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# SIMULATION OPTIMIZATION OF BUFFER ALLOCATIONS IN PRODUCTION LINES WITH UNRELIABLE MACHINES

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# Simulation Optimization of Buffer Allocations in Production Lines with Unreliable Machines

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**Abstract:** We use a recent simulation-based optimization method, sample-path optimization, to find optimal buffer allocations in tandem production lines where machines are subject to random breakdowns and repairs. We explore some of the functional properties of throughput of such systems and exploit these properties to prove the almost sure convergence of our optimization technique, under a regularity condition on the steady-state. Utilizing a generalized semi-Markov process (GSMP) representation of the system, we derive recursive expressions to compute one-sided directional derivatives of throughput, from a single simulation run. Finally, we give computational results for lines with up to 50 machines. We also compare results for smaller lines with the results from a more conventional method, stochastic approximation, whenever applicable. In these numerical studies, our method performed quite well on problems that are difficult by current computational standards.

**Key Words:** Stochastic optimization, buffer allocation, tandem manufacturing lines, steady-state throughput, sample-path optimization, generalized semi-Markov processes

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# 1 Problem Description and Related Literature

Finding the optimal buffer allocations continues to be an important problem in the design and analysis of production systems that operate under uncertainty, especially in systems where most of the uncertainty is centered around the availability of machines which are subject to unpredictable and lengthy breakdowns. In capital intensive industries like automotive, even a simple redistribution of total existing buffer capacity may lead to significant savings in spending, see Ho *et al.* (1983) or Wei *et al.* (1989) for example. One of the aims of the work reported here is to enhance the set of available tools for optimizing the performance of tandem production lines, operating under such uncertainty, which can be viewed as the building blocks of more general production systems.

Tandem queues consist of a number of servers in series with buffers of possibly finite sizes between the servers. Jobs start at the first server, pass through each server in sequence, and finally leave the system after being served by the last server. These queues have been widely used as models for several manufacturing and communication systems; see for example Buzacott and Shanthikumar (1992) and Yamashita and Önvural (1994). We focus on a particular tandem queue where service rates are deterministic and the servers are subject to random breakdowns with associated random repair times. It is common to use this type of queues to model tandem production lines in which machines are the servers.

Tandem lines are a class of production lines which are extensively used for mass production of various products. The study of such lines has drawn much attention of engineers and business managers who want to improve an existing line or design a new one. In any case, one is faced with an optimization problem in a complex stochastic system: to optimize the performance of the line under various financial and/or non-financial constraints. Possible decision variables in tandem production lines include buffer capacities, cycle times of machines, and repair rates of machines. Recently, there has been progress towards the optimization of steady-state throughput, the amount of production per unit time by the last machine in steady-state, with respect to machine *cycle times*. Plambeck *et al.* (1996) used sample-path optimization, a simulation-based optimization method, to optimize lines with up to 50 machines under various linear equality and inequality constraints on the cycle times. The aim of the current paper is to go one step further and to optimize the steady-state throughput with respect to *buffer capacities*. Under certain conditions, the existence of a steady-state in tandem queues is guaranteed by regeneration theorems. We do not go into any detail about such conditions; we refer the reader to, for example Loynes (1962), Nummelin (1981), and Gershwin and Schick (1983).

Consider a tandem production line with $m$ processing machines $(M_1, \ldots, M_m)$ connected by $m-1$ buffers $(B_1, \ldots, B_{m-1})$. The material processed may be discrete entities (e.g. assemblies in an automobile factory), in which case we speak of a discrete tandem (DT) line, or it may be fluid-like (e.g. chemical production), in which case we refer to a continuous tandem (CT) line. The time it takes a machine to process one unit of product is called the cycle time. Notice that in a CT line the natural description for processing rate of a machine is the flow rate which is the reciprocal of cycle time. The product, discrete or continuous, enters from one end of the line, goes through each machine in sequence,

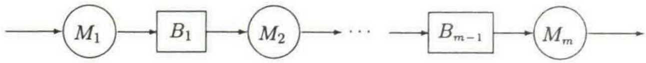


Figure 1: The tandem production line

finally emerges from the other end as a final product. Since many real world systems can be modeled by DT lines, they have received a lot of attention in the literature; see Suri and Fu (1994) and the references therein. One approach to model and analyze DT lines is to approximate them by CT lines, which is also the approach we will employ in this work.

Our basic model has some additional features: There is infinite supply to the first machine and infinite demand from the last machine. There are no transfer delays from machines to buffers, within buffers, or from buffers to machines. A machine can fail only when it is operational. The amount of product processed by each machine between its failures, i.e. the operating quantity between failures for each machine, is a random variable. The repair time for each machine is also a random variable. In the DT line model, cycle times of machines are deterministic and machines are blocked via manufacturing blocking. In the CT line model analogous to this DT line, each machine has a deterministic maximum flow rate, $C_i$; so machine $i$ can work at a rate anywhere between 0 and $C_i$. See Suri and Fu (1994) for a translation of various input parameters and performance measures between these CT and DT lines. It is argued in Suri and Fu (1994) that a natural failure model for CT lines, that are approximations for DT lines, is one in which the next failure of a machine is determined by the *quantity produced* since the last failure (as opposed to being determined by the *time of operation* since the last failure). The failure model we used in our CT line model and simulation is therefore based on the quantity produced by each machine.

A few words about the dynamics of the line are in order: Consider a machine $M_i$. In addition to its own failures, sometimes $M_i$ may have to reduce its production rate or even completely stop because of the interactions with other machines. For example, if the buffer $B_i$ is full, $M_i$ cannot produce at a rate larger than the current rate of $M_{i+1}$; in such a case $M_i$ is said to be *blocked*. Similarly, if the buffer $B_{i-1}$ is empty, $M_i$ cannot produce at a rate larger than the current rate of $M_{i-1}$; in that case $M_i$ is said to be *starved*.

As a result of such interactions between the machines, one would like to increase the buffer capacities to make the machines more independent of each other to increase the throughput. However, due to financial and spatial limitations, increasing the buffer capacities arbitrarily is not feasible in practice. Finding optimal buffer capacities that maximize the performance of the line and yet do not violate the financial/spatial constraints, is still an open question in the study of tandem production lines. Analytical results based on Markov chain representations of the model are available for only 2- and 3-machine DT lines (Gershwin and Schick (1983)) and for 2-machine CT lines (Gershwin and Schick (1980)).

To find the optimal buffer allocation in DT lines, a heuristic method based on a Markov chain representation was used in Hillier and So (1991); since the number of states of the Markov chain grows very rapidly with increasing number of machines and buffer capacities (e.g. a line with four stations and a buffer capacity of three at each station gives rise to a Markov chain with 19,402 states), they could only consider lines with up to 5 machines.

The intractability of analytical models for long production lines makes simulation an attractive approach. A method to estimate the sensitivity of line throughput with respect to buffer capacities in DT lines was introduced in Ho *et al.* (1979), and these gradient estimates were then used in a heuristic "hill climbing" algorithm to find the optimal buffer allocation. As for CT lines, an algorithm based on generalized Benders' decomposition was developed to optimize steady-state throughput and in-process inventory with respect to buffer capacities in Caramanis (1987). To compute the necessary gradients, the approach of Ho *et al.* (1979) was adopted for CT lines; but there was no justification for using a deterministic optimization technique with noisy function and gradient values to solve a stochastic optimization problem.

There are many trade-offs, arising from the complex dynamics of the system, and people who study these lines usually focus on two common performance measures: line throughput (the amount of production per unit time) and in-process inventory. Since the throughput is random, typically one is concerned with ***steady-state throughput***. We also focus on that performance measure, since one can put bounds on parts of or on the total in-process inventory, via additional constraints, as illustrated by the numerical examples in §5. We are actually interested in optimizing the steady-state throughput of a tandem line with many machines under possibly several constraints. Because of the lack of analytical results for lines longer than two machines, we use a simulation-based optimization method. Furthermore, as mentioned earlier, we follow the approach of Suri and Fu (1994) in using CT line approximations to model and analyze DT lines. It is proven in Fu (1996) that the continuous production case is the limit of the discrete production case, in a certain sense, as the piece size approaches zero while the production rate remains constant.

There are several reasons why working with CT line approximations is useful. Using CT lines instead of DT lines brings considerable increase in computational efficiency. Extensive numerical results on the substantial time advantage of CT simulations over DT simulations are reported in Suri and Fu (1994). For example, in an extensive study of 192 15-machine lines, in 2/3 of the 192 cases the ratio of DT simulation time to CT simulation time was at least 10, whereas in 10 of the cases the ratio was more than 80. Furthermore, extensive numerical results on both DT and CT lines in Suri and Fu (1994) indicate that approximation of DT lines via CT lines is quite accurate. For example, for fairly small lines (up to six machines), the throughput values obtained from CT line approximations in Suri and Fu (1994) were very close to the throughput of the original DT line (relative errors ranging from 0.0% to −2.3%); in an extensive study of 192 15-machine lines, in 90% of the cases the difference between the DT line throughput and the equivalent CT line throughput was less than 4%. We decided to approximate the DT lines by CT lines, since CT line simulations are substantially faster than DT line simulations, the approximations are quite accurate, and we are interested in optimizing systems of large size (for which the increase in the computational efficiency is expected to be even higher).

When addressing the buffer allocation problem, using CT line simulations is very beneficial from optimization point of view as well. First, techniques for continuous parameter optimization are much more advanced than those for discrete parameter optimization. Second, dealing with continuous parameters enables us to compute directional derivatives of throughput, using infinitesimal perturbation analysis (IPA), which are valuable for optimization purposes. (In Appendix B, utilizing a GSMP representation developed for the CT lines in Suri and Fu (1994), we derive recursive expressions for directional derivatives of throughput with respect to buffer capacities.)

Clearly, the buffer allocation problem in production lines is one instance of a generic simulation optimization problem: given that one can obtain a function and a gradient value at a parameter setting, locate an optimizer of the performance function. When faced with this problem, people often used some form of the stochastic approximation method; see Robbins and Monro (1951) or the single-run optimization variant Meketon (1987). These methods are known to have a number of drawbacks. First, their empirical performance is very sensitive to the choice of a predetermined step size. Fu and Healy (1992), L'Ecuyer *et al.* (1994), and Glasserman and Tayur (1995) contain a number of examples which demonstrate this sensitivity. Second, since they are mainly first-order gradient methods, they are often thought to experience more difficulties on large problems than on small problems. Third, in case of constrained optimization, these methods handle inequality constraints –even linear inequalities– via projection onto the feasible set. In general, this can retard the performance of an algorithm immensely, as is illustrated by an example in Appendix 6 of Plambeck *et al.* (1996). In that example, such a method requires nearly $10^{43}$ steps to find the minimizer (the origin) of a linear function on the nonnegative orthant $\mathbf{R}^2_+$. Notice that this difficulty does not arise in case of linear equality constraints since one can reduce this to an unconstrained problem by appropriate affine transformations. Finally, if the function being optimized is non-differentiable, then the stochastic approximation method becomes a variant of subgradient optimization; see Correa and Lemaréchal (1993) for example. That method is known to be very slow and it also suffers from other drawbacks such as the lack of a good stopping criterion and the difficulty in enforcing feasibility as mentioned above.

Recently a new method called *sample-path optimization* that overcomes some of these difficulties was proposed in Plambeck *et al.* (1996) and analyzed in Robinson (1996). The method exploits the fact that the performance function we want to optimize is the almost-sure limit of a sequence of approximating functions (outputs of simulations of runs of increasing lengths, all using the same random number streams). That is, if we go out far enough along the sample path we get a good estimate of the limit function. Being a deterministic function, this resulting estimate can then be optimized using deterministic optimization techniques. One of the most powerful features of sample-path optimization is the availability of superlinearly convergent (fast) deterministic optimization methods that can handle constraints explicitly. Using these methods we can often optimize the approximating function to high accuracy in relatively few function and gradient evaluations. This is particularly important when function and gradient evaluations are expensive. The method can be used even when the performance function or the sample functions are non-differentiable (convexity of the functions is required in this case), this time using methods of non-smooth convex minimization, such as bundle algorithms, in the optimization scheme. Another useful feature of this approach is its modularity; the computation of function and gradient values is separated from the optimization. This enables the use of already existing simulation codes (if they also provide gradient values or can be modified to do so) together with optimization codes that call external subroutines for function and gradient evaluations. If the system simulated is large and complex, and the optimization code is sophisticated, then the advantage of modularity becomes more substantial.

Since the optimization problem we are facing is a difficult one with possibly several variables and constraints, we believe that sample-path optimization can be an appropriate technique to solve this problem. Roughly speaking, the method we propose consists of fixing a single sample point (by the method of common random numbers) and a relatively long run (i.e. we do not make multiple runs or "batch", for the purposes of averaging or constructing a distribution) and working with the resulting *deterministic* function. Using the recursive expressions we derive in Appendix B, we compute an *exact* directional derivative of this deterministic function. Therefore our simulation procedure should be viewed as a subroutine providing a function and a gradient evaluation of a deterministic function at a given parameter setting. We then connect this to a standard nonlinear programming solver which requires an external subroutine providing function and gradient values, and solve the problem. Furthermore, by showing certain functional properties of throughput along with a niceness condition, we can verify that a set of sufficient conditions, mentioned in §2, that guarantee almost-sure convergence of our procedure holds. In other words, we show that under a regularity condition on the steady-state, the optimizer computed using the scheme just described, converges almost surely to the correct optimizer as we go far enough on the sample-path.

