# Seasonalities in foreign exchange markets

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# SEASONALITIES IN FOREIGN EXCHANGE MARKETS

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# SEASONALITIES IN FOREIGN EXCHANGE MARKETS

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Abstract

The paper investigates the existence of seasonalities in ex-ante and ex-post profits from forward speculation in foreign exchange markets. Ex-ante profits are constructed using a nonparametric recursive technique. Descriptive statistics and frequency domain methods are used to show that seasonalities are present and significant in all the time series considered. A model economy is constructed to show that these findings are not inconsistent with optimization and the absence of arbitrage opportunities. Simulated estimators are computed for a version of the model using a criterion which minimizes the distance at seasonal frequencies between simulated and actual time series in the metric defined by the covariance of the two periodograms.

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## 1) INTRODUCTION

Efficiency in financial markets is often identified either with the inexistence of predictable variations in the return of assets or with absence of arbitrage trading strategies which yield nonzero profits. The discovery of seasonalities or systematic variations in the ex-ante profits from speculation is therefore troublesome for certain formulations of the efficient market hypothesis.

This paper investigates whether seasonalities exist in the ex-ante and the ex-post profits from forward speculation in foregin exchange markets. Ex-ante profits are constructed using a nonparametric recursive technique. A bootstrap algorithm is used to construct the density of linear forecasts, conditional on the available information set at each t. The difference between the mean of the conditional density of forecasted future spot rates and the forward rate is then taken to represent an estimate of the ex-ante profits. The value of the US dollar in terms of six currencies ( French franc, Swiss franc, German mark, English pound, Canadian dollar and Japanese yen) are used and their weekly rates are considered. The paper shows that there are seasonal variations in ex-ante and ex-post profits for all the currencies considered and that the results are robust to changes in the unit of account selected.

This evidence can be interpreted in either of two ways. If one believe that ex-ante profits from forward speculation should be constant, either because agents are risk neutral or because the time interval is short ( Sims (1984), Lehman(1987)), then the presence of seasonal variations in the ex-ante profits series represent inefficiencies. In this case, in fact, it is possible to devise arbitrage trading strategies which yield systematic profits.

Alternatively, one may be led to think that seasonalities in the ex-ante profits from forward speculation are the result of seasonal patterns in the underlying economic structure and that they are not inconsistent with efficiency.

The second part of the paper presents a monetary equilibrium model with production where agents are risk adverse and face exogenous liquidity constraints on their purchase of goods. The model is similar to the one of Hodrick (1988) and it has the potential to qualitatively generate seasonalities in the ex-ante profits from forward speculation as the result of agents´ optimization and of the specific assumptions on the shocks to the primitives of the model. In this model no arbitrage opportunities exist despite the presence of seasonalities in the ex-ante profit series. One version of the model is then estimated using an "estimation by simulation" technique (Ingram-Lee(1986)). The estimated parameters are such that the time series properties of actual and simulated data at seasonal frequencies are close in the metric given by the covariance of the two periodograms. Estimates for the free parameters are quite reasonable and much lower than those reported in Mark (1985) and Prescott-Merha (1985).

The paper is organized as follows: the next section describes the data and the forecasting procedure . Section 3 demonstrates the existence of seasonalities in ex-ante and ex-post profits and tests for their significance. Section 4 introduces a general equilibrium model which can generate seasonalities in ex-ante profits without implying the existence of arbitrage opportunities. Section 5 provides estimates of the free parameters of the model. Section 6 outlines the conclusions. A technical description of the bootstrap algorithm and of the estimation of the density of the linear forecasts is contained in Appendix A. Some asymptotic properties for the simulation estimators are presented in Appendix B.

## 2) THE DATA, THE MODEL AND THE FORECASTING PROCEDURE.

The data set employed spans an eight years interval, from the first week of 1979 to the last week of 1987. For all variables weekly samplings (at Wednesday) of daily values are constructed. Spot rates are taken from the New York market for six different currencies [French franc (FF), Swiss franc (SF), German mark (DM), English pound (L), Canadian dollar (Can$), and Japanese yen (Yen)] in terms of the US dollar. Cross rates in terms of the DM and the Yen are also computed to check the robustness of the results to changes in the unit of measurement. The forward rates are arithmetic averages of the bid-ask spread in the New York market for the same currencies. Interest rates are 13 weeks Euromarket rates computed as averages of the bid-ask spread.

Ex-post profits from forward speculation as an annualized percentage of the spot rate on a contract, quoted at t for execution at t+13, are defined as:

$$PR_{t,t+13} = 400*(S_{t+13} - F_{t,t+13})/S_t \qquad (2.1)$$

$S_{t+13}$ is the spot rate prevailing at t+13 and $F_{t,t+13}$ is the forward rate quoted at t for transactions to be delivered at t+13. Ex-ante profits are similarly defined as:

$$PR_{t,t+13}{}^{e} = 400*(S_{t+13}{}^{e} - F_{t,t+13})/S_t \qquad (2.2)$$

where the supescript "e" indicates expected values. Since $PR_{t,t+13}{}^{e}$ is not observable, an auxiliary assumption of how agents form expectations of the future spot rates is necessary to make (2.2) operative. There are several way of proceeding in this case. One is to use survey data on expectations of future spot rates as in Dominguez (1986), Frenkel-Froot (1987), and Ito (1988,b). Another is to assume that expectations can be approximated by linear projections on the available information set as in Ito (1988,a), Canova-Ito (1987) . Finally, one can construct conditional means nonparametrically as in Diebold-Nason (1989),Gallant-Hsieh-Tauchen (1988).[1] This paper adopts a modifies version of the second approach.

The forecasting specification used here is a vector autoregressive (VAR) model which includes, for each currency considered, the spot exchange rate and two k-period Euromarket rates. Although there are other series available at a weekly frequency, the presumption is that these financial variables contain all the information that is useful in predicting spot rates because they react more quickly than any other variable to news.

In formulating the VAR model two basic questions have to be faced. The first concerns the treatement of trending variables. The second the lag length of the model. Since the VAR model is used for prediction and since I adopt the Bayesian approach of viewing forecasting as the construction of the posterior conditional distribution of future data conditional on available data, I follow Sims (1988) and DeJong-Whiteman (1988) and specify the VAR model in trend stationary form.

The choice of the lag length of the model is crucial in generating forecasts, representing the trade-off between spanning a larger information set and producing less accurate estimates for a given a sample size. It is known that forecasts from a VAR model are very volatile. But this volatility tends to be spurious and either due to a small sample bias or to an inadequate treatement of the nonlinearities of the system.

To exemplify these problems, consider the simpler problem of generating forecasts from the univariate model:

$$Y_t = f(a,Y_{t-1}) + u_t \qquad u_t \sim (0,\sigma^2_u) \qquad (2.3)$$

where f is a Borel measurable function $R^k \to R$ and a is a kx1 vector of parameters. The optimal point forecast of $Y_{t+1}$ with a quadratic loss is $E(Y_{t+1}|\ I_t)=f(a,Y_t)$, where $I_t = \{ X_t \text{ in } L^2: X_t=g(Y_t)$ for some Borel measurable function $g\}$ is the information set. The variance of the one step ahead forecast error is $\sigma^2_u$. In practical problems two types of approximations to this optimal predictor are usually taken:

(1) a first order Taylor expansion is constructed with $f(a,Y_t)=\alpha Y_t+e_t$ ; $e_t \sim (0,\sigma^2_t)$); where $\alpha$ is a kx1 vector.

(2) an estimate of $\alpha$ is used in place of the true value.

With the two approximations the variance of the one step ahead forecast error from a linear projection model is $\sigma^2_u+\sigma^2_t+\sigma^2_u*Y_{t+1}(Y'Y)^{-1}Y_{t+1}$. Here $Y=(1,Y_1....Y_t)$, the second term is due to the Taylor approximation, and the last term is due to parameter uncertainty. The forecast error of a j-period ahead linear predictor is $\Sigma_{0<i<j}\ \alpha^{2(j-i-1)}(\sigma^2_u+\sigma^2_t)$ + $var[(\alpha^j-\hat{\alpha}^j)Y_t]$. A large amount of parameter uncertainty or an inappropriate linear approximation therefore creates a large sampling error variance. Parameter variability is large if the inital estimates are based on a small sample size and declines over time as better estimates are constructed. In the limit the variance of the forecast error will contain only the cumulative effect of the innovations in the model and of the linear approximation.

Bayesian methods are often succesful in dealing with this problem. In this paper I try to control and correct for the error term due to small sample biases in an alternative way. Instead of generating point forecasts with an optimally chosen lag length, I construct the density of recursive linear projections for each date in the forecasting sample from an arbitrary chosen length for the VAR model using a bootstrap algorithm and a kernel estimator. The conditional mean of the density [2]is then taken to represent $S_{t+13}{}^{e}$.

The basic approach is as follows. Using a VAR model where each right hand side variable enters with eight lags, a trend, and a constant, I estimate the model over the period 79,1-80,10. Then I generate a 13-step forecasts for the exchange rate, and recursively update and forecast for each of the data points up to 87,51. Bootstraping on the residuals for the period 79,1-80,10, I obtain another set of initial estimated coefficients, produce a 13-step ahead forecasts and repete the recursive updating and forecasting procedure starting from these initial estimates. Independently drawing 100 bootstrap samples over the period 79,1-80,10, I generate 100 independent initial estimates and 100 independent point forecasts for each date in the forecasting sample. Then, for each of these dates, I construct the histogram of the linear prediction, provide an estimate of the density function and compute a measure of location [3]. Details on the algorithm appear in appedix A.

