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Abstract

In this paper we first of all show that contemporaneous aggregation of independent univariate GARCH processes yields a process which satisfies the weak GARCH conditions introduced by Drost and Nijman (1992). Subsequently we analyze the dependence of the parameters in the aggregate on the parameters in the underlying models and present numerical results for the aggregation of two GARCH(1,1) processes with identical "persistence" parameters, and a GARCH(1,1) process with conditionally homoskedastic white noise. We show that the variance parameters after aggregation depend on the underlying variance and kurtosis parameters. Subsequently, we generalize the results by showing that a linear combination of variables generated by a multivariate GARCH process will also be weak GARCH and analyzing the parameters in this weak GARCH process. We also derive the marginal (weak GARCH) processes implied by several multivariate GARCH processes. The results explain why GARCH is found in univariate as well as multivariate series and can be used to facilitate the specification of multivariate GARCH processes.

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1. Introduction

It is a well known stylized fact that many financial time series such as stock returns, exchange rates or interest rates exhibit conditional heteroskedasticity. For univariate time series, the ARCH model proposed by Engle (1982) or the GARCH model proposed by Bollerslev (1986) are usually used to parametrize the conditional heteroskedasticity. Multivariate extensions of these models have been proposed e.g. by Engle (1987), Attanasio and Edey (1987), Bollerslev, Engle and Wooldridge (1988), Diebold and Nerlove (1989), Baba, Engle, Kraft and Kroner (1989), Engle, Ng and Rothschild (1990) and King, Sentana and Wadhwani (1991). An excellent survey of the many applications of GARCH processes is provided by Bollerslev, Chou and Kroner (1992).

In the existing literature univariate or multivariate GARCH models are typically assumed for the time series under consideration, without wondering whether these assumptions are consistent with GARCH assumptions for other time series and/or at other frequencies. GARCH models have been estimated e.g. for the (log) returns in the Deutsche Mark / US Dollar exchange rate, the US Dollar / Japanese Yen rate and the Deutsche Mark / Japanese Yen rate. As the returns on the third exchange rate are simply the sum of the returns on the first two exchange rates, the GARCH models for these exchange rates implicitly specify a model for the third exchange rate as well. Similarly GARCH models have been fitted to returns on individual stocks as well as to returns on portfolios, but once more the relation between the models for the individual stocks and the one for the portfolio has not been considered explicitly. Also multivariate GARCH models are used nowadays to describe e.g. the joint behaviour of interest rates, exchange rates and stock returns. Little attention has been paid however to the implication of such a multivariate model for the univariate processes of the components. In this paper we derive the properties of linear combinations of variables generated by a multivariate GARCH process as well as the properties of marginal processes implied by a multivariate GARCH assumption. We show that the parametric structure of the commonly used GARCH models is lost by taking linear combinations or by marginalizing. Nevertheless we show that the linear combinations and marginal processes will still be weak GARCH processes as defined by Drost and Nijman (1992). These results explain why GARCH is found in univariate as well as multivariate series and can be used to facilitate the specification of multivariate GARCH processes. Throughout this paper we restrict ourselves to bivariate models with GARCH(1,1) variances in order to keep the algebra simple. Our general framework however applies equally well to more general cases.

Consider a univariate time series $\{y_t\}$ which is stationary and symmetric with finite fourth moments. Drost and Nijman (1992) have defined $\{y_t\}$ to be weak GARCH(1,1) if in the recursion

$$\sigma_t^2 = \psi + \beta \sigma_{t-1}^2 + \alpha y_{t-1}^2 \qquad (1)$$

the parameters ψ, α and β can be chosen such that

$$P[\, y_t \mid y_{t-1}, y_{t-2}, \ldots .\,] = 0 \qquad (2)$$

and

$$P[\, y_t^2 \mid y_{t-1}, y_{t-2}, \ldots .\,] = \sigma_t^2 \qquad (3)$$

where $P[\, x_t \mid z_{t-1}, z_{t-2}, \ldots .\,]$ denotes the best linear predictor of x_t in terms of a constant and values of z_{t-i} and z_{t-i}^2 ($i = 1, 2, \ldots .$). Drost and Nijman (1992) have shown that GARCH models which assume that y_t/σ_t is i.i.d. as proposed by Bollerslev (1986), which they refer to as strong GARCH models, are not closed under temporal aggregation. A strong GARCH assumption e.g. at the daily frequency is inconsistent with a strong GARCH assumption at the weekly frequency. In addition they have shown that the class of weak GARCH processes is closed under temporal aggregation. In this paper we complement their results by showing that contemporaneous aggregation of independent univariate GARCH processes yields a weak GARCH process. We analyze the dependence of the parameters in the aggregate process on the parameters in the underlying models. Subsequently, we generalize this result by showing that a linear combination of variables generated by a multivariate GARCH process will also be weak GARCH and analyzing the parameters in this weak

GARCH process. Finally we derive the marginal weak processes implied by multivariate GARCH processes. The parameters in weak GARCH processes can easily be estimated consistently. Moreover simulation experiments suggest that in many cases the commonly used estimators, which are ML under the assumption that the model is strong GARCH and the conditional distribution normal, converge to values close to the weak GARCH parameters in large samples.