The remainder of this paper is divided into four main sections. In §2, we discuss the basic ideas behind the sample-path optimization method and cite a set of sufficient conditions that guarantee the almost-sure convergence of the method. In §3, we discuss some functional properties of throughput and show how they can be used to prove that the conditions cited in §2 hold, guaranteeing the convergence of the sample-path optimization method when applied to the buffer allocation problem. In §4, we explain and discuss several issues related to the way we compute directional derivatives of throughput from a single simulation run. In §5, we present some computational results. Finally, §6 contains some concluding remarks. At the end, there are two appendices containing additional technical details. Appendix A has some of the technical results used in §3. Appendix B contains a brief description of the GSMP representation of Suri and Fu (1994) and our derivation of recursive expressions for computing directional derivatives of throughput, based on this GSMP representation.

In our view, the present work makes the following contribution:
- To present a solution approach for a difficult stochastic optimization problem which naturally arises in manufacturing systems. Some recent developments in modeling stochastic systems and optimization via simulation (such as modeling fluid systems using GSMP's and sample-path optimization) are assembled together and applied to a well-studied but not well-solved problem. The approach is proven to be convergent under mild conditions and its empirical effectiveness is demonstrated by succesfully solving some numerical examples

which are considered very difficult by current standards.
- To demonstrate that as a simulation-based optimization method, sample-path optimization, has a number of features that makes it both theoretically and practically a user-friendly and effective method for solving difficult problems, unlike more conventional methods such as stochastic approximation. The conditions for convergence can be verified on non-trivial, difficult problems. Exact gradients simply computed by IPA, even for threshold-type parameters, can be succesfully used in connection with it. It is a numerically robust procedure; several variables and various constraints can be handled easily.
- To show how path-wise functional properties of performance measures can be useful aside from providing qualitative guidelines and insights, and give references to work reported elsewhere, for additional exposition and details regarding properties of throughput in buffer capacities.

Furthermore, the approach we use here is clearly applicable to several other difficult stochastic optimization problems in the manufacturing and operations area. Broadly speaking, the same approach is used in Gürkan and Karaesmen (1997) for finding optimal hedging points of a production flow controller, in the sense of Bielecki and Kumar (1988). There, the fluid version of the problem is considered and the dynamics of the controller are modelled as a GSMP; GSMP framework lets us work with multiple states, and not only with exponential distribution but with fairly general distributions. Utilizing the GSMP representation, the sensitivity estimates of a cost function, consisting of inventory and backlog costs, with respect to hedging points are computed. These sensitivity estimates are then used in connection with sample-path optimization for finding the optimal solution. To this end, we also hope that the present work makes this solution procedure accessible, both theoretically and operationally, to other researchers trying to solve other difficult problems which this approach could be applicable.

## 2 Sample-path optimization method

In this section we describe the basic ideas behind sample-path optimization, a simulation-based method, for optimizing performance functions in certain stochastic systems; we also mention a set of sufficient conditions that guarantee the convergence of the method. We do not go into any technical detail and refer the interested reader to Robinson (1996), which also contains a brief survey of related techniques and ideas similar to sample-path optimization that have appeared in the literature. A comprehensive summary of the properties of the method is given in Gürkan *et al.* (1994) which also reports the performance of the method on a small closed queueing network. An alternative set of conditions to those developed in Robinson (1996) for proving the convergence of the method are provided in Gürkan *et al.* (1996, 1997). This new set of conditions substantially broadens the class of problems to which the method is applicable; in particular it enables the solution of stochastic variational inequalities using the sample-path technique.

Many problems in simulation optimization can be modeled by an extended-real-valued stochastic process $\{L_n(x) \mid n = 1, 2, \dots\}$. The $L_n$ take values that may be real numbers or $\pm\infty$, whereas the parameter $x$ takes values in $\mathbf{R}^k$. For each $n \geq 1$ and each $x \in \mathbf{R}^k$, $L_n(x)$

are random variables defined on a common probability space $(\Omega, \mathcal{F}, P)$. Using extended-real-valued random variables is very convenient for modeling constraints, since one can always set $L_n(x) = +\infty$ for those $x$ that do not satisfy the constraints. In what follows, we use the term "proper" for an extended-real-valued function $f$. It means that $f$ never takes the value $-\infty$ and it is not identically $+\infty$.

The method assumes the existence of a limit function $L_\infty$ such that the $L_n$ almost surely converge pointwise to $L_\infty$ as $n \to \infty$. For many systems, such existence and convergence can often be inferred from regeneration theorems and/or the strong law of large numbers. In the following we refer to $L_n(x)$ as the sample function and we write $L_n(\omega, x)$ when we want to emphasize the dependence of $L_n(x)$ on the sample point $\omega$.

Let us demonstrate this setup with a simple example. Suppose that we are analyzing an $M/M/1$ queue and we are interested in the steady-state system time of a customer, denoted by $L_\infty$. Let $L_n$ be the average of the system times of $n$ customers, i.e. $L_n$ is the output of a simulation of run length $n$ ($n$ service completions in this case). From the regeneration theorems we know that under certain conditions on the parameters of the system $L_\infty$ exists and the $L_n$ converge pointwise to $L_\infty$ along almost every sample path.

We are interested in finding the infimum and, if it exists, a minimizer of $L_\infty$. In general we can only observe $L_n$ for finite $n$. Therefore we approximate minimizers of $L_\infty$ using such information about $L_n$. The method is simple: fix a large $n$ and $\omega \in \Omega$, compute a minimizer $x_n^*(\omega)$ of $L_n(\omega, .)$, and take $x_n^*(\omega)$ as an approximate minimizer of $L_\infty(\omega, .)$. Note that minimizers of $L_\infty(\omega, \cdot)$ may generally depend on the sample point $\omega$. However, in many practical problems for which one would anticipate using this technique $L_\infty$ is a deterministic function, for example a steady-state performance function or an expected value, i.e. it is independent of $\omega$.

As shown in Robinson (1996), the conceptual method of sample-path optimization converges with probability one under three hypotheses: the approximating functions $L_n(\omega, \cdot)$ are lower semicontinuous and proper; they *epiconverge* to the limit function $L_\infty(\omega, \cdot)$; and the limit function $L_\infty(\omega, \cdot)$ almost surely has a nonempty, compact set of minimizers. For a precise statement of this result, see Theorem 3.7 and Proposition 3.8 of Robinson (1996). Note that since numerical methods used in practice find solutions that are approximate, the behavior of the method when $\epsilon$-minimizers (points yielding an objective function value within some positive tolerance $\epsilon$ of the minimum value) are computed is quite important from a practical point of view; results in Section 4 of Robinson (1996), especially Theorem 4.2, show that the behavior of the method remains unchanged in that case.

Since *epiconvergence* is possibly an unfamiliar kind of convergence, we give its definition below.

**Definition 1** *A sequence $L_n$ of extended-real-valued functions defined on $\mathbf{R}^k$* epiconverges *to an extended-real-valued function $L_\infty$ defined on $\mathbf{R}^k$ (written $L_n \xrightarrow{e} L_\infty$) if for each $x \in \mathbf{R}^k$ the following hold:*

*a. For each sequence $\{x_n\}$ converging to $x$, $L_\infty(x) \leq \liminf_{n\to\infty} L_n(x_n)$.*

*b. For some sequence $\{x_n\}$ converging to $x$, $L_\infty(x) \geq \limsup_{n\to\infty} L_n(x_n)$.*

Note that in (b) we actually have $L_\infty(x) = \lim_{n\to\infty} L_n(x_n)$, because of (a).

It is known that epiconvergence is independent of pointwise convergence in the sense that neither implies the other. For a very readable elementary treatment of the relationships between different types of convergence, see Kall (1986). The forthcoming book by Rockafellar and Wets (1997) contains comprehensive treatment of epiconvergence and related issues; we thank the authors of that book for making the extracts of a draft version available to us.

Notice that once we fix $n$ and a sample point $\omega$, $L_n(\omega, x)$ becomes a deterministic function of $x$. With this observation, very powerful methods of constrained and unconstrained deterministic optimization are available to use on $L_n$. In the smooth case we can apply superlinearly convergent methods like the BFGS algorithm (or a variant of it in case of constraints) to minimize $L_n$ to high accuracy in few function and gradient evaluations. For more information on these algorithms see Fletcher (1987) or Gill *et al.* (1981). Use of superlinearly convergent methods enables us to be confident about the location and the accuracy of the minimizer of $L_n$; i.e. we can differentiate between the errors due to the approximation of $L_\infty$ by $L_n$ and those due to the inaccurate computation of a minimizer of $L_n$. With slower algorithms like stochastic approximation this is difficult, if not impossible. If the sample functions and/or the performance function we want to minimize are nondifferentiable and convex, then we can use the Bundle-Trust method; see Kiwiel (1990). We emphasize that in both the smooth and the non-smooth case, the deterministic solution methods available can handle constraints explicitly and without any difficulty.

An unanswered question about this method is how large a sample should be chosen to get a good estimate of the limit function $L_\infty$, and hence to get a good estimate of the solution. When $L_\infty$ is an expectation in a static system and a sample mean construction is used to estimate it, then under certain regularity conditions one can use a certain type of central limit theorem to choose $n$ so as to achieve a good estimate of $L_\infty$; see Rubinstein and Shapiro (1993). In other situations, such as the dynamic setting here, one can solve the problem for increasing values of $n$ and observe the convergence behavior of the solutions. As employed in Plambeck *et al.* (1996) and examples in §5, this approach has produced good results for large problems and complicated systems.

In this section, we summarized the basic ideas behind sample-path optimization and mentioned some of the potential advantages it has over more conventional simulation optimization techniques. We also cited one set of sufficient conditions which guarantee the convergence of the method with probability one. In the next section, we will show how to verify that these conditions hold for a large family of problems.

## 3 Convergence of the method

We focus on minimizing a combination of the reciprocal of steady-state throughput (the amount of production per unit time by the last machine in steady-state) and a cost function. We use this functional form to model a problem where one wants to maximize the throughput but there are costs associated with increasing the buffer capacities. Since the sample-path optimization method can easily handle additional constraints, one can put bounds on parts of or on the total in-process inventory as well (as illustrated by the examples in §5). In this section, we mention some properties of throughput as a function of buffer capacities and discuss their implications on the convergence of sample-path optimization.

We fix a sample path of length $T$ and let $\Theta_T(b)$ be the sample throughput when $b = (b_1, \dots, b_{m-1})$ is the vector of buffer capacities. In Gürkan and Özge (1997), we provide a mathematical framework to model the dependence of throughput on the buffer capacities and the maximum machine flow rates. By exploiting that framework it is possible to prove certain functional properties of throughput. Below, we show how to use these properties together with a regularity condition on the steady-state to verify that the sufficient conditions (mentioned in §2) for the almost-sure convergence of the method hold. This is not common in simulation optimization literature. In a few cases where convexity (of the functions involved) was present, it was possible to verify the assumptions of sample-path optimization or stochastic counterpart method (a closely related technique); see Plambeck *et al.* (1996) and Shapiro and Homem-de-Mello (1996). On the other hand, the assumptions needed to guarantee the convergence of stochastic approximation are usually numerous, tedious, and difficult to validate in practice; see Haurie *et al.* (1994) or Andradóttir (1996) for example. Therefore, in the past people usually had to resort to simulation optimization without much theoretical support and rely on indirect, numeric verification techniques.

In the next result, we deal with the upper semicontinuity of sample throughput. This is important for two reasons: convergence analysis of sample-path optimization for our problem requires upper semicontinuity of sample throughput, and lack of upper semicontinuity in a function to be maximized may cause great difficulties when doing practical optimization.

**Theorem 1** *For $T \in [0, \infty)$, $\Theta_T$ is an upper semicontinuous function of $b$ with probability one.*

*Proof.* See Gürkan and Özge (1997).

As can be seen from Figure 2, $\Theta_T$ for $T \in [0, \infty)$ cannot be lower semicontinuous; see Gürkan and Özge (1997) for a discussion on this. Fortunately, as mentioned in §2 and can be seen in Theorem 3.7 and Proposition 3.8 of Robinson(1996), the upper semicontinuity of $\Theta_T$ suffices to prove the convergence of the conceptual method; the discontinuity of the sample functions does not constitute a problem from the theoretical point of view.

It is also possible to show the monotonicity of throughput in buffer capacities. Results of this nature have appeared in the literature for DT lines, see for example Meester and Shanthikumar (1990). Although the monotonicity of throughput in buffer capacities has been part of the folklore, see for example Ho *et al.* (1983), as far as we are aware a formal proof has not appeared in the literature for CT lines before. Below we use the term "non-decreasing" for a function $f : \mathbf{R}^k \to \mathbf{R}$, by which we mean that $f(x_1, \dots, x_k) \geq f(y_1, \dots, y_k)$ whenever $x_i \geq y_i$ for $i = 1, \dots, k$.

**Theorem 2** *For $T \in [0, \infty]$, $\Theta_T$ is a non-decreasing function of $b$ with probability one.*

*Proof.* See Gürkan and Özge (1997).

This monotonicity result together with a technical lemma about epiconvergence of monotone functions can be used to show the following result.

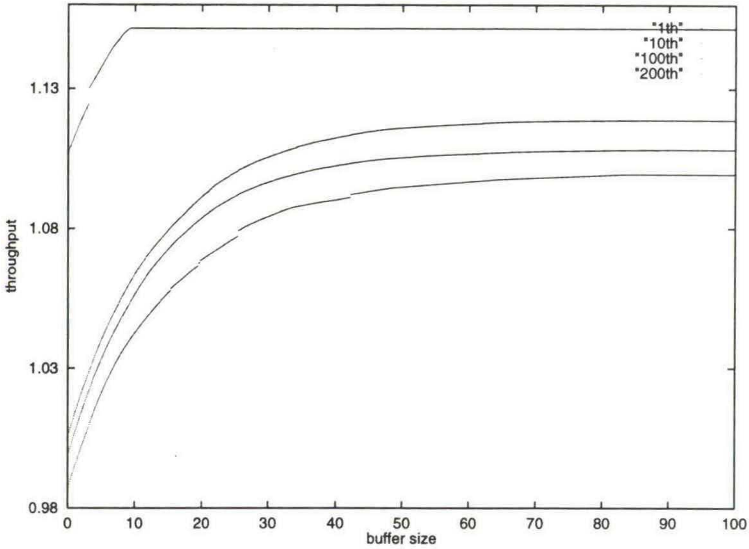


Figure 2: The throughput of a 2-machine CT line for different run lengths.

