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A BARGAINING GAME WITH COALITION FORMATION

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A Bargaining Game with Coalition Formation

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Abstract

This paper considers a game in which coalitions form in order to have a stronger position in a bargaining process. Both coalition formation and bargaining are non-cooperative. The players are a seller and two potential buyers with different reservation prices who bargain over the allocation of a good and the payments to be made. Players may form coalitions before this bargaining process. In equilibrium, each two-player coalition is formed with probability one third regardless of the relative strength of the players, and expected payoffs coincide with the Shapley value of a related cooperative game.

Keywords: Noncooperative bargaining, coalition formation, random proposers.

JEL classification: C78, C72.

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1. Introduction

This paper studies endogenous coalition formation when coalitions are formed in order to have a stronger position in a bargaining process with other players. It is assumed that there is a given value to be divided among the players, and the allocation of that value is determined by noncooperative bargaining. Before the bargaining process starts, players may form (disjoint) coalitions; if a coalition forms, it will act as a single unit in the bargaining process. The coalition formation process is modelled in a similar way to the subsequent bargaining process.

The idea of forming coalitions to have a stronger bargaining position is developed in Hart and Kurz (1983), who approach the problem in an axiomatic way. The present paper takes a strategic approach, using an extension of the Rubinstein (1982) two-person alternating-offer model to n players. In this extension proposers are randomly selected following Okada (1996).

The game is divided in two stages: coalition formation and bargaining between coalitions. In the bargaining stage, a coalition is randomly selected to make a proposal to other coalitions[1]; in the coalition formation stage, a player is selected randomly, and he proposes a coalition and a division of the payoff that the coalition will attain in the subsequent bargaining stage. Players are patient, but negotiations may break down when a proposal is rejected. The solution concept is stationary perfect equilibrium.

Although the problem can be formulated more generally, the paper focuses on an economic application. In this application, a seller can sell an object to one of two buyers with different reservation prices. The allocation of the good and payments to be made are determined by bargaining among the players. The reservation prices (and the fact that the seller attaches no value to the good) are common knowledge, and reselling is feasible.

The main question we investigate is whether the buyer with the lower reservation price can expect a positive payoff. Efficient bargaining implies that this buyer cannot expect to get the object; however, his presence in the market benefits the seller, who can presumably ask a higher price when there are two buyers, whereas his exit from the market would benefit the other buyer, who would then pay a lower price.

[1] Not necessarily to all; see the next section.

If coalitions can be formed prior to the bargaining process, the weak buyer may benefit from his influence on the price. He can negotiate with the seller and get paid to be in the market, or he can negotiate with the other buyer and get paid to be out of the market. The question is then what coalition will form and what are the expected payoffs for the three players (will the weak buyer "capture" the whole price difference or not?).

The results of the model are as follows: the weak buyer has a positive expected payoff, but he cannot capture the whole price difference, as the seller and the strong buyer would then form a coalition themselves. Perhaps surprisingly, for any values of the parameters, each two-player coalition forms with probability one third. No matter how weak the weak buyer is, he will be part of a coalition two thirds of the time. His expected payoff will be affected by how weak he is, but not his probability of being in a coalition. The players' expected payoffs correspond to the Shapley value of a related cooperative game, and the possibility of forming coalitions makes the two buyers better-off. Finally, the one-seller-two-buyer situation illustrates that assuming random proposers instead of a fixed protocol may yield very different results (see Section 3.2).

The rest of the paper is organized as follows. Section 2 is devoted to the one-seller-two-buyer game. Section 3 discusses the results as well as the assumptions of the model. Section 4 is devoted to the related literature. Section 5 contains the concluding remarks. Proofs are provided in the Appendix.

2. The Model

The set of agents is N = {1,2,3}. Agent 1 is a potential seller who owns one unit of some good and derives zero utility from keeping it; agents 2 and 3 are potential buyers, whose reservation prices for the good are respectively u_2 and u_3, $0 < u_2 < u_3$. All players are risk-neutral and valuations are common knowledge. The characteristic function associated to this situation is then $v(\{1,2,3\}) = v(\{1,3\}) = u_3$; $v(\{1,2\}) = u_2$; $v(\{2,3\}) = v(\{i\}) = 0$ for $i = 1,2,3$.

The question we investigate is whether player 2 can expect a positive payoff in this situation. On the one hand, he cannot expect to buy the good, as for any price player 2 is prepared to pay, player 3 is prepared to pay more. On the other hand, player 2 may affect the price of the good: if player 2 were not in the game, the "intuitive" outcome would be player 3 buying the good for $\frac{u_3}{2}$; if player 2 is in the game, the price cannot be lower than u_2. It can be argued that player 2 will somehow exploit this power and, for example, form a cartel with player 3 and get a share of the gains derived from the cartel.

To address this question, we model the situation described above as a two-stage game. The allocation of the good and payments are determined in a bargaining process with random proposers and exogenous probability of breakdown. Prior to this bargaining players may form coalitions: if two players form a coalition, they will bargain with the third player as a single unit. It is assumed that contracts specifying the division of the coalitional payoff can be enforced. The game is now described in more detail.

2.1. The coalition formation stage

The game starts with Nature selecting a proposer; each of the three players is selected with probability $\frac{1}{3}$. A proposal consists of a coalition to which the proposer belongs and a division of the coalitional payoff.

The coalitional payoff can be a monetary payment (e.g., the payoff for coalition $\{1\}$ would be the price) a consumer's surplus (e.g., for coalition $\{3\}$ the payoff would be the difference between his reservation price and the price actually paid) or a sum of payments and consumer's surplus (for coalitions $\{1,3\}$ the payoff would be the total value to be created, u_3).

The coalitional payoff is determined at the end of the game and players can anticipate it by the usual backwards induction argument. Because the game includes chance moves, this payoff is not deterministic. Since all players are risk neutral only expected payoffs matter, thus there are many ways of dividing the coalitional payoff depending on how players in the coalition share the risk, all of them leading to the same expected payoffs for the players.

We will assume for concreteness that the proposer bears all the risk. A proposal is then a pair (S, y), where S is a coalition to which the proposer belongs, and y is a $|S-1|$-dimensional vector describing the (deterministic) payoffs to the remaining players in S. The proposer is understood to get the residual payoff[2].

The expected coalitional payoff depends on which other coalitions form; we will denote by $\varphi(S;\pi)$ the expected payoff for coalition S when the coalition structure is π. Given a coalition structure $\pi = (S_1, ..., S_n)$ we will denote by $\varphi(\pi)$ the payoff vector whose j-th entry is the expected payoff for coalition S_j This function is usually called a *partition function*, as opposed to the characteristic function in which the value of a coalition does not depend on what other coalitions form.

[2]If a player proposes to stay alone, he does not have to specify any payoff division or accept his own proposal.

Because there are only three players, the coalition structure is determined given S unless S is a singleton; this greatly simplifies the analysis.

Once a proposal is made, the rest of the players in S accept or reject the proposal sequentially[3]. If the proposal is accepted the coalition forms and its players retire from this stage (thus coalitions can not be enlarged). If the proposal is rejected, nature chooses a new proposer with probability δ, and breakdown occurs with probability $1-\delta$. If breakdown occurs, all players play the bargaining stage as single units.

The coalition formation stage lasts until breakdown occurs or a coalition structure (a partition of the set of players) is formed. Because there are only three players, stage 1 ends once a (nonsingleton) coalition is formed.

There are two cases in which the game ends after the first stage: if the grand coalition forms, the division of the value of the grand coalition has already been decided (as a proposal to form a coalition includes a payoff division) and nothing remains to be settled; if coalition $\{1,3\}$ forms, it can achieve a payoff of u_3 by itself.

2.2. The bargaining stage

In the bargaining game between coalitions each coalition acts as a unit (for example, each coalition sends a representative). Depending on the outcome of the first stage, bargaining takes place between a two-player coalition and a single player, or among three single players.

The bargaining process runs as follows: first, a coalition is chosen by nature to be the proposer (all coalitions are chosen with equal probability). This coalition makes a proposal about the allocation of the good and transfers between coalitions. The coalitions affected by the proposal accept or reject sequentially. If a proposal is rejected, nature selects a new proposer with probability δ. With probability $1-\delta$, a breakdown of the negotiations takes place and each coalition gets the payoff it can get by itself, $v(S)$. We will think of δ as being close to 1.

Notice the difference between $v(S)$ and the expected payoff of a coalition in this bargaining process, $\varphi(S\ ;\pi)$. $v(S)$ represents the payoff the coalition gets "by itself", i.e., when it is isolated from other players. However, the coalition will often be able to do better than this, and can improve upon $v(S)$ by reaching an

[3]The order in which players accept or reject does not affect the results, since the first player to reject has no advantage over other players when proposers are randomly selected.

agreement with other coalitions. For example, $v(\{2,3\}) = 0$, as neither 2 nor 3 own the good, but $\varphi(\{2,3\}; \{\{1\}, \{2,3\}\}) = \frac{u_3}{2} > 0$, as the coalition $\{2,3\}$ can act as a single buyer and reach an agreement with 1 about the price of the good.

Once an agreement is reached, it is implemented and the game ends, except in one case: if the three players are playing the bargaining game as single units, and player 1 agrees to sell the good to player 2, it is allowed for player 2 to resell the good to player 3. The game continues then until players 2 and 3 reach an agreement or breakdown occurs. If breakdown occurs, player 2 keeps the good. This responds to the idea of allowing for bargaining between the players until all gains from contracting are exhausted.