The plan of this paper is as follows. In section 2 we will consider aggregation of independent GARCH processes. Numerical results are presented on the GARCH parameters that arise through aggregation of a GARCH process with conditional homoskedastic noise, as well as through the aggregation of two GARCH processes with identical "persistence" parameters. In section 3 we introduce an unrestricted bivariate GARCH(1,1) model and check that this structure is preserved under linear transformation. Moreover we show how marginal processes can be derived from multivariate representations. In section 4, diagonal models, conditionally orthogonal models and factor GARCH models, which all are special cases of the general set-up in section 3, are analyzed in more detail. Numerical results are presented on the impact of marginalization on the properties of linear combinations of variables generated by these models. Section 5 contains the results of a simulation experiment on the properties of the commonly used quasi maximum likelihood estimator in cases where data are generated by aggregating independent univariate GARCH processes or by marginalizing from multivariate GARCH processes. Finally section 6 concludes. Some technicalities are outlined in appendices.

2. Contemporaneous aggregation of independent univariate GARCH processes

In this section we consider the simple case of aggregation of two independent GARCH(1,1) processes. The results derived in this section are special cases of those in sections 3 and 4, but are derived here in a more straightforward and intuitive manner. Moreover the assumption of conditional normality which will be made in sections 3 and 4 to simplify the algebra can be avoided here.

Consider variables y_{1t} and y_{2t} which are both generated by (strong) GARCH(1,1) models i.e.,

$$y_{it} = \sigma_{it} \xi_{it} \quad (i = 1,2), \tag{4}$$

$$\xi_{it} \text{ i.i.d.}, \quad E\, \xi_{it} = 0, \quad E\, \xi_{it}^2 = 1, \quad E\, \xi_{it}^4 = \kappa_i,$$

$$\sigma_{it}^2 = \nu_i + \beta_i \sigma_{i,t-1}^2 + \alpha_i y_{it-1}^2. \tag{5}$$

It is well known that substitution of $\sigma_{it}^2 = y_{it}^2 - \eta_{it}$ yields an ARMA(1,1) model for y_{it}^2,

$$y_{it}^2 = \sigma_i^2 + [1-(\alpha_i+\beta_i)L]^{-1} [1-\beta_i L]\, \eta_{it} , \tag{6}$$

where L is the lag-operator defined by $L\, y_{it} = y_{it-1}$ and where $\sigma_i^2 = E\, y_{it}^2 = \nu_i(1-\alpha_i-\beta_i)^{-1}$ and

$$\eta_{it} = y_{it}^2 - E[y_{it}^2 | I_{t-1}] = (\xi_{it}^2 - 1)\, \sigma_{it}^2. \tag{7}$$

From (6) one easily obtains

$$(y_{1t}+y_{2t})^2 = \sigma_1^2 + \sigma_2^2 + [1-(\alpha_1+\beta_1)L]^{-1}[1-\beta_1 L]\, \eta_{1t} +$$
$$+ [1-(\alpha_2+\beta_2)L]^{-1}[1-\beta_2 L]\, \eta_{2t} + 2\, y_{1t} y_{2t}. \tag{8}$$

As η_{1t}, η_{2t} and $y_{1t}y_{2t}$ are mutually uncorrelated and none of these three variables is autocorrelated, (8) implies that the sum of two independent strong GARCH(1,1) processes is weak GARCH(2,2). This result is obviously related to the well known result (see e.g. Lütkepohl (1984)) that the sum of two independent ARMA(1,1) processes is ARMA(2,2). One important difference with the ARMA case is the presence of the cross product term in the right hand side of (8), which complicates the derivation of the GARCH parameters for the aggregate series as we will see below.

Obviously equation (8) only yields an upper bound on the orders of the weak GARCH process. In two important special cases, though, aggregation of

GARCH(1,1) processes for which $\alpha_1+\beta_1 = \alpha_2+\beta_2$ and aggregation of a GARCH(1,1) process with conditionally homoskedastic white noise, the aggregate will in fact be weak GARCH(1,1). For these special cases we will derive the weak GARCH parameters in this section. The general case is discussed in section 4. The value of $\alpha_1+\beta_1$ has often been referred to as the persistence of shocks to the volatilty in the GARCH process. As this terminology can easily be critized we will put this terminology between quotation marks whenever it is used. In fact the mean lag of the ARMA model in (6) is probably a much better measure of persistence.

If the "persistence" parameters in the two independent GARCH processes which are added coincide, i.e. $\alpha_1 + \beta_1 = \alpha_2 + \beta_2 = \gamma$, equation (8) reduces to

$$[1-\gamma L]\ (y_{1t}+y_{2t})^2 = \tilde{\psi} + [1-\beta_1 L]\ \eta_{1t} + [1-\beta_2 L]\ \eta_{2t} + 2\ [1-\gamma L]\ y_{1t}y_{2t}, \quad (9)$$

with $\tilde{\psi} = (\psi_1+\psi_2)$. Because of the properties of η_{1t}, η_{2t} and $y_{1t}y_{2t}$ the right hand side will be a first order moving average process, say $(1-\lambda L)\omega_t$. Rewriting (9) yields

$$(1-\gamma L)\ (y_{1t}+y_{2t})^2 = \tilde{\psi} + (1 - \lambda L)\ \omega_t, \quad (10)$$

where

$$\lambda\ (1+\lambda^2)^{-1} = - E\ u_t u_{t-1}\ /\ E\ u_t^2,$$

$$u_t = [1-\beta_1 L]\ \eta_{1t} + [1-\beta_2 L]\ \eta_{2t} + 2\ [1-\gamma L]\ y_{1t}y_{2t},$$

$$\omega_t = (1 -\lambda L)^{-1}\ u_t.$$

Once λ is known, it is clear from the comparison with (6) that the weak GARCH parameters for $y_{1t}+y_{2t}$ are simply $(\tilde{\beta},\tilde{\alpha}) = (\lambda, \gamma-\lambda)$. Note that the value of the parameter $\alpha+\beta$ is the same in the aggregate process as in the two underlying processes. The remaining problem is the determination of λ. From (10) one has

$$E\ u_t^2 = (1+\beta_1^2)\ E\ \eta_{1t}^2 + (1+\beta_2^2)\ E\ \eta_{2t}^2 + 4\ (1+\gamma^2)\ E\ y_{1t}^2 y_{2t}^2 \tag{11}$$

and

$$-\ E\ u_t u_{t-1} = \beta_1\ E\ \eta_{1t}^2 + \beta_2\ E\ \eta_{2t}^2 + 4\ \gamma\ E\ y_{1t}^2 y_{2t}^2. \tag{12}$$