**Proposition 1** *Assume that with probability one,*

*a.* $\Theta_T \to \Theta_\infty$.

*b.* $\Theta_\infty$ *is upper semicontinuous.*

*Then with probability one,* $1/\Theta_T \xrightarrow{e} 1/\Theta_\infty$.

*Proof.* Use Theorem 2 with Proposition 2 of Appendix A.

Proposition 1 shows that $1/\Theta_T \xrightarrow{e} 1/\Theta_\infty$, provided $\Theta_\infty$ is upper semicontinuous. Intuitively, one even expects it to be continuous: the steady-state throughput of a line should not be very sensitive to arbitrarily small changes in the buffer capacities. In a 2-machine line, the continuity of steady-state throughput is provided by the analytical formula derived in Gershwin and Schick (1980). For longer lines we are not aware of results of this nature, although computational evidence strongly indicates that steady-state throughput is indeed a *continuous* function of buffer capacities, see Figure 2. Figure 2 displays the throughput of a 2-machine CT line, where operating quantities to failures and repair times are exponentially distributed, for different run lengths $T$. In extensive numerical experiments (also for longer lines) we observed the same kind of behavior: a discontinuous function with frequent jumps of large sizes when $T$ is small, a smooth function when $T$ is large.

To verify the convergence conditions, we also need to show that an extended-real-valued function derived from $1/\Theta_T$ (a finite-real-valued function), by restricting it to a nonempty closed set, is proper. We will do this via the next result, Theorem 3. It basically states that (independent of the observation length $T$) the throughput of a system with infinite buffer capacities is bounded above (by the maximum flow rate of the last machine) and a system with no buffer capacities would still produce a positive throughput.

Let $W_p^i$ be the operating quantity between the $(p-1)st$ and the $pth$ failures at $M_i$ and $R_p^i$ be the repair time of $M_i$ after the $pth$ failure. The analysis in this paper does not depend on the particular distributions chosen for the random variables $W_p^i$ and $R_p^i$ (as long as they are not deterministic), except in Theorem 3. This is the only result in which we will make a distributional assumption: We assume that for each $i$ and $p$, random variables $W_p^i$ and $R_p^i$ are exponentially distributed with means $w_i$ and $1/r_i$ respectively, and show that $\Theta_T(0) > 0$ for any $T$, which means that a CT line with no buffer capacity has still positive throughput. We also used exponential distribution in the numerical experiments reported in §5. Essentially any distribution whose support is on $(0, \infty)$ can be used; the choice of the exponential distribution is made for ease of exposition, in both cases.

**Theorem 3** *Assume that for each $i$ and $p$, random variables $W_p^i$ and $R_p^i$ are exponentially distributed with means $w_i$ and $1/r_i$ respectively. Then for $T \in [0, \infty]$, $1/\Theta_T$ is a uniformly bounded and strictly positive function of $b$ with probability one; that is* $0 < 1/C_m \leq 1/\Theta_T(b) \leq 1/\Theta_T(0) < \infty$ *for any $b$.*

*Proof.* $0 < 1/C_m \leq 1/\Theta_T(b) \leq 1/\Theta_T(0)$ is obvious, since the throughput is bounded by $C_m$, the maximum flow rate of the last machine, in any case. We now show that for any $T$, $\Theta_T(0) > 0$.

When $b = 0$, the line operates at the rate of the slowest machine, say $C_{min}$ and it stops (i.e. fails) whenever one of the machines fails. Since there is no buffer between the machines and the product is continuous, this particular $m-$machine line degenerates to a $1-$machine line but with possibly more complicated failure and repair distributions. We have $\Theta_T(0) = Q_T/T$ where $Q_T$ is the amount produced by this $1-$machine line in $[0, T]$. For this equivalent $1-$machine line, define

$X_i$: operating quantity between the $(i-1)st$ and $ith$ failures of the machine,

$Y_i$: repair time after the $ith$ failure.

Observe that the $X_i$ are exponentially distributed random variables with rate $w_1^{-1} + \cdots + w_m^{-1}$ and the probability density function (pdf) of $Y_i$ is given by

$$f(t) = \frac{w_1^{-1}}{w_1^{-1} + \cdots + w_m^{-1}} r_1 \cdot exp(-r_1 t) + \ldots + \frac{w_m^{-1}}{w_1^{-1} + \cdots + w_m^{-1}} r_m \cdot exp(-r_m t),$$

by conditioning on which machine has failed. Since $Q_T \geq \min\{X_1, C_{min}T\}$ and $X_1 > 0$ with probability one, we have $Q_T/T > 0$ with probability one for any finite $T$.

Let $t_0 = 0$ and $t_n =$ time of the $nth$ repair for $n \geq 1$. Then $t_n = \sum_{i=1}^n (C_{min}^{-1} X_i + Y_i)$ and the amount produced at time $t_n$ is $\sum_{i=1}^n X_i$. For any $T \in [t_{n-1}, t_n]$, the ratio $Q_T/T$ is smallest either at $T = t_{n-1}$ or at $T = t_n$ (the quantity produced remains constant between $t_{n-1} + C_{min}^{-1} X_i$ and $t_n$). So

$$\inf_T \frac{Q_T}{T} = \inf_n \frac{\sum_{i=1}^n X_i}{\sum_{i=1}^n (C_{min}^{-1} X_i + Y_i)}.$$

By the strong law of large numbers,

$$\frac{1}{n}\sum_{i=1}^n X_i \to (w_1^{-1} + \cdots + w_m^{-1})^{-1} \text{ and } \frac{1}{n}\sum_{i=1}^n Y_i \to \frac{(r_1 w_1)^{-1} + \cdots + (r_m w_m)^{-1}}{w_1^{-1} + \cdots + w_m^{-1}} \text{ as } n \to \infty.$$

Hence

$$\lim_{n\to\infty} \frac{\sum_{i=1}^n X_i}{\sum_{i=1}^n (C_{min}^{-1} X_i + Y_i)} > 0.$$

So $K = \inf_T Q_T/T > 0$, and we conclude that $\Theta_T(0) \geq K > 0$, for any $T$. ∎

As mentioned earlier, in our computations we minimized a combination of reciprocal of throughput and a cost function subject to a set of constraints. The cost function usually captures the information regarding costs of buffer capacities whereas the constraints specify the admissible allocations in terms of technological, financial, or spatial restrictions. Let $h_T := \alpha/\Theta_T + \beta f$ be the finite-real-valued function that we are interested in minimizing where $\alpha$ and $\beta$ are scalars and $f$ is a cost function of $b$. Let A be a nonempty closed constraint set and define a new family of extended-real-valued functions $\{g_T\}$ by taking $g_T = h_T$ on $A$ and $g_T = +\infty$ off $A$. In Proposition 3 of Appendix A, we show that if $\{h_T\}$ has certain properties, then $\{g_T\}$ also has the same properties. We use this technical result in the proof of our main convergence result, Theorem 4, which shows that the sample-path optimization method converges when applied to the optimization of throughput with respect to buffer capacities.

**Theorem 4** *Suppose that, with probability one, $\Theta_\infty$ is an upper semicontinuous function of $b$ and $f$ is a continuous, non-decreasing, non-negative function that is norm-coercive: i.e., $f(b) \to \infty$ as $||b|| \to \infty$. Let $A$ be a nonempty closed set and $\{f_T\}$ be a sequence of lower semicontinuous cost functions associated with buffer capacities that converge uniformly on compact sets to $f$. Then for sufficiently large $T$ and any positive scalars $\alpha$ and $\beta$, the set of minimizers of $\frac{\alpha}{\Theta_T} + \beta f_T$ on $A$ is nonempty and any point in it is close to some minimizer of $\frac{\alpha}{\Theta_\infty} + \beta f$ on $A$, with probability one.*

*Proof.* The function $\frac{\alpha}{\Theta_\infty} + \beta f$ is lower semicontinuous, and it is norm-coercive because $f$ is norm-coercive and $\frac{\alpha}{\Theta_\infty}$ is bounded below by zero. Thus the set of minimizers of $\frac{\alpha}{\Theta_\infty} + \beta f$ on $\mathbf{R}_+^{m-1}$ is nonempty and bounded; it must also be closed by lower semicontinuity. Use Theorem 1 to see that each $\frac{\alpha}{\Theta_T} + \beta f_T$ is a lower semicontinuous function of $b$. Then apply Theorem 7.44 of Rockafellar and Wets (1997) and Proposition 1 to get $\frac{\alpha}{\Theta_T} + \beta f_T \xrightarrow{e} \frac{\alpha}{\Theta_\infty} + \beta f$. Let $g_T$ be $\frac{\alpha}{\Theta_T} + \beta f_T$ on $A$ and $+\infty$ off $A$. Use Theorem 3 and Proposition 3 of Appendix A to observe that $\{g_T\}$ satisfy the requirements of Theorem 3.7 and Proposition 3.8 of Robinson (1996). ∎

**Remark 1** Although Theorem 4 allows us to work with a sequence of functions $\{f_T\}$, a typical choice would be to use the constant sequence in which $f_T = f := \sum_{i=1}^{m-1} b_i$ for every $T$, as we did in the numerical examples in §5. This functional form models a problem in which one wants to maximize the throughput but there are costs associated with increasing the buffer capacities.

Theorem 4 tells us that, provided a regularity condition on the steady-state of the system holds, if we go out far enough in the sample-path, then each approximate optimization problem will have a solution, and each such solution we compute is close to a true solution of the limit problem. Note that if the approximate minimizer found by the optimization code is a local minimum then it is close to a local minimum of the limit function; if it is a global minimum then it is close to a global minimum of the limit function. We should emphasize that in the absence of convexity of the function being optimized, standard nonlinear optimization codes can only guarantee to find a local minimum which may or may not be a global minimum.

## 4 Gradient Estimation

In the previous section, we have discussed the theoretical issues that arise when using sample-path optimization to find optimal buffer allocations in tandem lines with unreliable machines. We now turn to issues that arise in practical implementation.

Recall that the solution methodology we use consists of fixing a long sample path and optimizing the resulting function using the most powerful deterministic optimization algorithms available to us. Most of these algorithms require that function evaluations at a given parameter setting be supplemented by sensitivity information. Modern simulation technology provides us with gradient estimation techniques such as the likelihood ratio or the score function method (Rubinstein and Shapiro (1993)), and perturbation analysis (Glasserman (1991) and Ho and Cao(1991)); these methods usually enable us to obtain (approximate) gradient evaluations from a single realization of the sample path.

In our computations, we use a simulation code written by Bor-Ruey Fu, based on a generalized semi-Markov process (GSMP) representation of CT lines developed in Suri and Fu (1994); we thank him for making this code available to us. In Appendix B, utilizing that GSMP representation, we derive recursive expressions for the one-sided directional derivatives of throughput with respect to the buffer capacities.

A GSMP can be thought as a mathematical framework which models the evolution of a discrete-event simulation. An excellent description of this framework can be found in Shedler (1993). The basic idea of a GSMP can be explained as follows: There is a set of states and a set of events. The GSMP jumps from one state to another upon the occurrence of an event; at each state there are some active events. At any time, each active event is associated with a clock representing the residual lifetime of that event and a speed at which the clock runs down. If the clock corresponding to event $e$ in state $s$ equals $k$ and the speed at which this clock runs is $r$, then $e$ is scheduled to occur after $k/r$ units of time. The next event and the time until it occurs are always determined by the smallest clock reading/clock speed ratio.

Upon the occurrence of an event, changes may occur in the physical state, clock settings, and clock speeds: If event $e$ occurs in state $s$, the process may move to a new state $s'$ with a certain probability $p(s'; s, e)$; the set of active events changes with the state; clocks for any old events which remain active continue to run in the new state; new clocks are initialized for all new active events and for the event which just occurred if it is also active in the new state. The initial value of each new clock for event $e$ in state $s$ is a random variable with a prespecified cumulative distribution function $F(\cdot; s, e)$. This goes on until a termination criteria is reached. Although, in the past GSMP's were mainly used to model systems with discrete entities, they can also be used to model fluid systems; see Suri and Fu (1994) and Gürkan and Karaesmen (1997).

In Appendix B, we first briefly outline the GSMP representation developed in Suri and Fu (1994) for CT lines. Next, utilizing this GSMP representation, we derive a recursive formula to compute *exact* one-sided directional derivatives of throughput with respect to buffer capacities, in a single simulation run. The resulting infinitesimal perturbation analysis algorithm in the form of a pseudo-code that can easily be incorporated into a simulator can be found in §5.4 of Gürkan (1996).

The estimators we derive are solely based on data from the operation of the system at a single set of parameter values. They are therefore easily computable from a single simulation run or even from real data. We believe that this type of sensitivity information would prove to be useful even if it is not used in connection with a sophisticated optimization algorithm. In the rest of this section, we explain a few key ideas and discuss some issues, related to the estimator we derive in Appendix B.

As mentioned earlier, we use infinitesimal perturbation analysis (IPA) to compute one-sided directional derivatives of throughput, $\Theta(b)$. Since the sample throughput is a discontinuous function, we cannot work with its gradient, instead we work with its one-sided directional derivatives; see for example Rockafellar (1970) for more on one-sided directional derivatives. Let $f : \mathbf{R}^{m-1} \to \mathbf{R}$ be a function which has one-sided directional derivatives at a point $b = (b_1, \ldots, b_{m-1})$ with respect to the positive unit vectors, then IPA computes an array whose $jth$ component is

$$\frac{d^+f}{db_j} = \lim_{\Delta b \downarrow 0} \frac{f(b + \Delta b \cdot y_j) - f(b)}{\Delta b}$$

where $y_j$ is the $jth$ unit vector in $\mathbf{R}^{m-1}$. If in addition $f$ is differentiable at the point $b$, then the array computed is the gradient. In either case, such information would be valuable for optimization purposes. In the following we use $d^+(\cdot)/db_j$ to denote the one-sided directional derivative of $(\cdot)$ at the point $b$ with respect to the vector $y_j$; at points where $(\cdot)$ is differentiable, it should be understood that $d^+(\cdot)/db_j$ stands for the derivative of $(\cdot)$ with respect to $b_j$.