The procedure is repeated for each of six VAR models considered. Figure 1 presents two of these densities. All the others have similar features and are omitted for reason of space. From that figure it is clear that the densities for some currencies and for several t´s tend to be multimodal and have relatively fat tails. Also, as more data points are added, the densities tend to get concentrated around the median [4]. For the sake of robustness densities of the forecasts are also computed using a multimarket VAR which includes 11 variables (6 exchange and 5 interest rates) using the same procedure. The Swiss and the French interest rates were excluded because collinear with the other interest rates in the system. The pictures were very similar to the previous ones and therefore omitted.

As a measure of location one could chose the median or the conditional mean of the distribution. For the six markets under consideration the two measures are practically indistinguishable. In what follows I use the conditional mean of the density of the linear forecasts as a best approximation to $S_{t+13}^{e}$.

There are two ways of assess the reasonableness of our estimates for the ex-ante profits. One is to see whether ex-post and ex-ante profits have similar statistical properties. Another is to compare it with ex-ante profits constructed from the multimarket forecasts. Table 1 shows that ex-post and the two measures of ex-ante profits have in fact similar statistical properties. For this reason, the tests of next section concentrate on ex-post profits and the single market measure of ex-ante profits.

## 3) THE TESTS

In this section I present two types of evidence concerning the existence of seasonality in profits from forward speculation. First, I compute weekly means and standard deviations of the variables of interest. Second, I calculate the percentage of the variance of the series due to each seasonal band and tests whether this value is significantly different from the percentage which would appear in that band if the process were a white noise. Under the null hypothesis that the process is a white noise, the ratio of the percentage value of the variance inside and outside each band is asymptotically F-distributed with degrees of freedom given by twice the number of periodogram ordinates inside and outside the band. (see Canova (1988)). Seasonal bands are defined as small neighborhoods around the seasonal frequencies $\theta=2\pi p/q$, $p=1..[q/2]$, where [.] is the largest integer less or equal to $q/2$, $q=52$. Each band is arbitrarily normalized to contain 9 periodogram ordinates. Since the interval $[0,\pi]$ is divided in 520 points, under the null hypothesis of white noise process, each seasonal band contains less that 2 % of the total variance. Cumulatively, seasonals should not significantly account for more than 50% of the variability of the series [5].

Figures 2 and 3 present an estimated 90 % confidence band for the weekly means of ex-ante and ex-post profits in the six different markets when the U.S. dollar is used as unit of account [6]. The band is constructed using

$$\hat{z}_{it} \pm [\ [1/v^2 * \Sigma_v \hat{\sigma}_{iv}] * t_{\alpha/2,v-1} \qquad (3.1)$$

where $\hat{z}_{it}$ is the mean of $x_t$ for week i=1,2...52; $1/v^2 * \Sigma_v \hat{\sigma}_{iv}$ is the standard error of the weekly mean, v is the number of observations for week i in the sample, $t_{\alpha/2,v-1}$ is the t-value for the confidence level $\alpha$ and v-1 degrees of freedom. For ex-ante profits both $\hat{z}_{it}$ and $\hat{\sigma}_{it}$ depend on $\hat{F}$, the bootstrap density of the forecasts. Here $\hat{\sigma}_{it}$ includes the standard error of the 13 step ahead forecast error and the standard error due to parameter uncertainty.

Table 2 contains the percentages of the variance in seasonal bands for ex-ante and ex-post profits which are significantly different from the ones of a white noise (at $\alpha \leq .10$) and the total variability appearing in seasonal bands. In order to check the robustness of the results the same statistics are computed using the DM and the Yen as units of account. These are included in tables 3-4.

The evidence from figures 2 and 3 is strong. For ex-ante profits there are weeks when the return from forward speculation is as much as twice the average return over the entire sample with a standard error of about one half of the standard deviation of the average return over the entire sample (Table 2 has the overall means and standard errors). This is true for $/DM, $/FF, and $/SF at the 43rd week, for the $/L rate at the 48th week, for the Yen/$ rate at the 34th week and for the $/can$ rate at the 30th week. In each of these weeks the entire 90% confidence band lies on one side of zero and it is smallest in size. Although the timing is not exact, a similar pattern occurs in the ex-post profits. For \$/DM, \$/SF and \$/FF markets the pattern is remarkedly similar with the weekly means becoming positive in the last 12 weeks of the year. For the \$/L market the mean is positive between the 16th and the 30th week; for the Yen/\$ two peaks are present at the 16th and 32nd week. The \$/can\$ profit series has a pattern similar to the first three European currencies but leads them in the movements of the weekly means by about 4 weeks.Since both ex-ante and ex-post profits show seasonal patterns, the existence of seasonalities in ex-ante profits is not due to the way exchange rate forecasts were computed.

The evidence emerging from tables 2-4 strengthens and refines the results. The behaviour of the ex-ante profits in the \$/DM,\$/SF and \$/FF market is once again similar. Cycles of 4 ($\theta=.5\pi$) and 3.71 ($\theta=.5384\pi$) weeks are significant for these currencies (and 2.08 weeks ($\theta=.9615\pi$) for \$/SF). For the British pound cycles of 3.71 are also very significant. For all currencies the percentage of the variance belonging to the band centered around cycles of 3.71 weeks is at least 3 or 4 times the percentage that would appear if the process were a white noise. For the Yen/\$ ex-ante profits cycles of 17.3 weeks ($\theta=.1153\pi$) and for the \$/can\$ cycles of 4 weeks are significant.

For the ex-post profits, the evidence is even stronger. In the band centered around cycles of 3.71 weeks the percentage of the variance in each series except for the can$/$ market exceeds 4%, with peaks at 10-11% for $/SF and $/L markets. These results are robust and substantially persist when DM or Yen is used to normalize exchange rates.

In conclusion, seasonalities are present in both the ex-ante and ex-post profits, are a phenomena common to all markets, are independent of the unit of account chosen and, in general, correspond to cycles of slightly less than 4 weeks (with the exception of the yen/$ ex-ante profits)

The presence of seasonalities in these variables poses several questions. If seasonalities in ex-ante profits exist, it must be true that the forward rate is not a good predictor of the (expected) future spot rate at seasonal frequencies. The unbiasedness of the forward rate has been rejected in several studies (Cumby (1988) for references). It is often argued that, apart from measurement errors or "peso" problems , this result implies either the existence of a risk premium (see e.g. Hodrick-Srivastava (1984)) or of unexploited opportunies in each market (see e.g.Frankel (1982)). One wonders if the extension of these conclusions to seasonal frequencies is warranted. The questions that naturally emerge are: are there seasonals that agents are unaware of or are these seasonals the result of a seasonally varying conditions in the economy? If the latter is true, what factors may have generated seasonalities in ex-ante profits? Is the magnitude of these seasonal profits consistent with the magnitude generated by a reasonable parametrization of a general equilibrium model? In the next two sections I attempt to provide an answer to these questions.

#### 4) A THEORETICAL FRAMEWORK OF ANALYSIS

The framework of analysis used in this section is very similar to Hodrick (1988). It is a two country, two good, cash-in-advance (CIA) model. Each country is specialized in the production of one good. Technologies are linear in labor:

$$Y_{it} = \alpha_{it} l_{it} \qquad i=1,2 \qquad (4.1)$$

where $\alpha_{it}$ is a productivity shock. Labor is immobile across countries and supplied inelastically so that production choices are trivial. Labor compensation is paid in local currency. Goods are storable overnight but perish if not consumed the following period.

Preferences of the agents are assumed to be homothetic and identical across countries. The objective function of the representative consumer in either country is to maximize expected lifetime utility given by:

$$E_o \Sigma_t \beta^t U(c_{1t}, c_{2t}, u_t) \qquad (4.2)$$

with $0<\beta<1$, where $c_{it}$ is the consumption of good i, i=1,2 and $u_t$ is a preference shock. The instantaneous utility function is assumed to possess all the concavity properties that are necessary for the Inada condition to be satisfied. Agents are endowed with one unit of time each period.

Information relevant for the decisions of the agents is obtained at the beginning of each period and summarized in the vector $x_t$. The timing of the model is as follows. At each t there are three subperiods: the goods market opens in the first subperiod, the asset market in the third subperiod and production occurs between the goods and asset market. Labor compensations are paid at the beginning of the asset market subperiod and goods are stored overnight to be offered next period in the goods market.