The coalition formation stage and the bargaining stage are formally very similar. In both stages players move sequentially, proposers are selected randomly and there is an exogenous probability of breakdown. However there are three differences: the stages differ in the players (the first stage is played among individual players, whereas the second is played among coalitions), in the content of proposals (in the first stage, proposals include a coalition and payoff division within the coalition, whereas in the second stage a proposal consists of a payoff division among coalitions) and in the consequences of breakdown (if breakdown occurs in the first stage, no coalitions form and the game proceeds to the second stage; if breakdown occurs in the second stage, the game ends).

2.3. The equilibrium concept

The history of the game at a given moment consists of all proposers, proposals, and responses so far. A strategy for a player in the first stage (analogously for a coalition in the second stage) assigns proposals to all nodes at which the player is a proposer and a response to all possible proposals at every node at which the player is a responder. A strategy is *stationary* if it is independent of the history except possibly the payoff-relevant aspects like the coalitions that have formed and the current proposal. A *stationary perfect equilibrium* is a subgame perfect equilibrium in which each player employs a stationary strategy. I focus on stationary perfect equilibria.

The game can be solved by backwards induction: first, the equilibrium of the bargaining stage is found for each possible coalition structure; this determines the expected payoffs (the function $\varphi(S, \pi)$) that are used as an input to solve the coalition formation stage.

2.4. Solving the bargaining stage

The bargaining process in the second stage depends on the outcome of the first stage. If either {1,2,3} or {1,3} have been formed, nothing remains to be settled as the total value has been divided. Thus, there are three bargaining processes to be considered, corresponding to coalition structures $(\{1\},\{2,3\})$, $(\{1,2\},\{3\})$ and $(\{1\},\{2\},\{3\})$.

2.4.1. The second stage with coalition structure $\pi = (\{1\},\{2,3\})$

If players 2 and 3 form a "buyer cartel", there is effectively only one buyer in the market. There is no asymmetry between the two coalitions, as both of them receive a payoff of zero if a breakdown occurs, so we can expect both coalitions to receive the same payoff. As the total value to be created is u_3, each coalition gets $\frac{u_3}{2}$.

Formally, the two coalitions participate in a bargaining process with random proposers and exogenous probability of breakdown $1-\delta$. Nature selects each of the coalitions to be the proposer with probability $\frac{1}{2}$, and this coalition proposes a price. If the other coalition accepts the price, the agreement is implemented; if it rejects, a new proposer is (randomly) determined with probability δ, and the game ends with probability $1-\delta$.

In a stationary perfect equilibrium, each coalition names a price so that the other coalition is indifferent between accepting and rejecting and agreement is reached in the first period. Define $v_{\{1\}}$ (analogously, $v_{\{2,3\}}$) to be the continuation payoff for coalition {1} (i.e., the expected payoff for player 1 given that he rejects an offer)[4]. We can find the continuation payoffs from the following system of equations:

$$v_{\{1\}} = \frac{\delta}{2}\left[u_3 - v_{\{2,3\}}\right] + \frac{\delta}{2} v_{\{1\}}$$

$$v_{\{2,3\}} = \frac{\delta}{2}\left[u_3 - v_{\{1\}}\right] + \frac{\delta}{2} v_{\{2,3\}}$$

The solution to this system is $v_{\{1\}} = v_{\{2,3\}} = \frac{\delta}{2}\, u_3$.

[4]Notice that $v_{\{1\}}$ is a price, whereas $v_{\{2,3\}}$ is the consumer's surplus for player 3. If the coalitions reach an agreement over the price, player 3 will keep the good and player 2 will receive a payment as agreed between players 2 and 3 in the coalition formation stage.

The price of the good depends on which coalition is the proposer: if it is coalition $\{1\}$, the price is $\frac{2-\delta}{2}\, u_3$; if it is coalition $\{2,3\}$,the price is $\frac{\delta}{2}\, u_3$. As each coalition has probability $\frac{1}{2}$ to be the proposer, the expected price is $\frac{u_3}{2}$. In the limit when $\delta \rightarrow 1$, the actual price of the good tends to $\frac{u_3}{2}$ regardless of which coalition is the proposer.

2.4.2. The second stage with coalition structure $\pi = (\{1\},\{2\},\{3\})$

If the coalition structure resulting from the first stage is $(\{1\},\{2\},\{3\})$, bargaining takes place among individual players. Bargaining starts by a chance move that determines the first proposer; each player is selected with probability $\frac{1}{3}$.

If the seller is selected, he can offer the good to one of the buyers for a price. If a buyer is selected, he can propose a price to the seller. All players have also the possibility to propose a global agreement in which 3 gets the good and makes a payment to 1 and 2.

In this subgame it makes a difference whether reselling is feasible. Through most of the paper I will assume feasible reselling, though I will briefly consider the case of unfeasible reselling as well.

Feasible reselling If the good is bought by player 2 and reselling is feasible, it is natural to assume that player 3 will buy the good from 2. The resell price is again determined by bargaining between the two players. Each player is selected with probability $\frac{1}{2}$ to be the proposer, and breakdown occurs with probability $1-\delta$ after a proposal is rejected. If breakdown occurs, player 2 keeps the good.

In a subgame perfect equilibrium, each player's offer makes the other player indifferent between accepting and rejecting. Call w_2 and w_3 the continuation values of players 2 and 3, i.e., their expected payoff from rejecting an offer. In equilibrium, player i makes an offer that yields player j an utility of w_j. The continuation values are thus given by

$$w_2 = \tfrac{\delta}{2}(u_3 - w_3) + \tfrac{\delta}{2}w_2 + (1-\delta)u_2$$

$$w_3 = \tfrac{\delta}{2}(u_3 - w_2) + \tfrac{\delta}{2}w_3$$

For player 2, w_2 is the price received for the good when player 3 is the proposer; for player 3, w_3 is the consumer surplus he gets when player 2 is the proposer.

Solving the system above yields

$$w_2 = \frac{\delta\, u_3 + (2-\delta)\, u_2}{2}$$

$$w_3 = \frac{\delta(u_3 - u_2)}{2}$$

As a proposer, 3 pays a price of w_2;as a responder, he pays a price of $u_3 - w_3$. Because each player becomes a proposer with probability $\frac{1}{2}$, expected price is $\frac{1}{2}w_2 + \frac{1}{2}(u_3 - w_3) = \frac{u_3 + u_2}{2}$.

If player 2 were not in the game, the seller would sell the good to player 3 for $\frac{u_3}{2}$; player 2 can obtain a higher price from player 3 because he has a positive valuation for the object.

The possibility of reselling may affect the price at which the seller sells the good in the first place. If 1 is selected to be the proposer and proposes to 2, he anticipates that 2 will resell the good, so that the total value to be divided between 1 and 2 is not u_2 but $\frac{u_3 + u_2}{2}$. Analogously, if 2 is selected to be the proposer and proposes to buy the good from 1, he anticipates that he will resell it. The continuation value of player 3 takes reselling into account as well: if player 3 rejects a proposal, with a certain probability (determined by the strategies of the players) the good will be sold to player 2. Player 2 will then resell the good at the expected price of $\frac{u_3 + u_2}{2}$; this implies an expected payoff for 3 of $\frac{u_3 - u_2}{2}$.

Taking this into account we can find the equilibrium of the three-player bargaining stage. To do this, notice that player 1 has three meaningful alternatives: he can either propose to sell the good to player 2, to player 3 or randomize; player 2 has three alternatives as well: either he proposes to buy the good from player 1, or he proposes an agreement between the three players in which player 3 receives the good and pays transfers to players 1 and 2, or he can randomize between those two proposals; player 3 can only propose to buy the good from player 1[5]. There are then nine candidate equilibria.

The equilibrium can be found as follows: starting from a candidate equilibrium, find the continuation payoffs (determined by the strategies of the players). Given the continuation payoffs, check whether any player can improve by deviating from

[5] A proposal of a global agreement by 1 or 3 can be part of an equilibrium only if the continuation value of 2 equals zero. This can never be the case: since the sum of the continuation values of players 1 and 3 cannot exceed u_3, player 2 can always get at least $(1-\delta)u_3$ from proposing a global agreement, therefore his continuation value must be at least $\frac{\delta}{3}(1-\delta)u_3 > 0$.

his strategy. The (unique) equilibrium found by this procedure is described in proposition 2.1.

Proposition 2.1. *In the bargaining stage with coalition structure* $(\{1\},\{2\},\{3\})$ *and possibility of reselling, the following strategies constitute the unique equilibrium for* δ *close to 1:*

a) If $u_3 \geq 3u_2$ *player 1 offers the good to player 3, and players 2 and 3 buy the good from player 1. If player 2 gets the good, he resells it to 3. Further, player* i *accepts any proposal that gives him at least his continuation payoff.*

b) If $u_3 < 3u_2$ *player 1 randomizes between offering the good to players 2 and 3, and players 2 and 3 buy the good from player 1. If player 2 gets the good, he resells it to 3. Further, player* i *accepts any proposal that gives him at least his continuation payoff.*

For all values of the parameters, the limit of the expected payoffs when δ *tends to 1 is*

$$\left(\frac{u_3+u_2}{2}, 0, \frac{u_3-u_2}{2}\right).$$

The results can be interpreted as follows: in the limit when $\delta \to 1$, player 1 always receives a price $\frac{u_3+u_2}{2}$, and player 3 gets the good and enjoys a consumer's surplus $\frac{u_3-u_2}{2}$. Player 2 does not receive anything, either because players 1 and 3 reach an agreement with each other, or because he resells the good at the same price that he bought it. The seller benefits from the presence of a second buyer, as he obtains a price $\frac{u_3+u_2}{2}$ instead of $\frac{u_3}{2}$, but the second buyer himself does not get anything.

