On dominance solvable games and equilibrium selection theories

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1 Introduction

Ever since it was discussed in LUCE AND RAIFFA (1957) (see especially p. 100), the iterated elimination of dominated strategies has been advanced as a necessary requirement for 'rational' play. Accordingly, many expositions of game theory begin with a discussion of dominated strategies and appeal to equilibrium notions only when the procedure of iterated deletion of dominated strategies has proved to be incapable of reducing the game to a single strategy combination. Examples of textbooks with this property are KREPS (1990), MOULIN (1982) and MYERSON (1990)). All these authors note the differences between iterative elimination of strictly dominated strategies and the iterative elimination of strategies that are only weakly dominated. There seems to be a consensus that

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1 Introduction

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Recently, KOHLBERG AND MERTENS (1986) have strongly argued in favor of the iterated dominance requirement in its strong form, i.e. that iteratively eliminating weakly dominated strategies should not affect the game's outcome. On p. 1014 of their paper they write

> 'One might argue that, since dominated strategies are never actually chosen, and since all players know this, then deletion of such strategies can have no impact on strategic stability. This would lead to requiring that a strategically stable equilibrium remain so when a dominated strategy is deleted (and hence, when the deletion is done iteratively).'

They go on to note that unfortunately, in this strong form this requirement is incompatible with existence, because of the fact that the outcome of the process may depend on the order in which the strategies are eliminated. Therefore, Kohlberg and Mertens only ask for inclusion. Specifically they list as one of their main requirements for strategic stability

> *Iterated-Dominance: A solution of a game G contains a solution of any game G' obtained by G by deletion of a dominated strategy.*[2]

This requirement implies that a solution should not vanish entirely when a dominated

[1] COOPER et.al. (1990) provide evidence that strictly dominated strategies may influence the behavior of subjects that participate in games that are played in the laboratory.

[2] Kohlberg and Mertens use the term 'dominated' in the meaning of 'weakly dominated' and I'll use the same convention in the remainder of the paper. Note that every Nash equilibrium survives interated elimination of strictly dominated strategies.

strategy is deleted, hence neither should it vanish when the deletion is done iteratively. Elsewhere (VAN DAMME (1989)) the author has given examples to show that requiring invariance under iterated dominance may produce highly counterintuitive results. The problems are caused by the fact that a strategy s_i of player i may be dominated (and hence eliminated) because of (and only because of) the presence of a strategy s_j of player j that itself is eliminated in a later stage of the procedure. If this situation occurs, then player i eventually concludes that player j will not play s_j, but why then shouldn't he play s_i, and why is it justified that player j eliminates s_i? A similar point has been made in SAMUELSON (1989) where it is also shown that iterated elimination of dominated strategies is not equivalent to the assumption of admissibility being common knowledge. Hence, one may say that there is some disagreement in the profession about whether satisfactory solution concepts should satisfy the iterated dominance requirement.

In this paper, I demonstrate that the theory of equilibrium selection that has been recently proposed in HARSANYI AND SELTEN (1988) prescribes, in some games, solutions that violate the iterated dominance requirement. The demonstration will be by means of three examples that are in no way pathological, specifically, the elimination procedure is very well-behaved, any order of elimination produces the same unique outcome. It is also shown that some modifications of the theory (which are suggested as possible alternatives by the authors) do not lead to different results. As an example of such a modification, let us mention that the conclusion would be unchanged if, in the Harsanyi/Selten theory, one would not insist on the Pareto dominance criterion, but rather would just make use of risk dominance comparisons. I also show that the modified theory that has been proposed in GÜTH AND KALKOFEN (1989) may simimlarly prescribe a solution that does not satisfy the iterated dominance requirement.