Obviously $E\ y_{1t}^2 y_{2t}^2 = \sigma_1^2 \sigma_2^2$ by independence. Moreover $E\ \eta_{it}^2$ can be expressed as a function of σ_i^2, α_i, β_i and the kurtosis of the error term $\kappa_i = E\ \xi_{it}^4$,

$$E\ \eta_{it}^2 = E\ (\xi_{it}^2 - 1)^2\ E\ \sigma_{it}^4$$

$$= (\kappa_i - 1)\ \sigma_i^4\ \{1-(\beta_i+\alpha_i)^2\}\ /\ (1-\beta_i^2-2\alpha_i\beta_i-\alpha_i^2\kappa_i), \tag{13}$$

where the second equality follows from a straightforward generalization of the way in which Bollerslev (1986) computed the fourth moment of y_{it}. As $\lambda(1+\lambda^2)^{-1} = \lambda^{-1}(1+\lambda^{-2})^{-1}$, the quadratic equation $\lambda(1+\lambda^2)^{-1} = -Eu_t u_{t-1}/Eu_t^2$ yields a unique solution for λ within the unit circle.

Some illustrative numerical results on the aggregation of independent GARCH(1,1) processes with coinciding "persistence" parameters are reported in Table 1. In the first five lines of the table the values α_i and β_i do not differ between the two processes. The results in (9)-(13) indicate that for $y_{1t}+y_{2t}$ the coefficient on the lagged variance term, β, will exceed $\beta_1 = \beta_2$ while $\alpha < \alpha_i$. The numerical values for α and β depend on the variance ratio and the kurtosis parameters. If the variances of the two processes differ a lot or if at least one of the processes has fat tails, the weak GARCH parameters in the aggregate will be close to those in the underlying processes. The last three lines in the table show that the variance ratio can in fact have a very large impact on the weak GARCH parameters. Note that in the case considered in the last three lines the fourth moments condition is violated if $\kappa_i = 9$.

Table 1 Weak GARCH parameters α and β for $y_t = y_{1t} + y_{2t}$ in (1)-(3) if y_{it} ($i=1,2$) is generated by (4)-(5), assuming $\alpha_1+\beta_1 = \alpha_2+\beta_2 = \bar{\alpha}+\bar{\beta}$.

β_1	α_1	κ_1	β_2	α_2	κ_2	σ_1^2/σ_2^2	β	α
0.90	0.05	3.00	0.90	0.05	3.00	1.00	0.920	0.030
0.90	0.05	3.00	0.90	0.05	3.00	4.00	0.912	0.038
0.90	0.05	9.00	0.90	0.05	3.00	1.00	0.910	0.040
0.90	0.05	9.00	0.90	0.05	3.00	4.00	0.903	0.047
0.90	0.05	9.00	0.90	0.05	9.00	1.00	0.907	0.043
0.50	0.35	3.00	0.80	0.05	3.00	1.00	0.569	0.281
0.50	0.35	3.00	0.80	0.05	3.00	4.00	0.516	0.334
0.50	0.35	3.00	0.80	0.05	3.00	0.25	0.705	0.145

A second special case of (8) that is worth considering explicitly is aggregation of a GARCH(1,1) variable and conditionally homoskedastic white noise. In this case (8) reduces to

$$[1-(\alpha_1+\beta_1)L]\ (y_{1t}+y_{2t})^2 = \tilde{\psi} + [\ 1-\beta_1 L\ ]\ \eta_{1t} + [\ 1-(\alpha_1+\beta_1)L\ ]\ \eta_{2t}$$

$$+\ 2\ [1-(\alpha_1+\beta_1)L]\ y_{1t}y_{2t}. \qquad (14)$$

Equation (14) shows that the agggregate will in this case be weak GARCH(1,1) and that the "persistence" parameter of the GARCH process is not affected by adding conditionally homoskedastic noise. Along the lines sketched above, the weak GARCH parameters can easily be determined as functions of (σ_1^2, α_1, β_1, σ_2^2, κ_1, κ_2). Table 2 shows, as could be expected on intuitive grounds, that the GARCH parameters are affected much more by adding noise if the signal to noise ratio is low than when it is high. Also fat tails in the GARCH process lead to a smaller impact of the noise, while fat tailed noise yields larger deviations from the underlying variance parameters.

Table 2 Weak GARCH parameters α and β for $y_t = y_{1t} + y_{2t}$ in (1)-(3) if y_{it} ($i=1,2$) is generated by (4)-(5), assuming $\alpha_2 = \beta_2 = 0$.

