have neutralized the force of that argument by introducing informed consumers (exogenously as in Stahl, 1989a, or endogenously via advertising as in this paper). Indeed, given a finite number of sellers and advertising costs not too high, the Nash Equilibrium always

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[12]Grossman and Shapiro (1984) examine a Butters-type advertising model with differentiated products and also find the possibility of over-advertising. They do not, however, examine the effects of seller entry since they focus exclusively on a continuum model.

[13]Recently, Robert and Stahl (1991) consider a similar model except that (i) each buyer demands one unit up to a choke price of $v > 0$, and (ii) the search cost is a transportation cost and *not* a cost of obtaining a price quotation (such as a telephone call). The effect of entry in their model also leads to more monopolistic seller pricing, but the limit is not monopoly pricing. Unlike the present model with zero transportation costs and positive price quotation costs, when transportation costs are positive but obtaining price quotations is costless (above the transportation cost), there is a gap (of size c) in the support of the price distribution and no seller advertises a price above $r-c$. This forces more competitive pricing and keeps the average price bounded below the monopoly price. Robert (1989) analyzed a similar model to Robert and Stahl, except he assumed that initial marginal advertising costs $\gamma'(0) = 0$. Also, he did not characterize the asymptotic behavior of the SNE price distribution, and he did not study the impact of entry on total surplus.

exhibits price dispersion. What is remarkable is that as the number of sellers increases, pricing becomes asymptotically monopolistic. By analyzing the asymptotic behavior in some detail, we have learned that advertising per seller is curtailed with seller entry because the net marginal benefits to advertising efforts shrink to zero. At the same time, aggregate industry-wide advertising stays bounded above zero, which reinforces the shrinking benefits to advertising by an individual seller, because that seller gains nothing unless he is the lowest-priced seller about whom a typical buyer is informed. In other words, our model demonstrates how a seemingly more competitive environment (i.e. more sellers) can actually lead to less competitive behavior because the incentives for competitive pricing is significantly lessened by the larger number of competitors.

It is also insightful to compare these results with results from repeated games in which monopoly pricing can be maintained as a dynamic Nash Equilibrium with, for example, trigger strategies [e.g. Friedman, 1986]. Essentially, the threat of reversion to competitive behavior is used to deter firms from deviating from tacit collusion. Our result has some of this flavor (in that the "heat" of competition acts to deter competitive advertising and pricing), but there are more important differences. First, a grim trigger strategy supporting collusion can succeed in deterring all competitive instincts. However, in our model, only individual competitive behavior is deterred; the aggregate level of advertising remains positive, the lower bound on the price distribution declines, and the distribution of "sales prices" remains dispersed. Second, the repeated game equilibrium can support joint profit maximization of the sellers, whereas in our model the sellers (in aggregate) "waste" money on advertising. Third, tacit collusion in a repeated game seems less realistic with a very large number of firms (because of practical detection problems), but it is the large number case of our model that has near monopoly pricing. Thus, the forces leading to monopolistic pricing in our model are very different from the forces in a repeated game setting.

APPENDIX

Proof of Lemma 1.

If there is an atom in $F(p;r)$, there is a chance that two or more sellers may be charging the lowest price. In that event, we assume that the buyer randomizes between the low-priced sellers with each having an equal chance of being chosen. For example, if there is an atom of mass $f > 0$ at price $\tilde{p}$, and if a buyer is informed about a price $\tilde{p} \le r$ from K sellers while the information about all the other sellers is that their prices are higher than $\tilde{p}$, then each of the K sellers have a 1/K change of selling to that buyer. The probability of this event is $[\alpha(\tilde{p})f]^K \cdot [1 - A(\tilde{p};r)]^{N-K}$. Thus, in eq(3)

$$\varphi(\tilde{p}) = \sum_{K=0}^{N-1} \frac{(N-1)!}{(N-1-K)!(K+1)!} [\alpha(\tilde{p})f]^K \cdot [1 - A(\tilde{p};r)]^{N-1-K} . \tag{A1}$$

