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CAPITAL ACCUMULATION AND ENTRY DETERRENCE: A CLARIFYING NOTE

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CAPITAL ACCUMULATION AND ENTRY DETERRENCE: A CLARIFYING NOTE*

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Abstract

This note clarifies some of the entry deterrence aspects of capital accumulation. Since accumulating capital takes time the focus of this note is on the importance of time in the analysis of entry deterrence. While the post-entry game is modelled as a capital accumulation differential game for which we solve for the feedback equilibrium, we also add a time dimension to the pre-entry game assuming that the entry decision is subject to entry preparation that also takes time. This preparation period affects the analysis of entry deterrence and the possibility and the attractiveness of entry deterrence.

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1. Introduction

In his seminal work on the supply side of new markets Spence (1977, 1979) analyses the strategic interaction of early entrants in their game with potential entrants. The result is in spirit similar to the von Stackelberg oligopoly model but the strategic asymmetries are now induced by the history of the market. The crucial point is that the incumbents can make irrevocable investments in product-specific capital which deter entry or which lead to a favourable position on the market after new entries. In the terminology of Schelling (1960) these investments are an advance commitment or credible threat. Commitments to future investments are ruled out, because these commitments are not credible and because otherwise the fundamental asymmetry between established firms and potential entrants disappears. Dixit (1980) argues that this also means that an excess capacity strategy cannot be sustained. After entry a Cournot/Nash game will be played between all the firms in the market. Dixit stresses in his paper that it is important to distinguish a pre-entry stage and a post-entry stage. Although the rules of the post-entry game are exogenous (such as a Cournot/Nash oligopoly), the established firm can influence the outcome of that game to its advantage by building favourable initial conditions in the pre-entry stage. Fudenberg and Tirole (1983) analyse the post-entry game in continuous time with an infinite horizon. They derive the set of subgame perfect equilibria for investment strategies which are only a function of the state (that is the current capital stocks). By first allowing the firms to coordinate their strategies and by then invoking some ad hoc argument they can reduce the set of perfect equilibria to one. This is necessary to be able to discuss entry deterrence. Reynolds (1987) derives the subgame-perfect (or feedback Nash) equilibrium for the game in which investments are reversible but capacity is subject to adjustment costs. This equilibrium is unique. Reynolds compares it with the open-loop Nash equilibrium in which the investment strategies are only a function of time.

In this note the post-entry game is modelled as the capital accumulation game in Reynolds' paper. However, since investment commitments in advance of actual investments are not credible, the feedback Nash or subgame-perfect Markov equilibrium is in our view the only reasonable solution concept. In Dixit's terminology the rules of the post-entry game lead to the feedback Nash equilibrium. The values of this game are a function of the initial capacity levels. Suppose there is one incumbent and one potential entrant. At the start of the post-entry game the capacity level of the entrant is zero and the capacity level of the incumbent depends on its investments before entry. The novelty in this note is to model explicitly a time-lag between the potential entrant's preliminary decision to enter and the actual entry to the market. The incumbent can use this period to invest in extra capacity in order to deter entry or to achieve a good starting position for the post-entry game. When the investment costs are convex, the length of the time-lag plays a crucial role in the decision problem of the incumbent. A second novelty in this note is the introduction of preparation costs for the potential entrant. It is shown that this gives the incumbent firm the possibility to realize monopoly profits in the pre-entry stage, even if it is not a natural monopoly but an artificial monopoly in the terminology of Eaton and Lipsey (1981). It is therefore better to speak in this case of a natural monopoly and a strategic natural monopoly.

The note is organized as follows. Section 2 sets out the framework of the analysis. In section 3 and the appendix the equilibrium of the post-entry capital accumulation differential game is derived. Section 4 analyses the pre-entry decision problem of the incumbent firm. In section 5 a strategic natural monopoly is introduced as a consequence of small but non-zero preparation costs and section 6 concludes the paper.