Agents face two cash-in-advance (CIA) constraints in purchasing the consumption goods. They are represented by:

$$c_{1t} \leq M_{it+1}^{\sim} p_{1t} \qquad (4.3)$$

$$\theta t\ c_{2t} \leq M_{2t+1}^{\sim} S_t p_{1t} \qquad (4.4)$$

Where $p_{it}$ is the purchasing power of currency i, $M_{it+1}^{\sim}$ the amount of cash balances i acquired in the asset market at time t by private agents of either country, i=1,2, $\theta_t$ the real term of trade of country 1 defined by $\theta_t = S_t p_{1t}/p_{2t}$ and $S_t$ the exchange rate of currency 2 in terms of currency 1

In the asset market agents can acquire a variety of assets subject to the constraint that their purchases are limited by their wealth at each t. I assume that agents do not issue private debt, that there are two monies and only two sets of contingent bonds of maturity k issued by the governments. Let $B_{ht+1,k}(v)$ be the amount of contingent claims issued by government h, purchased by consumers at time t , with maturity k, event v and let $\exp(-r_{ht+1,k}(v)) = Q(v_{ht+1,k})/\pi(v_{ht+1,k})$ be the discount price in terms of money quoted at t for a k-period contingent bond payable by government h if v occur. $Q(v_{ht+1,k})$ is the Arrow-Debreu price at t of a claim of country h with maturity k for event v and $\pi(v_{ht+1,j})$ is the probability that v occurs at t+1+k. The agents´ budget constraint in the asset market is:

$$
\begin{aligned}
&p_{1t}\,[\tilde{M}_{1t+1} + \tilde{M}_{2t+1} S_t + E_t B_{1t+1,k} \exp(-r_{1t+1,k}) \\
&\quad + E_t B_{2t+1,k} \exp(-r_{2t+1,k}) S_t] \\
&\quad \le M_{1t}p_{1t} + M_{2t}p_{1t}S_{1t} - 0.5T_{1t} - 0.5T_{2t}\theta_t + \\
&\quad p_{1t}\,[\, B_{1t-k,k} + B_{2t-k,k}\, S_t + \\
&\quad E_t\{\, \Sigma_{1\le n\le k-1} B1t\text{-}n,k \exp(-r_{1t+1,k-n}) + \\
&\quad [\Sigma_{1\le n\le k-1} B_{2t-n,k} \exp(-r_{2t+1,k-n})]\, S_t\}] \qquad (4.5)
\end{aligned}
$$

where $T_{it}$ are lump sum taxes levied by government i on the representative consumer of each country and paid in the asset market [7]. The term in braces represents the amount of bond purchased in the last k-1 periods which have not yet matured but are sold in the market by agents. The arbitrage discount price of a k-n pertiods, event v bond, issued by goverment h, is $\exp(-r_{ht+1,k-n}(v))=\exp(-r_{ht+1-n,k}(v))/\exp(-r_{ht+1-n,n}(v))$. Here $E_tB_{ht+1,k} \exp(-r_{ht+1,k})$ is given by $\Sigma_v B_{ht+1,k}(v) * \exp(-r_{ht+1,k}(v)) * \pi(v_{ht+1,k})$.

Agents receive their labor compensation between the goods and asset markets which implies that

$$M_{it}p_{1t} = \alpha_{it} + (\tilde{M}_{it}\, p_{it-1} - c_{it-1}) \qquad i=1,2 \qquad (4.6)$$

with $\alpha_{it}=0$ if the agent lives in country h, h unequal to i.

The two goverments purchase an exogenous amount $G_{hit}$ of good i and levy exogenous taxes $T_{hit}$ on the residents of country i subject to the following budget constraint:

$$\Sigma_i(G_{hit}-T_{hit})=[E_tB_{ht+1,k}\exp(-r_{ht+1,k})B_{ht-k,k}+M_{ht+1}-M_{ht}]\,P_{ht} \qquad h=1,2 \qquad (4.7)$$

I assume that $M_{ht+1}$ is either an exogenous stochastic process or a policy variable correlated with other exogenous processes and that the goverments issue contingent claims to satisfy their budget constraints.

Goverments are also subject to a CIA constraint in their purchases of goods, but not limited in their spending by previous accumulation of money . Their CIA´s are

$$G_{hit} \leq \tilde{N}_{hit+1}\,P_{it} \qquad h,i=1,2 \qquad (4.8)$$

where $\tilde{N}_{hit+1}$ is the amount of currency i purchased in the asset market by government h . Government purchases of domestic and foreign currencies in the asset market are subject to the following constraint:

$$\tilde{N}_{h1t+1}+\tilde{N}_{h2t+1}S_t \leq N_{h1t}+N_{h2t}S_t \qquad (4.9)$$

$$N_{hit}P_{it}=(N_{hit}P_{it-1}-G_{hit-1})+(M_{hit}-M_{hit-1})P_{it} \qquad i=1,2 \qquad (4.10)$$

$(M_{hit}-M_{hit-1})$ is the increment at time t in the money supply i made by goverment h. Without loss of generality assume that the government h can not issue currency of country i for i different than h [8].

The state vector $x_t$ includes $\{G_{it}, T_{it}, M_{it}, \alpha_{it}, u_t\}$. Let $W_{it}$ be the financial wealth of agents of country i at t. The value function for the agent's problem is :

$$V(W,x) = \max_{\{c_i, M_i^{\sim}, B_i(v)\}} [U(c_1,c_2,u)+\beta\int V(W',x')G(x'|x)dx' \quad (4.11)$$

Letting $\Phi_{jt}$ j=1,..5 be the Lagrangean multipliers associated with (4.3)-(4.6), a first order condition for the agent problem in each country is:

$$U_{1t} = \Phi_{4t}+\Phi_{1t}$$

$$U_{2t} =(\Phi_{5t}+\Phi_{2t})\theta_t$$

$$\Phi_{1t} = \Phi_{3t} - \Phi_{4t}$$

$$\Phi_{2t} = \Phi_{3t} -\Phi_{5t}$$

$$\Phi_{3t}p_{1t}\exp(-r_{1t+1,k}(v)) = E_t\beta^j(\Phi_{3t+k}p_{1t+k}+\Sigma_{1\le n\le k-1}\Phi_{3t+k}* \exp(-r_{1t+n+1,k-n}(v))*p_{1t+n}) \quad \text{all } v \qquad (4.12)$$

$$\Phi_{3t}p_{2t}\exp(-r_{2t+1,j}(v)) = E_t\beta^k(\Phi_{3t+k}p_{2t+k}+\Sigma_{1\le n\le k-1}\ \Phi_{3t,k}* \exp(-r_{2t+n+1,k-n}(v))*p_{2t+n}) \quad \text{all } v \qquad (4.13)$$

The transversality condition is given, for each i by:

$$\inf\ \{\ \beta^{t+k}\ W_{it+k}\} \dashrightarrow 0 \text{ as } t \dashrightarrow \infty \qquad (4.14)$$

The first four equations can be collapsed as follows:

$$U_{1t}\theta_t = U_{2t} \qquad (4.15)$$

The solution to the model can be found using (4.12), (4.13), (4.15), and the transversality condition (4.14).

## EQUILIBRIUM

An equilibrium for the economy is a set of initial conditions $(M_{io}, B_{io}, p_{io})$, a vector of exogenous stochastic processes for $(\alpha_{it}, G_{it}, T_{it}, M^s_{it+1}, u_t)$ and a vector of endogenous stochastic processes for $\{c_{it}, M_{it+1}^{\sim}, B_{it+1,k}(v)\}$ and prices $\{P_{it}, r_{it+1,k}, \theta_t\}$ such that:

a) the two government budget constraints are satisfied and the CIA constraints are satisfied with equality for all t.

b) Given the initial conditions, the vector of exogenous processes and prices, agents' choices maximize their utility function subject to their budget constraints.

c) $\Sigma_h(C_{hit} + G_{hit}) = \alpha_{it}$

$$M^s_{it+1} = \Sigma_h (N_{hit+1} + M_{hit+1}) \qquad (4.16)$$

$B^s_{iht+k}(v) = \Sigma_i B_{hit+k}(v)$ for each h,i=1,2; all v

so that market clearing prevails, where the superscript "s" indicates supply.

In a perfectly pooled equilibrium agents equally share the net of government consumption output of the two countries and no trading of unmatured contingent bond occur. Also given the assumptions on the timing of payments in the economy, the cash in advance constraints will all be satisfied with equality and all money multipliers will be positive.

## COMPUTATION OF THE PROFITS FROM FORWARD SPECULATION

From the CIA constraints a solution for the purchasing power of the two monies is given by:

$$P_{it} = \alpha_{it}/M_{it+1} \qquad i=1,2 \qquad (4.17)$$

An expression for the nominal exchange rate can be found using the solution for $\theta_t$ from the FOC as:

$$S_t = \theta_t P_{2t}/P_{1t} = \alpha_{2t}M_{1t+1}U_{2t}/\alpha_{1t}M_{2t+1}U_{1t} \qquad (4.18)$$

A solution for the two k-period (risk free) interest rates can be found using (4.12) and (4.13) and integrating over all possible v at each t to obtain, for all k:

$$\exp(-r_{it+1,k})=\beta^j E_t U_{it+k}\alpha_{it+k}(M_{it+k+1})^{-1}/U_{it}\alpha_{it}(M_{it+1})^{-1} \qquad (4.19)$$

From (4.18) and (4.19) using covered interest parity (CIP) relationship it is possible to price the k-period forward rate as:

$$F_{t,k}=S_t*\exp(r_{2t+1,k}-r_{1t+1,k})= E_t\ U_{2t+k}\alpha_{2t+k}(M_{2t+k+1})^{-1}/E_t\ U_{1t+k}\alpha_{1t+k}(M_{1t+k+1})^{-1} \qquad (4.20)$$

Finally, the k-period ahead conditional future spot rate $E_tS_{t+k}$ is given from (4.18) by

$$E_tS_{t+k}=E_t[\alpha_{2t+k}(M_{2t+k+1})^{-1}U_{2t+k}/\alpha_{1t+k}(M_{1t+k+1})^{-1}U_{1t+k}] \qquad (4.21)$$

If we let the relevant time interval to be a week and let k=13, annualized ex-ante profits from forward speculation as a percentage of the spot rate are defined as $PR_{t,13}=400*(E_tS_{t+13}-F_{t,13})/S_t$ which from (4.20) and (4.21) implies [9]:

$$PR_{t,13} =\{E_t\ [\alpha_{2t+13}(M_{2t+14})^{-1}U_{2t+13}/\alpha_{1t+13}(M_{1t+14})^{-1}U_{1t+13}]- E_t[\alpha_{2t+13}(M_{2t+14})^{-1}U_{2t+13}]/E_t[\alpha_{1t+13}(M_{1t+14})^{-1}U_{1t+13}]\}*400/ [\ (\alpha_{1t}M_{2t+1}U_{2t})/(\alpha_{2t}M_{1t+1}U_{1t})] \qquad (4.22)$$

I first examine the conditions required for (4.22) to be different than zero. Consider the case when all stochastic processes are conditionally independent This is a simplification which does not affetct the essence of the conclusions.