Let the sample path (obtained by fixing $\omega \in \Omega$) of the underlying stochastic system operated at the base point $b$ be called the nominal path; whereas the sample path (with the same fixed $\omega$) of the system operated at the perturbed point $b + \Delta b$ is called the perturbed path. Let $t_n$ be the time of $nth$ event. Two sample paths are said to be *similar* in $[0, t_n]$ if and only if the order of the events is the same for both paths. In other words, two sample paths are similar if the same sequence of states occur in the same order even though the state transition times of these sample paths may be different. "Similarity" of the nominal path and the perturbed path (or sometimes a weaker form of it) is a standard issue one needs to deal with when developing IPA algorithms. Our work is no exception. Let $t_0, t_1, \ldots$ be the event occurrence times in a sample path and $\tau_i = t_i - t_{i-1}$ be the time between the $(i-1)st$ and $ith$ events. We assume that along any sample path of finite length, say $n$ events, with $\min\{\tau_k | k = 1, 2, \ldots, n-1\} > 0$, there is always $\Delta b_j > 0$ (depending on the sample path) small enough such that increasing $b_j$ by $\Delta b_j$ does not cause any event order change; that is the perturbed path and the nominal path are similar. This similarity assumption is crucial for the IPA construction we will employ to compute $d^+\Theta(b)/db_j$. Unfortunately, checking the validity of the similarity condition rigorously for CT lines seems to be fairly burdensome. In Gürkan (1996), p. 80–82, we present a sufficient condition which guarantees the similarity of the perturbed path and the nominal path. Under this condition, it is possible to show that, given any finite $t_n$, there exists with probability one a small positive number $\delta$ (which may depend on the sample path) such that if $0 < \Delta b_j < \delta$ for each $j$, then the perturbed path is similar to the nominal one in $[0, t_n]$. The intuitive idea behind this is that on the sample path of this discrete event dynamic system, any two events are separated by a finite time interval with probability one, provided that the distribution functions of all the random variables involved in the GSMP (operating quantities between failures and repair times of machine $M_i$ for $i = 1, \ldots, m$ in our case) are independent of each other, concentrated on $(0, \infty)$, absolutely continuous, and have finite means. Hence, for a path of finite length we can always arrange to have the accumulated perturbations on the perturbed path be smaller than such finite time intervals; see Gürkan (1996) for a rigorous discussion on this.

The sample throughput can be defined as $\Theta(b) = Q/T(Q)$, where $Q$ is the prespecified volume to be produced by the last machine and $T(Q)$ is the time required to produce $Q$ when $b = (b_1, \ldots, b_{m-1})$ is the vector of buffer capacities. Without loss of generality, we assume $t_0 = 0$, $q_i(t_0) = 0$, and $U_i(t_0) = 0$ for $i = 1, \ldots, m$. If the $nth$ event is the event that the cumulative volume produced by $M_m$ equals $Q$, then $T(Q) = t_n$ and we have,

$$\frac{d^+\Theta(b)}{db_j} = \frac{d^+}{db_j}\left(\frac{Q}{T(Q)}\right) = -\frac{Q}{T^2(Q)}\frac{d^+T(Q)}{db_j} = -\frac{Q}{t_n^2}\frac{d^+t_n}{db_j}. \tag{4.1}$$

From (4.1) we see that $\Theta(b)$ has a one-sided directional derivative with respect to $y_j$ at $b$ if and only if $t_n$ has one. In Appendix B we show that $t_n$ has the desired property and derive a recursive expression for $d^+t_n/db_j$, which is a quantity computable from the simulation information generated up time $t_n$.

A very closely related work is reported in Fu (1996). There an IPA algorithm to compute the partial derivatives of throughput with respect to the flow rates of machines was developed. Some of our results in Appendix B have been adapted from Fu (1996), and this is noted where applicable. In several cases these adaptations involved additional proof and/or extension of the result.

**Remark 2** A GSMP is non-interruptive (in the sense of Schassberger (1976)) if a clock, once set, continues to run until the associated event occurs. The GSMP representation, of Suri and Fu (1994), for the CT line always violates this condition. For example, suppose $M_i$ is failed and $M_{i+1}$ is operational at time $t$, and a buffer empty event, say $\mathcal{B}E_i$, is scheduled to occur at time $t + \Delta t$. Let $v_i(t)$ be the flow rate of $M_i$ at time $t$. If $M_i$ is repaired at time $t'$,where $t' < t + \Delta t$ and $v_{i+1}(t') \leq v_i(t')$, then the event $\mathcal{B}E_i$, which was active at time $t$, is no longer scheduled at time $t'$. In this case $\mathcal{B}E_i$ is interrupted by the repair of $M_i$.

It is worth noting that violation of the non-interruption condition makes the generic IPA gradient estimation algorithm of Glasserman (1991) not directly applicable. It also rules out the applicability of the results, developed in Glasserman and Yao (1992a,1992b), for checking the first and second order properties of stochastic systems that can be modelled as non-interruptive GSMP's. Note that we are not ruling out the possibility of constructing another GSMP representation, for this system, which is non-interruptive or the possibility of modifying some results of Glasserman (1991) or Glasserman and Yao (1992a,1992b) so that they are applicable to interruptive GSMP's. However, both of these approaches would require further investigation which is not the subject of this paper.

**Remark 3** An important problem in gradient estimation literature has been the development of estimators with good asymptotic behavior. The convergence theorems and convergence rate results for stochastic approximation methods are concerned with the unbiasedness of this estimator; for example see Kushner and Clark (1978). An additional advantage of sample-path optimization is that the convergence of the conceptual method is independent of the asymptotic properties of this estimator. As mentioned earlier, when using sample-path optimization the only requirement from the practical point of view is an *exact* gradient or a directional derivative (whichever is available) of the sample function to be optimized. If IPA is applicable to a problem, then the resulting gradient estimator naturally satisfies this requirement.

## 5 Numerical Experiments

We now report the results of three set of numerical experiments which we used to test the empirical performance of sample-path optimization (SPO). In some of these problems, we compared the performance of SPO with the performance of stochastic approximation (SA). Although the single-run optimization (SRO) variant of SA appears to perform better than classical SA for some problems in the literature (Leung (1990)), we used SA in our comparisons. This is due to the fact that applying SRO to the buffer allocation problem would require certain *ad hoc* techniques which we did not want to go into. (For example, one would need to deal with issues such as: if $B_j$ is full at an iteration and the algorithm prescribes to reduce its capacity at the next iteration, should we throw away some of the contents of $B_j$? If so, how should we use such volumes in the calculation of line throughput, if at all? etc.)

In all the experiments conducted, the objective function was of the form

$$\min L_n(b) := c \cdot \Theta_n^{-1}(b) + \sum_{i=1}^{m-1} b_i$$

where $c$ is a scaling constant specific to each problem and $m$ is the number of machines in the line. As mentioned above, this functional form captured the tradeoff of cost (of increasing the buffer capacities) against throughput. In all cases, the operating quantities between failures and the repair times of the machines were exponentially distributed with specified means.

DESCRIPTION FOR SPO

As the optimizer for SPO, we used the deterministic nonlinear optimization code E04UCF of NAG Fortran library, Mark 16 (NAG (1993)). The routine E04UCF is essentially identical to NPSOL (Gill *et al.* (1986). This is a sequential quadratic programming method incorporating an augmented Lagrangian merit function and a BFGS quasi-Newton approximation to the Hessian of the Lagrangian.

The code determined the total number of simulation runs $K$ required to find an approximate minimizer using SPO; this decision was controlled by the "Optimality Tolerance (*OptTol*)". The parameter *OptTol* specifies the accuracy to which the user wishes the final iterate to approximate a solution of the problem. Broadly speaking, *OptTol* indicates the number of correct figures desired in the objective function at the solution. For example, if *OptTol* is $10^{-6}$, the final value of $L_n$ should have approximately six correct figures. When there are only linear constraints, E04UCF considers a point "optimal" if the current step length, the norm of the search direction, and the norm of the projected gradient become sufficiently small. Although we only imposed linear constraints in the numerical results reported here, E04UCF is capable of handling nonlinear constraints as well; in the presence of nonlinear constraints, to be considered "optimal", the point should also satisfy an additional stopping criteria; see NAG (1993) for additional details.

DESCRIPTION FOR SA

For the application of SA to problems with only simple bound constraints on the buffer sizes, we generated a sequence of points according to

$$b^{i+1} = \Pi_\Phi(b^i - \frac{a_0}{i} g^i)$$

where $g^i$ is an estimate of the gradient or the directional derivative (whichever is available) at $b^i$, $a_0$ is the predetermined step size constant, and $\Pi_\Phi(\cdot)$ is the projection onto the feasible set $\Phi$ determined by the bound constraints. It is well known that if one does not impose explicit bounds on the variables to ensure the boundedness of the iterates, the SA algorithm may diverge. Therefore in all the experiments reported here we used simple bound constraints to ensure that the SA algorithm did not suffer from unboundedness problems.

For problems with additional linear equality constraints on the buffer sizes, we took a different approach that exploits this special structure. The idea is to reduce the dimension of the problem via affine transformations and solve an unconstrained problem in a lower dimensional space. Let us demonstrate this approach on the following optimization problem: $\min f(x)$ subject to $Ax = d$ where $x \in \mathbf{R}^n$, $f : \mathbf{R}^n \to \mathbf{R}$, $A$ is an $m \times n$ matrix with full row rank $m$ ($\leq n$), and $d \in \mathbf{R}^m$. Applying QR factorization to $A^T$, we find an orthonormal $n \times n$ matrix $Q$ and an upper triangular $m \times m$ matrix $R$ such that

$$A^T = Q \begin{pmatrix} R \\ 0 \end{pmatrix}.$$

Let $L = R^T$ and partition $Q = [Q_1 \; Q_2]$, where $Q_1$ and $Q_2$ are $n \times m$ and $n \times (n-m)$ matrices respectively. Then we get $A[Q_1 \; Q_2] = [L \; 0]$. It is not difficult to check that any feasible point $x$ can be written as $x = Q_2 z + Q_1 L^{-1} d$ with $z := Q_2^T x \in \mathbf{R}^{n-m}$.

In other words, any $x \in \mathbf{R}^n$ can be transformed to a point in $\mathbf{R}^{n-m}$ (which can be thought as a model for *kernelA*) via $Q_2^T x$. Similarly, any $z \in \mathbf{R}^{n-m}$ can be transformed to a feasible point in $\mathbf{R}^n$ via $Q_2 z + Q_1 L^{-1} d$. Now if we write $\phi(z) = f(Q_2 z + Q_1 L^{-1} d)$, the problem we want to solve becomes

$$\min_{z \in \mathbf{R}^{n-m}} \phi(z).$$

Hence if we impose $m$ independent linear equality constraints on an $n$-dimensional problem, we will effectively solve an unconstrained optimization problem with $n-m$ variables. See for example Gill *et al.* (1981) or Fletcher (1987) for more on these issues.

Therefore when there are only linear equality constraints, we can summarize the steps of the SA algorithm as follows:

*Step 1:* Given $b^i$ and $g^i$, compute $z^i = Q_2^T b^i$ and $d\phi^i = Q_2^T g^i$.

*Step 2:* Compute $z^{i+1} = z^i - \frac{a_0}{i} d\phi^i$.

*Step 3:* Simulate at $b^{i+1} = Q_2 z^{i+1} + Q_1 L^{-1} d$ to get $g^{i+1}$, let $i = i+1$, and go to Step 1.

We could iterate according to this scheme if we did not have to impose the bounds on the variables. To handle this issue we transformed the bounds on $x$, $l \leq x \leq u$, into linear inequality constraints on $z$ as:

$$l - Q_1 L^{-1} d \leq Q_2 z \leq u - Q_1 L^{-1} d. \tag{5.1}$$

We can handle general linear inequality constraints on $x$ using the same idea; $Bx \leq e$ would be transformed to $z-$space as:

$$BQ_2z \leq e - BQ_1L^{-1}d. \tag{5.2}$$

After each step in the reduced space we had to project onto the region where the inequalities (5.1) and (5.2) are satisfied. Note that after the affine transformations, simple bound constraints on variable $x$ become general linear inequality constraints on variable $z$. Therefore, in the presence of either simple bounds or additional linear inequality constraints, to maintain feasibility one has to do a non-trivial projection in the $z-$space.

To solve problems with linear inequality constraints and/or bounds, we need to modify *Step 2* of the above algorithm as follows:

*Step 2':* Compute $z^{i+1} = \Pi_\Psi(z^i - i^{-1}a_0 d\phi^i)$ where $\Pi_\Psi(\cdot)$ is the orthogonal projection onto the feasible set

$$\Psi = \{z \mid l - Q_1L^{-1}d \leq Q_2z \leq u - Q_1L^{-1}d, \quad BQ_2z \leq e - BQ_1L^{-1}d\}.$$

The projection onto a feasible set defined by linear inequalities can be carried out by solving a minimization problem with quadratic objective function and linear inequality constraints. For this purpose we used E04NAF of NAG Fortran library, Mark 16 (NAG (1993)). This routine is essentially identical to SOL/QPSOL described in Gill *et al.* (1986).

EXAMPLE 1

The first set of experiments is with a 3-machine line. We considered two separate cases, a balanced (i.e. symmetric) (Problem 1) and an unbalanced (Problem 2) line, and compared the performance of SPO and SA. Table 1 contains the data for these problems.