2. The framework

Consider an industry in which there is one incumbent firm and one potential entrant. It is assumed that entry cannot be decided upon and carried out instantaneously. The lag between the entry decision and the actual entry is denoted by the preparation time $t^e > 0$. Moreover, the entry decision is not a commitment. A firm can reconsider this decision at the last moment before the preparation time has elapsed. Considering entry is not necessarily costless and we denote this cost as $f^e \geq 0$. Clearly, if $f^e = 0$, the potential entrant will announce its intention right away and leave the actual entry decision to time t^e or later. We start our analysis by considering the case of zero preparation costs. In section 5 we consider the case in which the potential entrant has to bear some preparation costs.

Both firms in our model accumulate some form of capital according to the standard capital accumulation dynamics

$$\dot{K}_i(t) = I_i(t) - \delta K_i(t), \; K_i(0) = K_{i0}, \; i = 1,2, \tag{1}$$

where K_i denotes the capital level, I_i the investment level and δ is a common depreciation factor. The entrant can start to accumulate capital only after it enters. There is no capital accumulation during the preparation period. Investment is costly and we let $C_i(I_i)$ be the cost of investment. C_i is a convex and increasing function.

We assume that the instantaneous profits at time t can be expressed as a function of the state variables $[K_1(t), K_2(t)]$. This is not to say that the firms do not compete through prices or quantities but that a reduced form can be used. It is assumed that the profit function $\Pi_i(K_1(t), K_2(t))$ is an increasing and concave function of K_i and a decreasing function of K_j, $i = 1,2$, $j \neq i$. The objective of each firm is to maximize its discounted stream of profits net of investment costs

$$\int_0^\infty \{\Pi_i(K_i(t), K_j(t)) - C_i(I_i(t))\} \, e^{-rt} \, dt, \tag{2}$$

where r is the common discount rate, subject to the capital accumulation dynamics (1).

Entry is associated with some fixed (sunk) cost of entry, denoted by F^e. That is f^e is paid at the beginning of the preparation period while F^e is paid at the time of the entry itself. Clearly, the potential entrant will enter the market, if, at the moment of entry, the discounted stream of profits net of investment costs at least covers the entry fee F^e. The post-entry game is described by (1)-(2), starting at time t^e. This can be called a capital accumulation differential game (see, e.g., Fershtman and Muller (1984) and Reynolds (1987)). Suppose that the incumbent firm is denoted as firm 1 and the entrant firm as firm 2. On the assumption that prior to entry the entrant does not accumulate capital, the initial condition of the capital accumulation game is $[K_1(t^e),0]$. If the value functions V_i of the post-entry game exist, the potential entrant will choose to enter at time t^e only if

$$V_2(K_1(t^e),0) - F^e > 0. \tag{3}$$

This implies that if the incumbent firm achieves at time t^e at least the capital level

$$K^d(F^e) := \inf \{K_1 | V_2(K_1,0) - F^e \le 0\} \tag{4}$$

entry is blocked. Therefore we define $K^d(F^e)$ as the capital deterrence level.

An incumbent firm may ignore the possibility of entry. This firm maximizes (2) subject to (1) with K_2 always equal to 0. We denote the optimal capital accumulation path of this firm as $K^m(.)$.

Definition 1: An incumbent firm will be called a natural monopoly, if

$$K^m(t) \ge K^d(F^e) \text{ for every } t \ge t^e. \tag{5}$$

Note that the position of a firm as a natural monopoly depends on both the entry cost F^e and the length of the preparation period t^e.

Given the deterrence level $K^d(F^e)$, the incumbent firm is facing a standard entry deterrence problem. The firm can accumulate capital up to $K^d(F^e)$ and block entry, or it can accommodate entry. Since the investment costs are convex, the decision of the incumbent depends on the time it has to reach the deterrence level. The shorter the preparation time t^e is the more costly it will be to reach $K^d(F^e)$. Moreover, the incumbent's decision problem is not just whether to deter entry or to accommodate entry. Even if this firm decides to accommodate, it is of importance which capital level it reaches by time t^e as this level affects its profits in the post-entry game.