Apart from the degenerate case when the conditional mean of some process is identically equal to zero, profits will be equal to zero only when the stochastic processes for country 1 are constant. Risk neutrality is neither necessary nor sufficient to make $PR_{t,13}$ equal to zero. If the distribution of $u_t$ is degenerate and agents are risk neutral, randomness in the money supplies or outputs can prevent $PR_{t,13}$ to be equal to zero. Conversely, if outputs and money supplies are constant, $PR_{t,13}$ will be different than zero even if agents are risk neutral if a preference shock affects the marginal utility of $c_1$. The inclusion of nonlinearities in the stochastic process for the exogenous variables of the type introduced by Hodrick (1988) and Abel (1988) creates a richer pattern of dynamics for $PR_{t,13}$ without affecting the basic message of the exercise.

In general, there is also no reason to expect $PR_{t,13}$ to be constant over time, unless conditional and unconditional means are equal. Finally, there is no reason to expect $PR_{t,13}$ to be of the same sign over time. By Jensen´s inequality, the terms in braces will in general be positive, but depending on the form chosen for the marginal utility of $c_2$, the sign of the expression may well change over time.

We are now in the position to address the question of how seasonalities in $PR_{t,13}$ could emerge. There are at least three different reasons why seasonalities in ex-ante profits may exist. Seasonalities may be determined by a seasonal conditional mean in one of the exogenous processes, by seasonalities in the conditional covariance of any two exogenous processes impinging on the economy of country 1; or by a combination of any of the above reasons. Conditional variances could enter in the determination of $PR_{t,13}$ as well for some particular distributions for the exogenous variables [10]. In this case seasonalities in the conditional variance may induce seasonalities in $RP_{t,j}$.

Therefore seasonalities in the ex-ante profits from forward speculation are consistent with optimizing agents and with the absence of unexploited arbitrage opportunities.

As far as discerning among all possible sources of seasonality, there is no reason to exclude any possible case from the list of causes. Also, it is impossible check empirically which variable may have induced the seasonal pattern we observed in the data since, except for data on certain money supplies, no data for the relevant variables is available at weekly frequency. The hope is that the simulation exercises of next section may provide an indication of the most likely cause of seasonalities.

## 5) SIMULATING THE MODEL

In this section I employ an " estimation by simulation" technique to quantitatively examine whether a version of the model described in the previous section is able to generate a time series for the ex-ante profits from forward speculation which matches the properties of the actual time series at seasonal frequencies.

The estimation by simulation technique was introduced by Ingram and Lee (1986) and it is similar in spirit to the method of simulated moments of McFadden(1988) and Pakes and Pollard (1986). Estimates are found by minimizing the distance between the moments of the actual and the simulated data, in a metric given by the covariance of the difference of the actual and the simulated moments.

Let $y_j(\beta)$ j=1,...N, N=nT, be the simulated time series for the ex-ante profits from forward speculation generated using (4.22). $\beta$ is a rx1 vector of unknown parameters which includes all the free parameters of the model. Let $x_t$ be the estimated time series for the ex-ante profits, t=1...T and define the periodograms of $y_j(\beta)$ and $x_t$ by:

$$I_x(\theta) = 2/T\ \Sigma_t\ |\ e^{-i\theta t}\ x_t\ |^2 \tag{5.1}$$

$$I_{y,\beta}(\theta) = 2/N\ \Sigma_j\ |\ e^{-i\theta j}\ y_j(\beta)|^2 \tag{5.2}$$

Let $g_{t,\beta}(\theta) = \{\ |e^{-i\theta t}\ x_t|^2 - \Sigma_{l\le j\le m}\ |e^{-i\theta j}\ y_j(\beta)|^2\ \}$ where $l= 1+(t-1)n$ ;$m=[nt]$ and [.] indicates the maximum integer less than or equal to nt. Define $G_{T,\beta}(\theta)$ by:

$$G_{T,\beta}(\theta) = 2/T\ \Sigma_t\ g_{t,\beta}(\theta) \quad (5.3)$$

The criterion function I employ is the following:

$$Q = ||\Omega||^{-1} \int_\Omega G_{T,\beta}(\theta)\ W_T(\theta)\ G_{T,\beta}(\theta)\ d\theta \quad (5.4)$$

where $\Omega = U_k\ \Gamma_k$ ; $\Gamma_k = [\ 2\pi k/T-\varepsilon,\ 2\pi k/T+\varepsilon\ ]$ , $k=1,2....[T/2]$ and $||.||$ indicates the number of periodogram ordinates contained in $\Omega$. Estimates for the unknown $\beta$'s are found by minimizing (5.4), i.e by minimizing the average distance at seasonal frequencies between the periodogram of the actual and the simulated data in the metric given by $W_T$.

Following Hansen (1982, theorem 3.2) an optimal choice for $W_T(\theta)$ is given by:

$$W_T(\theta) = \{\ (1+1/n^2)S(\theta)^2\}^{-1} \quad (5.5)$$

where $S(\theta) = \Sigma_{-\infty\le h\le\infty}\ R_x(h)\ e^{-i\theta h} =$

$$= \Sigma_{-\infty\le h\le\infty}\ R_{y,\beta}(h)\ e^{-i\theta h} \quad (5.6)$$

where the last equality holds under the null hypothesis that the $\beta$ are the correct ones . Substituting (5.3) and (5.5) into (5.4) estimates of $\beta$ are found by minimizing:

$$Q = ||\Omega||^{-1} \int_\Omega \Sigma_t\ g_{t,\beta}(\theta)[(1+1/n^2)S(\theta)^2]^{-1}\ \Sigma_t\ g_{t,\beta}(\theta)\ d\theta \quad (5.7)$$

A first order condition for the problem (after some algebraic manipulation) is:

$$\int_\Omega [I_x(\theta) - I_{y,\beta}(\theta)]\ W_T(\theta)F_y(\theta)F_{y'}(-\theta)\ \ d\theta = 0 \quad (5.8)$$

where $F_z(\theta)$ is the Fourier transform of z at frequency $\theta$ and $y' = dy(\beta)/d\beta$. The solution to (5.8) is an estimator which weighs the difference in the periodogram of simulated and actual data by a linear combination of the Fourier tranform of the convolution of $y(\beta)$ and $y'(\beta)$ and the variance of the periodogram of actual data at seasonal frequencies. Note that (5.8) is an rx1 vector of first order conditions, one for each free parameter.

Under some regularity conditions (which are provided in appendix B) the solution to (5.8) produces consistent and asymptotically normal estimators of the true parameters (Ingram-Lee (1986)). Also, since for each $\theta \in \Omega$ the expression in (5.8) is a chi-square variate with 2 degrees of freedom, an asymptotic goodness of fit test is available for the model as $Q^{1/2} - (4*||\Omega||)^{1/2} \sim N(0,1)$ (see Hastings -Peakock (1985),p.50).

Since the estimation procedure increasingly complicates with dimensionality of $\beta$, I adopt a simple specification for the driving forces in (4.24). Let all the shock be conditionally independent and the utility function be represented by the two-parameters family of functions:

$$U(c_{1t},c_{2t},u_t) = (c_{1t})^{1-\delta}/1-\delta \; +(c_{2t})^{1-\tau}/1-\tau \qquad (5.9)$$

Also, let the proportion of government consumption in total outputs be constant over time. Further let $z_{1t} = \{\alpha_{it} , M_{it}\}$ be conditionally log normally distributed with mean vector $\mu_t$ and variance $diag(\sigma_{jjt})$ where $\mu_t$ satisfies:

$$\mu_t = a(L)\, z_{1t-1} + (1-a(1))\, z_1^* \tag{5.10}$$

where $z_1^*$ is the unconditional mean of the process and $a(L)$ is assumed to have a block diagonal structure with entries $a_{jj}(L)$ $j=1,..4$. Since the dimensionality of the parameter space is still very large I proceed as follows. From the the money supplies data I estimate $a_{33}(L)$, $a_{44}(L)$, $\sigma^2_{33t}=\sigma^2_{33}$, $\sigma^2_{44t}=\sigma^2_{44}$ [11]. Further, I restrict a priori $a_{11s}$ and $a_{22s}$ to be zero for all s except for s=1 (or s=4). With these assumptions (4.25) reduces to:

$$\begin{aligned} RP_{t,k} = {} & C^*\exp\{(\delta-1)^*[a_{11}(L)/L_k]_+{}^*\alpha_{1t} + (1-\tau)^*[a_{22}(L)/L_k]_+{}^*\alpha_{2t} \\ & - [a_{44}(L)\hat{}/L^k]_+{}^*M_{2t} + [a_{33}(L)\hat{}/L^k]_+{}^*M_{1t}\ \}^* \\ & \exp\{0.5((1-\tau)^2\sigma^2_{22t} + \sigma^2\hat{}_{44})\}^*[\exp\{0.5(\sigma^2\hat{}_{33} + (\delta-1)^2\sigma^2_{11t})\} \\ & - \exp\{-0.5(\sigma^2\hat{}_{33} + (\delta-1)^2\sigma^2_{11t})\}] \end{aligned} \tag{5.11}$$

where $C=\{[2^*(1-2\Phi_1)]^{-\tau}\}^*\{[2^*(1-2\Phi_2)]^{\delta}\}$; $\Phi_i$ are the proportions of government consumption in GNPs, "^" indicates estimated values and the sign $[.]_+$ indicates the annihilation operator. The free parameters in (5.10) are ($\delta$, $\tau$, $a_{111}$, $a_{221}$, $\sigma^2_{11t}$, $\sigma^2_{22t}$, $\Phi_1$, $\Phi_2$). I estimate the constant share of government consumption in total output from National Accounts Tables as the average over the sample under consideration. Further I will assume that the remaining conditional variances are constant and arbitrarily set them equal to 0.4 [12]. Therefore, the minimization is undertaken over 4 free parameters ($\delta,\tau$, $a_{111}$, $a_{221}$).