Remark 1. *The presence of a second buyer always results in a higher price, regardless of whether* u_2 *is greater or smaller than the price when he is not in the market,* $\frac{u_3}{2}$.

The presence of player 2 always influences the price because he can resell the good to player 3 for $\frac{u_3+u_2}{2}$ (player 2 has an advantage over player 1 in a bilateral bargaining with 3 because he has a positive valuation for the good). The reservation price of player 2 becomes in practice $\frac{u_3+u_2}{2}$, which is higher than the price when he is not in the market $\frac{u_3}{2}$. Player 1 should then be able to obtain a price $\frac{u_3+u_2}{2}$, as it is the case.

Unfeasible reselling If reselling is not feasible, the outcome of the bargaining procedure may not be efficient, as it is possible that player 2 buys the good from 1 instead of proposing a global agreement in which player 3 gets the good and pays transfers to both 1 and 2.

It seems reasonable to expect that the presence of a second buyer will not affect the price if he is not prepared to pay more than $\frac{u_3}{2}$. On the other hand, if $u_2 > \frac{u_3}{2}$, we can expect that the seller sells the good to player 3 at a price equal to u_2. Proposition 2 states that this is indeed the case.

Proposition 2.2. *In the bargaining stage with coalition structure* $(\{1\},\{2\},\{3\})$ *and without the possibility of reselling, the following strategies constitute the unique equilibrium for* δ *close to 1:*

a) If $u_2 \leq \frac{u_3}{2}$, *player 1 offers the good to player 3, player 3 buys the good from player 1, and player 2 proposes a global agreement. Further, player* i *accepts any proposal that gives him at least his continuation payoff* v_i.

Expected payoffs when δ *tends to 1 are* $(\frac{u_3}{2}, 0, \frac{u_3}{2})$.

b) If $u_2 > \frac{u_3}{2}$, *1 offers the good to 3, 3 buys the good from 1, and 2 randomizes between buying the good from 1 and proposing a global agreement; in the limit when* δ *tends to 1, he proposes a global agreement with probability 1. Player* i *accepts any proposal that gives him at least* v_i.

Expected payoffs when δ *tends to 1 are* $(u_2, 0, u_3 - u_2)$

For $u_2 > \frac{u_3}{2}$, the outcome of the process may be inefficient, as player 2 receives the good with positive probability. However, this inefficiency vanishes as δ tends to 1.

Again, for $u_2 > \frac{u_3}{2}$, the seller benefits from the presence of a second buyer (and the first buyer is hurt to the same extent), but the second buyer himself does not get anything for $\delta \to 1$.

Remark 2. *The seller benefits from the possibility of reselling, as* $\frac{u_3 + u_2}{2} > \max(u_2, \frac{u_3}{2})$.

2.4.3. The second stage with coalition structure $\pi = (\{1,2\},\{3\})$

This case is equivalent to the case of player 2 having bought the good from player 1 and reselling it to player 3. Because of the asymmetry between the two coalitions (coalition $\{1,2\}$ obtains a payoff equal to u_2 in the event of breakdown, whereas coalition $\{3\}$ obtains a payoff of 0) expected price is not $\frac{u_3}{2}$ but $\frac{u_3+u_2}{2}$.

2.4.4. Summing up: the partition function

We have solved the bargaining game between coalitions for all possible coalition structures. The partition function associated with this game assigns a payoff for each coalition in each coalition structure. As it results from the equilibrium of the game, the partition function is given by[6]

$$\varphi(\{1,2,3\}) = u_3$$

$$\varphi(\{1,3\},\{2\}) = (u_3, 0)$$

$$\varphi(\{1,2\},\{3\}) = (\tfrac{u_3+u_2}{2}, \tfrac{u_3-u_2}{2})$$

$$\varphi(\{1\},\{2,3\}) = (\tfrac{u_3}{2}, \tfrac{u_3}{2})$$

$\varphi(\{1\},\{2\},\{3\}) = (\frac{u_3+u_2}{2}, 0, \frac{u_3-u_2}{2})$ if reselling is feasible, $(u_2, 0, u_3 - u_2)$ if reselling is not feasible and $u_2 > \frac{u_3}{2}$, and $(\frac{u_3}{2}, 0, \frac{u_3}{2})$ if reselling is not feasible and $u_2 \leq \frac{u_3}{2}$.

From this point on, I will assume that reselling is feasible unless otherwise indicated.

2.5. Solving the coalition formation stage

As in the bargaining stage, proposers are randomly selected. A proposal includes a coalition to which the proposer belongs and a fixed payoff for the members of the coalition other than the proposer. The proposer is understood to keep the remaining of the coalition's payoff; this payoff is not deterministic but its expected value is given by the partition function φ and it is anticipated by the players by the usual argument of backwards induction.

There are many candidate equilibria. Each player can propose a two-player coalition to any of the other two players, or he can propose the grand coalition, or he can propose to stay alone, or he can randomize between two or three of these alternatives. He can make proposals that will be accepted given the strategies of the other players, or he can make proposals that will not be accepted, causing a delay; he can randomize between acceptable an unacceptable proposals

[6] I take the limit when δ tends to 1. This simplifies the calculations and does not affect the results.

of several sorts (we will refer to acceptable proposals unless otherwise specified). The following lemmas restrict the candidate equilibria: in equilibrium, all players propose two-player coalitions and proposals are accepted[7].

We will denote the continuation payoff of player i (expected payoff given that a proposal is rejected) at this stage by V_i.

Lemma 2.3. *None of the players proposes to stay alone in equilibrium.*

Lemma 2.4. *None of the players proposes to form the grand coalition in equilibrium.*

Lemma 2.5. *None of the players makes unacceptable proposals in equilibrium.*

It is clear that the solution of this stage cannot imply a clear-cut prediction of the coalition structure, as it is not possible that all three players propose the same coalition in equilibrium: all three players proposing the grand coalition cannot be an equilibrium (see lemma 2.4), and not all three players can propose the same two-player coalition (as a player has to belong to the coalition he proposes, and anyway it would not be in his interest to propose that the other two players form a coalition against him). It could still be the case that there is a coalition that is proposed in equilibrium by all its members (for example, both buyers propose to form a buyer cartel). Proposition 2.6 states that this is not the case, and proposition 2.7 states that there is no pure-strategy equilibrium.

Proposition 2.6. *There is no stationary perfect equilibrium in which a coalition is proposed by all its members with probability 1.*

Very informally, the intuition for this result is as follows: a key feature of a bargaining game with random proposers is that the continuation payoff of the players depends on how often other players propose to him. If two players propose to each other with probability 1, they form part of a coalition with very high probability (at least two thirds), and this implies a high continuation value for

[7] I take into account that proposers may randomly choose coalitions, but I do not consider cases in which responders randomise between accepting or rejecting a proposal for technical convenience.

them. So high, that someone who proposes to one of these players in the candidate equilibrium would actually prefer to propose to somebody else.

From this result we can conjecture that the equilibrium strategies must somehow be "balanced", so that no player becomes a responder too often.

Proposition 2.7. *There is no equilibrium in pure strategies.*

In particular, and somewhat surprisingly, no "balanced" pure strategies (strategies that imply each player becoming a responder with equal probability) constitute an equilibrium.

Proposition 2.8. *There is a family of stationary perfect equilibria in which at least two players randomize between two-player coalitions.*

For any equilibrium in this family, the continuation payoffs of the players are

$$v_1 = \frac{(3\,u_3 + u_2)}{6}$$

$$v_2 = \frac{u_2}{6}$$

$$v_3 = \frac{(3\,u_3 - 2\,u_2)}{6}$$

In the limit when $\delta \to 1$, each two-player coalition forms with probability $\frac{1}{3}$.

There is no other equilibrium.

Particularly appealing equilibria are perhaps the ones in which one of the players proposes to each of the other players with probability $\frac{1}{2}$. These can be considered to be "focal " equilibria. When $\delta \to 1$, these equilibria converge to the same strategy combination in which every player proposes to each of the other two with probability $\frac{1}{2}$. These strategy combination has desirable properties (see section 3.1).

This result is coherent with Proposition 2.6 (as no player should receive a proposal "too often" in equilibrium). However, the result is surprising if we think that the size of u_2 does not play a role in determining how often player 2 is part of a coalition. It determines the extent to which the presence or absence of player 2 in the market influences price, and can be considered a measure of player 2's bargaining power.

Remark 3. *The expected payoff for player 2 is* $\frac{u_2}{6}$. *If coalitions are not allowed, the price of the good is* $\frac{u_3+u_2}{2}$ *when player 2 is in the market, and* $\frac{u_3}{2}$ *when player 3 is the only buyer. Thus, the change in the price player 2 can induce by entering or exiting the market is* $\frac{u_2}{2}$. *If coalitions are allowed, player 2 captures exactly* $\frac{1}{3}$ *of this value.*

This section concludes with two more remarks about how the possibility of forming coalitions influences the equilibrium price and the players' expected payoffs.