β_1	α_1	κ_1	κ_2	σ_1^2/σ_2^2	β	α
0.90	0.05	3.00	3.00	1.00	0.933	0.017
0.90	0.05	3.00	3.00	4.00	0.914	0.036
0.90	0.05	3.00	3.00	0.25	0.947	0.003
0.90	0.05	9.00	3.00	1.00	0.915	0.035
0.90	0.05	3.00	9.00	1.00	0.940	0.010
0.50	0.35	3.00	3.00	1.00	0.571	0.279
0.50	0.35	3.00	3.00	4.00	0.517	0.333
0.50	0.35	3.00	3.00	0.25	0.721	0.129

3. Marginalization and aggregation in multivariate GARCH models: the general case.

In the previous section we restricted ourselves to aggregation of independent GARCH processes. In many applications, e.g. to exchange rates or stock returns, the independence assumption is very strong. For that reason we therefore consider in this section the aggregation of two processes whose joint distribution belongs to a class of bivariate GARCH(1,1) models. We first check that the class is closed under linear transformation and discuss the properties of the weak GARCH processes that arise after marginalization.

The class of bivariate GARCH models that we consider is

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \Sigma_t^{1/2} \begin{bmatrix} \xi_{1t} \\ \xi_{2t} \end{bmatrix}, \qquad \begin{bmatrix} \xi_{1t} \\ \xi_{2t} \end{bmatrix} \sim \text{i.i.d. } N(0, I_2), \tag{15}$$

where

$$\Sigma_t = \begin{bmatrix} \sigma_{11t} & \sigma_{12t} \\ \sigma_{12t} & \sigma_{22t} \end{bmatrix},$$

and

$$\begin{bmatrix} \sigma_{11t} \\ \sigma_{12t} \\ \sigma_{22t} \end{bmatrix} = \begin{bmatrix} \psi_{11} \\ \psi_{12} \\ \psi_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} \sigma_{11t-1} \\ \sigma_{12t-1} \\ \sigma_{22t-1} \end{bmatrix} +$$

$$+ \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} y_{1t-1}^2 \\ y_{1t-1}y_{2t-1} \\ y_{2t-1}^2 \end{bmatrix}. \tag{16}$$

As we assume throughout that Σ_t is a positive-semidefinite matrix (see Baba et al. (1989) for sufficient conditions), it can be interpreted as $Var_{t-1}(y_t)$ where $y_t' = (y_{1t}, y_{2t})$. In the sequel we also assume that the parameters in (16) are such that fourth moments of y_t exist. In a more compact matrix notation (16) can be written as

$$s_t = \psi + B\, s_{t-1} + A\, z_{t-1}, \tag{17}$$

where $s_t' = (\sigma_{11t}, \sigma_{12t}, \sigma_{22t}) = \{vech(\Sigma_t)\}'$, $\psi' = (\psi_{11}, \psi_{12}, \psi_{22})$ and $z_t' = (y_{1t}^2, y_{1t}y_{2t}, y_{2t}^2) = \{vech(y_t y_t')\}'$. The model in (15) and (17) is a particular case of the multivariate GARCH(p,q) model considered in Bollerslev, Engle and Wooldridge (1988). If $b_{ij} = 0$ ($\forall$ i,j) one obtains the original bivariate ARCH(1) specification of Kraft and Engle (1982). Other interesting special cases are the diagonal and factor GARCH model, and models which yield conditional orthogonality. These special cases will be considered in the next section.

An important property of the bivariate GARCH model in (16) is that its structure is preserved under linear transformation (see Baba et al. (1989)). In appendix A we show that if P is a non-singular matrix, the vector $x_t = P\, y_t$ will also satisfy the bivariate GARCH model in (15) and (17) with the parameters ψ, B and A replaced by $\tilde{\psi}$, $\tilde{B}$ and $\tilde{A}$ defined as

$$\begin{aligned} \tilde{\psi} &= D^+ (P \otimes P)\, D\, \psi, \\ \tilde{B} &= D^+ (P \otimes P)\, D\, B\, D^+ (P^{-1} \otimes P^{-1})\, D, \\ \tilde{A} &= D^+ (P \otimes P)\, D\, A\, D^+ (P^{-1} \otimes P^{-1})\, D, \end{aligned} \tag{18}$$

where D is the duplication matrix defined by $vec(\Sigma_t) = D\ vech(\Sigma_t)$ and D^+ is its Moore-Penrose inverse.

A second important aspect in the analysis of the bivariate GARCH model is the derivation of the implied marginal processes. Marginalization is required as the second step in assesing the impact of contemporaneous aggregation on variables that are generated by the multivariate GARCH processes, but it is moreover of substantial interest in its own right and can be used to facilitate the specification of multivariate GARCH processes. In order to discuss marginalization define $\eta_t = z_t - E_{t-1}[z_t]$. Substitution in (17) yields the trivariate ARMA(1,1) process

$$[\ I - (A+B)\ L\ ]\ z_t = \psi + [\ I - B\ L\ ]\ \eta_t. \tag{19}$$

The marginal process for y_{1t} can be obtained from the first row of the final form of the model (see e.g. Zellner and Palm (1974)),

$$\det \{I-(A+B)L\}\ z_t = \psi^* + \text{adj}\ \{I-(A+B)L\}\ (I-BL)\ \eta_t \tag{20}$$

where $\psi^* = \text{adj}\ \{I-(A+B)L\}\ \psi$ and where det and adj refer to the determinant and adjoint matrix respectively. As the determinant is a third order polynomial in the lag operator and the elements of the adjoint matrix are second order polynomials in the lag operator, (20) shows that the marginal process for y_{1t} will be at most weak GARCH(3,3). As we shall see in the next section, though, in many cases of interest a reduction of these orders can be obtained. Of course a reduction through coincidental parameter cancellation is always a possibility.