In both problems, the bounds $l$ and $u$ had each element equal to zero and 200 respectively. We chose the upper bounds large enough so that they were inactive around the solution. The scaling constant $c$ was $10,000$ for Problem 1 and $5,000$ for Problem 2. We solved each problem from two different initial points and the random number seeds for the simulation were consistent for both methods. Since for small $n$ the resulting functions were discontinuous, we used $n = 2,000,000$ volumetric units per simulation call to get a good estimate of the limit function. To get two decimal digit accuracy in the objective function values, we set *OptTol* to $10^{-6}$ in Problem 1 and to $10^{-5}$ in Problem 2. As mentioned above, the code E04UCF determined $K$, the total number of simulation runs to find an approximate minimizer using SPO. In each case, for the SA algorithm we then made the same number $K$ of simulation runs, each run corresponding to one iteration of the algorithm.

We solved two versions of each problem: (A) only simple bounds, (B) an additional linear equality constraint. The additional linear equality constraints were $b_1 + b_2 = 100$ and $b_1 + b_2 = 40$ for Problems 1 and 2 respectively. For SA, in the presence of simple bound constraints only, we used 140 and 90 as $a_0$ for Problem 1 and 2 respectively; we changed the values of $a_0$ to 1.2 and 2.6 respectively after introducing the equality constraint. The results for the balanced line with only simple bounds and the additional linear equality constraint appear in Table 2 and 3 respectively. Corresponding results for the unbalanced line are summarized in Table 4 and 5.

Table 6 contains the "optimal" solutions, which we found by using SPO with a very large computational effort; 50,000,000 volumetric units per simulation run. This was feasible since these are small lines. In Table 6, $\|Z^T g\|$ represents the norm of the projected gradient which should be approximately zero around the solution. The "Error" column in Table 2–5 is the Euclidean distance between the final point and the optimal solution of the corresponding problem.

A few words about the $a_0$ values used for the results in Table 2-5 are in order. In all four cases, we needed to spend considerable effort to find a good value of $a_0$. This "sensitivity to the initial step size choice" is an important difficulty experienced by most SA-type methods, as our own experience confirmed. Each of the reported $a_0$ is found after trying at least 10 different values, the first few to find the correct magnitude, the rest to fine-tune the value so that correct convergence occurred in specified number of iterations. This was feasible since these were small problems and we knew where the "optimal" solution was. In a study in which one does not have a priori knowledge about the location of the optimizer or in which the problem size is not small, this sensitivity issue would constitute a more serious problem (as we see in the next set of experiments).

EXAMPLE 2

Problem 3, the second set of experiments which we compared the performance of SPO and SA, is with a 15-machine line. This problem is motivated by related research done at Ford Motor Company, see Wei *et al.* (1989). Table 7 contains the specifications of the line. As in the first set of experiments, the vectors $l$ and $u$ had each element equal to zero and 200 respectively. Again, these bounds were large enough so that none of them was active at the optimal solution. We chose $c$ to be $7,000$ and used $n = 1,000,000$ volumetric units per simulation run. We considered two versions of the problem: (A) linear equality constraints, (B) additional linear inequalities and bounds (some of which are active at the optimal solution). The equality constraints used are as follows:

$$b_3 + b_4 + b_5 = 60, \quad b_7 + b_8 + b_9 = 50, \quad b_{11} + b_{12} + b_{13} = 70, \quad b_{12} + b_{13} + b_{14} = 45.$$

Table 8 contains the results for (A). We set *OptTol* to $10^{-5}$ and started from the infeasible point $b = (50, 20, 100, 50, 50, 15, 70, 20, 10, 15, 25, 20.5, 24.5, 0)$ which is projected to the feasible initial point specified in the table. The SPO method converged after 11 iterations. For SA, since this is not a small line (it takes around 12 minutes to compute one function and gradient evaluation when $n = 1,000,000$, with a DEC 5000), it was not feasible to try many values for $a_0$. We tried $1, 5, 10, 20$; the resulting final points are reported. We found the "optimal" solution by SPO using $n = 5,000,000$ volumetric units per simulation run and $10^{-6}$ as *OptTol*. At the "optimal" point the objective function value is 4078.87 and the norm of the projected gradient $\|Z^T g\|$ is 2.0E-1.

For (B), we added the following inequality constraints:

$$b_1 + b_2 \leq 75, \qquad 25 \leq b_3, \qquad b_6 \leq 30, \qquad 40 \leq b_{10}, \qquad 20 \leq b_{12}. \tag{5.3}$$

Table 9 contains the results. We observed convergence to the final point by SPO in 11 iterations. For SA, we tried 7, 10, 12, 15 as $a_0$. We again found the "optimal" solution by using SPO with *OptTol* set to $10^{-6}$ and $n = 5,000,000$; the final objective function value is 4108.80, the norm of the projected gradient $\|Z^T g\|$ is 1.0E-1, the first three inequality constraints are active while all others are inactive at the "optimal" solution. As before, "Error" in Table 8 and 9 is the Euclidean distance between the final point and the optimal solution of the corresponding problem.

EXAMPLE 3

The last set of experiments is with a 50-machine line; as a result of our experience with SA for 15-machine problem, we only applied SPO in this case. Table 10 contains the data of the line. We chose $c$ to be 10,000 and set *OptTol* to $10^{-4}$. We first considered an unconstrained problem, Problem 4A. Due to the size of the line, we took a different approach to solve this problem. We first let $n = 200,000$, started from a vector of 10's, and found an approximate minimizer. Next, we took this point as our initial point and increased the run length to $n = 500,000$. Finally, we increased $n$ to 1,000,000. Table 11 shows the approximate minimizers found each time. Table 12 contains the number of iterations K, the objective function value and the norm of the gradient at each point. We decided to accept the point when $n = 1,000,000$ as "optimal" since the algorithm took only one iteration, the Euclidean distance between it and the previous point was small, and the norm of the gradient was close to zero. In Table 12, "Error" is the Euclidean distance between each point and the final point when $n = 1,000,000$; we also report the function value at each point when $n = 1,000,000$.

We also solved a constrained problem, Problem 4B, with the following constraints:

$$\sum_{i=1}^{10} b_i = 175, \qquad \sum_{i=11}^{25} b_i = 210, \qquad \sum_{i=26}^{30} b_i = 100, \qquad \sum_{i=31}^{40} b_i = 120, \tag{5.4}$$

$$\sum_{i=41}^{49} b_i = 150, \qquad b_1 \geq 10, \qquad b_{10} \leq 10, \qquad \sum_{i=11}^{14} b_i \leq 50, \tag{5.5}$$

$$\sum_{i=17}^{19} b_i \geq 20, \qquad b_{21} \geq 10, \qquad \sum_{i=22}^{24} b_i \leq 90, \qquad b_{33} \geq 10, \qquad \sum_{i=40}^{42} b_i \geq 40,$$
$$\sum_{i=3}^{7} b_i \geq 75, \qquad \sum_{i=34}^{39} b_i \geq 42, \qquad \sum_{i=29}^{32} b_i \geq 100, \qquad b_{45} \geq 10, \qquad \sum_{i=47}^{49} b_i \leq 60. \tag{5.6}$$

We started from the infeasible point $b = (5,5,5,5,5,20,20,20,20,20,15,15,15,15,15, 5,5,5,5,5,25,25,25,25,25,20,20,20,20,20,15,15,15,15,15,5,5,5,5,5,10,10,10,10)$ which is projected to the feasible initial point $b = (10,11.875,11.875,11.875,11.875, 26.875,26.875,26.875,26.875,10,12.5,12.5,12.5,12.5,12.5,2.5,13,4,4,4,24,24,24,24, 11.33,11.33,11.33,46,20,17,17,17,17,17,7,7,7,7,7,35,35,35,35,10,0,0,0,0)$. We repeated exactly the same experiment we did for the unconstrained problem; the corresponding results are shown in Table 13 and 14. The constraints in (5.4–5.6) are active while all others are inactive at the optimal point when $n = 1,000,000$.

Summary of Numerical Experience

The results of our numerical experience can be summarized as follows.

1. Since for small $n$, the sample throughput is a discontinuous function, as illustrated in Figure 2, it is necessary to use fairly long run-lengths to get a good estimate of the steady-state throughput.

2. The cost function appears to be quite flat near the optimum. In other words, near the optimum, the cost does not vary much. As a consequence, though our algorithm does not always terminate at the same buffer capacities, the costs at termination are consistently very close.

3. Although the results reported in Table 2-5 suggest that both methods work equally well on small problems, due to the several trials to find a good value of $a_0$, the computational effort spent in finding each SA solution was at least 10 times the effort spent for the corresponding SPO solution. In larger problems, the issue of finding a "good initial step size" becomes a more important difficulty.

4. In general we found SPO an efficient and robust method to work with: one can start with an initial run length and a desired accuracy, compute an approximate minimizer, and increase the run length and/or the desired accuracy until no considerable improvement in the solution is observed.

## 6 Conclusion

We have presented some results of using sample-path optimization to find optimal buffer allocations in tandem production lines with unreliable machines. We discussed some structural properties of throughput and how these properties together with a niceness condition on the steady-state can be used to prove the convergence of the conceptual method. We also discussed the basic ideas behind developing an IPA algorithm to compute directional derivatives of throughput with respect to buffer capacities. Finally, we have presented the results of some numerical experiments. In these numerical studies, our method performed quite well on problems that are difficult to solve by current computational standards.

Under a niceness condition on the steady-state of the system, the solution procedure we propose is guaranteed to provide accurate solutions for the buffer allocation problem in CT lines. On the other hand, when the main goal is to solve the buffer allocation problem in DT lines, depending on the particular problem of interest, our procedure can still be useful: One can start with the DT line and approximate it by a CT line, for example using the guidelines of Suri and Fu (1994), solve the resulting continuous parameter optimization problem and find the optimal buffer configuration using the procedure outlined here, and translate the results back to the DT line setting again. Alternatively, if the discrete nature of the problem is really important and have to be addressed, then our procedure can be used in connection with a discrete optimization method and speed up the solution procedure to a great extent, for example by providing a good starting point.

Possible directions for future research may include the following.

- In proving the convergence of sample-path optimization, we imposed the upper semi-continuity of $\Theta_\infty$ as a regularity condition. Although extensive numerical experiments we performed indicate that it is indeed a continuous function of buffer capacities, more theoretical work is needed to prove this.
- More experiments may bring more insight and better understanding to the problem and to the capabilities and strengths of the solution methodologies we considered. We hope that our numerical examples demonstrated that these tools can successfully be used to gain more insight about buffer allocation problems.
- It would be interesting to test this scheme of approximating a DT line by a CT line, finding its optimal buffer configuration using sample-path optimization, and translating the resulting CT line parameters back to the DT line setting, on a real-world problem. One can then compare the empirical performance of such a scheme with the performance of other heuristics especially developed for DT lines.
- Although the tandem lines we studied in this work were originally inspired by production/assembly systems, the underlying ideas could be useful in applications involving telecommunication and traffic systems where fluid models are commonly used, or in other difficult optimization problems in manufacturing/operations area. Working with fluid approximations, in the context of simulation optimization, has several advantages: potential gains in computational efficiency, possibility of computing exact directional derivatives of the performance measures of interest (e.g. by utilizing a GSMP representation of the fluid model), and using this sensitivity information in connection with simulation optimization methods.
- The work reported here is another encouraging sign that sample-path optimization will contribute to the solution of difficult problems; the method proves to be especially useful in solving problems with many variables and/or constraints. However, there is still more work to be done on both theoretical and operational issues involved in using the method successfully in optimization of more general production lines and/or other classes of problems.

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**Appendix A: Technical Results for Section 3**

We prove a general result about the epiconvergence of non-increasing functions; it is used in the proof of Proposition 1 in §3.

**Proposition 2** *Assume that with probability one,*

*a.* $L_n \to L_\infty$.

*b.* $L_\infty$ *is lower semicontinuous.*

*c. Each* $L_n (1 \leq n \leq \infty)$ *is a non-increasing function.*

*Then with probability one,* $L_n \xrightarrow{e} L_\infty$.

*Proof.* Construct a set $\Gamma$ of measure zero such that whenever $\omega \notin \Gamma$, $L_n \to L_\infty$ pointwise, $L_\infty$ is lower semicontinuous, and for each $n = 1, \ldots, \infty$, $L_n$ is a non-increasing function. Choose any $\omega \notin \Gamma$ and for brevity omit the sample point $\omega$ from $L_n$ and $L_\infty$.

We first prove that $L_n$ are (almost) equi-lower semicontinuous, i.e for any $x \in \mathbf{R}^n$ and $\epsilon > 0$ there exist a neighborhood $U(x, \epsilon)$ and a number $N(x, \epsilon)$ such that $L_n(y) > L_n(x) - \epsilon$ for each $y \in U(x, \epsilon)$ and $n \geq N(x, \epsilon)$. Fix $x$ and $\epsilon > 0$. Since $L_\infty$ is lower semicontinuous, we can find a $\delta > 0$ satisfying $L_\infty(y) > L_\infty(x) - \epsilon/3$ for $y \in \Pi_{i=1}^m [x_i - \delta, x_i + \delta]$. We also have $L_n(x + \delta) \to L_\infty(x + \delta)$ and $L_n(x) \to L_\infty(x)$ by pointwise convergence where $(x + \delta)$ means $(x_1 + \delta, \ldots, x_m + \delta)$. Hence we can choose $N$ such that $L_n(x + \delta) > L_\infty(x + \delta) - \epsilon/3$ and $L_\infty(x) > L_n(x) - \epsilon/3$ for $n \geq N$. Then for $n \geq N$,

$$L_n(y) \geq L_n(x+\delta) > L_\infty(x+\delta) - \epsilon/3 > L_\infty(x) - \epsilon/3 - \epsilon/3 \geq L_n(x) - \epsilon/3 - 2\epsilon/3 = L_n(x) - \epsilon.$$

Now, $L_n \xrightarrow{e} L_\infty$ follows from Theorem 5 of Kall (1986). ∎

The following result shows that when there is a sequence of finite-real-valued functions with certain properties, these properties are preserved by the extended-real-valued functions constructed by restricting the finite-real-valued functions to a nonempty closed constraint set.