3. The post-entry game: capital accumulation game

As was already stated in section 2, after entry the two firms are engaged in a capital accumulation differential game. Because in the general case such a game is not analytically tractable, a linear-quadratic structure is adopted here. The cost functions are given by

$$C(I_i) = \tfrac{1}{2} cI_i^2, \; c > 0, \; i = 1,2 \tag{6}$$

and the profit functions are given by

$$\Pi_i(K_i,K_j) = K_i(a - K_i - K_j), \; i,j = 1,2, \; i \neq j. \tag{7}$$

It follows that the firms try to maximize

$$\int_0^\infty \{K_i(t)(a - K_i(t) - K_j(t)) - \tfrac{1}{2} cI_i^2(t)\} \, e^{-rt} \, dt, \; i,j = 1,2, \; i \neq j, \tag{8}$$

subject to (1), where the initial time is taken to be 0 in order to simplify notation, although the actual initial time is t^e.

The capital accumulation differential game (8)-(1) is identical to the one in Reynolds (1987). Furthermore, the game is in structure very similar to the dynamic duopoly with sticky prices (Fershtman and Kamien, 1987) and to a model of competitive arms accumulation (van der Ploeg and de Zeeuw, 1990). Because it seems reasonable to assume that the firms can condition their investments on the current capital levels and that the firms can not commit themselves to future investments, the feedback Nash (Starr and Ho, 1969) or subgame-perfect Markov equilibrium has to be derived. The outcome can be found in Reynolds (1987), but the appendix of this paper presents this outcome and the derivation in a much more transparent way.

The capital accumulation game can therefore be summarized by a value function $V_i(K_1,K_2)$, which is continuous in its arguments with $\partial V_i/\partial K_i > 0$ and

$\partial V_i/\partial K_j < 0$, $i \neq j$, and which gives the value for player i of the capital accumulation game that starts with the initial condition (K_1, K_2).

4. The incumbent decision problem

At date t = 0 the incumbent has to decide whether to follow a capital accumulation path that prevents entry or a path that accommodates entry. The incumbent has to compare its profits given by the outcome of problem 1, when entry is accommodated, and its profits given by the outcome of problem 2, when entry is deterred.

Problem 1: Accommodating Entry

$$\underset{I_1(.)}{\text{maximize}} \int_0^{t^e} \{K_1(t)(a - K_1(t)) - \tfrac{1}{2} cI_1^2(t)\}\, e^{-rt}\, dt + V_1(K_1(t^e), 0)\, e^{-rt^e}, \tag{9}$$

subject to (1), $K_1(0) = K_{10}$ and $K_1(t^e) < K^d(F^e)$.

The trade-off for the incumbent in problem 1 is to realize monopoly profits in the pre-entry stage, on the one hand, and to reach a favourable initial position in the post-entry stage, on the other hand.

Problem 2: Deterring Entry

$$\underset{I_1(.)}{\text{maximize}} \int_0^{\infty} \{K_1(t)(a - K_1(t)) - \tfrac{1}{2} cI_1^2(t)\}\, e^{-rt}\, dt, \tag{10}$$

subject to (1), $K_1(0) = K_{10}$ and $K_1(t) \geq K^d(F^e)$ for every $t \geq t^e$.

When the last constraint is not binding, problem 2 corresponds to a natural monopoly as defined in definition 1. Otherwise, the incumbent is sometimes called an artificial monopoly.

Let $V^a(K_{10},t^e,K^d(F^e))$ be the value of the control problem for the incumbent when entry is accommodated (problem 1) and $V^d(K_{10},t^e,K^d(F^e))$ be the value when the incumbent chooses to deter entry (problem 2). Clearly, the incumbent will deter entry iff $V^d(K_{10},t^e,K^d(F^e)) > V^a(K_{10},t^e,K^d(F^e))$. Although we choose not to provide specific solutions to problems 1 and 2, we argue that both $V^a(.)$ and $V^d(.)$ increase with t^e. There are two reasons for this. Firstly, a higher t^e implies that the incumbent can enjoy a longer monopolistic period. Secondly, the incumbent now has more time either to achieve a favourable starting position for the post-entry game in case it chooses to accommodate entry, or to achieve the capital deterrence level K^d. Since the accumulation costs are convex, reaching these levels is less costly when there is more time. Clearly, the effects of changing t^e in $V^a(.)$ and $V^d(.)$ are not identical. Therefore, it is possible that for a given pair $(t^e,K^d(F^e))$ the incumbent will choose to deter entry whereas for a shorter preparation period, i.e. $\hat{t}^e < t^e$, the optimal strategy is to accommodate entry. This happens when the extra investment costs to reach the capital deterrence level $K^d(F^e)$ in a shorter period $\hat{t}^e$ outweigh the losses that are suffered in problem 1 due to a shorter preparation period.