Suppose $F(p;r)$ has an atom of mass $f > 0$ at $\tilde{p}$. Let $\alpha_j = \alpha(\tilde{p};r)$, and consider $p_j = \tilde{p}-\varepsilon$ for $\varepsilon > 0$. Then, $\lim_{\varepsilon\to 0} \varphi(p_j) = [1 - A(\tilde{p};r) + \alpha(\tilde{p};r)f]^{N-1} =$

$$\sum_{K=0}^{N-1} \frac{(N-1)!}{(N-1-K)!K!} [\alpha(\tilde{p};r)f]^K \cdot [1 - A(\tilde{p};r)]^{N-1-K} . \tag{A2}$$

Hence, using eq(3) and (A1-2), $\lim_{\varepsilon\to 0} [E\pi(p_j,\alpha_j) - E\pi(\tilde{p},\alpha_j)] =$

$$R(\tilde{p})\alpha(\tilde{p};r)\cdot \sum_{K=0}^{N-1} \frac{(N-1)!K}{(N-1-K)!(K+1)!} [\alpha(\tilde{p};r)f]^K \cdot [1 - A(\tilde{p};r)]^{N-1-K} ,$$

which is strictly positive if $\alpha(\tilde{p};r) > 0$, implying that undercutting $\tilde{p}$ is profitable: a contradiction. Therefore, if $f > 0$, then $\alpha(\tilde{p};r) = 0$. Q.E.D.

Proof of Lemma 2.

(a) If $\alpha = 1$ for all $p \le P$, then we have classic Bertrand price competition so F is concentrated at 0, implying that $E\pi = -\gamma(1) < 0$. Since this cannot be part of a SNE, we must have $\bar{\alpha} < 1$, in which case $E\pi(r,0) = R(r)(1-\bar{\alpha})^{N-1}/N > 0$. Therefore, $E\pi^* > 0$.

(b)(1) If $P > r$, then a sale could occur at P only if all other sellers have $p_k = P$, and market share would be f^{N-1}/N, where f is the atom at P. But since $P > r$, all buyers will have searched all sellers and be fully informed. Therefore, a deviation to $P-\varepsilon$ for some $\varepsilon > 0$ will capture the whole market. Given $E\pi^* > 0$, this is a profitable deviation. Hence, we must have $f = 0$, and $E\pi(P,\alpha) \le 0$, which is inconsistent with (a). Therefore, $P \le r$.

(2) Suppose $\hat{p} < P \le r$. In this case, for $p_j \in (\hat{p},P)$, sales revenues are decreasing and market share is decreasing in p, so $p_j = \hat{p}$ dominates P, a contradiction. Therefore, $P \le \min\{r,\hat{p}\}$.

(3) Suppose $P < \min\{r,\hat{p}\}$. Then, for $p_j \in (P,\min\{r,\hat{p}\})$, $E\pi(p_j,\alpha_j) = R(p_j)[\alpha_j + (1-\alpha_j)/N](1-\bar{\alpha})^{N-1} - \gamma(\alpha_j)$, which is increasing in p_j - a contradiction. Therefore, $P = \min\{r,\hat{p}\}$. Q.E.D.

Proof of Lemma 3.

Suppose to the contrary that there exists an open interval $(p',p'') \subset [B,P]$ that is not contained in the support of $F(p;r)$. Observe that $A(p;r)$ will be constant on this open interval, so

$$E\pi(p_j,\alpha_j) = R(p_j)\{\alpha_j[1-A(p';r)]^{N-1} + (1-\alpha_j)(1-\bar{\alpha})^{N-1}/N\} - \gamma(\alpha_j) ,$$

which is increasing in p_j, so p' cannot be in the support. Hence, we can extend the open interval in the direction of B, leading to the conclusion that B is not in the support - a contradiction. Q.E.D.