The reason why these selection theories yield a solution that does not survive when dominated strategies are iteratively eliminated is that, to find the solution of a game G, one should not apply the theory to G directly but rather to a sequence G^ε of (uniformly) perturbed games and then investigate the limit of this sequence as ε tends to zero. The theories adopt this roundabout way to guarantee that the solution of G is a perfect equilibrium of G. Note that a perfect equilibrium is one in undominated strategies so that the solution prescribed by Harsanyi/Selten or Güth/Kalkofen certainly has not vanished after one round of elimination. However, perturbing the game may convert weak inequalities into strict ones, so that G^ε need not necessarily be dominance solvable when G has this property. Indeed this is the case in the examples discussed below: Perturbing the game, changes some weak equilibria into strict ones. The solution theories pick for the perturbed game G^ε a strict equilibrium of G^ε that converges to an equilibrium of G that is only weak and that vanishes when weakly dominated strategies are deleted iteratively.[3]

2 Dominance and Perturbations

For convenience, attention will be restricted to 2-person normal form games. Let $G = (X_1, X_2, u_1, u_2)$ be such a game, where X_i is a finite set of strategies for player i and $u_i : X_1 \times X_2 \to \mathbf{R}$ is this player's payoff function. S_i denotes the set of mixed strategies of player i and u_i is extended bilinearly from $X_1 \times X_2$ to $S_1 \times S_2$. The strategy s_i is said to be (weakly) *dominated* if there exists some strategy s_i' such that

$$u_i(s_i', s_j) \geq u_i(s_i, s_j) \qquad \text{(all } s_j\text{), and} \qquad u_i(s_i', s_j) > u_i(s_i, s_j) \qquad \text{(some } s_j\text{)} \quad (2.1)$$

G' is said to *result from G by elimination of dominated strategies* if $G' = (X_1', X_2', u_1, u_2)$ with $X_i' \subset X_i$ and each strategy $X_i \backslash X_i'$ is dominated in G. (Note that we do not insist that all dominated strategies are eliminated simultaneously.) The game G is said to be *dominance solvable* if

[3]In this respect it is also worthwhile to draw attention to DEKEL AND FUDENBERG (1987) in which it is pointed out that the iterated dominance requirement relies on the (strong) assumption that players have no doubts about their opponents' payoffs. These authors show that, once such slight doubts exist, all that can be justified is one round of deletion of weakly dominated strategies followed by iterated deletion of strictly dominated strategies. Any perfect equilibrium (hence, in particular the HS solution) survives this elimination procedure.

(i) there exists a sequence $G^0, G^1, \ldots, G^n$ with

(a) G^{k+1} results from G^k by eliminating dominated strategies,

(b) $G^0 = G$,

(c) in G^n each player has exactly one pure strategy, and

(ii) if $G^0, \ldots, G^n$ and $\bar{G}^0, \ldots, \bar{G}^m$ satisfy (i), then $G^n = \bar{G}^m$.

Hence, we require that elimination yields a unique outcome and that all orders of elimination produce the same outcome. If the game G is dominance solvable, this unique outcome will be called the ID-solution of G. (It should be noted that our definition of dominance solvability is more restrictive (and less subject to criticism) than the one proposed in MOULIN (1982).)

We now turn to the concepts of risk dominance and of payoff dominance. Let s, s' be two equilibria of G. We say that s *payoff dominates* s' if

$$u_i(s) > u_i(s') \qquad \text{for} \qquad i = 1,2 \tag{2.2}$$

Assume that s and s' are strict equilibria, i.e. s_i is the unique best reply against s_j ($i \neq j$), and similarly for s'. Clearly, strict equilibria are necessarily in pure strategies. Suppose that the players find themselves in the situation, where it is common knowledge that the solution of G must be either s or s' without knowing which of both is the solution. Risk dominance tries to capture the idea that in this state of confusion players enter a process of expectation formation that may lead them to conclude that one equilibrium is less risky than the other. Let player i assign probability λ to the event that player j chooses s'_j, hence, he attaches probability $1-\lambda$ to the event that j chooses s_j. Then player i will play a best response against the mixed strategy $(1-\lambda)s_j + \lambda s'_j$. Denote the set of pure best responses against this mixed strategy by $B_i(1-\lambda)s_j + \lambda s'_j$. Let us assume that

$$B_i((1-\lambda)s_j + \lambda s_j') \subset \{s_i, s_i'\} \qquad \text{for all } \lambda \in [0,1], \text{ all } i \tag{2.3}$$

and define

$$r_i(s,s') = \max\{\lambda \in [0,1],\ B_i((1-\lambda)s_j + \lambda s_j') = \{s_i\}\} \tag{2.4}$$

Note that, if $r_i(s,s')$ is large, then player i can attach a relatively high probability to player j playing s_j' and still be justified in playing s_i. HARSANYI AND SELTEN (1988) define risk dominance in an elaborate way by using the tracing procedure. It is beyond the scope of this paper to provide that definition here. Fortunately for our purpose an alternative characterization of *risk dominance* is available in the special case where (2.3) is satisfied. We provide this characterization as Proposition 1.