**Proposition 3** *Let* $A$ *be a nonempty closed subset of* $\mathbf{R}^k$, *and let* $\{h_n | 1 \leq n \leq \infty\}$ *be a sequence of finite-real-valued random functions on* $A$. *Assume that with probability one the following hold:*

*a. Each* $h_n$ *is lower semicontinuous and domain of* $h_n$ *(the set on which* $h_n$ *is finite-real-valued) is nonempty.*

*b.* $h_n \xrightarrow{e} h_\infty$.

*For* $1 \leq n \leq \infty$ *define a function* $g_n$ *to be* $h_n$ *on* $A$ *and* $+\infty$ *off* $A$. *Then the family of functions* $g_n$ *almost surely satisfies the following:*

*i) Each* $g_n$ *is lower semicontinuous and proper.*

*ii)* $g_n \xrightarrow{e} g_\infty$.

*Proof.* Properness is immediate. The rest of the proof is similar to Proposition 2.4 of Robinson (1996); we skip it to save space.

## Appendix B: Recursions Based on the GSMP Representation for CT Lines

We first briefly outline the GSMP representation developed in Suri and Fu (1994) to model CT lines. In what follows, when $t$ is the time of an event, take $f(t) = f(t^+)$ for every function $f$ of $t$. We start by defining the variables which correspond to the physical state of the system.

Let $\alpha_i(t)$ be the state of $M_i, i = 1, \ldots, m$, where

$$\alpha_i(t) = \begin{cases} \mathbf{D} & \text{if } M_i \text{ is down (failed)}, \\ \mathbf{O} & \text{if } M_i \text{ is operational at full capacity } C_i, \\ \mathbf{S} & \text{if } M_i \text{ is starved}, \\ \mathbf{B} & \text{if } M_i \text{ is blocked.} \end{cases}$$

Let $v_i(t) \in [0, C_i]$ be the flow rate of $M_i, i = 1, \ldots, m$ and $x_j(t) \in [0, b_j]$ be the level of $B_j, j = 1, \ldots, m-1$ at time $t$.

The set of event types is $E = \{\mathcal{F}_i, \mathcal{R}_i, \mathcal{BF}_j, \mathcal{BE}_j, \mathcal{T}_f \ : i = 1, \ldots, m, j = 1, \ldots, m-1\}$, where

$\mathcal{F}_i$ represents the failure of $M_i, i = 1, \ldots, m$

$\mathcal{R}_i$ represents the repair of $M_i, i = 1, \ldots, m$

$\mathcal{BF}_j$ represents the becoming full of $B_j, j = 1, \ldots, m-1$

$\mathcal{BE}_j$ represents the becoming empty of $B_j, j = 1, \ldots, m-1$

$\mathcal{T}_f$ represents the termination of simulation.

We assume that the simulation is terminated when the last machine produces $Q$ units of product. Now, we can describe the clocks and associated clock speeds. Let $k(e,t)$ be the reading of the clock for event $e$ at time $t$ and $r(e,t)$ be the speed at which that clock runs down. $E(t)$ is the current set of active events, i.e. the set of events with $r(e,t) \neq 0$. Then the next event to occur is given by

$$e^*(t) = argmin\{\frac{k(e,t)}{r(e,t)}|e \in E(t)\}$$

When more than one event satisfies the "argmin" on the RHS, in principle, any consistent tie breaking rule may be used; the authors of Suri and Fu (1994) suggest using the lexicographic order by event name.

Define $W_i(t)$ as the remaining volume to be produced by $M_i$ until failure whenever it is operational, $U_i(t)$ as the remaining time to the repair of $M_i$ whenever it is failed, and $q_i(t)$ as the volume produced by $M_i$ up to time $t$. Recall that $b_j$ was defined to be the capacity of buffer $B_j$, for $j = 1, \ldots, m-1$. Then the clock readings and the clock speeds are defined as in Table 15.

Note that the units for the clock readings are different: $W_i(t)$ is in volume units whereas $U_i(t)$ is in time units. The speed associated with each clock essentially converts these units into standard time units. To understand why these choices of clocks and speeds are appropriate and to see a detailed account of changes that take place upon the occurrences of events in different states, we refer the reader to Suri and Fu (1994).

| $e$ | $\mathcal{F}_i$ | $\mathcal{R}_i$ | $\mathcal{BF}_j$ | $\mathcal{BE}_j$ | $\mathcal{T}_f$ |
|---|---|---|---|---|---|
| $k(e,t)$ | $W_i(t)$ | $U_i(t)$ | $b_j - x_j(t)$ | $x_j(t)$ | $Q - q_m(t)$ |
| $r(e,t)$ | $v_i(t)$ | $I_{\{\alpha_i(t)=\mathbf{D}\}}$ | $[v_j(t) - v_{j+1}(t)]_+$ | $[v_{j+1}(t) - v_j(t)]_+$ | $v_m(t)$ |

Table 1: The clock readings and associated speeds

**Remark 4** If the termination criteria were simulating the CT line for a prespecified amount of time $T$ instead of a prespecified quantity $Q$, we would need to modify $k(e,t)$ and $r(e,t)$ for $e = \mathcal{T}_f$: $k(e,t) = T - t$ and $r(e,t) = 1$ for all $t$. Furthermore, although it may require additional modifications, a non-deterministic termination time or quantity rule can also be handled in this framework, as long as it is based on a *stopping time.*

Next, by utilizing this GSMP representation, we show how to compute *exact* one-sided directional derivatives of sample throughput with respect to buffer capacities, in a single simulation run. Let $t_0, t_1, \ldots$ be the event occurrence times in a sample path and $\tau_i = t_i - t_{i-1}$ be the time between the $(i-1)st$ and $ith$ events. Recall that we assume $t_0 = 0$, $q_i(t_0) = 0$, and $U_i(t_0) = 0$ for $i = 1, \ldots, m$. If the $nth$ event is the event that the cumulative volume produced by $M_m$ equals $Q$, then $T(Q) = t_n$ and we have,

$$\frac{d^+\Theta(b)}{db_j} = \frac{d^+}{db_j}\left(\frac{Q}{T(Q)}\right) = -\frac{Q}{T^2(Q)}\frac{d^+T(Q)}{db_j} = -\frac{Q}{t_n^2}\frac{d^+t_n}{db_j}.$$

In the following we show that $t_n$ has the desired property and derive a recursive expression for $d^+t_n/db_j$, which turns out to be a quantity computable from the simulation information generated up time $t_n$.

For $n = 0, 1, \ldots$ let $e_{n+1}$ be the $(n+1)st$ event; then

$$e_{n+1} = e^*(t_n) = argmin\{\frac{k(e,t_n)}{r(e,t_n)} | e \in E(t_n)\} \text{ and}$$

$$t_{n+1} = t_n + \tau_{n+1} \qquad \text{where} \qquad \tau_{n+1} = \frac{k(e_{n+1},t_n)}{r(e_{n+1},t_n)}.$$

Let $y_j$ be the $jth$ unit vector in $\mathbf{R}^{m-1}$. Here is the main theorem.

**Theorem 5** *For $n \geq 0$, $t_{n+1}$ has a one-sided directional derivative at $b$ with respect to $y_j$ for all $b$ and $j = 1, \ldots, m-1$ given by*

$$\frac{d^+t_{n+1}}{db_j} = \frac{d^+t_n}{db_j} + \frac{1}{r(e_{n+1},t_n)}\frac{d^+k(e_{n+1},t_n)}{db_j}. \tag{6.1}$$

The proof of Theorem 5 is by induction. First we prove a few technical lemmas and then proceed with the proof of the theorem.

**Lemma 1** *For all $n = 0, 1, \ldots$, $r(e_{n+1}, t_n)$ has a one-sided directional derivative at $b$ with respect to $y_j$ for all $b$ and $j = 1, \ldots, m-1$ that is 0.*

*Proof.* Observe that at any event time $t_n$, $v_i(t_n)$ is equal to either one of $C_1, \ldots, C_m$ or to 0. Thus $v_i(t_n)$ is a function of $C_1, \ldots, C_m$, and $t_n$. Of those variables only $t_n$ depends on the buffer capacities. As a consequence of similarity for small enough $\Delta b > 0$, $t_n(b+\Delta b y_j)$ is still the occurrence time of the $n$th event. Hence $v_i(t_n(b+\Delta b y_j))$, the flow rate of $M_i$ at time $t_n(b+\Delta b y_j)$, must be the same as $v_i(t_n(b))$. The same argument applies to $\alpha_i$ to give $\alpha_i(t_n(b+\Delta b y_j)) = \alpha_i(t_n(b))$. The clock speeds of events are functions of either $\alpha_i$ or $v_i$; therefore we must have $r(e_{n+1}, t_n(b+\Delta b y_j)) = r(e_{n+1}, t_n(b))$. So

$$\frac{d^+ r(e_{n+1}, t_n)}{db_j} = \lim_{\Delta b \downarrow 0} \frac{r(e_{n+1}, t_n(b+\Delta b y_j)) - r(e_{n+1}, t_n(b))}{\Delta b} = 0. \blacksquare$$

The next lemma will start the induction.

**Lemma 2** *$t_1, \tau_1, U_i(t_0)$ and $q_i(t_0)$ for $i = 1, \ldots, m$, have one-sided directional derivatives at $b$ with respect to $y_j$ for all $b$ and $j = 1, \ldots, m-1$. These are given by $d^+U_i(t_0)/db_j = d^+q_i(t_0)/db_j = 0$ for all $i$ and $j$, and*

$$\frac{d^+t_1}{db_j} = \frac{d^+\tau_1}{db_j} = \frac{d^+k(e_1, t_0)}{db_j}\frac{1}{r(e_1, t_0)} = \begin{cases} I_{\{i=j\}} \frac{1}{[v_i(t_0) - v_{i+1}(t_0)]^+} & \text{if } e_2 = \mathcal{BF}_i, \\ 0 & \text{otherwise.} \end{cases}$$

*Proof.* Since we start to operate the CT-line at time $t_0 = 0$, we have $q_i(t_0) = 0$ and $U_i(t_0) = 0$ for all $i = 1, \ldots, m$. Obviously $q_i(t_0)$ and $U_i(t_0)$ are independent of $b_j$; hence they are differentiable functions of $b_j$. Since $t_1 = \tau_1 = k(e_1, t_0)/r(e_1, t_0)$, to prove the assertion about $t_1$ and $\tau_1$ it is enough to show the following facts about $k(e_1, t_0)$ and $r(e_1, t_0)$ (the result will then follow from elementary properties of one-sided directional derivatives): (a) $r(e_1, t_0) \neq 0$; (b) $r(e_1, t_0)$ has a one-sided directional derivative at $b$ with respect to $y_j$ for all $b$ and $j = 1, \ldots, m-1$ that is finite; (c) $k(e_1, t_0)$ has a one-sided directional derivative at $b$ with respect to $y_j$ for all $b$ and $j = 1, \ldots, m-1$ that is finite. Since $e_1 \in E(t_0)$, (a) is immediate from the definition of $E(t)$ and (b) follows from Lemma 1. To see (c), observe that $E(t_0) = \{\mathcal{F}_i, \mathcal{BF}_i, \mathcal{T}_f \ : i = 1, \ldots, m\}$. So we have

$$k(e_1, t_0) = \begin{cases} W_1^i & \text{if } e_1 = \mathcal{F}_i, \\ b_i & \text{if } e_1 = \mathcal{BF}_i, \\ Q & \text{if } e_1 = \mathcal{T}_f. \end{cases}$$

Thus $k(e_1, t_0)$ is a differentiable function of $b_j$ for $j = 1, \ldots, m-1$ whose derivative is either 0 or 1. $\blacksquare$

**Remark 5** We have assumed that when we start to observe the CT-line all machines are up. Our results would go through with slight modifications for a different set of initial conditions.