Proof of Lemma 4.

Suppose to the contrary that there exists two prices $p' < p'' \leq \hat{p}$, both in the support of $F(p;r)$ such that $\alpha(p') = 0 = \alpha(p'')$. But then $E\pi(p',0) = R(p')(1-\bar{\alpha})^{N-1}/N < R(p'')(1-\bar{\alpha})^{N-1}/N = E\pi(p'',0)$, a contradiction. Q.E.D.

Proof of Lemma 5.

Suppose there exists an atom at $p' < P$. By Lemma 1, $\alpha(p') = 0$, and by Lemma 4, there can be no atom at P. Since $\alpha(P)$ must be chosen optimally, it follows that $E\pi(P,\alpha(P)) \geq R(P)(1-\bar{\alpha})^{N-1}/N$, which in turn is strictly greater than $R(p')(1-\bar{\alpha})^{N-1}/N = E\pi(p',0)$, which contradicts the optimality of p'. Q.E.D.

Proof of Lemma 6.

From Lemma 4, $\alpha(p) > 0$ for all $p < P$, so the l.h.s. of eq(7) must be non-negative for all $p < P$. Suppose the l.h.s. of eq(7) is negative at $p = P$ and $\alpha_P = 0$. Then since $\gamma()$ is strictly convex and the l.h.s. of eq(7) is continuous in p and α_j, it follows that $\alpha(p) = 0$ for some interval below P, which contradicts Lemma 4. Therefore, $\partial E\pi(p,\alpha_j)/\partial\alpha_j \geq 0$ for all $p \in [B,P]$. Next, suppose the l.h.s. of eq(7) is strictly positive at P. Then, $\alpha(P) = 1$, and by eq(9), $\alpha(p) = 1$ for all $p \in [B,P]$, implying that $\bar{\alpha} = 1$. But then using eq(7), $\partial E\pi(P,1)/\partial\alpha_j = -\gamma(1) < 0$, a contradiction. [Also, $E\pi(P,1) = -\gamma(1) < 0$.] Q.E.D.

Proof of Lemma 7.

Note that if $\gamma'(0) > [(N-1)/N]R(P)$, then eq(9) cannot be satisfied, so there is no solution for $\alpha(p)$ that is compatible with a SNE. If $\gamma'(0) = [(N-1)/N]R(P)$, then it must be that $\bar{\alpha} = 0$, in which case $E\pi(\cdot,0)$ is strictly increasing up to $\hat{p}$; hence $P = \hat{p}$, which violates the premise that $\gamma'(0) < [(N-$

$1)/N]R(\hat{p})$. Therefore, a SNE must have $\gamma'(0) < [(N-1)/N]R(P)$. The constraint on r then follows from Lemma 2. Q.E.D.

Proof of Theorem 1.

(a) Given a pure-strategy SNE at price P, then by eq(1), $r > P$, so by Lemma 2 we must have $P = \min\{r,\hat{p}\} = \hat{p}$. Moreover, by Lemma 1, $\alpha(\hat{p};r) = 0$, and, by eq(4), $\bar{\alpha} = A(\hat{p};r) = 0$. Hence, $E\pi(p_j,\alpha_j) = R(p_j)\{\alpha_j + (1-\alpha_j)/N\} - \gamma(\alpha_j)$. Then $\partial E\pi/\partial p_j = R'(p_j)\{\alpha_j + (1-\alpha_j)/N\}$, which is increasing up to $\hat{p}$. Further, $\partial E\pi/\partial\alpha_j = [(N-1)/N]R(\hat{p}) - \gamma'(\alpha_j)$. Hence, $\alpha(\hat{p};r) = 0$ is optimal iff $\gamma'(0) \geq [(N-1)/N]R(\hat{p})$.