To find the weak GARCH parameters, we proceed as in (10) and write the ARMA(3,3) model for y_{1t}^2 as

$$(1-\pi_1L-\pi_2L^2-\pi_3L^3)\ y_{1t}^2 = c + (1-\lambda_1L-\lambda_2L^2-\lambda_3L^3)\ \omega_t; \quad E\ \omega_t^2 = \sigma_\omega^2. \tag{21}$$

The autoregressive coefficients in the ARMA model in squares, which correspond to the persistence parameters in the weak GARCH model, can easily be found using a well known result for characteristic equations,

$$\det\{I-(A+B)L\} = -\det(A+B)\ L^3 + p(A+B)\ L^2 - tr(A+B)\ L + 1 \qquad (22)$$

with

$$p(A+B) = \det\ (A^-_{11}+B^-_{11}) + \det\ (A^-_{22}+B^-_{22}) + \det\ (A^-_{33}+ B^-_{33}) \qquad (23)$$

where A^-_{ij} is the 2x2 matrix obtained from A by eliminating row i and column j. The determination of σ^2_ω and of the MA coefficients λ_i is less straightforward. These coefficients can be obtained by equating the variance and first three autocovariances of $(1-\lambda_1 L-\lambda_2 L^2-\lambda_3 L^3)\omega_t$ with those of the first element of the right hand side of (20). Given that η_t is serially uncorrelated with zero mean, only its contemporaneous variance-covariance matrix is required. This is derived in appendix B.

4. Marginalization and aggregation in multivariate GARCH models: special cases.

Given the large number of parameters involved, restrictions have usually been imposed in order to estimate the bivariate GARCH model. Three special cases will be analyzed in three subsections: the diagonal model, models in which y_{1t} and y_{2t} are conditionally orthogonal and the factor GARCH model.

4.1 The diagonal GARCH model

A first special case that has been popular is the diagonal GARCH model estimated e.g. by Attanasio and Edey (1987) and Bollerslev, Engle and Wooldridge (1988) where A and B are both assumed to be diagonal. If moreover $a_{22} = b_{22} = 0$ the independent case that has been analyzed in section 2 is obtained. Equation (20) shows that the marginal process for y_{1t} arising from a diagonal GARCH model will be weak GARCH with parameters $(\alpha,\beta) = (a_{11},b_{11})$, irrespective of the fact whether the two processes are independent, i.e. whether $a_{22} = 0$ and $b_{22} = 0$, or not. The stronger result that the marginal process arising from a diagonal model will be strong GARCH can easily be verfied directly.

The general results in section 3 show that linear combinations $\delta_1 y_{1t}+\delta_2 y_{2t}$ will be weak GARCH(2,2) if y_{1t} and y_{2t} are generated by independent GARCH(1,1) processes, which is the result that has been obtained in section 2 along different lines. Linear combinations of variables generated by the diagonal GARCH model on the other hand will be weak GARCH(3,3) in general as there is no reason to expect cancelling roots in (21) in this case. This is caused by the fact that the class of diagonal GARCH processes is not closed under linear transformation.

4.2 The conditionally orthogonal case

A second special case arises when y_{1t} and y_{2t} are conditionally orthogonal, so that $\sigma_{12t} = 0$ $(\forall t)$ i.e. $a_{21} = a_{22} = a_{23} = b_{21} = b_{22} = b_{23} = b_{12} = b_{32} = \psi_{12} = 0$. The model that arises is (15) with $\sigma_{12t} = 0$,

$$\sigma_{11t} = \psi_{11} + b_{11}\sigma_{11t-1} + b_{13}\,\sigma_{22t-1} + $$

$$+ a_{11}\,y^2_{1t-1} + a_{12}\,y_{1t-1}y_{2t-1} + a_{13}\,y^2_{2t-1} \tag{24}$$

and an analogous expression for σ_{22t}. It is not hard to check that in general the marginal process for y_{1t} will be weak GARCH(2,2). Obviously the model reduces to GARCH(1,1) if $b_{13} = a_{12} = a_{13} = 0$. Interestingly this arises in other cases as well, e.g. if y_{2t} is homoskedastic, i.e. $b_{31} = b_{33} = a_{31} = a_{32} = a_{33} = 0$. Also, if y_{2t} only depends on its own squared values $(b_{31} = a_{31} = a_{32} = 0)$ and moreover $b_{13} = a_{13} = 0$, weak GARCH(1,1) is obtained as the marginal process for y_{1t} whether or not its conditional variance depends on lagged values of $y_{1t}y_{2t}$. The results in section 3 can be used to show that linear combinations $\delta_1 y_{1t}+\delta_2 y_{2t}$ of conditionally orthogonal processes y_{1t} and y_{2t} will be weak GARCH(2,2), as in the independent case. Remember that in the general case the linear combination is GARCH(3,3). In the independent case the marginal process will be GARCH(1,1), while it is GARCH(2,2) in the conditionally orthogonal case. Similar the order of the GARCH process of linear combinations of the two variables distinguishes conditionally orthogonal processes from the general case. The above results can be used in principle to distinguish between independent and conditionally orthogonal processes using univariate methods only, although this may be difficult to do in practice.