Remark 4. *The equilibrium price is lower when players are allowed to form coalitions.*

If players are not allowed to form coalitions, they have to play the bargaining game as single units; the price is then $\frac{u_3+u_2}{2}$. If players are allowed to form coalitions, the expected price of the good depends on the concrete equilibrium being considered. This price equals $\frac{u_3+u_2}{2}$ when coalition $\{1,2\}$ forms and $\frac{u_3}{2}$ when coalition $\{2,3\}$ forms. However, when coalition $\{1,3\}$ forms, the price depends on which of the two players was the proposer, and that depends on the concrete equilibrium being considered (this is related to the fact that players get a higher payoff when they are proposers). Taking this into account, we can conclude that the expected price ranges from $\frac{u_3}{2}+\frac{2u_2}{9}$ (expected price when 3 is always the proposer for coalition $\{1,3\}$) to $\frac{u_3}{2}+\frac{5u_2}{18}$ (expected price when 1 is always the proposer for coalition $\{1,3\}$)[8].

Remark 5. *The possibility of forming coalitions makes the two buyers better-off, whereas the seller is worse-off.*

Expected payoffs corresponding to the no coalition case are $\frac{u_3+u_2}{2}$ for player 1, 0 for player 2, and $\frac{u_3-u_2}{2}$ for player 3. Comparing these payoffs with the ones in proposition 2.8, we see that both buyers see their payoff increased by $\frac{u_2}{6}$.

The intuition for these results is that the seller cannot get much from the possibility of forming coalitions: if he forms a coalition with 3, there is nothing this coalition can gain from bargaining with player 2; if he forms a coalition with 2, the coalition gets the same payoff player 1 gets in the bargaining process with no coalitions.

[8]This may seem surprising, since proposition 2.8 states that expected payoffs do not depend on the equilibrium considered. However, expected payoffs *conditional on a coalition being formed* do depend on the concrete equilibrium, because a player receives a higher payoff when he is the proposer (this proposer's advantage does not vanish as δ tends to 1).

2.5.1. The case of unfeasible reselling

As we have seen in section 2.4.2., the infeasibility of reselling does not change the payoff of a two-player coalition but it affects the payoffs players get when no coalitions are formed. The presence of player 2 in the market is then irrelevant if $u_2 \leq \frac{u_3}{2}$, and leads to a price of only u_2 (instead of $\frac{u_3+u_2}{2}$) if $u_2 > \frac{u_3}{2}$.

Somewhat surprisingly, the equilibria described in proposition 2.8 exist regardless of whether reselling is feasible.

Proposition 2.9. *There is a family of stationary perfect equilibria in which both strategies and payoffs are as described in proposition 2.8.*

Remark 6. *The weak buyer benefits from the possibility of forming coalitions even if his presence was irrelevant for the original market (i.e., even if $u_2 \leq \frac{u_3}{2}$).*[9]

Remark 7. *The expected equilibrium price may be higher or lower when players are allowed to form coalitions, depending on how large is u_2.*

As in the reselling case, the expected equilibrium price ranges from $\frac{u_3}{2}+\frac{2u_2}{9}$ to $\frac{u_3}{2}+\frac{5\,u_2}{18}$. For a small u_2, this price is always higher than the price when coalitions are not allowed, for high values of u_2 is always lower, and for intermediate values it depends on the concrete equilibrium considered.

Remark 8. *The possibility of forming coalitions always makes player 2 better-off. Player 1 is better-off for small values of u_2 relative to u_3 ($u_2 < \frac{3u_3}{5}$) whereas player 3 is better-off for large values of u_2 ($u_2 > \frac{3u_3}{4}$).*

We can then conclude that the results in the no-reselling case are similar to the results in the reselling case provided that u_2 is large enough relative to u_3.

3. Discussion

3.1. The results

In a bargaining game with a seller and two buyers, we have studied the position of the "weak" buyer who can not get the good if bargaining is efficient, but who

[9] In the reselling case, coalition $\{1,2\}$ could be interpreted as "2 gets paid to be in the market", (as opposed to coalition $\{2,3\}$ in which "2 gets paid to be out of the market") since the payoff is the same for coalition $\{1,2\}$ and for player 1 when no coalitions are formed. That interpretation is not possible for the no-reselling case.

can benefit from his influence in the price by colluding with either the other buyer or the seller. We have found that the weak buyer can indeed get a positive payoff from playing this game, even though he can not capture the whole influence he has on the price.

It is worth to be noticed that the players' continuation payoffs coincide with the Shapley value of the cooperative game given by the characteristic function we have used as an input of our bargaining game[10].

A surprising result of the model is that each coalition forms with probability $\frac{1}{3}$. We can then say that the model *generates* a random matching procedure. Even though we allow players to choose their partners, the final outcome looks as if matching was random.

We can compare the results of this strategic model with the results of the axiomatic model of Hart and Kurz. They compute the payoffs for each player in the following way: given a coalition structure, they find the average marginal contribution (as in the Shapley value) assuming that players arrive randomly, but that players in the same coalition arrive successively. This yields the following payoff vectors for our game:

$(\frac{2u_3+u_2}{4}, \frac{u_2}{4}, \frac{u_3-u_2}{2})$ for coalition structure $(\{1,2\},\{3\})$
$(\frac{u_3}{2}, \frac{u_2}{4}, \frac{2u_3-u_2}{4})$ for coalition structure $(\{1\},\{2,3\})$
$(\frac{2u_3+u_2}{4}, 0, \frac{2u_3-u_2}{4})$ for coalition structure $(\{1,3\},\{2\})$
$(\frac{3u_3+u_2}{6}, \frac{u_2}{6}, \frac{3u_3-2u_2}{6})$ for coalition structures $(\{1,2,3\})$ and $(\{1\},\{2\},\{3\})$.

Hart and Kurz consider the possibility of deviations of groups of players. A group of players can abandon the coalitions to which they belong and organize themselves in any way, not necessarily as a single coalition. All two-player coalitions are stable in this sense in our game: there is no nonempty coalition that can deviate and make all its members better-off.

The results in this paper are consistent with Hart and Kurz's in two ways: with regard to stability of coalition structures, all coalitions that form in equilibrium are stable; with regard to value, it can be checked that the payoffs for the players *conditional on a coalition structure* coincide (in the limit when δ tends to 1) with the Owen value for the "focal equilibrium" in which (again in the limit when δ tends to 1) each player proposes to each of the other two with probability $\frac{1}{2}$.

Consider, for example, coalition structure $(\{1\},\{2,3\})$. Given that $\{2,3\}$ forms, player 1's payoff is $\frac{u_3}{2}$, corresponding to Owen's value. The payoff for

[10]The same result is found by Osborne and Rubinstein in page 184 of *Bargaining and Markets*. Their model is in turn based on Gul (1989). The main difference with the present model is that they *assume* random matching between the agents.

coalition $\{2, 3\}$ is $\frac{u_3}{2}$, divided in the following way: with probability $\frac{1}{2}$, player 2 offers to player 3 his continuation payoff $\frac{3\,u_3 - 2\,u_2}{6}$ and gets $\frac{u_2}{3}$; with probability $\frac{1}{2}$, player 3 offers to 2 his continuation payoff $\frac{u_2}{6}$ and gets $\frac{3\,u_3 - u_2}{6}$. Expected payoffs coincide then with the Owen value.

3.2. The assumptions

In dynamic models of bargaining it is usually assumed that there is a force that motivates the players to reach an agreement as soon as possible: this force may be players impatience or an exogenous probability of breakdown. The reason why I have chosen a model with probability of breakdown instead of a model with a discount factor is that it yields more intuitive results. Suppose player 2 has bought the good from player 1, and $u_2 > \frac{u_3}{2}$. It is common knowledge that there are gains from trade, as player 3 has a higher valuation for the good. If players share a common discount factor δ, and each player can opt out of the game after having rejected a proposal, in equilibrium player 3 offers to player 2 exactly u_2. If player 2 anticipates that and he is indeed impatient, he will prefer to keep the good to himself instead of starting to bargain with player 3 in the first place. Thus, even though it is common knowledge that there are gains from trade, no transaction takes place. If instead we assume that there is an exogenous probability of breakdown, players 2 and 3 will trade. However, an exogenous probability of breakdown is perhaps a less natural assumption than players' impatience.

The model with random proposers was preferred to the model with a fixed protocol for similar reasons: in particular, in a model with a fixed protocol the price does not depend on the number of buyers!

To illustrate this point, consider a seller (player 1) who owns a unit of a given good and n buyers with identical valuation u; all valuations are common knowledge. At equilibrium, each of the buyers will propose to the seller a price equal to his continuation value. Thus, continuation values are identical for all buyers and given by $v_i = \delta\ (u - v_1)$; all continuation values are equal regardless of the strategy of the seller. The continuation value for player 1 is then $v_1 = \delta\ (u - v_i)$, no matter to which of the buyers he proposes or how he randomizes between them. Solving this system of equations, we get $v_1 = \frac{\delta\, u}{1 + \delta}$ and an expected price of $\frac{u}{2}$, as if there was only one buyer, though there are possibly many, their reservation price u is common knowledge and they are not colluding.

In a model with random proposers and two buyers we get (in the limit when δ

tends to 1) a Bertrand result in which buyers do not have any surplus. The seller offers the good to each of the two buyers with equal probability, and continuation values are such that v_1 tends to 1, and v_2 and v_3 tend to zero.