The table below provides the results obtained for the \$/DM and for the Yen/\$ market. I choose to match the seasonal properties of these two profit series on the presumption that these are the most important markets since European currencies are linked to the DM through the EMS and profits in the \$/can\$ market are small. As alternative, one could match the average ex-ante profits in the six markets.

Several local minima were found and the criterion function is very flat around the local minima. The criterion function is relative insensitive to small changes in the autoregressive parameters, but it has relatively narrow contours in the $(\delta,\tau)$ dimensions. The minimum minimorum for the two markets are:

| market | $a_{111}$ | $a_{221}$ | $\delta$ | $\tau$ | goodness of fit sign. level |
|---|---|---|---|---|---|
| \$/DM | 1.035382 | .9821020 | .9077807 | .8212576 | .01 |
| Yen/\$ | 1.014869 | .9700879 | .7343542 | .8039577 | .00 |

Few features deserve some comments. First, the values for $(\delta,\tau)$ are low when compared with the typical estimates of risk adversion parameters needed to match other financial data (see Prescott-Merha(1985)) or similar data over a different sample period (see Mark (1985)). They imply that agents have less than logarithmic curvature in their preferences. Second, the stochastic processes for outputs in country 1 are nonstationary but close to random walks. Given the constancy of the share of goverment consumptions and the almost logaritmic specification for the utility, this result implies that the marginal utility of consumption is approximately a random walk as one would expect with weekly data. Third, the goodness of fit test rejects the model at 5% confidence in each of the two markets, but the rejections are not extreme.

Several robustness tests have been carried out. For example, I allowed in turn the AR representation of outputs to be a purely seasonal process. Alternatively, I restricted $\tau$ to be equal to $\delta$. Finally, I roughly tested for the sensitivity of the estimates to changes in the assummed variance for the processes. In the first case the fit improves (the significance levels of the goodness of fit test are .01 and .02) and the values of $\tau$ and $\delta$ needed to minimize the function slightly decrease.In the second case, the common value for $\tau$ and $\delta$ was between the two estimated values of the table and required a substantial nonstationary behaviour for the output of country 2 in both markets.In the third case, increasing the variance of the processes caused the values of $\tau$ and $\delta$ to exceed 1, while the autoregressive parameters were always greater than one.

## 6) CONCLUSIONS

This paper presents evidence on the existence of seasonalities in the ex-ante and ex-post profits from forward speculation in exchange markets. Ex-ante profits are constructed as a difference between the conditional mean of the density of recursive forecasts of a linear VAR model and the forward rate. Pictures of the weekly means and frequency domain tests demonstate the existence and the significance of seasonalities in all markets. The evidence is shown to be consistent with optimization and the absence of inefficiencies. It is shown that there are at least three different sets of conditions have the ability to qualitatively produce pattern of seasonalities as the one observed in the actual data. A version of the model is simulated and parameters estimated using an estimation by simulation technique. The results show that the model can quantitatively generate seasonal patterns in ex-ante profits which are close to observed pattern of seasonalities with reasonable parameter values.

## APPENDIX A

This appendix provides a detailed description of the bootstrap algorithm used to generate distribution of forecasts for various exchange rates.

Let the VAR model for each currency be:

$$Y_t = \Sigma_j a_j Y_{t-j} + c + t + e_t \qquad e_t \sim (0,\Omega) \quad \text{(A.1)}$$

where $Y_t$ is a 3x1 vector, c a constant and t a linear time trend and $a_j$ is a 3x3 matrix for each j=1,...p. Let (A.1) be estimated by any consistent method on the interval [0,s]; estimates $\hat{a}_j(s)$, $\hat{c}(s)$ derived and the residual vector $\hat{e}_t$ computed. Then at s, compute $P[Y_{s+k} | I_s]$ from (A.1) where $I_s$ is the information set available at s, given the estimates obtained, update $\hat{a}_j(s)$, $\hat{c}(s)$ and $\hat{\Omega}(s)$ with the Kalman filter formulas to obtain $\hat{a}_j(s+1)$, $\hat{c}(s+1)$ and $\hat{\Omega}(s+1)$, compute $P[Y_{s+1+k}|I_{s+1}]$ and so on until the whole sample is exhausted. This creates the first set of point estimate of $P[Y_{t+k}|I_t]$ for all $t \in [s,T]$.

The bootstrap algorithm used to create independent point estimates for $P[Y_{t+k}|I_t]$ amounts to the followings :

I) From the vector $\hat{e}_t$, construct an estimate of the joint density of the residuals on [0,s] as :

$$\hat{\Gamma} = \text{mass}\{1/(s+1)\} \text{ at } \hat{e}_0, \hat{e}_1, \hat{e}_2 \ldots\ldots \hat{e}_s$$

II) From $\hat{\Gamma}$ draw a bootstrap data set for $e_t^*$ for each $t \in [0,s]$ and construct a new estimate for $Y_t$, $t \in [j,s]$ as:

$$Y^*_t = \Sigma_j \hat{a}_j(s) Y_{t-j} + \hat{c}(s) + t + e^*_t \qquad \text{(A.2)}$$

where $e^*_t$ satisfies $E[e^*_t | I_{t-1}] = 0$ ;
$E[e^*_{ti}\, e^*_{tj} | I_{t-1}] = \hat{\Omega}_t$
and where $\hat{a}_j(s)$ and $\hat{c}(s)$ are the same parameter estimates obtained from the original data up to s.

III) Repeat the estimation algorithm on [0,s] to get new estimates $a^*_j(s)$ and $c^*(s)$ for the true parameters of the model.

IV) Compute $P[Y_{s+k}|I_s]$ from (A.2) with the estimates in III), update $a^*_j(s)$, $c^*(s)$ and $\Omega^{*\hat{}}(s)$ using the Kalman filter, compute $P[Y_{s+1+k}|I_{s+1}]$ and update v times until s+v+k= T.

V) Independently repeat steps II)-IV) m times.

After m times, we will have m independent point estimate of the stochastic process $x_t = P[Y_{t+k}|I_t]$ for each t. We can construct histograms of $x_t$ at each t and numerically evaluate the density function $f(x_t)$. The density of $x_t$ can be constructed using a window estimator of the ordered statistics. Let $x_{1t} \ldots x_{mt}$ be a random sample of size m for fixed t and let $z_{1t} \ldots z_{mt}$ be the ordered statistics of the x's. Let $B_{mt}$ be the σ-algebra generated by the x's. Then for any $w_t = w \in R$, let a(1,m,w) and a(2,m,w) be two random sequences, measurable for every m with respect to $B_{mt}$ for fixed t and let

$$P[\ 1 \le a(2,m,w) - a(1,m,w) \le k_m\ ;\ 1 \le a(1,m,w) \le a(2,m,w) \le k_m\ ;$$
$$z_{a(1,m,w)} \le w \le z_{a(2,m,w)}\ ] = 1 \qquad \text{(A.3)}$$

for every m, where $k_m$ is sequence of positive numbers to be chosen. Then, for $z_{a(1,m,w)} \le w \le z_{a(2,m,w)}$

$$f_m(w) = \frac{a(2,m,w) - a(1,m,w)}{m[z_{a(1,m,w)} - z_{a(2,m,w)}]} \qquad \text{(A.4)}$$

and for $w \le z_{a(1,m,w)}$ or $w \ge z_{a(2,m,w)}$, $f_m(w)=0$

For the specific case under consideration I follow Rao (1981, p.102-103) is choosing $k_m$ to be a constant sequence of integers and define:

$$a(1,m,w) = \max\ [R_m(w) - 0.5*k_m,\ 1]$$
$$a(2,m,w) = \max\ [R_m(w) + 0.5*k_m,\ m]$$

$$R_m = m*F_m(w) \qquad \text{(A.5)}$$

and $F_m(w)$ is the empirical distribution function. With these choices $f_m(w)$ reduces to:

$$= \frac{k_m}{m[z_{j+0.5km/2} - z_{j-0.5km/2}]} \qquad j=0.5k_m+1,..m+0.5k_m.$$

$$f_m(w) \qquad = \frac{k_m+j-1}{m[z_{j+0.5km} - z_1]} \qquad j=1....\ \ 0.5k_m$$

$$= \frac{m-j+0.5k_m}{m[z_m - z_{j-0.5km}]} \qquad j=m+1+0.5k_m,...m-1$$

(A.6)

if $z_j \le w \le z_{j+1}$ and $f_m(w)=0$ otherwise.
Note that $k_m$ is the smoothing sequence. For figures 1-5 $k_m$ is set to 11 ,independent of t, while in figure 6 $k_m$ is equal to 5.