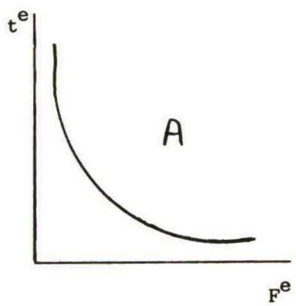

Figure 1

We can conclude that the length of the preparation period is an important factor in determining whether the incumbent's optimal strategy is to deter entry or not. In particular we expect that the set of possible pairs (t^e,F^e) will be divided as in Figure 1, such that for $(t^e,F^e) \in A$ entry is deterred whereas for $(t^e,F^e) \notin A$ entry is accommodated.

Intuitively, for a high F^e the level of deterrence capital K^d is low and thus the incumbent only needs a short preparation period to accumulate this level. Reducing F^e leads to a higher K^d, which implies that the incumbent will only accumulate this level if the preparation period is sufficiently long. The area A covers both the situations that correspond to a natural monopoly and to an artificial monopoly.

Likewise, whether the incumbent firm is a natural monopoly or not also depends on the length of the preparation period t^e. It can very well be that $K^m(t) \geq K^d(F^e)$, $t \geq t^e$, which implies that the incumbent firm is a natural monopoly for the preparation period t^e, whereas $K^m(\hat{t}^e) < K^d(F^e)$ holds for a shorter preparation period $\hat{t}^e < t^e$, which implies that in that case the incumbent firm is not a natural monopoly.

5. The strategic natural monopoly

The standard definition of natural monopoly describes the case where a firm, by acting as a monopolist that maximizes profits, while ignoring the possibility of entry, in fact deters entry. That is the profit maximizing capital accumulation path $K^m(.)$ lies on or above the capital deterrence level $K^d(F^e)$ from time t^e onwards, so that entry is automatically deterred.

Consider now the case in which the preparation costs $f^e = \varepsilon > 0$, and suppose that the incumbent firm is not a natural monopolist. The question is whether the potential entrant will announce entry in such a case and thus pay $f^e > 0$. The answer depends on what the potential entrant expects the incumbent's reaction to be to an entry announcement. If the announcement is done at time $t = 0$ and if $V^d(K_{10},t^e,K^d(F^e)) > V^a(K_{10},t^e,K^d(F^e))$, then the incumbent will react to such an announcement by accumulating the capital deterrence level $K^d(F^e)$ by time t^e, so that entry is deterred. Given such a reaction the optimal strategy of the potential entrant is not to start the preparations for entry at all and thus to avoid the costs $f^e > 0$. The only subgame perfect equilibrium in this case is that the incumbent firm acts as a monopolist, accumulating capital according to $K^m(.)$. That is not to say that the incumbent firm ignores the possibility of entry. Since there is a preparation period t^e, the incumbent realizes that if necessary once entry will be announced it can start to accumulate capital to the level $K^d(F^e)$ in order to deter entry. Moreover, since preparation to enter is not costless and entry will be deterred, the potential entrant will not start to prepare for entry, and the incumbent can make monopoly profits all the way.

This analysis implies that in the case of non-zero preparation costs the incumbent firm's capital accumulation path follows the path $K^m(.)$, even if the incumbent firm is not a natural monopoly. We denote such a market as a strategic natural monopoly (SNM). The part of the area A in Figure 1, that corresponds to an artificial monopoly in the case of zero preparation costs, now becomes the area of a strategic natural monopoly with monopoly profits for the incumbent firm at all times. Clearly, the position of a firm as a SNM depends again on both the entry cost F^e and the length of the preparation period t^e.

6. Conclusion

The role of capital in entry deterrence is well documented in the literature. The standard setting for such an analysis has been a two-stage game where in the first stage capital is built, usually with linear costs.