The models that assume a protocol give to a player who rejects a proposal the power to make the next proposal for sure; the situation described above illustrates that this power may be excessive.

A coalition in this paper is understood as a *bargaining unit*. This means that when the weak buyer forms a coalition with either the strong buyer or the seller, they play the bargaining game together against the other player. Alternatively, we could assume that the weak buyer can get paid to play the bargaining game by the seller (or not to play it by the other buyer), but that, if he plays this game, he will do it noncooperatively (so he cannot collude with any of the other parties during the bargaining process). This restriction makes no difference for the case in which reselling is allowed, as the seller is indifferent between having the weak buyer in the market as an independent unit and colluding with him against the strong buyer (i.e., $\varphi(\{1\};(\{1\},\{2\},\{3\}) = \varphi(\{1,2\};(\{1,2\},\{3\}) = \frac{u_3+u_2}{2}$), and the strong buyer is indifferent between having the weak buyer exiting the market and forming a buyer cartel (in both cases the price is $\frac{u_3}{2}$) .The restriction makes a difference if reselling is not allowed (if the weak buyer can get paid to be in the market, but he cannot collude with the seller once he is there, the price will be u_2 or $\frac{u_3}{2}$, whereas if they can act together also during the bargaining process, the price will be $\frac{u_3+u_2}{2}$).

4. Related Literature

The idea that coalitions may form to get a better position in a bargaining process has been posed by Hart and Kurz (1983). They criticize the usual approach to cooperative games, in which a characteristic function assigns a worth to each coalition, coalitions form in order to get this worth and leave the game, and the formation of subcoalitions is inefficient given a superadditive characteristic function. Instead, they consider coalitions that form in order to bargain as a unit with the rest of the players over the division of the value of the grand coalition. The issue here is not efficiency, which is always assumed, but distribution. The worth of a coalition depends now on the whole coalition structure, so that the characteristic function is substituted by a partition function.

Hart and Kurz approach the problem in an axiomatic way, and obtain a value (Owen's value, a variant of the Shapley value for games with a priori unions) for each player in each coalition structure. They do not present a dynamic process according to which coalition structures form, but they specify which ones will be stable, using the notion of strong equilibrium. They conclude that a stable coalition structure may fail to exist.

The present paper takes a strategic approach, using an extension of the Rubinstein (1982) two-person alternating-offer model to n players. The natural extension of that model, in which a protocol determines the proposer and the order in which players accept or reject, with the first rejector becoming the next proposer, is considered by Chatterjee et al. (1993). The extension considered here, in which a proposer is randomly selected every period, has been considered by Okada (1996). Okada's model has desirable properties (no delay of agreement in equilibrium for super-additive games) and yields more intuitive results (see section 3.2.).

Both Chatterjee et al. (1993) and Okada (1996) allow for any coalition to form. In Hart and Mas-Colell (1996) it is assumed that only the grand coalition can form (thus efficiency is guaranteed). However, subcoalitions matter because of the possibility of a partial breakdown: with a certain probability, one of the players is thrown out of the game and the game continues with a smaller set of players. They show that this game implements the Shapley value.

Chatterjee et al. (1993), Okada (1996) and Hart and Mas-Colell (1996) consider games in characteristic function form, that is, games in which the value of a coalition does not depend on what other coalitions form. Strategic models of coalition formation in which a partition function is used are found in Bloch (1996) for symmetric players and fixed payoff division inside the coalitions, and Ray and Vohra (1996) in a general setting (thus players may be asymmetric and payoff division is endogenously determined). Both papers assume a protocol.

The present paper is related to Hart and Kurz (1983) in that it incorporates two stages: coalition formation and bargaining between coalitions. Proposers are randomly selected following Okada. The model shares the use of a partition function with Ray and Vohra (1996), with the difference that proposers are randomly determined every period, so that a player who rejects a proposal cannot be sure that he will become the next proposer. Breakdown may take place as in Hart and Mas-Colell (1996), though the consequences of a breakdown are different.

Gul (1989) considers a bargaining model that implements the Shapley value. Agents own valuable resources that can be combined and produce utility according to a characteristic function, and may buy each other's resources. He points out that his model may be interpreted as a model of coalition formation. Since the utility of a coalition depends on how the remaining resources are partitioned, there are externalities among coalitions. The main difference with the current paper is that Gul assumes random matching.

A market with one seller and two buyers is studied by von Neumann and Morgenstern (1944). They point out that there are many undominated imputations; in some of them the price lies between u_2 and u_3 whereas in others it lies under u_2. An outcome under u_2 is related to the existence of a coalition between the two buyers. The present paper provides a model in which the seller and the weak buyer may form a coalition as well[11].

5. Concluding Remarks

In a game with a seller and two potential buyers, we have found that the weaker buyer can exploit his influence on the price by forming a coalition with one of the other players, and that each two-player coalition forms with equal probability in equilibrium regardless of the values of the parameters. Expected payoffs correspond to the Shapley value of the game, and all coalitions that form in equilibrium are stable in the sense of Hart and Kurz. For one of the equilibria, the players' payoffs conditional on an equilibrium coalition structure converge to the Owen's value[12].

The use of a partition function instead of a characteristic function to describe

[11] The results in this paper differ from the results of von Neumann and Morgenstern. They attribute a price smaller than u_2 to a coalition of the two buyers; this is not always the case in the present paper. If coalition $\{1,3\}$ forms, the price of the good is either $\frac{3u_3+2u_2}{6}$ or $\frac{3u_3-u_2}{6}$; these values may be smaller than u_2 depending on the parameters. Thus the mere possibility of a buyers' coalition may drive the price below u_2.

If coalition $\{1,2\}$ forms, the imputation that arises is dominated according to von Neumann and Morgenstern. For example, if 1 proposes to 2, 2 receives a payoff of $\frac{u_2}{6}$. 1 and 3 could find a division of u_3 that would make them both better-off. However, if 1 would propose to 3 he would have to offer him his continuation payoff; this payoff is such that 1 is indifferent between proposing to 2 and to 3.

Stone (1948) argues that there is nothing the seller can get from a coalition with the weak buyer, unless there is imperfect information about players' valuations for the good. This holds in the reselling case since 1 and 2 together get the same payoff 1 would get if all players were alone (though coalition $\{1,2\}$ may form in equilibrium) but in the no reselling case player 1 may "push up" the price by forming a coalition with player 2, even with perfect information.

[12] The fact that these results do not depend on whether reselling is feasible is not casual: actually the results generalize to a class of three-person bargaining games. This more general case is left for a companion paper.

the coalitions' payoffs is a useful tool to study situations in which a game is played between coalitions (for example, in environmental coalitions, trading blocks or cartels, it is clear that the payoff of a coalition should depend on the whole coalition structure). The use of random proposers instead of a fixed protocol may prove to be a useful tool as well.

A model of coalition formation in which proposers are random and there is an exogenous probability of breakdown can be used to study other bargaining games, like apex games, or other games that can be played among coalitions, like oligopoly games. The study of these games remains a question for further research.

6. Appendix

6.1. Proof of proposition 2.1

The strategy of the proof is as follows: first, the continuation values are found given the strategies described in the proposition; second, given the continuation values, we check that the strategies are indeed an equilibrium, i.e., that none of the players wants to deviate.

a) The continuation value v_1 is found as follows: if player 1 rejects a proposal, with probability δ the game will continue, and each of the players will be selected with probability $\frac{1}{3}$ to be the next proposer. If 1 is selected, he proposes to 3, and thus receives $u_3 - v_3$. If 2 or 3 are selected, they propose to 1, so that he receives his continuation value v_1. With probability $1-\delta$ the game ends and he receives 0. Analogous reasoning can be made for player 2: if he rejects a proposal, with probability $\frac{\delta}{3}$ he becomes the next proposer and receives $\frac{u_3+u_2}{2} - v_1$(taking reselling into account); in all other cases his payoff is zero, either because the game ends or because players 1 and 3 propose to each other. Finally, if player 3 rejects an offer he gets to make the next proposal with probability $\frac{\delta}{3}$, in which case he proposes to 1 and gets $u_3 - v_1$; with probability $\frac{\delta}{3}$ player 1 is selected and offers to 3 his continuation payoff v_3; with probability $\frac{\delta}{3}$, 2 becomes the next proposer, buys the good from 1 and resells it to 3, so that 3 receives an expected payoff of $\frac{u_3-u_2}{2}$; with probability $1-\delta$ the game ends and he receives 0. The continuation payoffs are thus found from the following system of equations:

$$v_1 = \frac{\delta}{3}\left(u_3 - v_3\right) + \frac{2\delta}{3}v_1$$
$$v_2 = \frac{\delta}{3}\left(\frac{u_3+u_2}{2} - v_1\right)$$
$$v_3 = \frac{\delta}{3}\left(u_3 - v_1\right) + \frac{\delta}{3}v_3 + \frac{\delta}{3}\frac{u_3-u_2}{2}$$

The solution of this system is

$v_1 = \frac{\delta\,[(6-5\delta)\,u_3 + \delta\,u_2]}{2\,(d^2 - 9\delta + 9)}$

$v_2 = \frac{\delta\,(1-\delta)\,[(3-2\delta)\,u_3 + 3\,u_2]}{2\,(d^2 - 9\delta + 9)}$

$v_3 = \frac{\delta\,[(9-8\delta)\,u_3 - (3-2\delta)\,u_2]}{2\,(d^2 - 9\delta + 9)}$

As δ tends to 1, v_1 tends to $\frac{u_3 + u_2}{2}$, v_2 tends to 0, and v_3 tends to $\frac{u_3 - u_2}{2}$.