**Lemma 3** [adapted from Proposition 3.4 of Fu (1996)] *Suppose that $U_i(t_{n-1})$ for $i = 1, \ldots, m$ and $\tau_n$ have one-sided directional derivatives at $b$ with respect to $y_j$ for all $b$ and*

$j = 1, \ldots, m-1$. *Then* $U_i(t_n)$ *has a one-sided directional derivative at* $b$ *with respect to* $y_j$ *for all* $b$ *and* $j = 1, \ldots, m-1$ *given by*

$$\frac{d^+U_i(t_n)}{db_j} = \begin{cases} \frac{d^+U_i(t_{n-1})}{db_j} - \frac{d^+\tau_n}{db_j} & \textit{if } \mathcal{R}_i \in E(t_n), \\ 0 & \textit{otherwise.} \end{cases}$$

*Proof.* Recall that

$$U_i(t_n) = \begin{cases} U_i(t_{n-1}) - \tau_n & \text{if } \mathcal{R}_i \in E(t_n), \\ 0 & \text{otherwise.} \end{cases} \tag{6.2}$$

The result follows by taking the one-sided directional derivative of the right-hand side of equation (6.2) at $b$ with respect to $y_j$. ∎

**Lemma 4** [adapted from Proposition 3.2 of Fu (1996)] *Suppose that* $q_i(t_{n-1})$ *for* $i = 1, \ldots, m$ *and* $\tau_n$ *have one-sided directional derivatives at* $b$ *with respect to* $y_j$ *for all* $b$ *and* $j = 1, \ldots, m-1$. *Then* $q_i(t_n)$ *has a one-sided directional derivative at* $b$ *with respect to* $y_j$ *for all* $b$ *and* $j = 1, \ldots, m-1$ *given by*

$$\frac{d^+q_i(t_n)}{db_j} = \frac{d^+q_i(t_{n-1})}{db_j} + v_i(t_{n-1}) \cdot \frac{d^+\tau_n}{db_j}.$$

*Proof.* Since $v_i(t)$ is constant between adjacent events, we have $q_i(t_n) = q_i(t_{n-1}) + v_i(t_{n-1}) \cdot \tau_n$. From the proof of Lemma 1 it is clear that $dv_i(t_{n-1})/db_j$ is 0; hence the result follows. ∎

**Lemma 5** [adapted from Proposition 3.5 of Fu (1996)] *Suppose that* $U_i(t_n)$ *and* $q_i(t_n)$ *for* $i = 1, \ldots, m$ *have one-sided directional derivatives at* $b$ *with respect to* $y_j$ *for all* $b$ *and* $j = 1, \ldots, m-1$. *Then* $k(e_{n+1}, t_n)$ *has a one-sided directional derivative at* $b$ *with respect to* $y_j$ *for all* $b$ *and* $j = 1, \ldots, m-1$ *given by*

$$\frac{d^+k(e_{n+1}, t_n)}{db_j} = \begin{cases} -\frac{d^+q_i(t_n)}{db_j} & \textit{if } e_{n+1} = \mathcal{F}_i, \\ \frac{d^+U_i(t_n)}{db_j} & \textit{if } e_{n+1} = \mathcal{R}_i, \\ I_{\{i=j\}} - \frac{d^+q_i(t_n)}{db_j} + \frac{d^+q_{i+1}(t_n)}{db_j} & \textit{if } e_{n+1} = \mathcal{BF}_i, \\ \frac{d^+q_i(t_n)}{db_j} - \frac{d^+q_{i+1}(t_n)}{db_j} & \textit{if } e_{n+1} = \mathcal{BE}_i, \\ -\frac{d^+q_m(t_n)}{db_j} & \textit{if } e_{n+1} = \mathcal{T}_f. \end{cases} \tag{6.3}$$

*Proof.* Let $W_r^i$ be the random variable denoting the operating volume between the $(r-1)st$ and the $rth$ failures at $M_i$. If the $(n+1)st$ event is the $pth$ failure of $M_i$, i.e. $e_{n+1} = \mathcal{F}_i$, then

$$k(e_{n+1}, t_n) = W_i(t_n) = \sum_{r=1}^{p} W_r^i - q_i(t_n).$$

Hence

$$k(e_{n+1}, t_n) = \begin{cases} \sum_{r=1}^{p} W_r^i - q_i(t_n) & \text{if } e_{n+1} = \mathcal{F}_i, \\ U_i(t_n) & \text{if } e_{n+1} = \mathcal{R}_i, \\ b_i - [q_i(t_n) - q_{i+1}(t_n)] & \text{if } e_{n+1} = \mathcal{BF}_i, \\ q_i(t_n) - q_{i+1}(t_n) & \text{if } e_{n+1} = \mathcal{BE}_i, \\ Q - q_m(t_n) & \text{if } e_{n+1} = \mathcal{T}_f. \end{cases}$$

The result follows immediately. ∎

Now we are ready to prove the main theorem.

*Proof of Theorem 5.* Lemma 2 provides the start of the inductive argument. Suppose that $t_n, \tau_n, q_i(t_{n-1})$ and $U_i(t_{n-1})$ for $i = 1, \ldots, m$ have one-sided directional derivatives at $b$ with respect to $y_j$ for all $b$ and $j = 1, \ldots, m-1$. First apply Lemma 3 and Lemma 4 to get the one-sided directional differentiability of $q_i(t_n)$ and $U_i(t_n)$. Then Lemma 5 implies that $k(e_{n+1}, t_n)$ has a one-sided directional derivative at $b$ with respect to $y_j$ for all $b$ and $j = 1, \ldots, m-1$. Using elementary properties of one-sided directional derivatives along with Lemma 1 we conclude that $\tau_{n+1}$ has a one-sided directional derivative at $b$ with respect to $y_j$ for all $b$ and $j = 1, \ldots, m-1$. Since $t_{n+1} = t_n + \tau_{n+1}$, the one-sided directional derivative of $t_{n+1}$ at $b$ with respect to $y_j$ is given by

$$\frac{d^+ t_{n+1}}{db_j} = \frac{d^+ t_n}{db_j} + \frac{1}{r(e_{n+1}, t_n)} \frac{d^+ k(e_{n+1}, t_n)}{db_j}. \blacksquare$$

Notice that equation (6.1) provides a recursive representation for $dt_{n+1}/db_j$.

In the terminology of perturbation analysis, Theorem 5 and Lemmas 1–5 describe how perturbations at the time of an event affect perturbations at the time of the next event. To make this complete we need three more results which describe how the perturbations at one machine affect the perturbations at adjacent machines.

**Proposition 4** [adapted from Proposition 3.5 of Fu (1996)] *If $M_i$ fails at $t_n$, then*

$$\frac{d^+ q_i(t_n)}{db_j} = 0 \quad \textit{for} \quad j = 1, \ldots, m-1.$$

*Proof.* If the $n$th event is the $p$th failure of $M_i$, then the total volume produced by $M_i$ up to $t_n$, $q_i(t_n)$, is equal to $W_1^i + W_2^i + \ldots + W_p^i$, which does not depend on the buffer capacities. The result follows immediately. ∎

The following two propositions are related to the buffer events $\mathcal{BF}_i$ and $\mathcal{BE}_i$.

**Proposition 5** [adapted from Proposition 3.6 of Fu (1996)] *If $M_i$ is blocked by $M_{i+1}$ at $t_n$, then*

$$\frac{d^+ q_i(t_n)}{db_j} = \frac{d^+ q_{i+1}(t_n)}{db_j} + I_{\{i=j\}} \quad \textit{for} \quad j = 1, \ldots, m-1.$$

*Proof.* Since $M_i$ is blocked by $M_{i+1}$, the buffer $B_i$ must be full at $t_n$. Therefore the difference between the volume produced by $M_i$ and $M_{i+1}$ must be equal to the buffer capacity $b_i$, i.e. $q_i(t_n) - q_{i+1}(t_n) = b_i$. The result follows immediately. ∎

**Proposition 6** [adapted from Proposition 3.7 of Fu (1996)] *If $M_i$ is starved by $M_{i-1}$ at $t_n$, then*

$$\frac{d^+q_i(t_n)}{db_j} = \frac{d^+q_{i-1}(t_n)}{db_j} \quad \textit{for} \quad j = 1, \ldots, m-1.$$

*Proof.* Since $M_i$ is starved by $M_{i-1}$, the buffer $B_{i-1}$ must be empty at $t_n$. Therefore the volume produced by $M_i$ up to time $t_n$ must be the same as the volume produced by $M_{i-1}$ up to time $t_n$, i.e. $q_i(t_n) - q_{i-1}(t_n) = 0$. ▮

**Remark 6** Again, if our termination criteria were simulating the CT line for a prespecified amount of time $T$, we would define the throughput as $\Theta(b) = q_m(T)/T$. Then when $t_n = T$ for some $n$, we have

$$\frac{d^+\Theta(b)}{db_j} = \frac{d^+}{db_j}\left(\frac{q_m(T)}{T}\right) = \frac{d^+q_m(T)}{db_j}\frac{1}{T} = \frac{d^+q_m(t_n)}{db_j}\frac{1}{t_n}.$$

We also need to modify (6.3) as $\frac{d^+k(e_{k+1},t_k)}{db_j} = -\frac{d^+t_k}{db_j}$ if $e_{k+1} = \mathcal{T}_f$, since $k(e_{k+1}, t_k)$ is $T - t_k$ in that case.

The above results completely specify the perturbation generation and propagation rules and provide a recursive formula for $d^+\Theta(b)/db_j$. Using this IPA algorithm we can compute the exact one-sided directional derivative of $\Theta(b)$ at any $b$ in a single simulation run. §5.4 of Gürkan (1996) contains the pseudo-code, excerpted from Fu (1996), to simulate CT lines, and the pseudo-code of the IPA algorithm we developed in this appendix; the steps (needed for IPA algorithm) added to the basic simulation algorithm are marked as "IPA".

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| problem | m | cycle times of machines | mean volume to failure | mean time to repair |
|---|---|---|---|---|
| 1 | 3 | 0.2 0.8 0.2 | 100 100 100 | 10 10 10 |
| 2 | 3 | 0.045 0.07 0.03 | 160 240 120 | 2.0 1.2 1.0 |

Table 2: Specifications of 3-machine CT lines

| Initial point | K | SPO | | SA | |
|---|---|---|---|---|---|
| | | Final point | "Error" | Final point | "Error" |
| 95 105 | 9 | 56.85 56.60 | 8.0E-1 | 56.45 55.80 | 3.2E-1 |
| 30 30 | 7 | 56.35 56.83 | 7.8E-1 | 56.71 53.63 | 2.5E0 |

Table 3: Solutions generated by SPO and SA for Problem 1A

| Initial point | K | SPO | | SA | |
|---|---|---|---|---|---|
| | | Final point | "Error" | Final point | "Error" |
| 80 20 | 7 | 49.93 50.07 | 1.6E-1 | 50.06 49.94 | 2.8E-2 |
| 10 90 | 11 | 49.95 50.05 | 1.3E-1 | 49.88 50.12 | 2.3E-1 |

Table 4: Solutions generated by SPO and SA for Problem 1B

| Initial point | K | SPO Final point | SPO "Error" | SA Final point | SA "Error" |
|---|---|---|---|---|---|
| 10 10 | 11 | 37.19 24.13 | 6.6E-1 | 41.58 23.16 | 3.8E0 |
| 83 17 | 7 | 38.20 24.06 | 3.5E-1 | 37.35 23.14 | 1.1E0 |

Table 5: Solutions generated by SPO and SA for Problem 2A

| Initial point | K | SPO Final point | SPO "Error" | SA Final point | SA "Error" |
|---|---|---|---|---|---|
| 20 20 | 5 | 23.42 16.58 | 4.2E-2 | 23.23 16.77 | 3.1E-1 |
| 5 35 | 8 | 23.60 16.40 | 2.1E-1 | 23.54 16.46 | 1.3E-1 |

Table 6: Solutions generated by SPO and SA for Problem 2B

| Problem | $b^*$ | $L_n(b^*)$ | $\|Z^T g\|$ |
|---|---|---|---|
| 1A | 56.26 56.06 | 9141.94 | 4.0E-3 |
| 1B | 50.04 49.96 | 9146.18 | 0.0 |
| 2A | 37.85 24.11 | 868.13 | 7.0E-2 |
| 2B | 23.45 16.55 | 874.94 | 5.0E-4 |

Table 7: "Optimal" solutions to Problem 1 and 2

| machine | cycle time | mean volume to failure | mean time to repair |
|---|---|---|---|
| 1 | 0.230 | 100 | 10.0 |
| 2 | 0.430 | 90 | 4.5 |
| 3 | 0.306 | 100 | 6.0 |
| 4 | 0.250 | 90 | 4.5 |
| 5 | 0.350 | 90 | 4.5 |
| 6 | 0.400 | 100 | 5.0 |
| 7 | 0.200 | 90 | 5.4 |
| 8 | 0.333 | 90 | 5.4 |
| 9 | 0.280 | 90 | 4.5 |
| 10 | 0.320 | 90 | 5.4 |
| 11 | 0.308 | 120 | 6.0 |
| 12 | 0.400 | 100 | 8.0 |
| 13 | 0.300 | 90 | 1.8 |
| 14 | 0.360 | 90 | 4.5 |
| 15 | 0.240 | 90 | 4.5 |

Table 8: Specifications of the 15-machine CT line

| | | SPO | SA | | | | |
|---|---|---|---|---|---|---|---|
| | Initial point | Final point | $a_0 = 1$ Final point | $a_0 = 5$ Final point | $a_0 = 10$ Final point | $a_0 = 20$ Final point | "Optimal" solution |
| $b_1$ | 50.0 | 58.34 | 49.35 | 50.87 | 53.34 | 54.36 | 55.92 |
| $b_2$ | 20.0 | 47.68 | 21.47 | 31.90 | 47.02 | 82.57 | 47.14 |
| $b_3$ | 53.3 | 14.01 | 43.26 | 23.26 | 15.90 | 10.00 | 13.32 |
| $b_4$ | 3.3 | 19.43 | 5.86 | 11.66 | 14.89 | 16.69 | 19.45 |
| $b_5$ | 3.3 | 26.57 | 10.88 | 25.08 | 29.20 | 33.32 | 27.23 |
| $b_6$ | 15.0 | 45.72 | 22.41 | 38.97 | 49.00 | 63.78 | 47.90 |
| $b_7$ | 50.0 | 16.26 | 34.37 | 10.14 | 12.62 | 13.91 | 15.66 |
| $b_8$ | 0.0 | 18.69 | 11.11 | 34.83 | 29.37 | 26.78 | 19.06 |
| $b_9$ | 0.0 | 15.05 | 4.51 | 5.03 | 8.01 | 9.31 | 15.29 |
| $b_{10}$ | 15.0 | 35.32 | 21.67 | 34.49 | 44.82 | 61.41 | 37.35 |
| $b_{11}$ | 25.0 | 38.12 | 28.48 | 34.83 | 37.63 | 38.39 | 38.72 |
| $b_{12}$ | 20.5 | 17.13 | 18.86 | 16.33 | 16.23 | 18.42 | 16.35 |
| $b_{13}$ | 24.5 | 14.74 | 22.66 | 18.84 | 16.14 | 13.19 | 14.93 |
| $b_{14}$ | 0.0 | 13.12 | 3.48 | 9.83 | 12.63 | 13.39 | 13.72 |
| "Error" | | 4.23 | 61.25 | 30.66 | 16.35 | 47.44 | – |