## APPENDIX B

Ingram-Lee (1986) showed consistency and asymptotic normality of simulation estimators.Their assumptions and their proofs are similar to Hansen's (1982). Here I report a set of conditions for problem. under consideration. Gallant (1987) provides a more general set of conditions and proofs.

For consistency assume:

i) $x_t$ and $y_j(\beta)$ are independent, stationary and ergodic stochastic processes.

ii) $(S,\sigma)$ is a compact separable metric space $S \in R^r$ and $\beta^* \in S$.

iii) $I_{y,\beta}(\theta)$ is Borel measurable for each $\beta \in S$ and continuous for each $y \in R$ and $\theta \in [-\pi,\pi]$. $I_x(\theta)$ is continuous for each $x \in R$ and $\theta \in [-\pi,\pi]$.

iv) $I_{y,\beta}(\theta)$ is continuous in the mean, i.e $\lim_{\delta \to 0} E\{\sup [|I_{y,\beta}(\theta)-I_{y,\beta^*}(\theta)| : \beta,\beta^* \in S, |\beta-\beta^*|<\delta]=0$

v) $E\Sigma_\theta(g_{t,\beta}(\theta))$ exists and it is finite for all $\beta \in S$ and $E\,\Sigma_\theta\,(g_{t,\beta^*}(\theta))=0$

vi) $W_T(\theta) \to W(\theta)$ a.s.

Then if $\Sigma_\theta g_{t,\beta}(\theta)$ has a unique zero at $\beta^*$, then $\beta_{TN}$, the simulation estimator exists and is consistent.

For asymptotic normality assume:

i)-iii)-vi) above

vii) S is an open subset of R containing $\beta_0$.

viii) $Z(\theta)=dI_{y,\beta}(\theta)/d\beta$ is Borel measurable for each $\beta \in S$ and continuous for each $\beta$ and $\theta$ and continuous in the mean..

ix) $E\Sigma_\theta[Z(\theta)|\beta=\beta^*] = B$ exists is finite and of full rank

x) $E\Sigma_\theta\{g_{t,\beta}(\theta)\, g_{t,\beta}(\theta)'\}$ exists and it is finite, $E\{g_t(\theta)|g_{t-i}(\theta), i=1,2..\}^2 \to 0$ and is a martingale difference process with square summable residuals.

xi) $\beta_{TN} \to \beta^*$ in probability

Then $\sqrt{T}^*(\beta_{TN}-\beta^*) \to N(0, (AB)^{-1}ASA'(AB)^{-1\prime})$

where $A=B'W$ , $S=\text{cov}\Sigma_\theta[I_{y,\beta}(\theta)-I_x(\theta)]$

**FOOTNOTES**

1 Survey data are not a very reliable source to tests for efficiency because they are likely to report the mode or the median instead of conditional mean forecast. If the distribution of the underlying variable is asymmetric the median and the mode do not correspond to the conditional expectation. The linera projection approximation is exact if the variables in the informations set are jointly normal and approximately correct if the variables in the information set are asymptotically normal. On the other hand the nonparameteric construction of conditional means is quite complicated if the conditioning set is large.

2 One can show the direction of the bias introduced by approximating $E_t[S_{t+k} | I_t]$ with $E_t[P_t(S_{t+k} | I_t)]$. By Chung(1974,theorem 9.1.4,p.302)if G is a convex function on R and x and G(x) are integrable then $G[E(x|I_t)] \leq E[G(x)|I_t]$. Since $P_t$ is an integrable convex function, it follows that $P[S_{t+k}|I_t] = P_t[E_t(S_{t+k}|I_t)|I_t] \leq E_t[P_t(S_{t+k}|I_t)|I_t\}$. Also

$$E_t[S_{t+k}|I_t]=E_t[P_t(S_{t+k}|I_t)|I_t]+\int \varepsilon^{\#}_{t+k} f[\{P_t(S_{t+k}|I_t)\}]d\varepsilon^{\#}_{t+k}$$

where $\varepsilon^{\#}_{t+k} = S_{t+k} - P_t[S_{t+k}|I_t]$. Therefore

$$P[S_{t+k}|I_t\} \leq E[P(S_{t+k}|I_t)|I_t] \leq E[S_{t+k}|I_t]$$

with equality holding if $S_t$ is a linear process of if variables in $I_t$ are jointly normal. In general one should expect a downward bias in the measure of $S^e_{t+k}$ employed, but also an improvement over single point forecasts.

3 It should be noted that this procedure do not generate a bootstrap estimate for the forecasts. A true bootstrap forecasts estimate requires the application of the recursive bootstrap algorithm at each data point in the sample. This procedure is straightforward to implement but computationally very expensive.

4 From these observations one can show that construction of the density of the forecasts by direct Monte Carlo methods assuming normal residuals would have provided poor estimates of the location and spread parameters. Calzolari-Panattoni (1988) argue that the mode of the density of forecasts provides an appropriate measure of location in situations where the density is multimodal, skewed and/or leptokurtic. However, the mode is sensitive to the the smoothing technique employed to compute the density and can changes depending on the smoothing parameter chosen.

5 This test can detect both deterministic and stochastic seasonals. Deterministic seasonals should be modelled as harmonic series.

6 Similar pictures for spot rates, forward rates, interest rates and ex-ante profits using the Yen or the DM as a unit of account are available on request by the author.

7 The assumption that Goverments tax agents of both countries is made for the sake of symmetry and it is ineffectual for the results that follow. Agents pay taxes in the asset market period because in this case no precautionary money holding will exist and CIA are satisfied with equality

8 Alternatively, it is possible to assume that each government has an endownment of the currency of the other country at time 0 and it is allowed to inject currency of the other country in the asset market period at t constrained only by amount of reserves available at each t. This extension however does not produce any modification in the results presented.

9 The results that follows are independent of the standardization of the profit series by $400/s_t$. However, the division by $S_t$ may introduce further patterns of seasonality in the data generated by the model.

10 For example, if $\varepsilon_t \sim \text{Log } N(\mu,\sigma^2)$, then $E(\varepsilon_t)=\exp(\mu+.5\sigma^2)$. If $\varepsilon_t$ has an extreme value distribution with (a,b) as location and spread parameters, then $E_t(\varepsilon_t)= b*\Gamma'(1)+a$ where $\Gamma'(1)$ is the derivative of a gamma distribution.

11 Since only weekly data for the money supply are available only in the US, I constructed weekly estimates for the money supply in the other two countries by inputting monthly values to each week of the month and then estimating the autoregressive parameters imposing a smoothness prior. The conditional variance of the weekly series was then computed using the difference between the original and the predicted values of the series.

12 Assuming constant conditional variances may be a gross simplification. Also fixing the variances at certain values may strongly affect the fit of the model. Nevertheless this procedure provides a firmer discipline in choosing the remaining parameter values. A sensitivity analysis is conducted to check the effects of this assumption on the results.

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## TABLE 1: STATISTICS FOR THE PROFIT SERIES
## US $ AS UNIT OF ACCOUNT

**EX-ANTE PROFITS (SINGLE MARKET ESTIMATES) ANNUALIZED PERCENTAGES**

| PROFIT SERIES | MEAN | SE | MIN | MAX | SKEWNESS TEST | KURTHOSIS TEST | F.O. CORR. |
|---|---|---|---|---|---|---|---|
| $/DM | -10.64 | 21.09 | -67.82 | 77.87 | .05 | .00 | .95 |
| $/SF | -4.28 | 24.26 | -58.81 | 68.00 | .54 | .21 | .96 |
| $/FF | -12.75 | 18.81 | -70.07 | 30.81 | .00 | .31 | .97 |
| $/L | -2.93 | 24.39 | -50.81 | 97.06 | .00 | .00 | .96 |
| YEN/$ | 5.15 | 21.50 | -56.24 | 82.97 | .00 | .00 | .90 |
| $/CAN$ | -.58 | 4.88 | -10.67 | 16.72 | .00 | .00 | .80 |

**EX-POST PROFITS ANNUALIZED PERCENTAGES**

| PROFIT SERIES | MEAN | SE | MIN | MAX | SKEWNESS TEST | KURTHOSIS TEST | F.O. CORR. |
|---|---|---|---|---|---|---|---|
| $/DM | -2.72 | 26.09 | -59.32 | 62.19 | .13 | .00 | .94 |
| $/SF | -3.58 | 28.83 | -68.03 | 78.14 | .00 | .04 | .94 |
| $/FF | -.53 | 25.58 | -58.04 | 63.30 | .58 | .00 | .94 |
| $/L | .03 | 26.97 | -65.01 | 83.46 | .05 | .31 | .94 |
| YEN/$ | .88 | 27.39 | -64.44 | 60.05 | .25 | .00 | .95 |
| $/CAN$ | -.51 | 8.15 | -22.56 | 24.68 | .01 | .17 | .91 |

**EX-ANTE PROFITS (MULTIMARKET ESTIMATES) ANNUALIZED PERCENTAGES**

| PROFIT SERIES | MEAN | SE | MIN | MAX | SKEWNESS TEST | KURTHOSIS TEST | F.O CORR. |
|---|---|---|---|---|---|---|---|
| $/DM | -11.48 | 27.31 | -123.08 | 62.20 | .05 | .05 | .96 |
| $/SF | 8.14 | 54.75 | -118.51 | 276.3 | .00 | .00 | .97 |
| $/FF | -12.01 | 34.15 | -125.95 | 82.44 | .04 | .05 | .96 |
| $/L | -1.43 | 29.9 | -92.25 | 89.81 | .00 | .00 | .96 |
| YEN/$ | -6.44 | 29.5 | -102.81 | 62.31 | .00 | .00 | .97 |
| $/CAN$ | -1.49 | 7.86 | -21.35 | 23.72 | .00 | .00 | .94 |

Note: In the skewness and kurthosis column, significance levels are reported. The statistics compare the estimated skewdness and kurthosis to the ones of a normal distribution using a two sided test. The last column reports first order serial correlation coefficients.