Proposition 1 . *(cf. HARSANYI AND SELTEN (1988, Thm 5.4.2.) If condition (2.3) is satisfied, then s risk dominates s' (written $s \succ s'$) if and only if*

$$r_1(s,s') + r_2(s,s') > 1 \tag{2.5}$$

For our purposes it suffices to take the characterization provided by the proposition as the definition of risk dominance. Note that (2.5) is equivalent to

$$r_1(s,s')r_2(s,s') > r_1(s',s)r_2(s',s) \tag{2.6}$$

a condition that says that, in the game where only s and s' are available, the stability region of the equilibrium s (i.e. the region where this equilibrium is a best response) has a larger area than the stability region of equilibrium s'.

Let m_i be the number of strategies in X_i and let $\varepsilon > 0$ be such that $m_i\varepsilon < 1$. If s_i is a mixed (or pure) strategy of player i, then s_i^ε is the completely mixed strategy defined by

$$s_i^\varepsilon(x_i) = (1 - m_i\varepsilon)s_i(x_i) + \varepsilon \qquad (x_i \in X_i) \tag{2.7}$$

The interpretation of s_i^ε is that, if player i intends to play s_i, but due to mistakes also chooses each pure strategy with probability ε, then player i will actually choose x_i with probability $s_i^\varepsilon(x_i)$. The uniformly perturbed game G^ε is the game $G^\varepsilon = (X_1, X_2, u_1^\varepsilon, u_2^\varepsilon)$ where[4]

$$u_1^\varepsilon(x_1, x_2) = u_i(x_1^\varepsilon, x_2^\varepsilon) \qquad ((x_1, x_2) \in X_1 \times X_2) \tag{2.8}$$

Hence, the perturbed game is an ordinary 2-person normal form game which models the trembles in the players' actions by slightly modifying the payoffs from the original game. We are finally in the position to outline the main steps of the Harsanyi/Selten theory, at least as far as they apply to the examples to be discussed below. (For more detail the reader is referred to the flowchart on p. 222 of HARSANYI AND SELTEN (1988).)

HS1 Perturb the game. The solution $f(G)$ is found as $f(G) = \lim_{\varepsilon \downarrow 0} f(G^\varepsilon)$

HS2 In the perturbed game G^ε, eliminate the inferior pure strategies. (In a 2-person normal form game a pure strategy is inferior iff it is dominated.)

HS3 The initial solution candidates are the strict equilibria of G^ε.

[4]As usual we identify the pure strategy x_i with the mixed strategy that assigns probability 1 to x_i.

HS4 If there is a solution candidate in G^ε, say s, that payoff dominates all others, then $s = f(G^\varepsilon)$.

HS5 If there is a solution candidate, say s, that risk dominates all others, then $s = f(G^\varepsilon)$.

Clearly payoff dominance is a transitive relationship. Risk dominance may be intransitive, however (see the example in Sect. 4). To break such ties Harsanyi and Selten propose

HS6 If the solution candidates of G^ε are s, s' and s'', and we have $s \succ s' \succ s'' \succ s$, then $s = f(G^\varepsilon)$ if for each player i, strategy s_i is the unique best response against $(s_j + s_j' + s_j'')/3$.