For these strategies to be an equilibrium, it must be the case that none of the players can be better-off by deviating from his prescribed strategy. First, it is clear that player i should accept any offer that gives him an expected payoff of at least v_i. Second, it can be easily checked that the payoff a player gets when he makes the prescribed proposal is higher than his continuation payoff, thus no player has an incentive to deviate to make unacceptable proposals. It remains to be checked that players can not be better-off by making (acceptable) proposals different from the prescribed ones. Because proposing a global agreement is dominated for players 1 and 3, we only have to check two possible deviations: player 1 proposing to player 2, and player 2 proposing a global agreement. We prove now that neither deviation is profitable.

Suppose that player 1 is selected to be the proposer. If he sticks to his prescribed strategy, he gets $u_3 - v_3$, whereas if he proposes to player 2 he gets $\frac{u_3 + u_2}{2} - v_2$. The difference between those expressions equals

$\frac{(1-\delta)\,\left[\left(9 - 6\delta - 2\delta^2\right)u_3 + 3\,(2\delta - 3)\,u_2\right]}{2\left(\delta^2 - 9\delta + 9\right)}$.

This expression is positive for $u_3 \geq \frac{3\,(3-2\delta)}{9 - 6\delta - 2\delta^2}\,u_2 = \Phi(\delta)\,u_2$

Notice that $\Phi'(\delta) > 0$, or the condition is more restrictive for higher values of δ. $\Phi(1) = 3$, thus if $u_3 \geq 3u_2$, the inequality is satisfied for all values of δ.

We conclude that, for $u_3 \geq 3u_2$, player 1 has no incentive to deviate.

Consider now player 2. If he sticks to his prescribed strategy and proposes to player 1, he gets $\frac{u_3 + u_2}{2} - v_1$. If he proposes a global agreement he receives $u_3 - v_1 - v_3$. The difference between these two expressions equals

$\frac{3\,(1-\delta)\,[(3-\delta)\,u_2 - 3\,(1-\delta)\,u_3]}{2\left(\delta^2 - 9\delta + 9\right)}$

This expression is positive provided that $u_3 \leq \frac{(3-\delta)}{3\,(1-\delta)}\,u_2$ (and this is indeed the case for δ close enough to 1).

b) Suppose player 1 offers the good to player 2 with probability λ and to player 3 with probability 1 - λ, whereas players 2 and 3 always propose to player 1. The continuation payoffs given these strategies are given by the following system of equations (the last one being an indifference condition for player 1)

$v_1 = \frac{\delta}{3}(u_3 - v_3) + \frac{2\delta}{3}v_1$

$v_2 = \frac{\delta}{3}\left(\frac{u_3 + u_2}{2} - v_1\right) + \frac{\delta}{3}\ \lambda\ v_2$
$v_3 = \frac{\delta}{3}\ (u_3 - v_1) + \frac{\delta}{3}\ (1 - \lambda)\ v_3 + \frac{\delta}{3}(1 + \lambda)\frac{u_3 - u_2}{2}$
$u_3 - v_3 = \frac{u_3 + u_2}{2} - v_2$
The solution of this system is
$v_1 = \frac{\delta\ [(3 - 2\delta)\ u_3 + u_2]}{2\ (6 - 5\delta)}$
$v_2 = \frac{(1-\delta)\ [\ (4\delta - 3)\ u_3 + 3\ u_2]}{2\ (6 - 5\delta)}$
$v_3 = \frac{\left(3 + 2\delta - 4\delta^2\right) u_3 - (3 - 2\delta)\ u_2}{2\ (6 - 5\delta)}$
$\lambda = \frac{(9 - 6\delta)\ u_2 - \left(9 - 2\delta^2 - 6\delta\right) u_3}{\delta\ [\ 3u_2 + (4\delta - 3)\ u_3\ ]}$

Notice that λ increases with δ. For $\delta = 1$, $\lambda = \frac{3u_2 - u_3}{3u_2 + u_3}$, thus we need $3u_2 > u_3$ to keep λ nonnegative.

The limit of the continuation payoffs when δ tends to 1 is the same as in the previous case: v_1 tends to $\frac{u_3 + u_2}{2}$, v_2 tends to 0, and v_3 tends to $\frac{u_3 - u_2}{2}$.

For the limit case $u_2 = u_3$ (identical buyers), λ equals $\frac{1}{2}$ and $v_2 = v_3 = \frac{2\delta\ (1 - \delta)\ u_3}{(6 - 5\delta)}$, whereas

$v_1 = \frac{\delta\ (2 - \delta)\ u_3}{(6 - 5\delta)}$ (thus in the limit when δ tends to 1, the seller gets all the surplus).

It can be easily checked that none of the players has an incentive to deviate to making unacceptable proposals. We check now deviations to acceptable ones: this reduces to check that player 2 has no incentives to deviate, since 1 is by definition indifferent between proposing to 2 and to 3. If 2 sticks to his prescribed strategy, his payoff as a proposer is $\frac{u_3 + u_2}{2} - v_1$; alternatively, he can propose a global agreement and get $u_3 - v_1 - v_3$. The difference between these two payoffs is $\frac{(1 - \delta)\ [\ (4\delta - 3)\ u_3 + 3u_2]}{2\ (6 - 5\delta)} \geq 0$, thus player 2 has no incentive to deviate from his strategy.

To prove that the equilibrium is unique, one has to check that none of the (eight) other possible strategy combinations constitutes an equilibrium. This is tedious but straightforward.

Finally, one should prove that there is no stationary perfect equilibrium in which (at least) one player makes unacceptable proposals. This can be proved by contradiction:

First we prove that, should player 2 buy the good, he will resell it without delay to player 3.

Suppose at least one player makes unacceptable offers. If only player 2 makes unacceptable offers, player 3's continuation payoff can be written as follows:

$w_3 = \frac{\delta}{2}(u_3 - w_2) + \frac{\delta}{2}w_3$

Because $\delta < 1$, either $u_3 - w_2 = w_3 = 0$ (and this is impossible since w_2 can not exceed δu_3) or $u_3 - w_2 > w_3$, that is, $u_3 - w_3 > w_2$, and player 2 could be better-off by offering the price $u_3 - w_3$.

If player 3 makes unacceptable proposals, his continuation payoff is (regardless of whether 2's proposals are acceptable)

$w_3 = \frac{\delta}{2}w_3 + \frac{\delta}{2}w_3$

Because $\delta < 1$,this is only possible if $w_3 = 0$, but then player 3 could offer a price of δu_3 to player 2 and be better-off.

Now we prove that there is no delay in the bargaining among the three players. Suppose there is a stationary perfect equilibrium in which a player $i \in N$ makes an unacceptable proposal. We will distinguish two cases depending on the other players' strategies: either player i *receives* some acceptable proposal in equilibrium, or he does not.

If he does not and $i = 1$ or 2, his expected payoff equals zero. Then player i has a profitable deviation: he can propose a global agreement and offer each of the two players his continuation payoff: because the sum of their expected payoffs is (strictly, because of delay) smaller than u_3,the sum of their continuation payoffs is smaller than δu_3, so that player i would get at least $(1-\delta)u_3 > 0$. If $i = 3$, $v_3 \leq \frac{\delta}{3}\frac{u_3 - u_2}{2} < \frac{u_3 - u_2}{2}$. Then this strategy combination can not be an equilibrium: by assumption, player 1 is not proposing to player 3; however, by doing so he can get more than $\frac{u_3 + u_2}{2}$, more than what he can get by either proposing to 2 or making unacceptable proposals.

If the strategies of the other players are such that at least one player $j \in N$ makes an acceptable proposal to which i is a responder, call k the value to be divided between i and j (for example, if $j = 1$ and $i = 2$, and player 1 proposes a global agreement in equilibrium, k equals $u_3 - v_3$). The payoff for player j when he becomes a proposer is $k - v_i$ (regardless of whether he follows a pure or a mixed strategy). It must be the case that $k - v_i \geq v_j$ (otherwise it would not be in the interest of player j to make such a proposal). If $k - v_i > v_j$, player i would be better-off by offering to j his continuation value (since $k - v_j > v_i$). We argue here that this is indeed the case.

The continuation payoff for player j, v_j, equals δ times a convex combination of $k - v_i, v_j$ itself, and whatever payoff j gets when the other two players reach an agreement without him. For $j = 1$, it can not be the case that the other two players reach an agreement, thus the convex combination only includes $k - v_i$ and v_j, and, since $\delta < 1$, $k - v_i > v_j$. For $j = 2$, the payoff given that 1 and 3 reach an agreement is zero, thus the strict inequality must hold as well. For $j = 3$, the payoff given that 1 and 2 reach an agreement is $\frac{u_3 - u_2}{2}$. Either this value has weight zero (i.e., all agreements include player 3), or $\frac{u_3 - u_2}{2} \leq v_3$. In both cases the strict inequality holds. The reason why $\frac{u_3 - u_2}{2} > v_3$ implies a zero weight is that 1 and 2 have no incentives to reach an agreement without 3: by including player 3 in the agreement they can earn $\frac{u_3 - u_2}{2} - v_3$. ■

6.2. Proof of proposition 2.2

a) The continuation payoffs can be found from the following system of equations

$v_1 = \frac{\delta}{3}\,(u_3 - v_3) + \frac{2\delta}{3}v_1$
$v_2 = \frac{\delta}{3}\,(u_3 - v_1 - v_3)$
$v_3 = \frac{\delta}{3}\,(u_3 - v_1) + \frac{2\delta}{3}v_3$

This implies $v_1 = v_3 = \frac{\delta\, u_3}{3 - \delta}$ and $v_2 = \frac{\delta\,(1 - \delta)\,u_3}{3 - \delta}$

We prove now that both players 1 and 2 stick to these strategies provided that $u_2 \leq \frac{u_3}{2}$.