TABLE 2: SEASONALITY TESTS (US$ AS UNIT OF ACCOUNT)
EX-POST PROFITS

| RATE | FREQUENCY | % OF THE VARIANCE | SIGNIFICANCE | TOTAL% |
|---|---|---|---|---|
| $/DM | .2307π | 2.89 | .097 | |
| | .5000π | 2.89 | .096 | |
| | .5384π | 9.07 | .000 | 48 |
| $/SF | .2307π | 3.06 | .066 | |
| | .5000π | 3.26 | .041 | |
| | .5384π | 11.06 | .000 | |
| | .6923π | 3.82 | .010 | 42 |
| $/FF | .0384π | 2.89 | .097 | |
| | .4230π | 2.91 | .092 | |
| | .5384π | 7.10 | .000 | |
| | .6538π | 3.03 | .071 | |
| | .8846π | 3.32 | .036 | 44 |
| $/L | .0384π | 3.00 | .075 | |
| | .5384π | 10.43 | .000 | |
| | .8076π | 2.89 | .096 | 39 |
| YEN/$ | .0768π | 3.04 | .069 | |
| | .5384π | 4.69 | .000 | |
| | .6923π | 4.83 | .000 | |
| | .8461π | 4.60 | .001 | 41 |
| CAN$/$ | .2307π | 4.64 | .001 | |
| | .5000π | 4.23 | .003 | 30 |
| | | **EX-ANTE PROFITS** | | |
| $/DM | .5000π | 3.33 | .025 | |
| | .5384π | 4.27 | .001 | |
| | .9615π | 2.96 | .064 | 38 |
| $/SF | .5000π | 4.09 | .003 | |
| | .5384π | 5.30 | .000 | |
| | .9615π | 3.89 | .005 | 37 |
| $/FF | .5000π | 3.59 | .012 | |
| | .5384π | 3.37 | .023 | |
| | .9615π | 2.90 | .074 | 34 |
| $/L | .5384π | 5.61 | .000 | 40 |
| YEN/$ | .1153π | 3.27 | .030 | |
| | .2692π | 2.81 | .090 | |
| | .3461π | 2.96 | .063 | |
| | .9230π | 2.86 | .081 | 38 |
| CAN$/$ | .5000π | 3.91 | .005 | |
| | .5769π | 2.95 | .065 | 35 |

TABLE 3: SEASONALITY TESTS (DM AS UNIT OF ACCOUNT)

EX-POST PROFITS

| RATE | FREQUENCY | % OF THE VARIANCE | SIGNIFICANCE | TOTAL% |
|---|---|---|---|---|
| SW/DD | .1153π | 8.23 | .000 | |
| | .7694π | 11.09 | .000 | |
| | .9616π | 24.72 | .000 | 53 |
| FF/DD | .1537π | 3.51 | .016 | |
| | .1923π | 8.85 | .000 | |
| | .2305π | 3.98 | .004 | |
| | .2693π | 3.17 | .038 | |
| | .4232π | 4.74 | .000 | |
| | .6921π | 16.25 | .000 | |
| | .7300π | 4.24 | .000 | 56 |
| L/DM | .0381π | 3.44 | .019 | |
| | .1153π | 4.01 | .003 | |
| | .6336π | 5.08 | .000 | 28 |
| Y/DM | .2693π | 2.95 | .055 | |
| | .4232π | 4.31 | .001 | |
| | .5762π | 3.31 | .027 | |
| | .6154π | 4.45 | .001 | 39 |
| CAN$/DM | .7698π | 3.26 | .031 | |
| | .9235π | 3.41 | .020 | 27 |

EX-ANTE PROFITS

| RATE | FREQUENCY | % OF THE VARIANCE | SIGNIFICANCE | TOTAL% |
|---|---|---|---|---|
| SF/DM | | | | 42 |
| FF/DM | | | | 40 |
| L/DM | | | | 41 |
| YEN/DM | .1819π | 3.17 | .048 | |
| | .4322π | 3.58 | .013 | |
| | .8468π | 5.54 | .000 | |
| | .9235π | 3.31 | .035 | 49 |
| CAN/DM | .0385π | 3.26 | .039 | |
| | .0769π | 3.28 | .038 | |
| | .1151π | 3.13 | .047 | |
| | .1525π | 3.09 | .050 | 43 |

Note: Total refers to the total percentage seasonal variability.

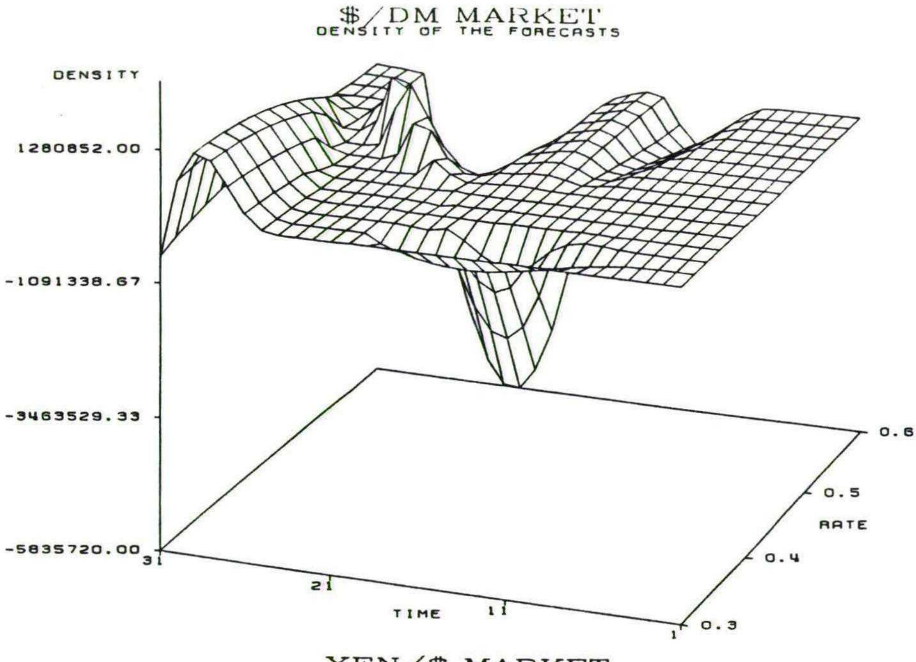
**TABLE 4: SEASONALITY TESTS (YEN AS UNIT OF ACCOUNT)**