(b) Suppose $\gamma'(0) \geq [(N-1)/N]R(\hat{p})$ and there exists a strictly mixed-strategy SNE. By Lemma 3, the support is a connected interval $[B,\hat{p}]$. Hence, first-order conditions must hold at $\hat{p}$. Differentiating eq(5) with respect to α_j and evaluating at $\hat{p}$ yields: $[(N-1)/N]R(\hat{p})(1-\bar{\alpha})^{N-1} = \gamma'[\alpha(\hat{p};r)] \geq \gamma'(0)$, which violates the premise unless $\bar{\alpha} = 0 = \alpha(\hat{p};r)$. But then, $\partial E\pi/\partial p_j = R'(p_j)\{\alpha_j + (1-\alpha_j)/N\}$, which is increasing up to $\hat{p}$. Thus, contradicting the supposition that $B < \hat{p}$. Q.E.D.

Proof of Theorem 2.

First, consider the case with $r > R^{-1}[\gamma'(0)N/(N-1)]$. The discussion following eq(9) and Figure 1 established that $\alpha_P \in [0,a_L]$. Since $a_L < 1$, by eq(9), $\alpha(p;r,\alpha_P) > \alpha_P$ for all $p < P$. Observing that $\bar{\alpha}(a_L) = a_L$, if $\alpha_P = a_L$, then $\alpha(p;r,a_L) > \bar{\alpha}(a_L)$ for all $p < P$. Hence, $F' < A'/\bar{\alpha}(a_L)$, and integrating: $F(p;r,a_L) < A(p;r,a_L)/\bar{\alpha}(a_L) = 1$. Therefore, there must be an atom at P. But by Lemma 1, α_P must be zero, which it is not. Therefore, $\alpha_P = a_L$ is not a SNE solution, and $F(p;r,a_L) < 1$. By continuity of $F(P;r,\cdot)$, there exists an $\varepsilon > 0$, such that $\alpha_P \in (a_L-\varepsilon,a_L)$ is also not a solution and $F(p;r,\alpha_P) < 1$. As α_P

decreases from a_L towards zero, if there is an $\alpha_P \in (0, a_L)$ such that $F(p;r,\alpha_P) = 1$, then that α_P defines a SNE. If no such α_P exists, then $\alpha_P = 0$ defines the SNE with an atom at P, consistent with Lemma 4.

Second consider the case with $r = R^{-1}[\gamma'(0)N/(N-1)]$. If $P = \hat{p}$, which exceeds r, then the argument above applies. Suppose $P = r$. Then, from eq(9) it must be that $\alpha_P = \bar{\alpha} = 0$. But from Lemma 4, $\alpha(p) > 0$ for all $p < P$, which implies that $F(p) = 0$ for all $p < P$; that is, $B = P$. Also, from eq(6), $[1 - A(p;r,0)]^{N-1} = \gamma'(0)\{1/R(p) + 1/[N-1)R(P)]\} \geq N\gamma'(0)/[(N-1)R(P)] = 1$, where the strict inequality applies for all $p < P$, which implies that $B = P$. Hence, in this case eqs(7 - 14) uniquely generate a pure-strategy SNE at $R^{-1}[\gamma'(0)N/(N-1)]$. [Of course, the given r is not consistent when $c > 0$, but we are not imposing consistency in this theorem.] Q.E.D.

<u>Proof of Theorem 3</u>.

Because of the potential non-uniqueness of the α_P in Theorem 2, we will find a consistent α_P^* and r* simultaneously. Let $P' = R^{-1}[\gamma'(0)N/(N-1)]$, and take $\gamma'(0) < [(N-1)/N]R(\hat{p})$, so $P' < \hat{p}$, and no pure-strategy SNE exists. Consider the following mappings:

(i) $P_n = \min\{\hat{p}, \max\{P', P_{n-1} - H[P_{n-1}, c, F_0(\cdot;P_{n-1}, \alpha_{P,n-1})]\}\}$, and

(ii) $\alpha_{P,n} = \min\{\max\{\alpha_{P,n-1} + F_0(\cdot;P_{n-1}, \alpha_{P,n-1}) - 1, 0\}, a_L(P_{n-1})\}$.