Player 1 gets $u_3 - v_3$ if he follows his prescribed strategy, whereas he gets $u_2 - v_2$ if he proposes to sell the good to player 2. The difference between the two payoffs is

$\frac{3 - \delta^2 - \delta}{3 - \delta}\, u_3 - u_2 \geq 0$ for $u_3 \geq \frac{3 - \delta}{3 - \delta^2 - \delta}\, u_2 = \Psi(\delta)\, u_2$

Because $\Psi'(\delta) > 0$, the condition is more restrictive for higher values of δ. $\Psi(1) = 2$, thus we need $u_3 \geq 2u_2$ for the inequality to be satisfied for δ arbitrarily close to 1.

Player 2 gets $u_3 - v_1 - v_3$ if he proposes a global agreement, and $u_2 - v_1$ if he buys the good for himself. The difference between these two payoffs is

$\frac{3 - 2\,\delta}{3 - \delta}\, u_3 - u_2 \geq 0$ for $u_3 \geq \frac{3 - \delta}{3 - 2\,\delta}\, u_2$.

Again the inequality is more restrictive for higher values of δ,and it holds for δ arbitrarily close to 1 given that $u_3 \geq 2u_2$.

It is easily checked that no player has an incentive to deviate to making an unacceptable proposal.

b) Suppose player 2 proposes to buy the good from 1 with probability λ, and a global agreement with probability $1 - \lambda$. The continuation payoffs can be found from the following system of equations, the fourth being an indifference condition for player 2:

$v_1 = \frac{\delta}{3}\,(u_3 - v_3) + \frac{2\delta}{3}v_1$
$v_2 = \frac{\delta}{3}\,(u_3 - v_1 - v_3)$
$v_3 = \frac{\delta}{3}\,(u_3 - v_1) + \frac{(2 - \lambda)\,\delta}{3}\, v_3$

$u_3 - v_1 - v_3 = u_2 - v_1 \Rightarrow v_3 = u_3 - u_2$

The solution to this system is

$v_1 = \frac{\delta\, u_2}{3 - 2\delta}$

$v_2 = \frac{\delta\,(1 - \delta)\, u_2}{3 - 2\delta}$

$v_3 = u_3 - u_2$

$\lambda = \frac{3\,(1-\delta)\,[\,(3-\delta)\,u_2 - (3 - 2\delta)\,u_3]}{\delta\,(u_3 - u_2)\,(3 - 2\delta)} \in (0, 1)$ provided that $u_2 < u_3$[13] $< 2u_2$ and δ close enough to 1.

We now check that none of the players has an incentive to deviate from his prescribed strategy. As in the previous cases, no player has an incentive to deviate to unacceptable proposals. Regarding acceptable ones, player 2 is by definition indifferent between proposing to buy the good from 1 and proposing a global agreement. Player 1 can stick to his prescribed strategy and get u_2, or he can propose to sell the good to player 2 and get $u_2 - v_2 \leq u_2$. Thus, we have indeed found an equilibrium.

It can be checked that no other strategies constitute an equilibrium. ■

6.3. Proof of lemma 2.3

Consider, for example, player 1. If he decides to stay alone, players 2 and 3 will form a coalition, so that player 1's expected payoff if he proposes to stay alone equals $\frac{u_3}{2}$; this is also the limit of V_1 when $\delta \to 1$. For this to be an equilibrium, V_2 must equal at least $\frac{u_2}{2}$ (otherwise 1 would have proposed to 2 instead of proposing to stay alone) and V_3 must equal at least $\frac{u_3}{2}$ (or 1 would have proposed to 3). However, V_2 is strictly smaller than $\frac{u_2}{2}$: player 2's payoff as a proposer will be close to $\frac{u_2}{2}$ for a large δ (since his best alternative is to propose to 1); his payoff when 1 is selected as a proposer will be $\frac{u_2}{4}$ (players 2 and 3 form then a coalition and split the gains equally), and his payoff when 3 is selected is zero (since the best alternative for 3 given the continuation payoffs is to propose to 1). Then $V_2 \simeq \frac{\delta}{3}\frac{u_2}{2} + \frac{\delta}{3}\frac{u_2}{4} + \frac{\delta}{3}\,0 + (1-\delta)\,0 < \frac{u_2}{2}$.

It can be proven analogously that neither 2 nor 3 will propose to stay alone in equilibrium. ■

[13] There is a discontinuity of the equilibrium strategies at $u_3 = u_2$. Player 1's strategy changes from a pure strategy to a mixed strategy that places a weight of $\frac{1}{2}$ on each buyer. Continuation payoffs then equal $\frac{2\,\delta\,(1-\delta)\,u_3}{(6 - 5\delta)}$ for players 2 and 3 and $\frac{\delta\,(2-\delta)\,u_3}{(6-5\delta)}$ for player 1, as in the reselling case.

6.4. Proof of lemma 2.4

Notice that it does not make much sense for 1 or 3 to propose the grand coalition: because they can attain u_3 without the consent of player 2, this strategy is weakly dominated and can only be part of an equilibrium if the continuation payoff of player 2 is zero[14]. This will never happen in equilibrium, since player 2 has a positive expected payoff in the event of breakdown. Even if we take the limit of the breakdown payoffs when $\delta \to 1$ for simplicity, player 2's continuation payoff must still be strictly positive: a continuation payoff of zero would imply continuation values for players 1 and 3 of at least $\frac{u_3 + u_2}{2}$ and $\frac{u_3}{2}$ respectively (otherwise 2 could get a positive payoff by proposing to 1 or 3); this would in turn imply that continuation payoffs sum up to at least $u_3 + \frac{u_2}{2}$, but continuation payoffs can not sum up to more than u_3.

If player 2 proposes the grand coalition, it must be the case that $u_3 - V_1 - V_3 \geq \frac{u_3}{2} - V_3$, or $V_1 \leq \frac{u_3}{2}$ (otherwise proposing a buyer cartel would be more profitable than proposing the grand coalition) and $V_3 \leq \frac{u_3 - u_2}{2}$ (otherwise a coalition with the seller only would be more profitable than the grand coalition). If we now consider player 1, he can get at least $\frac{u_3 + u_2}{2}$ when he is selected as a proposer by proposing to player 3. If he is not a proposer, the worst that can happen to him is that the other two players form a coalition, in which case he will get $\frac{u_3}{2}$. Thus $V_1 \geq \frac{\delta}{3}[\frac{u_3 + u_2}{2}] + \frac{2\delta}{3}\frac{u_3}{2} + (1-\delta)\frac{u_3 + u_2}{2} > \frac{u_3}{2}$, a contradiction. ■

6.5. Proof of lemma 2.5

Consider player 1. If he makes unacceptable proposals in equilibrium, he must make them every time he is a proposer because of stationarity (except possibly after some of the other players chooses to stay alone, but we have argued that this can not be part of an equilibrium). The continuation payoff for player 1, V_1, can be written as $\delta\,[\alpha V_1 + (1-\alpha)\,\frac{u_3}{2}] + (1-\delta)\frac{u_3 + u_2}{2}$, where $\alpha \geq \frac{1}{3}$. Thus, $V_1 = \frac{u_3\,(\alpha\delta - 1) + u_2(\delta - 1)}{2(\alpha\delta - 1)}$. The limit of this expression is $\frac{u_3}{2}$, except for $\alpha = 1$, in which case is $\frac{u_3 + u_2}{2}$. Consider first $\alpha < 1$. For player 1's strategy to be a best response, we need $V_2 \geq \frac{u_2}{2}$ (otherwise it would be better for player 1 to propose to 2) and $V_3 \geq \frac{u_3}{2}$ (or it would be better for 1 to propose to 3). The sum of the continuation payoffs for the three players would be then higher or equal than

[14] Since a player can guarantee himself a nonnegative payoff by rejecting all proposals and making unacceptable proposals himself, none of the players can have a negative continuation payoff in equilibrium.

$u_3 + \frac{u_2}{2}$, a contradiction since it can never be greater than u_3. For $\alpha = 1$, V_1 converges to $\frac{u_3+u_2}{2}$. This implies $V_2 > 0$ and $V_3 > \frac{u_3-u_2}{2}$, or 1 would be better-off by proposing to 2 or 3. Because the sum of the continuation payoffs can not exceed u_3, when $\delta \rightarrow 1$ V_2 must converge to 0 and V_3 must converge to $\frac{u_3-u_2}{2}$. The fact that $\alpha = 1$ means that players 2 and 3 never propose coalition $\{2,3\}$, but they make unacceptable proposals or they propose to player 1. However, given the continuation payoffs, this is not a best response for either of them.