Table 9: Solutions generated by SPO and SA for Problem 3A

| | | SPO | SA | | | | |
|---|---|---|---|---|---|---|---|
| | Initial point | Final point | $a_0 = 7$ Final point | $a_0 = 10$ Final point | $a_0 = 12$ Final point | $a_0 = 15$ Final point | "Optimal" solution |
| $b_1$ | 50.0 | 42.86 | 45.24 | 43.85 | 43.79 | 42.71 | 42.31 |
| $b_2$ | 20.0 | 32.14 | 29.76 | 31.15 | 31.21 | 32.29 | 32.69 |
| $b_3$ | 53.3 | 25.00 | 25.00 | 25.00 | 25.00 | 25.00 | 25.00 |
| $b_4$ | 3.3 | 13.52 | 11.34 | 14.36 | 15.24 | 15.11 | 13.60 |
| $b_5$ | 3.3 | 21.48 | 23.66 | 20.64 | 19.76 | 19.89 | 21.40 |
| $b_6$ | 15.0 | 30.00 | 30.00 | 30.00 | 30.00 | 30.00 | 30.00 |
| $b_7$ | 50.0 | 19.99 | 13.21 | 16.30 | 17.49 | 19.20 | 19.51 |
| $b_8$ | 0.0 | 17.24 | 31.14 | 25.47 | 23.62 | 20.38 | 17.61 |
| $b_9$ | 0.0 | 12.77 | 5.65 | 8.23 | 8.89 | 10.43 | 12.88 |
| $b_{10}$ | 15.0 | 40.00 | 42.66 | 42.85 | 42.20 | 42.17 | 41.47 |
| $b_{11}$ | 25.0 | 37.20 | 33.50 | 34.88 | 35.38 | 36.12 | 36.88 |
| $b_{12}$ | 20.5 | 20.03 | 20.00 | 20.00 | 20.02 | 20.71 | 20.01 |
| $b_{13}$ | 24.5 | 12.77 | 16.50 | 15.12 | 14.59 | 13.17 | 13.12 |
| $b_{14}$ | 0.0 | 12.20 | 8.50 | 9.88 | 10.38 | 11.12 | 11.88 |
| "Error" | | 1.87 | 18.39 | 10.65 | 8.55 | 4.56 | – |

Table 10: Solutions generated by SPO and SA for Problem 3B

| machine | cycle time | mean volume to failure | mean time to repair | machine | cycle time | mean volume to failure | mean time to repair |
|---|---|---|---|---|---|---|---|
| 1 | 0.850 | 116.65 | 4.82 | 26 | 0.300 | 102.58 | 5.67 |
| 2 | 0.440 | 101.44 | 8.79 | 27 | 0.570 | 98.44 | 9.85 |
| 3 | 0.360 | 101.86 | 7.02 | 28 | 0.460 | 109.22 | 5.89 |
| 4 | 0.900 | 112.07 | 6.91 | 29 | 0.530 | 117.36 | 8.27 |
| 5 | 0.490 | 112.42 | 9.69 | 30 | 0.960 | 96.46 | 8.55 |
| 6 | 0.920 | 116.08 | 9.11 | 31 | 0.810 | 110.98 | 6.80 |
| 7 | 0.620 | 97.69 | 9.75 | 32 | 0.950 | 118.29 | 5.12 |
| 8 | 0.990 | 95.06 | 7.25 | 33 | 0.170 | 90.25 | 6.39 |
| 9 | 0.980 | 115.51 | 6.59 | 34 | 0.270 | 103.57 | 6.87 |
| 10 | 0.880 | 99.05 | 5.29 | 35 | 0.160 | 88.05 | 8.15 |
| 11 | 0.400 | 111.69 | 9.18 | 36 | 0.690 | 113.83 | 5.77 |
| 12 | 0.870 | 101.34 | 9.82 | 37 | 0.390 | 84.58 | 4.06 |
| 13 | 0.180 | 115.08 | 5.16 | 38 | 0.860 | 87.14 | 6.55 |
| 14 | 0.410 | 98.38 | 9.22 | 39 | 0.500 | 113.94 | 4.27 |
| 15 | 0.650 | 82.09 | 4.51 | 40 | 0.910 | 118.64 | 9.38 |
| 16 | 0.680 | 101.59 | 5.71 | 41 | 0.425 | 89.88 | 5.22 |
| 17 | 0.140 | 86.58 | 7.87 | 42 | 0.475 | 108.54 | 6.36 |
| 18 | 0.250 | 105.86 | 4.15 | 43 | 0.280 | 101.22 | 9.33 |
| 19 | 0.220 | 104.92 | 5.49 | 44 | 0.200 | 95.23 | 4.60 |
| 20 | 0.290 | 81.22 | 4.19 | 45 | 0.450 | 96.02 | 4.75 |
| 21 | 0.710 | 96.88 | 4.94 | 46 | 0.750 | 103.29 | 7.48 |
| 22 | 0.630 | 95.89 | 4.27 | 47 | 0.310 | 114.63 | 4.91 |
| 23 | 0.150 | 99.93 | 8.33 | 48 | 0.970 | 98.60 | 4.34 |
| 24 | 1.000 | 87.48 | 9.83 | 49 | 0.940 | 87.42 | 8.47 |
| 25 | 0.930 | 100.32 | 6.45 | 50 | 0.370 | 93.68 | 6.51 |

Table 11: Specifications of 50-machine CTline

| | Optimal solution for $n$=2E5 | Optimal solution for $n$=5E5 | Optimal solution for $n$=10E5 | | Optimal solution for $n$=2E5 | Optimal solution for $n$=5E5 | Optimal solution for $n$=10E5 |
|---|---|---|---|---|---|---|---|
| $b_1$ | 5.35 | 5.21 | 5.85 | $b_{26}$ | 14.54 | 13.67 | 13.66 |
| $b_2$ | 3.23 | 3.70 | 3.97 | $b_{27}$ | 9.17 | 8.40 | 8.37 |
| $b_3$ | 8.50 | 11.26 | 11.70 | $b_{28}$ | 9.97 | 9.35 | 9.27 |
| $b_4$ | 17.14 | 16.88 | 17.10 | $b_{29}$ | 30.56 | 31.96 | 32.07 |
| $b_5$ | 19.51 | 18.62 | 19.00 | $b_{30}$ | 26.02 | 26.91 | 26.96 |
| $b_6$ | 24.09 | 22.67 | 22.80 | $b_{31}$ | 14.35 | 14.64 | 14.75 |
| $b_7$ | 41.12 | 40.10 | 40.10 | $b_{32}$ | 18.59 | 19.22 | 19.53 |
| $b_8$ | 31.51 | 30.65 | 30.60 | $b_{33}$ | 5.27 | 3.94 | 4.06 |
| $b_9$ | 24.28 | 24.59 | 24.50 | $b_{34}$ | 4.48 | 4.08 | 4.14 |
| $b_{10}$ | 16.46 | 18.15 | 18.10 | $b_{35}$ | 10.40 | 11.45 | 11.67 |
| $b_{11}$ | 16.09 | 20.16 | 20.10 | $b_{36}$ | 7.22 | 6.46 | 6.57 |
| $b_{12}$ | 13.98 | 18.77 | 18.70 | $b_{37}$ | 10.32 | 9.79 | 10.01 |
| $b_{13}$ | 10.46 | 13.52 | 13.30 | $b_{38}$ | 11.88 | 11.23 | 11.40 |
| $b_{14}$ | 8.05 | 10.35 | 10.20 | $b_{39}$ | 12.56 | 12.43 | 12.60 |
| $b_{15}$ | 5.52 | 6.70 | 6.51 | $b_{40}$ | 20.50 | 19.76 | 19.85 |
| $b_{16}$ | 8.58 | 10.55 | 10.50 | $b_{41}$ | 6.76 | 5.78 | 5.85 |
| $b_{17}$ | 3.28 | 3.75 | 3.59 | $b_{42}$ | 4.09 | 3.94 | 4.12 |
| $b_{18}$ | 2.67 | 3.03 | 2.88 | $b_{43}$ | 1.89 | 2.00 | 2.15 |
| $b_{19}$ | 2.51 | 2.65 | 2.54 | $b_{44}$ | 4.17 | 4.58 | 4.73 |
| $b_{20}$ | 6.59 | 8.09 | 8.13 | $b_{45}$ | 7.18 | 8.25 | 8.47 |
| $b_{21}$ | 4.71 | 5.07 | 5.01 | $b_{46}$ | 11.52 | 9.58 | 9.54 |
| $b_{22}$ | 9.96 | 10.29 | 10.20 | $b_{47}$ | 17.58 | 16.45 | 16.48 |
| $b_{23}$ | 47.36 | 48.71 | 48.80 | $b_{48}$ | 17.33 | 18.35 | 18.34 |
| $b_{24}$ | 45.60 | 47.30 | 47.50 | $b_{49}$ | 8.77 | 12.31 | 12.09 |
| $b_{25}$ | 21.29 | 21.33 | 21.40 | | | | |

Table 12: Solutions generated by SPO for Problem 4A

| | Optimal solution for $n$=2E5 | Optimal solution for $n$=5E5 | Optimal solution for $n$=10E5 |
|---|---|---|---|
| $K$ | 16 | 6 | 1 |
| $L_n(b_n^*)$ | 11989.57 | 12001.51 | 11997.54 |
| $\|\nabla L_n(b_n^*)\|$ | 8E-1 | 6E-1 | 5E-1 |
| "Error" | 10.55 | 1.27 | – |
| $L_{10^6}(b_n^*)$ | 12020.23 | 11998.49 | 11997.54 |

Table 13: Summary of SPO results for Problem 4A

| | Optimal solution for $n$=2E5 | Optimal solution for $n$=5E5 | Optimal solution for $n$=10E5 | | Optimal solution for $n$=2E5 | Optimal solution for $n$=5E5 | Optimal solution for $n$=10E5 |
|---|---|---|---|---|---|---|---|
| $b_1$ | 10.00 | 10.00 | 10.00 | $b_{26}$ | 20.76 | 20.80 | 20.89 |
| $b_2$ | 0.00 | 0.09 | 0.36 | $b_{27}$ | 6.57 | 6.52 | 6.57 |
| $b_3$ | 11.59 | 11.91 | 12.44 | $b_{28}$ | 7.74 | 7.69 | 7.68 |
| $b_4$ | 16.26 | 16.31 | 16.29 | $b_{29}$ | 44.86 | 44.81 | 44.69 |
| $b_5$ | 18.07 | 18.14 | 18.25 | $b_{30}$ | 20.07 | 20.18 | 20.17 |
| $b_6$ | 21.08 | 21.19 | 21.22 | $b_{31}$ | 17.71 | 17.81 | 17.81 |
| $b_7$ | 33.60 | 33.62 | 33.47 | $b_{32}$ | 17.36 | 17.38 | 17.34 |
| $b_8$ | 26.67 | 26.43 | 26.19 | $b_{33}$ | 10.00 | 10.00 | 10.00 |
| $b_9$ | 27.73 | 27.30 | 26.78 | $b_{34}$ | 9.14 | 9.06 | 8.98 |
| $b_{10}$ | 10.00 | 10.00 | 10.00 | $b_{35}$ | 10.23 | 10.37 | 10.46 |
| $b_{11}$ | 22.47 | 22.16 | 22.00 | $b_{36}$ | 4.34 | 4.39 | 4.49 |
| $b_{12}$ | 14.11 | 14.36 | 14.61 | $b_{37}$ | 8.54 | 8.59 | 8.66 |
| $b_{13}$ | 8.08 | 8.06 | 7.95 | $b_{38}$ | 11.12 | 11.06 | 11.05 |
| $b_{14}$ | 5.33 | 5.42 | 5.44 | $b_{39}$ | 12.55 | 12.49 | 12.46 |
| $b_{15}$ | 6.26 | 6.46 | 6.71 | $b_{40}$ | 19.01 | 18.87 | 18.74 |
| $b_{16}$ | 11.86 | 11.91 | 12.01 | $b_{41}$ | 10.52 | 10.58 | 10.65 |
| $b_{17}$ | 10.47 | 10.45 | 10.41 | $b_{42}$ | 10.48 | 10.55 | 10.61 |
| $b_{18}$ | 2.26 | 2.25 | 2.22 | $b_{43}$ | 0.84 | 0.85 | 0.86 |
| $b_{19}$ | 7.27 | 7.31 | 7.36 | $b_{44}$ | 1.64 | 1.65 | 1.67 |
| $b_{20}$ | 4.22 | 4.26 | 4.38 | $b_{45}$ | 11.12 | 11.11 | 11.13 |
| $b_{21}$ | 10.00 | 10.00 | 10.00 | $b_{46}$ | 63.47 | 63.43 | 63.39 |
| $b_{22}$ | 8.55 | 8.64 | 8.58 | $b_{47}$ | 7.40 | 7.38 | 7.34 |
| $b_{23}$ | 43.39 | 43.71 | 43.90 | $b_{48}$ | 32.74 | 32.70 | 32.66 |
| $b_{24}$ | 37.63 | 37.65 | 37.52 | $b_{49}$ | 11.78 | 11.74 | 11.70 |
| $b_{25}$ | 18.08 | 17.36 | 16.90 | | | | |

Table 14: Solutions generated by SPO for Problem 4B

| | Optimal solution for $n$=2E5 | Optimal solution for $n$=5E5 | Optimal solution for $n$=10E5 |
|---|---|---|---|
| $K$ | 10 | 1 | 1 |
| $L_n(b_n^*)$ | 12058.14 | 12081.55 | 12076.73 |
| $\|\|Z^T g\|\|$ | 7E-1 | 8E-1 | 7E-1 |
| "Error" | 2.19 | 1.13 | – |
| $L_{10^6}(b_n^*)$ | 12079.39 | 12078.19 | 12076.73 |

Table 15: Summary of SPO results for Problem 4B

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