4.3 The factor GARCH model

The restrictions in the third important special case, the factor GARCH model, are more subtle than in the previous two subsections. In the one factor case, the factor GARCH model as defined by Engle (1987) and Engle, Ng and Rothschild (1990) imposes

$$\Sigma_t = \Psi - \varphi\lambda'\Psi\varphi'\lambda + \varphi \, Var_{t-1}(\lambda' y_t)\varphi', \tag{25}$$

and

$$Var_{t-1}(\lambda' y_t) = \lambda'\Psi\lambda + \tilde{\beta}\, Var_{t-2}(\lambda' y_{t-1}) + \tilde{\alpha}\,(\lambda' y_{t-1})^2, \tag{26}$$

where $\varphi' = (\varphi_1, \varphi_2)$ and $\lambda' = (\lambda_1, \lambda_2)$ are two dimensional vectors which are normalized such that $\varphi'\lambda = 1$. It is not difficult to see that this model is a special case of the bivariate GARCH model in (16) with

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \tilde{\alpha} \begin{bmatrix} \varphi_1^2\lambda_1^2 & 2\varphi_1^2\lambda_1\lambda_2 & \varphi_1^2\lambda_2^2 \\ \varphi_1\varphi_2\lambda_1^2 & 2\varphi_1\varphi_2\lambda_1\lambda_2 & \varphi_1\varphi_2\lambda_2^2 \\ \varphi_2^2\lambda_1^2 & 2\varphi_2^2\lambda_1\lambda_2 & \varphi_2^2\lambda_2^2 \end{bmatrix}, \tag{27}$$

and $B = \tilde{\beta}\tilde{\alpha}^{-1} A$. According to (26), the "factor representing portfolio" $\lambda' y_t$ is generated by a strong GARCH(1,1) model, with parameters $\lambda'\Psi\lambda$, β and α. The marginal process for any linear combination $\delta' y_t$ where y_t is generated by a factor GARCH model can be derived using the results in appendices A and B.

An alternative extension of standard (i.e. conditionally homoskedastic) factor analysis models to time-varying variances was introduced by Diebold and Nerlove (1989) and extended by King, Sentana and Wadhwani (1991). Their model is a standard latent factor model with unobservable common factors which are generated by strong GARCH processes,

$$y_t = \varphi f_t + w_t \tag{28}$$

$$\begin{bmatrix} f_t \\ w_t \end{bmatrix} = \begin{bmatrix} \omega_t & 0 \\ 0 & \Phi \end{bmatrix} \begin{bmatrix} f_t^* \\ w_t^* \end{bmatrix} \text{ with } \begin{bmatrix} f_t^* \\ w_t^* \end{bmatrix} \sim \text{i.i.d. } N(0, I_3) \tag{29}$$

and

$$\omega_t = \vartheta + \tilde{\beta}\, \omega_{t-1} + \tilde{\alpha}\, f_{t-1}^2 . \tag{30}$$

The conditionally heteroskedastic latent factor model in (28)-(30) is not quite the same as the factor GARCH in (15), (25) and (26). However, both models are very closely related. In Sentana (1993) it is shown that factor GARCH models can be written as conditionally heteroskedastic latent factor models in which the latent factors are weak GARCH processes. Moreover it is shown there that the conditionally heteroskedastic factor model can be written in the form of the factor GARCH model, provided the assumption in the factor GARCH model that the factor is generated by a strong GARCH model is replaced by the assumption that it is generated by a weak one.

In order to illustrate the results on marginalization and linear transformation in factor GARCH models, we computed the weak GARCH parameters for several linear combinations $\delta_1 y_{1t} + \delta_2 y_{2t}$ for a number of values of the parameters in the factor GARCH model. First of all we normalized in such a way that $E\, f_t^2 = 1$ and $E\, y_{it}^2 = 1$ $(i=1,2)$. It can be checked that given these normalizations all parameters in (25) and (26), and thereby the weak GARCH parameters, are fully determined by $\tilde{\alpha}$ and $\tilde{\beta}$ in (26) and the signal to noise ratios $R_i = Var(\varphi_i f_t) / Var(y_{it}) = \varphi_i^2$ in (28). The results are presented in table 3. For two specifications of the conditional heteroskedasticity in the factor and two pairs of signal to noise ratios, the weak GARCH parameters for the marginal processes for y_{1t} and y_{2t} as well as for linear combinations are presented. We have already noted that $\lambda' y_t$ will be strong GARCH with parameters $\tilde{\beta}$ and $\tilde{\alpha}$. The value of λ_2/λ_1 which corresponds to $R_1 = .75$ and $R_2 = .70$ is .913, while a value of .236 is implied by $R_1 = .95$ and $R_2 = .10$. The results in table 3 show that when the weight on y_{2t} is increased keeping the weight on y_{1t} fixed to unity, the weak GARCH parameter

Table 3. Weak GARCH parameters α and β for $\delta_1 y_{1t}+\delta_2 y_{2t}$ in (1)-(3) if the data are generated by the factor GARCH model in (15), (25) and (26).

GARCH param. of factor		signal to noise ratios		weight in linear comb.		weak GARCH parameters	
$\tilde{\beta}$	$\tilde{\alpha}$	R_1	R_2	δ_1	δ_2	β	α
.80	.10	.75	.70	1	0	.834	.067
				1	.5	.804	.096
				1	.913	.800	.100
				1	1	.800	.100
				.5	1	.807	.093
				0	1	.841	.059
.10	.40	.75	.70	1	0	.216	.284
				1	.5	.113	.387
				1	.913	.100	.400
				1	1	.100	.400
				.5	1	.123	.377
				0	1	.240	.260
.80	.10	.95	.10	1	0	.807	.093
				1	.236	.800	.100
				1	.5	.806	.094
				1	1	.833	.067
				.5	1	.868	.032
				0	1	.898	.002
.10	.40	.95	.10	1	0	.122	.378
				1	.236	.100	.400
				1	.5	.121	.379
				1	1	.211	.289
				.5	1	.349	.152
				0	1	.491	.009

α will increase upto the value $\tilde{\alpha}$, which is reached when the weight on y_{2t} equals λ_2/λ_1. Further increases in the weight on y_{2t} appear to yield smaller values for α. This property can in principle be used in finding factor representing portfolios using univariate analysis only. Note that e.g. in Engle, Ng and Rothschild (1990) the relative weights in the factor representing portfolios are assumed to be known a priori.