In this note we claim that the pre-entry stage should also be modelled with a specific attention to the role of time. Under the standard assumption of convex investment costs the length of the pre-entry period proves to be an important factor in the analysis of entry deterrence. Furthermore, it is shown that in the case of non-zero preparation costs the artificial monopoly can in fact realize monopoly profits in the pre-entry stage because of its credible threat to accumulate extra capital in order to deter entry and to burden the potential entrant with these preparation costs when it starts to prepare for entry.

These observations may lead to a more detailed analysis of the entry deterrence problem in which the preparation period can also be one of the firm's strategic variables. This analysis is, however, beyond the scope of this note and will be subject of further research.

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Appendix

Consider the differential game (i = 1,2)

$$\underset{I_i(.)}{\text{maximise}} \int_0^\infty \{-\tfrac{1}{2}\, cI_i^2(t) + \tfrac{1}{2}\, x'(t)Q_i x(t) + q_i' x(t)\} e^{-rt}\, dt \tag{A.1}$$

$$\text{subject to } \dot{x}(t) = Ax(t) + B_1 I_1(t) + B_2 I_2(t),\ x(0) = x_0, \tag{A.2}$$

where the state x consists of the capital stocks $[K_1, K_2]'$, and where

$$A := \begin{bmatrix} -\delta & 0 \\ 0 & -\delta \end{bmatrix};\ B_1 := \begin{bmatrix} 1 \\ 0 \end{bmatrix};\ B_2 := \begin{bmatrix} 0 \\ 1 \end{bmatrix};\ Q_1 := \begin{bmatrix} -2 & -1 \\ -1 & 0 \end{bmatrix};\ Q_2 := \begin{bmatrix} 0 & -1 \\ -1 & -2 \end{bmatrix};\ q_1 := \begin{bmatrix} a \\ 0 \end{bmatrix};\ q_2 := \begin{bmatrix} 0 \\ a \end{bmatrix}.$$

The value functions are denoted as V. The feedback Nash equilibrium for this differential game results from the dynamic programming equations (i = 1,2)

$$rV_i(x) = \max \{-\tfrac{1}{2}\, cI_i^2 + \tfrac{1}{2}\, x'Q_i x + q_i' x + V_{ix}'(x)(Ax + B_1 u_1 + B_2 u_2)\}. \tag{A.3}$$

The equilibrium strategies are given by

$$I_i(x) = (1/c) B_i' V_{ix}(x). \tag{A.4}$$

The dynamic programming equations become (i,j = 1,2; i ≠ j)

$$rV_i(x) = -\tfrac{1}{2}\, (1/c) V_{ix}'(x) B_i B_i' V_{ix}(x) + \tfrac{1}{2}\, x'Q_i x + q_i' x +$$
$$V_{ix}'(x)(Ax + (1/c) B_i B_i' V_{ix}(x) + (1/c) B_j B_j' V_{jx}(x))\}. \tag{A.5}$$

With the quadratic value functions

$$V_1(x) = \tfrac{1}{2}\, x' \begin{bmatrix} p_1 & p_3 \\ p_3 & p_2 \end{bmatrix} x + \ldots ;\ V_2(x) = \tfrac{1}{2}\, x' \begin{bmatrix} p_2 & p_3 \\ p_3 & p_1 \end{bmatrix} x + \ldots \tag{A.6}$$

equation of the quadratic terms of (A.5) yields the system of equations

$$p_1^2 + 2p_3^2 - (2\delta+r)cp_1 - 2c = 0, \tag{A.7}$$

$$2p_1p_3 + p_3p_2 - (2\delta+r)cp_3 - c = , \tag{A.8}$$

$$p_3^2 + 2p_1p_2 - (2\delta+r)cp_2 = 0. \tag{A.9}$$

When the equilibrium strategies of (A.4) are substituted in the system (A.2) the closed-loop system results with the state-transition matrix

$$A^{cl} := \begin{bmatrix} -\delta+p_1/c & p_3/c \\ p_3/c & -\delta+p_1/c \end{bmatrix}. \tag{A.10}$$