3 An Example Using Payoff Dominance

Consider the game G_1 from Figure 1

[insert Figure 1 about here]

G_1 is clearly dominance solvable with only $C = (C_1, C_2)$ being the ID-solution. We will show that the HS-solution of this game, is $B = (B_1, B_2)$. To prove the claim we follow the steps HS1 - HS4 from the previous section. We first compute the perturbed game. Note that, for example

$$\begin{aligned} u_1^\varepsilon(A_1, A_2) = \ & (1-2\varepsilon)\left((1-2\varepsilon)u_1(A_1, A_2) + \varepsilon u_1(A_1, B_2) + \varepsilon u_1(A_1, C_2)\right) \\ & + \varepsilon\left((1-2\varepsilon)u_1(B_1, A_2) + \varepsilon u_1(B_1, B_2) + \varepsilon u_1(B_1, C_2)\right) \\ & + \varepsilon\left((1-2\varepsilon)u_1(C_1, A_2) + \varepsilon u_1(C_1, B_2) + \varepsilon u_1(C_1, C_2)\right). \end{aligned}$$

To simplify notation we will neglect terms of order ε^2, a simplification that is justified since all dominance comparisons in G_1^ε are by means of strict inequalities, hence, they will remain to be satisfied also when the terms with ε^2 are included. Keeping the simplification in mind, the reader may verify that the payoff matrix of G_1^ε is given as in Figure 1ε

[insert Figure 1ε about here]

One sees that A_i is strictly dominated in G_1^ε, hence, HS2 says that we should eliminate A_1 and A_2, since these will not be chosen by intention. However, these actions may be chosen by mistake and the fact that B_i does better against A_j as C_i does implies that $B = (B_1, B_2)$ is a strict equilibrium of G_1^ε. Condition HS3 says that the solution candidates of G_1^ε are B and C and HS4 implies that B is the HS-solution of G_1^ε since B payoff dominates C. By HS1, therefore, B is the HS-solution of G_1.

Note that in the game G_1^ε, the equilibrium C risk dominates the equilibrium B. Namely we have

$$r_1(C,B) = r_2(C,B) = (1-8\varepsilon)/(1-6\varepsilon)$$

so that $r_1(C,B) + r_2(C,B) > 1$ if ε is sufficiently small. Hence, the example may lead the reader to think that the discrepancy between the ID-solution and the HS-solution is caused by the fact that the latter makes use of the Pareto dominance criterion. As Harsanyi and Selten point out there is also some interest in a theory that uses only risk dominance comparisons. In the next section we show that this change in the theory (that is dropping requirement HS4) will not change the negative result obtained in this section. We will denote the solution obtained by applying the HS-postulates from Sect. 2 except HS4 by the HS*-solution.

4 An Example Using Only Risk Dominance

Consider the game G_2 from Figure 2 which is simply the game from Figure 1 enlarged with another strategy D_i. Again this game is dominance solvable with ID-solution $C = (C_1, C_2)$. Neglecting terms of order ε^2, the payoff matrix of G_2^ε is given in Figure 2ε.

[insert Figures 2 and 2ε about here]

In Game G_2^ε, the strategy D_i is strictly dominated, hence, it may be deleted. The equilibria A, B and C are all strict equilibria for $\varepsilon > 0$, ε sufficiently small. Let us investigate the risk dominance relationships between these equilibria. It is easily seen that, for $s, s' \in \{A, B, C\}$ and for each $\varepsilon > 0$, the condition from (2.3) is satisfied. (Note that this condition is no longer satisfied if $\varepsilon = 0$, for example B_i is a best response against a mixture of A_j and C_j.) Let $r_i(s, s')$ be defined as in (2.4) and let $r_i^\varepsilon(s, s')$ be the corresponding quantity for the game G_2^ε. It is also easily seen that, for $s, s' \in \{A, B, C\}$, the map $r_i^\varepsilon(s, s')$ is continuous in ε. Now, for $i = 1, 2$ we have

$$\begin{aligned} r_i(A, B) &= 1 \\ r_i(B, C) &= 1 \\ r_i(C, A) &= 3/4 \end{aligned}$$

so that, for each $\varepsilon > 0$ sufficiently small

$$A \succ B \succ C \succ A,$$

hence, the risk dominance relationship is cyclic. To determine the solution we, therefore, have to turn to HS6 and we obviously find $f(G_2^\varepsilon) = B$. Hence, the HS*-solution of G_2 is equilibrium B.