5. Probability limit of the standard QMLE estimator

In the previous sections we have described how weak GARCH parameters can be obtained for sums of independent strong GARCH processes as well as for marginal processes implied by a number of multivariate GARCH processes. The weak GARCH parameters can of course easily be estimated consistently from the autocorrelations of the squared observations. The estimators obtained in this way are probably not very efficient. Moreover many empirical studies in the literature are based on the use of a quasi maximum likelihood estimator (QMLE) under the assumption of a strong GARCH process and conditional normality. For both reasons we consider the properties of this QMLE estimator in this section. In Drost and Nijman (1992) it was noted that if data were generated by strong GARCH processes at one frequency and QMLE estimates were computed at lower frequencies, the parameter estimates tended to be very close to the weak GARCH parameters. In tables 4 and 5 we report results on a similar simulation experiment where we concentrate on aggregation and marginalization. Note that Weiss (1986) has shown that the QMLE estimator is consistent under weaker conditions that the validity of conditionally normal strong GARCH assumption, which are referred to as the semi-strong GARCH conditions in Drost and Nijman (1992). The properties of the QMLE estimator under weak GARCH have not yet been established, so that we have to rely on Monte Carlo results.

In the first row of table 4 e.g. we consider QMLE estimation of the sum of independent conditionally normal GARCH processes with variance parameters $\beta_i = 0.9$ and $\alpha_i = 0.05$ and equal (unconditional) variances. In the last two columns of the table we present the averages over 20 samples of the QMLE estimates obtained from 80,000 generated data. Given the very low

Table 4: Weak GARCH parameters and estimated plim of standard QMLE estimator for $y_t = y_{1t} + y_{2t}$ if y_{it} (i=1,2) is generated by (4)-(5)

β_1	α_1	κ_1	β_2	α_2	κ_2	σ_1^2/σ_2^2	β	α	plim $\hat{\beta}$	plim $\hat{\alpha}$
0.90	0.05	3.00	0.90	0.05	3.00	1.00	0.920	0.030	0.920	0.029
0.90	0.05	3.00	0.90	0.05	3.00	4.00	0.912	0.038	0.910	0.037
0.90	0.05	9.00	0.90	0.05	3.00	1.00	0.910	0.040	0.916	0.036
0.90	0.05	9.00	0.90	0.05	3.00	4.00	0.903	0.047	0.909	0.041
0.90	0.05	9.00	0.90	0.05	9.00	1.00	0.907	0.043	0.909	0.040
0.50	0.35	3.00	0.80	0.05	3.00	1.00	0.569	0.281	0.690	0.125
0.50	0.35	3.00	0.80	0.05	3.00	4.00	0.516	0.334	0.581	0.234
0.50	0.35	3.00	0.80	0.05	3.00	0.25	0.705	0.145	0.775	0.062
0.50	0.35	3.00	0	0	3.00	1.00	0.571	0.274	0.685	0.112
0.50	0.35	3.00	0	0	3.00	4.00	0.517	0.333	0.582	0.232
0.50	0.35	3.00	0	0	3.00	0.25	0.721	0.129	0.783	0.038

Table 5: Weak GARCH parameters and estimated plim of standard QMLE estimator for $\delta_1 y_{1t} + \delta_2 y_{2t}$ if data are is generated by the factor GARCH model in (17), (27) and (28)

GARCH param. of factor		signal to noise ratios		weight in linear comb.		weak GARCH parameters			
$\tilde{\beta}$	$\tilde{\alpha}$	R_1	R_2	δ_1	δ_2	β	α	plim $\hat{\beta}$	plim $\hat{\alpha}$
.80	.10	.75	.70	1	0	0.834	0.067	0.820	0.075
				1	.5	0.804	0.096	0.803	0.096
				1	.913	0.800	0.100	0.800	0.100
				1	1	0.800	0.100	0.799	0.100
				.5	1	0.807	0.093	0.805	0.094
				0	1	0.841	0.059	0.828	0.070
.80	.10	.95	.10	1	0	0.807	0.093	0.808	0.092
				1	.236	0.800	0.100	0.804	0.098
				1	.5	0.806	0.094	0.807	0.092
				1	1	0.833	0.067	0.829	0.067
				.5	1	0.868	0.032	0.869	0.031
				0	1	0.898	0.002	0.807	0.003

variability of the estimates over the 20 samples these must be very precise estimates of the true probability limit of the QMLE estimators. The estimated probability limit is 0.920 for $\hat{\beta}$ and 0.029 for $\hat{\alpha}$, which is very close to the weak GARCH parameters that are presented in table 1 and in columns eight and nine of table 5. The same holds true for the cases presented in lines 2 to 5 in the table. These result are all in line with the evidence on temporal aggregation in Drost and Nijman (1992). Table 4 also contains a number of cases however where the estimated probability limit of the QMLE estimators differs substantially from the weak GARCH parameters. The probabiliy limit of the QMLE estimate of the "persistence" parameter $\hat{\alpha} + \hat{\beta}$ is roughly in line with the value suggested by the weak GARCH parameters, although the underestimation is clearly significant. The plim of the estimated individual coefficients differs considerably from the weak GARCH parameters. Of course the question why the QMLE estimators is approximately consistent in some cases and clearly inconsistent in others is an important topic for future research, that is however outside the scope of this paper.