The eigenvalues of A^{cl} are $-\delta + p_1/c \pm p_3/c$, so that the closed-loop system is stable if and only if

$$p_1 + p_3 - \delta c < 0 \text{ and } p_1 - p_3 - \delta c < 0. \tag{A.11}$$

Equation (A.7) describes an ellipse in the (p_1,p_3)-plane. Furthermore, for each value of p_2 equation (A.8) describes a hyperbola in the (p_1,p_3)-plane and equation (A.9) a parabola. Defining $\rho := \surd(2c + \frac{1}{4}(2\delta+r)^2c^2)$, the use of polar coordinates

$$p_1 = \tfrac{1}{2}(2\delta+r)c + \rho\sin\varphi;\; p_3 = \tfrac{1}{2}\surd 2\,\rho\cos\varphi,\; -\pi < \varphi \le \pi, \tag{A.12}$$

and the elimination of p_2 leads to the equation

$$\rho^2\cos^3\varphi - 8\rho^2\sin^2\varphi\cos\varphi + 4\surd 2\,c\sin\varphi = 0. \tag{A.13}$$

Defining $\alpha := \surd 2\,\rho^2/c = 2\surd 2 + \frac{1}{4}\surd 2\,(2\delta+r)^2c$, (A.13) yields eventually

$$\tan^3\varphi - \alpha\tan^2\varphi + \tan\varphi + (1/8)\alpha = 0. \tag{A.14}$$

Consider the function f given by

$$f(y) := y^3 - \alpha y^2 + y + (1/8)\alpha, \quad \alpha \geq 2\sqrt{2}. \tag{A.15}$$

Some straightforward calculus shows that the function f has one negative root y_1 and two positive roots y_2 and y_3 with $\frac{1}{4}\sqrt{2} < y_2 < y_3$. Consider y_2 as a function of α. It is easy to see that $y_2(\alpha) = \frac{1}{2}\sqrt{2}$ for $\alpha = 2\sqrt{2}$. Implicit differentiation yields

$$y_2'(\alpha) = [y_2^2(\alpha) - (1/8)]/f'(y_2(\alpha)) < 0. \tag{A.16}$$

It follows that the smallest positive root y_2 of the function f satisfies $\frac{1}{4}\sqrt{2} < y_2 < \frac{1}{2}\sqrt{2}$. Since $f(-\frac{1}{2}\sqrt{2}) < 0$, the negative root y_1 of the function f satisfies $y_1 > -\frac{1}{2}\sqrt{2}$.
The largest positive root y_3 of the function f is given by

$$y_3(\alpha) = 2\sqrt{\{(\alpha^2/9) - (1/3)\}} \cos(\psi/3) + (\alpha/3), \tag{A.17}$$

where

$$\psi = \arccos\, [(\alpha^3/27) - (11\alpha/48)]/[\sqrt{\{(\alpha^2/9) - (1/3)\}}]^3, \quad 0 < \psi < (\pi/2).$$

Claim
The largest positive root y_3 of the function f satisfies the stability constraints (A.11), but the negative root y_1 and the smallest positive root y_2 of the function f do not satisfy the stability constraints (A.11).

Proof
With the polar coordinates (A.12) the stability constraints (A.11) are given by $-\pi < \varphi < 0$ and

$$|\tan \varphi| > [\tfrac{1}{2}\sqrt{2}\, z + \tfrac{1}{2}\, rc]/z \text{ with } z > 0 \text{ given by } z^2 + (\tfrac{1}{2}\sqrt{2}\, z + \tfrac{1}{2}\, rc)^2 = \rho^2.$$

It follows that the stability constraints are given by

$$|\tan \varphi| > [2\alpha + r\sqrt{(3\sqrt{2}\, \alpha c - r^2c^2)}]/[2\sqrt{2}\, \alpha - r^2c]. \tag{A.18}$$

Because

$$|\tan \varphi| = [2\alpha + r\surd(3\surd 2\ \alpha c - r^2c^2)]/[8 + 4\delta c(\delta+r)] >$$

$$2\alpha/[8 + 4\delta c(\delta+r)] > \tfrac{1}{2}\surd 2,$$

and because $y_1 > -\tfrac{1}{2}\surd 2$ and $\tfrac{1}{4}\surd 2 < y_2 < \tfrac{1}{2}\surd 2$, these roots of the function f do not satisfy the stability constraints. Furthermore,

$$y_3(\alpha) > \surd\{(\alpha^2/9) - (1/3)\} + (\alpha/3). \tag{A.19}$$

The right-hand side of (A.19) is minimal for $\delta = 0$ and the right-hand side of (A.18) is maximal for $\delta = 0$. It is tedious but straightforward to show that this minimal value is larger than this maximal value.
It follows that the largest positive root y_3 of the function f satisfies the stability constraints. Q.E.D.