It is clear that, in the above example, the reason that the HS*-solution vanishes when dominated strategies are eliminated, lies in the fact that condition HS6 does not reflect the intensity of the risk dominance comparison. Once there are intransitivities among three equilibria, HS6 stipulates that players expect each of these equilibria to occur with probability $1/3$ even though the dominance relationship may be very asymmetric. In game G_2^ε one might argue that, since C very strongly risk dominates B, B very strongly risk dominates A and A only is just 'slightly' stronger than C, players will put high prior probability on C, an of course in this case they will come to conclude that they actually should play C. (The unique best response against $p_1 A_i + p_2 B_i + p_3 C_i$ is C_j is p_3 is large enough.) (On pp. 228-229 of their book, Harsanyi and Selten motivate their use of HS6, the reader can verify that the alternative of working with the 'multilateral prior' instead of the 'centroid' would not lead to different results in game G_2^ε.) In the next section we provide another example that does not rely on intransitivities of the risk dominance relation to show that the HS*-solution need not be identical to the ID-solution, hence, making use of intensities of risk dominance cannot eliminate the discrepancy.

5 An Example Not Relying on Intransitivity

The suggestion to make use of the intensity of the risk dominance relation has first been made by GÜTH AND KALKOFEN (1989) in their modification of the HS-theory. There are three main differences between these two theories:

(i) Harsanyi/Selten allow correlated strategies as beliefs whereas Güth/Kalkofen only allow independent mixed strategies.

(ii) Güth/Kalkofen do not give preference to 'primitive' equilibria, they do not insist on HS3. Instead they add a logarithmic term to the game, which they claim, will ensure that the perturbed game has only strict equilibria and, moreover, a finite number of them (See GÜTH AND KALKOFEN (1989, p. 111)).

(iii) Güth/Kalkofen replace the intransitive risk dominance relation by the transitive resistance avoidance relation.

For our purposes, difference (i) is not important since it does not arise in 2-person games. As far as (ii) is concerned we will not follow Güth/Kalkofen since the logarithmic term appears ad hoc and since the functioning of the procedure rests on an unproven conjecture. Hence, we concentrate on (iii). Güth/Kalkofen call the quantity $r_i(s, s')$ defined in (2.4) player i's resistance against s' in view of s and they define the (normalized) resistance of s against s' as

$$r(s, s') = \prod_i \frac{r_i(s, s')}{r_i(s', s)} \tag{5.1}$$

Note that $r(s, s') > 0$ if s and s' are strict equilibria. Güth/Kalkofen propose to look for that equilibrium against which the maximal resistance is minimal. Hence, let $R(s')$ be the vector $< r(s, s') >_{s \neq s'}$ arranged in nonincreasing order, i.e. the highest resistance value first, etc. Güth/Kalkofen propose to replace HS6 by

GK6 In G^ε choose the solution candidate s for which the resistance vector $R(s)$ is lexicographically minimal. (This equilibrium s is called the resistance dominant equilibrium.)

Note from (2.6) that if s risk dominates any other equilibrium, then

$$r(s, s') > 1 > r(s', s) \qquad \text{for all } s' \neq s,$$

hence, the first component of $R(s)$ is smaller than the first component of $R(s')$ if $s' \neq s$. Consequently, resistance dominance is in agreement with risk dominance if there is a unique risk dominant equilibrium, and GK6 can be viewed as a way to break intransitivities of the risk dominance relationship.

Returning to the example from the previous section, we see that we have, as $\varepsilon \to 0$

$$\begin{aligned} R^\varepsilon(A) \to &\quad (9,0) \\ R^\varepsilon(B) \to &\quad (\infty,0) \\ R^\varepsilon(C) \to &\quad (\infty,{}^1/_9) \end{aligned}$$

and we see that, if ε is sufficiently small, $R^\varepsilon(A)$ is lexicographically minimal. Hence, A is resistant dominant in this example. Our next example, however, shows that resistance dominance does not always produce the outcome that remains after all dominated strategies have been iteratively deleted. Specifically we give an example of a game with a risk dominant equilibrium that is not the ID-solution. Hence, we show that a solution theory based on HS1 - HS3 and HS5 does not single out the stable equilibrium for dominance solvable games.