[From the definition of a_L, it depends only on R(P).] Each mapping is continuous from a compact convex set into itself: $[P', \hat{p}]$ and [0,1] respectively. Therefore, by Brower's Fixed Point Theorem there exists a fixed point (P^*, α_P^*). (a) Note from (i) that if $H[P^*, c, F_0(\cdot;P^*, \alpha_P^*)] > 0$, then $P^* = P'$; but by Theorem 2, $\alpha_P^* = 0$ and $F_0(\cdot;P^*, \alpha_P^*)$ has all mass concentrated at P', so $H[0, c, F_0(\cdot;0, \alpha_P^*)] = -c$, a contradiction. Thus, we have $H[P^*, c, F_0(\cdot;P^*, \alpha_P^*)] \leq 0$

and $P^* > P'$; and observe that if $H[P^*, c, F_0(\cdot; P^*, \bar{\alpha}^*)] < 0$, then $P^* = \hat{p}$. (b) Note from (ii) that if $F_0(P^*; P^*, \alpha_P^*) > 1$, then $\alpha_P^* = a_L(P^*) > 0$; but this contradicts the fact that $F(P^*; P^*, a_L) < 1$ [from the proof of Theorem 2]. Thus, we have $F_0(P^*; P^*, \alpha_P^*) \leq 1$; and observe that if $F_0(P^*; P^*, \alpha_P^*) < 1$, then $\alpha_P^* = 0$ as required. If $P^* < \hat{p}$, so $H[P^*, c, F_0(\cdot; P^*, \alpha_P^*)] = 0$, then $r^* = P^*$ is a consistent buyer reservation price. If $H[P^*, c, F_0(\cdot; P^*, \alpha_P^*)] < 0$, then the consistent buyer reservation price r^* exceeds $P^* = \hat{p}$, and that is consistent. Q.E.D.

Proof of Lemma 8. That $\bar{\alpha}$ and α_P converge to 0 can be easily seen from Figure 1 or eq(9). Then, using eq(9), $(1-\bar{\alpha})^{N-1} \to \gamma'(0)/R(P)$. Observe that for sufficiently large N, $(1-\bar{\alpha})^{N-1}$ is arbitrarily close to $\exp(-N\bar{\alpha})$. Q.E.D.

Proof of Lemma 9.

That $\alpha(p) \to 0$ follows because by Lemma 8 the right-hand-side of eq(8) converges to zero. For the second assertion, note from eq(8) that for $\alpha \approx 0$, $\gamma''\alpha^2 \approx [1 - R(P)/R(p)]\gamma'(0)/[NR(P)]$ for $p < P$. Therefore, $N\alpha(p) \to \infty$. Q.E.D.

Proof of Lemma 10.

(a) Referring to eq(6), it will suffice to show that the right-hand side converges to $\gamma'(0)/R(p) > 0$. By Lemma 9, $\alpha(p) \to 0$, so for sufficiently large N, we can write $\gamma(\alpha) \approx \alpha\gamma'(0)$. Thus, the second term on the r.h.s. of eq(6) is approximately $[(\alpha - \alpha_P)\gamma'(0)]/[R(p)\alpha]$. In the first term, we first use eq(9) to substitute for $(1-\bar{\alpha})^{N-1}$, and then using Lemma 9, all the terms converge to zero as $N \to \infty$ except for one which asymptotically approaches $(\alpha_P/\alpha)\cdot\gamma'(0)/R(p)$. Hence, together the r.h.s. of eq(6) asymptotically approaches $\gamma'(0)/R(p) > 0$ as claimed.

(b) Next observe that if there is a scalar $A > 0$ such that $A(p) \geq A$ for all N, then $[1 - A(p)]^{N-1}$ will converge to zero, a contradiction. Q.E.D.