The conclusion of this analysis is that the parameters p_1 and p_3 of the value functions are given by

$$p_1 = \tfrac{1}{2}\ (2\delta+r)c + \surd\{[\tfrac{1}{2}\surd 2\ \alpha c\ y_3^2(\alpha)]/[1 + y_3^2(\alpha)]\}, \tag{A.20}$$

$$p_3 = \surd\{[\tfrac{1}{4}\surd 2\ \alpha c]/[1 + y_3^2(\alpha)]\}, \tag{A.21}$$

where $\alpha = 2\surd 2 + \tfrac{1}{4}\surd 2\ (2\delta+r)^2c$, and $y_3(\alpha)$ is given by (A.17).
The parameter p_2 of the value functions can then be calculated from (A.9), which yields

$$p_2 = -\tfrac{1}{4}\ \surd\{[\tfrac{1}{2}\surd 2\ \alpha c]/[y_3^2(\alpha) + y_3^4(\alpha)]\}. \tag{A.22}$$

The linear terms of the quadratic value functions

$$V_1(x) = \ldots. + [p_4\ p_5]x + \ldots. \ ;\ V_2(x) = \ldots. + [p_5\ p_4]x + \ldots. \tag{A.6}$$

can be found by equation of the linear terms of (A.5), which yields the simple system of equations

$$\{p_1 + p_3 - (\delta+r)c\}p_4 + p_3p_5 + ac = 0, \tag{A.23}$$

$$(p_2 + p_3)p_4 + \{p_1 - (\delta+r)c\}p_5 = 0.$$

Finally, the equilibrium strategies become

$$I_i(x) = (1/c)\{p_1K_i + p_3K_j + p_4\}, \; i,j = 1,2, \; i \neq j. \tag{A.4}$$

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9025	K. Kamiya and D. Talman	Variable Dimension Simplicial Algorithm for Balanced Games
9026	P. Skott	Efficiency Wages, Mark-Up Pricing and Effective Demand
9027	C. Dang and D. Talman	The D_1-Triangulation in Simplicial Variable Dimension Algorithms for Computing Solutions of Nonlinear Equations
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9030	Th. van de Klundert and R. Gradus	Optimal Government Debt under Distortionary Taxation
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9032	J. Osiewalski and M. Steel	Robust Bayesian Inference in Elliptical Regression Models
9033	C. R. Wichers	The Linear-Algebraic Structure of Least Squares
9034	C. de Vries	On the Relation between GARCH and Stable Processes
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9036	J. Driffill	The Term Structure of Interest Rates: Structural Stability and Macroeconomic Policy Changes in the UK
9037	F. van der Ploeg	Budgetary Aspects of Economic and Monetary Integration in Europe
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9039	A. Robson	An "Informationally Robust Equilibrium" for Two-Person Nonzero-Sum Games
9040	M.R. Baye, G. Tian and J. Zhou	The Existence of Pure-Strategy Nash Equilibrium in Games with Payoffs that are not Quasiconcave
9041	M. Burnovsky and I. Zang	"Costless" Indirect Regulation of Monopolies with Substantial Entry Cost

No.	Author(s)	Title
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9059	F. van der Ploeg	Channels of International Policy Transmission

No.	Author(s)	Title
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No.	Author(s)	Title
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No.	Author(s)	Title
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9136	H. Bester and E. Petrakis	The Incentives for Cost Reduction in a Differentiated Industry
9137	L. Mirman, L. Samuelson and E. Schlee	Strategic Information Manipulation in Duopolies
9138	C. Dang	The D_2^*-Triangulation for Continuous Deformation Algorithms to Compute Solutions of Nonlinear Equations
9139	A. de Zeeuw	Comment on "Nash and Stackelberg Solutions in a Differential Game Model of Capitalism"
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