[insert Figure 3 about here]

The game G_3 from Figure 3 is similar to game G_2, the difference being that (B_1, B_2) is not an equilibrium in G_3. Again G_3 is dominance solvable with ID-solution $C = (C_1, C_2)$. The uniformly perturbed game G_3^ε is given in Figure 3ε. One sees that G_3^ε admits two strict equilibria, viz. A and C, and that (2.3) is satisfied for these equilibria. In addition G_3^ε has three (symmetric) equilibria in mixed strategies, viz. players randomize among A and B, or among B and C, or among A, B and C. Because of HS3 only A and C are solution candidates, however. Now we have that, as $\varepsilon \to 0$, $r_i^\varepsilon(A, C) \to {}^3/_4$ for $i = 1, 2$, hence, for ε small enough A risk dominates C. (Hence, A also resistance dominates C.) Therefore, the solution obtained for G_3 by applying HS1 - HS3 and HS5 is A, and this is different from the ID-solution of G_3.

[insert Figure 3ε about here]

6 Conclusion

The above examples have given additional insights into important features of the Harsanyi/Selten solution theory. One way to paraphrase the results is by saying that the order in which the solution steps involved in the Harsanyi/Selten theory are applied matters: If the risk dominance (or the Pareto dominance) criterion is applied directly to the unperturbed game, one finds a different solution in each of the examples above. Harsanyi/Selten use the uniformly perturbed game to guarantee perfectness of the HS solution. What the above examples demonstrate is that the HS solution need not be a proper equilibrium (MYERSON (1978)): In each of the games G_i discussed above only $C = (C_1, C_2)$ is a proper equilibrium. Furthermore, all games given above are symmetric so that the notion of ESS (evolutionary stable strategies, MAYNARD SMITH (1982)) is well-defined for them. In each game only C is an ESS, hence, the HS-solution need not be an ESS. (See VAN DAMME (1987a, Thm 9.3.4.) for the proof that, if s is an ESS, then (s, s) is a proper equilibrium.) As is well-known, evolutionary stability is a local stability notion. On the contrast, iterated dominance is a kind of global relationship. Although in the above examples the ID-solution coincides with the ESS it should, therefore, not come as a surprise that examples can be constructed in which the ID-solution is not an ESS. (NACHBAR (1990, Example 2), VAN DAMME (1987b, Fig. 4).)

An important class of dominance solvable games is the class of normal form games arising from generic extensive forms with perfect information (MOULIN (1979)). The reader should not conclude from the above examples that the HS solution for such extensive form games differs from the subgame perfect equilibrium. The point is that Harsanyi/Selten do not solve a game via its normal form but rather via the so-called standard form. The latter basically is the agent normal form supplemented with information about which agents belong to the same player. Indeed, the Harsanyi/Selten solution of an extensive form game may differ from the solution of the normal form of this game. We conclude with the game from Figure 4 that gives an example where this happens.

[insert Figure 4 about here]

In the extensive form there is a subgame starting with the second decision made of player 1. Using HS1 – HS5 one sees that the solution of this subgame is (r, R_2). Harsanyi/Selten adopt the principle of subgame consistency, implying that player 1 realizes that, if he decides to play the subgame, he is forced to play according to the solution (r, R_2). Player 1, therefore, chooses L_1 and the HS-solution of the extensive form of the game is $(L_1 r, R_2)$. Figure 4ε gives the uniformly perturbed game associated with the normal form. The strategy $R_1 r$ is strictly dominated, hence, it may be eliminated. The strategies $L_1 l$ and $L_1 r$ are duplicates, hence, they may be replaced by their equivalence class L_1. (See HARSANYI AND SELTEN (1988, Fig. 3.29).) The resulting 2×2 game has 2 strict equilibria, viz. (L_1, R_2) and $(R_1 l, L_2)$. Neither of these equilibria payoff dominates the other, but $(R_1 l, L_2)$ risk dominates (L_1, R_2). Hence, the HS-solution of the normal form is $(R_1 l, L_2)$.