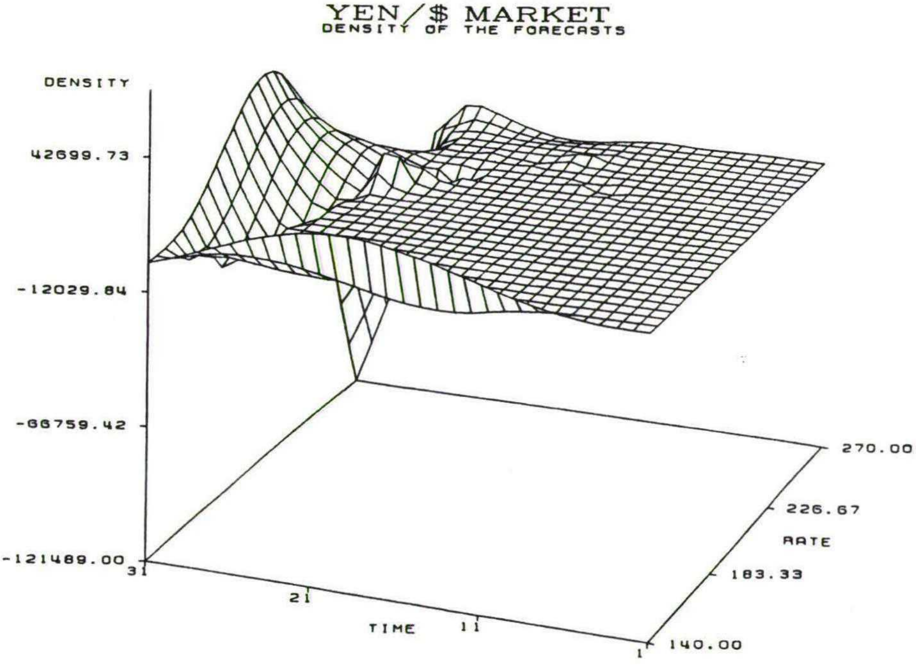
**EX-POST PROFITS**

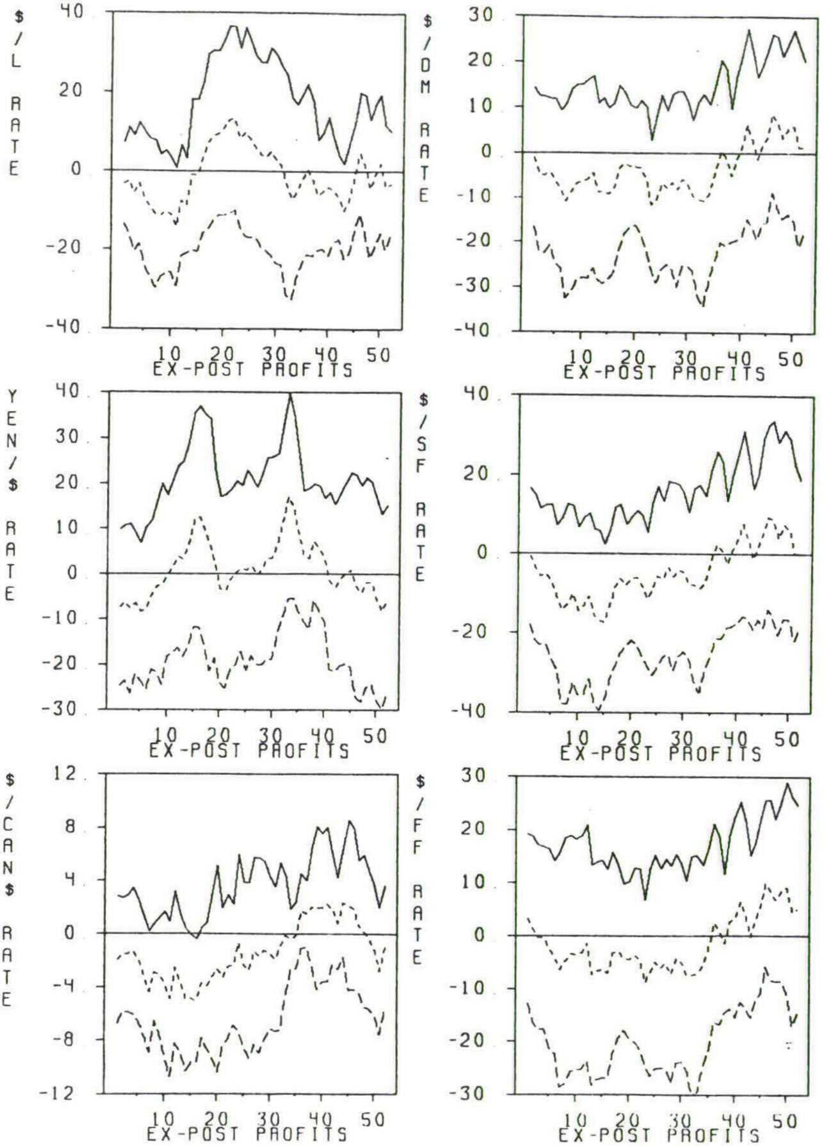
| RATE | FREQUENCY | % OF THE VARIANCE | SIGNIFICANCE | TOTAL% |
|---|---|---|---|---|
| SW/Y | .9615π | 19.99 | .000 | 25 |
| FF/Y | .1924π | 6.81 | .000 | |
| | .6924π | 4.31 | .001 | 22 |
| L/Y | .6532π | 5.35 | .000 | |
| | .7698π | 4.74 | .000 | 19 |
| CAN$/Y | .3075π | 4.33 | .001 | |
| | .3874π | 3.53 | .015 | 19 |

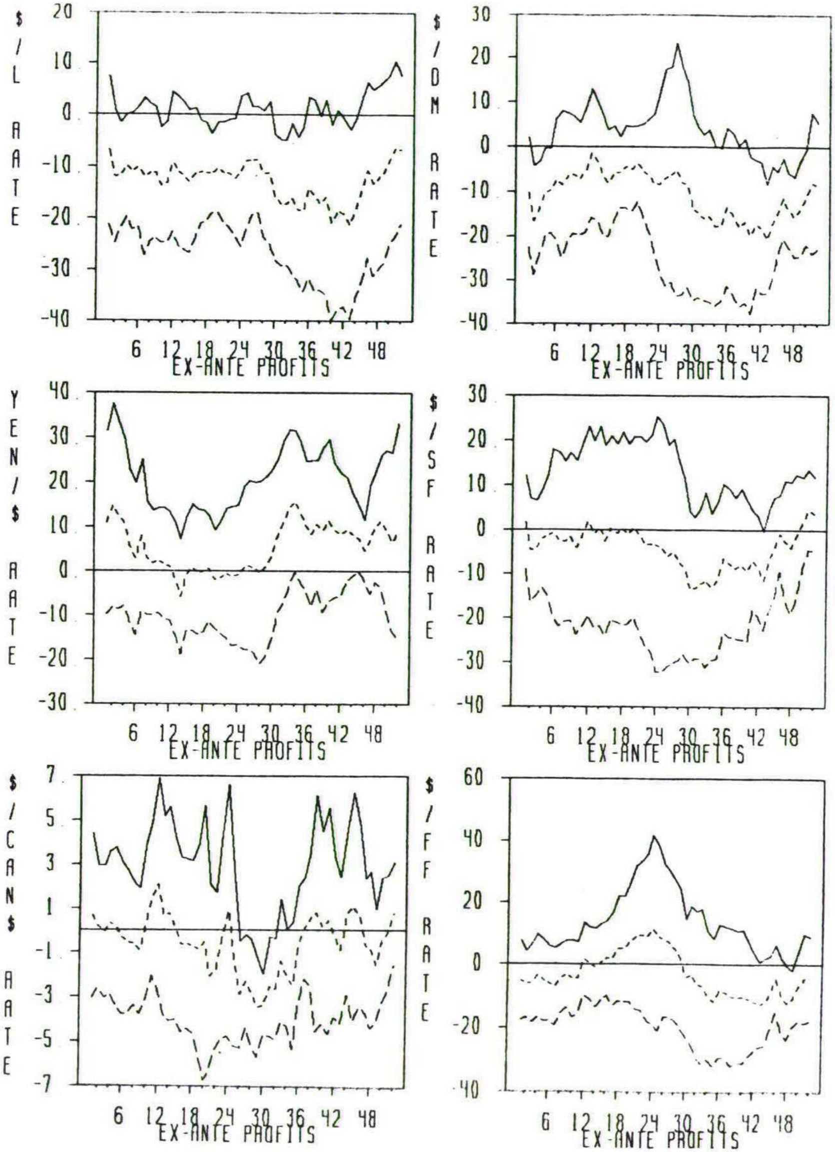
**EX-ANTE PROFITS**

| RATE | FREQUENCY | % OF THE VARIANCE | SIGNIFICANCE | TOTAL% |
|---|---|---|---|---|
| SF/YEN | | | | 41 |
| FF/YEN | .1814π | 3.00 | .057 | |
| | .2198π | 3.08 | .049 | 42 |
| L/YEN | | | | 39 |
| CAN$/YEN | .1524π | 3.78 | .011 | |
| | .2308π | 3.41 | .023 | |
| | .2665π | 3.06 | .053 | |
| | .5764π | 3.33 | .036 | |
| | .6528π | 3.08 | .051 | |
| | .9623π | 3.16 | .044 | 45 |


$/DM MARKET
DENSITY OF THE FORECASTS
DENSITY
1280852.00
-1091338.67
-3463529.33
-5835720.00
31
21
TIME
11
1
0.3
0.4
RATE
0.5
0.6


YEN/$ MARKET
DENSITY OF THE FORECASTS
DENSITY
42699.73
-12029.84
-66759.42
-121489.00
31
21
TIME
11
1
140.00
183.33
RATE
226.67
270.00


$/L RATE
40
20
0
-20
-40
10 20 30 40 50
EX-POST PROFITS
$/DM RATE
30
20
10
0
-10
-20
-30
-40
10 20 30 40 50
EX-POST PROFITS
YEN/$ RATE
40
30
20
10
0
-10
-20
-30
10 20 30 40 50
EX-POST PROFITS
$/SF RATE
40
20
0
-20
-40
10 20 30 40 50
EX-POST PROFITS
$/CAN$ RATE
12
8
4
0
-4
-8
-12
10 20 30 40 50
EX-POST PROFITS
$/FF RATE
30
20
10
0
-10
-20
-30
10 20 30 40 50
EX-POST PROFITS


$/L RATE
20
10
0
-10
-20
-30
-40
6 12 18 24 30 36 42 48
EX-ANTE PROFITS
$/DM RATE
30
20
10
0
-10
-20
-30
-40
6 12 18 24 30 36 42 48
EX-ANTE PROFITS
YEN/$ RATE
40
30
20
10
0
-10
-20
-30
6 12 18 24 30 36 42 48
EX-ANTE PROFITS
$/SF RATE
30
20
10
0
-10
-20
-30
-40
6 12 18 24 30 36 42 48
EX-ANTE PROFITS
$/CAN$ RATE
7
5
3
1
-1
-3
-5
-7
6 12 18 24 30 36 42 48
EX-ANTE PROFITS
$/FF RATE
60
40
20
0
-20
-40
6 12 18 24 30 36 42 48
EX-ANTE PROFITS

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