Proof of Theorem 4.

By definition, $\bar{\alpha} = \int_B^P \alpha(p)dF(p)$. Now multiply both sides by N. From Lemma 8, the left-hand side converges to $\ln[R(P)/\gamma'(0)]$. Given Lemma 8, if F(p) does not converge to zero for some $p < P$, then the right-hand side diverges to ∞, a contradiction. For the second assertion, use Lemmas 6 and 8 in eq(10) to get that in the limit $R(B) = \gamma'(0)$. Q.E.D.

Proof of Theorem 5.

(a) Using Theorem 4 and eq(1), it follows that $H(r,c,F_0) \rightarrow -c$ as $N \rightarrow \infty$. Therefore, there is an N' such that for all $N > N'$, $H(r,c,F_0) < 0$ for all $r \in [0,P]$, so the reservation price $r* > P$. Then, by Lemma 2, $P = \hat{p}$.

(b) By Theorem 4, $B \geq R^{-1}[\gamma'(0)]$, so R'/R is finite for all $p \in [B,P]$. Referring to eq(12), we see that first term in {} is $(1-A)/(N-1)$. Given Lemmas 6 and 8, the second term in {} asymptotically approaches $(1-\alpha)/[(N-1)^2\alpha]$. Recalling that $F'(p) = [\partial A/\partial p]/\alpha$, it follows that F' asymptotically approaches $[R'(p)/R(p)] \cdot \{N\alpha+1-\alpha\}/(N\alpha)^2$, which by Lemma 9 converges to zero for all $p < P$. Therefore, $\int_B^{P-} F'(p)dp \rightarrow 0$, as $N \rightarrow \infty$, implying that $F\{P\} > 0$ for sufficiently large N. It follows from Lemma 5 and eq(9) that $\alpha_P^* = 0$, and hence, there is a unique SNE for sufficiently large N. Q.E.D.

Proof of Theorem 6.

As $\lambda \rightarrow 0$, Figure 1 reveals that both a_L and a_H converge to one, so $\bar{\alpha} \rightarrow 1$ and $\gamma'(\alpha_P) \rightarrow 0$. Since $\bar{\alpha} \leq 1 - F\{P\}$, it follows that $F\{P\} \rightarrow 0$. Using eq(9) in eq(8), the λ's can be eliminated by division, so the solution $\alpha(p;r,\alpha_p)$ is independent of λ. Next, using eq(9) in eq(6), as $\lambda \to 0$, the right-hand-side of eq(6) becomes arbitrarily close to zero; hence, $A(p;r,\alpha_p)$ becomes arbitrarily close to 1 for all p, implying that $\partial A(p;r,\alpha_p)/\partial p$ is arbitrarily close to zero for all $p > B$. Consequently, using eq(13-14) and the fact that $F(P) \to 0$, for any α_p and any $p > B$, $1-F(p;r,\alpha_p)$ is arbitrarily close to zero. Further, from eq(10), one can see that $B \to 0$. Therefore, $F(p;r,\alpha_p) \to 1$ for all $p > 0$. Q.E.D.

Proof of Theorem 7.

(a) By Lemma 7, $P \geq \hat{r}$. Suppose $P \to P' > \hat{r}$. If there is no atom at P', then clearly $B \to B' < P'$; if there is an atom at P', then $\alpha_{P'} = 0$, so by eq(10) and the hypothesis that $P' > \hat{r}$, we have $B \to B' < P'$. Given $\gamma()$ is strictly convex, it follows that $B_0' < P'$; hence, $H(P',F_0) = \int_{B'}^{P'} D(p)F_0(p)dp > 0$, which implies that $r < P'$, a contradiction. Therefore, $P \to \hat{r}$. Using eq(8-9), it can be shown that $B \to \hat{r}$.

(b) From Figure 1, it is clear that α_p and $\bar{\alpha} \to 0$, since $P \to \hat{r}$. Q.E.D.

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