[insert Figure 4ε about here]

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	A_2	B_2	C_2
A_1	3, 3	0, 3	0, 0
B_1	3, 0	2, 2	0, 2
C_1	0, 0	2, 0	1, 1

Figure 1: Game G_1

	A_2	B_2	C_2
A_1	$3-9\varepsilon$, $3-9\varepsilon$	7ε, $3-7\varepsilon$	4ε, 9ε
B_1	$3-7\varepsilon$, 7ε	$2-3\varepsilon$, $2-3\varepsilon$	6ε, $2-5\varepsilon$
C_1	9ε, 4ε	$2-5\varepsilon$, 6ε	$1-2\varepsilon$, $1-2\varepsilon$

Figure 1ε: Game G_1^ε

	D_2	A_2	B_2	C_2
D_1	4, 4	0, 4	0, 0	0, 0
A_1	4, 0	3, 3	0, 3	0, 0
B_1	0, 0	3, 0	2, 2	0, 2
C_1	0, 0	0, 0	2, 0	1, 1

Figure 2: Game G_2

	D_2	A_2	B_2	C_2
D_1	$4-20\varepsilon$, $4-20\varepsilon$	10ε, $4-17\varepsilon$	8ε, 13ε	5ε, 11ε
A_1	$4-17\varepsilon$, 10ε	$3-11\varepsilon$, $3-11\varepsilon$	11ε, $3-13\varepsilon$	8ε, 9ε
B_1	13ε, 8ε	$3-13\varepsilon$, 11ε	$2-7\varepsilon$, $2-7\varepsilon$	6ε, $2-9\varepsilon$
C_1	11ε, 5ε	9ε, 8ε	$2-9\varepsilon$, 6ε	$1-4\varepsilon$, $1-4\varepsilon$

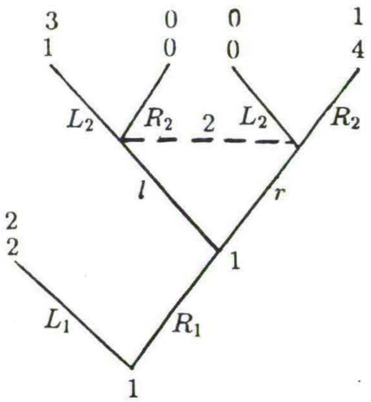
Figure 2ε: Game G_2^ε

	D_2	A_2	B_2	C_2
D_1	4, 4	0, 4	0, 0	0, 0
A_1	4, 0	3, 3	0, 3	0, 0
B_1	0, 0	3, 0	1, 1	0, 2
C_1	0, 0	0, 0	2, 0	1, 1

Figure 3: Game G_3

	D_2	A_2	B_2	C_2
D_1	$4-20\varepsilon$, $4-20\varepsilon$	10ε, $4-17\varepsilon$	7ε, 12ε	5ε, 11ε
A_1	$4-17\varepsilon$, 10ε	$3-11\varepsilon$, $3-11\varepsilon$	10ε, $3-14\varepsilon$	8ε, 9ε
B_1	12ε, 7ε	$3-14\varepsilon$, 10ε	$1-\varepsilon$, $1-\varepsilon$	5ε, $2-10\varepsilon$
C_1	11ε, 5ε	9ε, 8ε	$2-10\varepsilon$, 5ε	$1-4\varepsilon$, $1-4\varepsilon$

Figure 3ε: Game G_3^ε



extensive form

	L_2	R_2
L_1l	2, 2	2, 2
L_1r	2, 2	2, 2
R_1l	3, 1	0, 0
R_1r	0, 0	1, 4

normal form

Figure 4

	L_2	R_2
L_1l	$2-\varepsilon$, $2-3\varepsilon$	$2-3\varepsilon$, 2
L_1r	$2-\varepsilon$, $2-3\varepsilon$	$2-3\varepsilon$, 2
R_1l	$3-8\varepsilon$, 1	7ε, 5ε
R_1r	8ε, 9ε	1, $4+2\varepsilon$

Figure 4ε

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