It can be proven analogously that neither 2 nor 3 will make unacceptable proposals in equilibrium. ■

6.6. Proof of proposition 2.6

Lemmas 2.3 and 2.4 state that neither the grand coalition nor a singleton coalition can form in a stationary perfect equilibrium. It remains to be proved that no two-player coalition can form with probability 1 in a stationary perfect equilibrium.

Consider coalition $\{1,2\}$ first. Suppose that both players 1 and 2 propose coalition $\{1,2\}$; player 3 has then three alternatives: he can propose coalition $\{1,3\}$, coalition $\{2,3\}$, or he can randomize. Neither of these strategies would yield an equilibrium.

If 3 proposes to 1, the continuation payoffs are (in the limit when δ tends to 1) $V_1 = \frac{u_3+u_2}{2}$, $V_2 = 0$,and $V_3 = \frac{u_3-u_2}{2}$. Player 3 could better propose to 2 and get $\frac{u_3}{2}$ instead of proposing to 1 and get $\frac{u_3-u_2}{2}$ as his strategy prescribes.

If 3 proposes to 2, the continuation payoffs are (in the limit when δ tends to 1) $V_1 = \frac{u_3}{2}$, $V_2 = \frac{u_2}{2}$ and $V_3 = \frac{u_3-u_2}{2}$. Now player 3 would be better off proposing to 1 and getting $\frac{u_3}{2}$ instead of proposing to 2 and getting $\frac{u_3-u_2}{2}$.

If 3 randomizes between $\{1,3\}$ and $\{2,3\}$ continuation payoffs are (in the limit) $V_1 = \frac{5u_3+2u_2}{10}$, $V_2 = \frac{2u_2}{10}$ and $V_3 = \frac{5u_3-4u_2}{10}$. Player 2 would then prefer to propose to 3 and get $\frac{4u_2}{10}$ instead of proposing to 1 and getting $\frac{3u_2}{10}$.

It can be proved analogously that coalitions $\{2,3\}$ and $\{1,3\}$ can not be proposed by both members with probability 1. ■

6.7. Proof of proposition 2.7

The result can be proved as follows: take all possible pure strategy combinations, and show that at least one of the players can profitably deviate. Lemmas 2.3, 2.4 and 2.5 and proposition 2.6 reduce the possible cases to the two "cycles": 1 proposes to 2, 2 to 3 and 3 to 1, or 3 proposes to 2, 2 to 1 and 1 to 3.

In the first case, the continuation values can be found from the following system of equations:

$V_1 = \frac{\delta}{3}\left(\frac{u_3+u_2}{2} - V_2\right) + \frac{\delta}{3}V_1 + \frac{\delta}{3}\frac{u_3}{2} + (1-\delta)\frac{u_3+u_2}{2}$

$V_2 = \frac{\delta}{3}\left(\frac{u_3}{2} - V_3\right) + \frac{\delta}{3}V_2$

$V_3 = \frac{\delta}{3}\left(u_3 - V_1\right) + \frac{\delta}{3}V_3 + \frac{\delta}{3}\frac{u_3-u_2}{2} + (1-\delta)\frac{u_3-u_2}{2}$

The solution of the system of equations above is

$V_1 = \frac{u_3}{2} + \frac{\left(4\delta^2 - 12\delta + 9\right)u_2}{6\left(\delta^2 - 3\delta + 3\right)}$

$V_2 = \frac{\delta\left(3 - 2\delta\right)u_2}{6\left(\delta^2 - 3\delta + 3\right)}$

$V_3 = \frac{u_3}{2} - \frac{\left(2\delta^2 - 9\delta + 9\right)u_2}{6\left(\delta^2 - 3\delta + 3\right)}$

These strategies can not constitute an equilibrium because player 1 has a profitable deviation: if he proposes to player 3 instead of proposing to player 2, his payoff increases by $\frac{\delta\left(1-\delta\right)u_2}{2\left(\delta^2 - 3\delta + 3\right)} > 0$.

For the second alternative, we can set up a similar system of equations and find the continuation values

$V_1 = \frac{u_3}{2} + \frac{\left(4\delta^2 - 12\delta + 9\right)u_2}{6\left(\delta^2 - 3\delta + 3\right)}$

$V_2 = \frac{\delta^2 u_2}{6\left(\delta^2 - 3\delta + 3\right)}$

$V_3 = \frac{u_3}{2} - \frac{\left(5\delta^2 - 12\delta + 9\right)u_2}{6\left(\delta^2 - 3\delta + 3\right)}$

Again player 1 has a profitable deviation: if he would propose to player 2 rather than to player 3, his payoff would increase by $\frac{\delta\left(1-\delta\right)u_2}{2\left(\delta^2 - 3\delta + 3\right)} > 0$. ■

6.8. Proof of proposition 2.8

We first compute the continuation payoffs given the strategies of the players. We will use the following notation for the probabilities

	1	2	3
1		λ	$1-\lambda$
2	μ		$1-\mu$
3	θ	$1-\theta$	

The row indices indicate the proposers and the column indexes indicate the responders. Thus, player 1 proposes to 2 with probability λ and to 3 with probability 1 - λ , player 2 proposes to 1 with probability μ, and so on. Because by assumption players only propose two-player coalitions, entries in all rows sum up to one. Using this notation, the continuation payoffs are then given by

$V_1 = \frac{\delta}{3}(u_3 - v_3) + \frac{\delta(\mu+\theta)}{3} v_1 + \frac{\delta(2-\mu+\theta)}{3}\frac{u_3}{2} + (1-\delta)\frac{u_3+u_2}{2}$

$V_2 = \frac{\delta}{3}\left(\frac{u_3}{2} - v_3\right) + \frac{\delta(1-\theta+\lambda)}{3} v_2$

$V_3 = \frac{\delta}{3}(u_3 - v_1) + \frac{\delta(2-\lambda-\mu)}{3} v_3 + \frac{\delta(\lambda+\mu)}{3}\frac{u_3-u_2}{2} + (1-\delta)\frac{u_3-u_2}{2}$

Because each player is randomizing, he must be indifferent between proposing to each of the other two players. Thus we have to add three indifference conditions:

$\frac{u_3+u_2}{2} - V_2 = u_3 - V_3$

$\frac{u_3+u_2}{2} - V_1 = \frac{u_3}{2} - V_3$

$u_3 - V_1 = \frac{u_3}{2} - V_2$

We have in principle six equations and six unknowns. However, we have only five linearly independent equations (as any two of the three indifference conditions implies the third one). Taking θ as a parameter, the solution of the system is:

$V_1 = \frac{3u_3+u_2}{6}$

$V_2 = \frac{u_2}{6}$

$V_3 = \frac{3u_3-2u_2}{6}$

$\mu = \frac{[\delta(7-\theta)-6]}{\delta}$

$\lambda = \frac{[3-\delta(3-\theta)]}{\delta}$

$0 < \theta < 1$

In the limit when $\delta \to 1, \lambda = \theta = 1-\mu$. This means that each two-player coalition is formed with probability $\frac{1}{3}$[15].

It is easy to check that these strategies constitute an equilibrium. Players 1 and 3 are by construction indifferent between proposing to each of the other two players, and they are worse-off by proposing the grand coalition, as $u_2 > 0$. Player 2 is by construction indifferent between proposing to players 1 and 3. It remains to be checked that player 2 can not profit by proposing the grand coalition.

If player 2 would propose the grand coalition, his payoff would be $u_3 - \frac{3u_3+u_2}{6} - \frac{3u_3-2u_2}{6} = \frac{u_2}{6} < \frac{u_3}{2} - \frac{3u_3-2u_2}{6} = \frac{u_2}{3}$

Remark 9. *Two more equilibria in which not all players are randomizing are possible, namely*

$\mu = 0, \theta = \frac{7\delta-6}{\delta}, \lambda = \frac{4\delta-3}{\delta}$

and

$\theta = 0, \lambda = \frac{3(1-\delta)}{\delta}, \mu = \frac{7\delta-6}{\delta}$

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[15] Take, for example, coalition {1,2}. The probability that this coalition is formed equals $\frac{\lambda+\mu}{3}$, equal to $\frac{1}{3}$ in the limit.

For both equilibria the continuation values of the three players are the same as in the equilibria where all players play mixed strategies. Further, when $\delta \to 1$ we obtain $\lambda = 0 = 1 - \mu$, so that in the limit all two-player coalitions are formed with the same probability.

To prove that the equilibrium payoffs are unique, we start by noting that the condition $V_1 + V_2 + V_3 = u_3$ together with any two indifference conditions implies that equilibrium payoffs must be as in proposition 2.8. We also know from proposition 2.7 that there is no equilibrium in pure strategies. The only strategy combinations that could yield an equilibrium with different payoffs are those in which only one player randomizes. Lemmas 2.3 and 2.4 reduce the number of strategy combinations to twelve (as in equilibrium players propose only two-player coalitions), and proposition 2.6 reduces it further to nine. It can be proved that none of these nine strategy combinations constitute an equilibrium.

Since any two indifference conditions imply the third one, strategy combinations in which two players randomize do not have to be considered separately. Thus, we have exhausted all candidate equilibria. We can then conclude that equilibrium payoffs are unique and that in any equilibrium each two-player coalition is formed with probability $\frac{1}{3}$ (in the limit when $\delta \to 1$). ■

6.9. Proof of proposition 2.9

This proof is analogous to the proof of proposition 2.8., and is therefore omitted.

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