In table 5 results similar to those in table 4 are reported for the factor GARCH model. The estimated probability limits are, for the cases considered, usually close to the weak GARCH parameters. In particular the fact that α is largest at the linear combination which yields a strong GARCH process is reflected in the simulated results. The results in the last line of the table are probably caused by the fact that β is hardly identified if α is very close to zero.

6. Conclusions

In this paper we have discussed contemporaneous aggregation of independent univariate GARCH processes as well as marginalization and contemporaneous aggregation in more general multivariate GARCH processes. We have shown that the class of strong GARCH processes, which is typically assumed in applied work, is not closed under these transformations. We have also shown that the weak GARCH conditions are satisfied in all these cases and have discussed the relation between the various models and their parametervalues. The results explain why GARCH will be found e.g. when analyzing exchange rate returns which are the sum of two underlying conditionally heteroskedastic exchange rate returns or when analzying the return on a portfolio of stocks which are conditionally heteroskedastic.

Appendix A The impact of linear transformations on the parameters in the bivariate GARCH model

In order to derive the impact of linear transformation on the parameters in the bivariate GARCH model we use the duplication matrix D defined by $vec(\Sigma_t) = D\ vech(\Sigma_t)$. It is straightforward to check that $vech(\Sigma_t) = D^{+}vec(\Sigma_t)$. Equation (17) can now be rewritten as

$$vec(\Sigma_t) = D\gamma + D\ B\ D^{+}\ vec(\Sigma_{t-1}) + D\ A\ D^{+}\ vec(y_{t-1}y'_{t-1}) \qquad (A.1)$$

If $x_t = P\ y_t$ where P is a non-singular matrix, and $\Sigma^{*}_t = Var_{t-1}(x_t)$ the properties of the vec operator imply

$$vec(\Sigma^{*}_t) = vec(P\Sigma_t P') = (P\otimes P)\ vec(\Sigma_t)$$

and

$$\begin{aligned} vec(\Sigma^{*}_t) &= (P\otimes P)D\gamma + (P\otimes P)DBD'Vec(\Sigma_{t-1}) + (P\otimes P)\ DAD^{+}\ vec(y_{t-1}y'_{t-1}) \\ &= (P\otimes P)D\gamma + (P\otimes P)DBD^{+}(P^{-1}\otimes P^{-1})vec(\Sigma^{*}_{t-1}) \\ &\qquad + (P\otimes P)\ DAD^{+}\ (P^{-1}\otimes P^{-1})\ vec(x_{t-1}x'_{t-1}). \end{aligned} \qquad (A.2)$$

Premultiplying (A.2) with the Moore-Penrose inverse of D yields the required result.

Appendix B Determination of the variance of η_t in (19).

In this appendix we will derive the variance-covariance matrix of η_t defined in (19) which is required to determine the weak GARCH parameters in the marginal process for y_{1t} in (21). Note first of all that $Var(\eta_t) = E\, \eta_t\eta_t'$ as $E\, \eta_t = 0$. Moreover as $E_{t-1}z_t = s_t$ the law of iterated expectations yields

$$Var(\eta_t) = E\, \eta_t\eta_t' = E\, z_tz_t' - E\, s_ts_t'. \quad (A.3)$$

The typical element of $Var(\eta_t)$ therefore is $E(E_{t-1}(y_{it}y_{jt}y_{kt}y_{\ell t})-\sigma_{ijt}\sigma_{k\ell t})$. But because the conditional distribution is assumed to be normal in sections 3 and 4 (see (15)) we can use a well known result on fourth moments of a bivariate normal variable $E_{t-1}(y_{it}y_{jt}y_{kt}y_{\ell t}) = \sigma_{ijt}\sigma_{k\ell t} + \sigma_{ikt}\sigma_{j\ell t} + \sigma_{i\ell t}\sigma_{jkt}$ to show that

$$E\, \eta_{ijt}\eta_{k\ell t} = E\, (\sigma_{ikt}\sigma_{j\ell t} + \sigma_{i\ell t}\sigma_{jkt}). \quad (A.4)$$

The remaining problem is to determine $h_{ijk\ell} = E\, \sigma_{ijt}\sigma_{k\ell t}$. We solve this problem by constructing a six dimensional linear system with six unknowns, $h = E\, vech(s_ts_t')$. From (17) we know that

$$E\, s_ts_t' = \psi\psi' + \psi\, Ez_{t-1}'A' + A\, Ez_{t-1}\, \psi' + \psi\, Es_{t-1}'B' + B\, Es_{t-1}\, \psi' + \quad (A.6)$$

$$+ A\, Ez_{t-1}z_{t-1}'\, A' + B\, Es_{t-1}s_{t-1}'\, B' + A\, Ez_{t-1}s_{t-1}'\, B + B\, Es_{t-1}z_{t-1}'\, A'.$$

Now note first of all that $Ez_{t-1} = E\, s_{t-1} = [I-(A+B)]^{-1}\psi$. Moreover, by the law of iterated expectations, $E\, s_{t-1}z_{t-1}' = E\, s_{t-1}s_{t-1}$. As finally $E\, z_{t-1}z_{t-1}'$ is also a linear function of the six unknowns in h, as we have seen above, (A.9) yields a six dimensional linear system for $vech(E\, s_ts_t')$ in the six unknown elements of h.

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