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The fill rate service measure in an (s, Q) inventory system with order splitting

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Abstract

In this paper we derive expressions for the well-known fill rate service measure (P_2-service measure) and the fraction of the time the physical stock is positive (P_3-service measure) for the (s, Q) inventory model with order splitting. When order splitting is applied, replenishment orders are split equally among n suppliers. Demand is modelled as a compound renewal process. Lead times of the suppliers are independent and identically mixed Erlang distributed random variables. An approximation algorithm is derived to compute the optimal value of the reorder point s subject to a service level constraint. The algorithm is verified by simulation.

1 Introduction

Since the introduction of order splitting by Sculli and Wu (1981) many papers considering this vendor management strategy have appeared. The order splitting strategy or multiple sourcing strategy is the partitioning of a replenishment order among two or more suppliers. Order splitting is advocated for the purpose of reducing lead time uncertainties, whereby safety stocks are reduced (see, for example, Sculli and Wu (1981), Pan and Liao (1989), Kelle and Silver (1990a, 1990b), and Guo and Ganeshan (1995)). Other papers show that due to delayed partial deliveries order splitting can decrease the inventory holding cost

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(see, for example, Zhao and Lau (1992), Lau and Zhao (1993), Lau and Lau (1994), and Chiang and Chiang (1996)).

When order splitting is applied, replenishment orders consist of n partial deliveries, each defining its own sub-replenishment cycle. When the lead times of the partial deliveries are independent and (non)-identically distributed random variables, the time until the first partial delivery arrives is equal to the first order statistic of n independent and (non-)identically distributed random variables.

In this paper we will consider the (s, Q) inventory model with order splitting. In a practical situation it is often difficult to determine the value of the shortage costs. Then to find values for the optimal control variables often the expected ordering plus holding costs are minimized subject to a service level constraint (see, e.g, Hadley and Whitin (1963, p. 217), Tijms and Groenvelt (1984), Chiang and Chiang (1996)). Consequently, the service level contraint can be used to determine the reorder point s for given values of Q and n. In this paper we vary for the service level constraint between the fraction of the demand delivered directly from stock (P_2) and the fraction of the time the physical stock is positive (P_3).

The P_2 service measure is well-studied and widely applied in practice. However, in the order splitting literature the service measure is largely unexplored. The P_2 service measure is only discussed in Chiang and Benton (1994) and Chiang and Chiang (1996). The expressions derived by Chiang and Chiang (1996) are based on a model with deterministic lead times and normally distributed demand. Chiang and Benton (1994) consider at most two suppliers with shifted exponential lead times, and normally distributed demand.

The P_3 service measure or ready rate finds common application in the case of equipment used for emergency purposes (Silver and Peterson (1985, p.265)). In this paper we will derive an expression for this measure in an order splitting environment. Moreover, this service measure naturally arises from a necessary condition when minimizing the sum of the expected ordering, holding and shortage costs, where the shortage costs are proportional to the expected average backlog level (see, Janssen and de Kok (1997)).

The above discussed performance measures are derived under very general assumptions for the demand and lead time process. Demand is modelled as a compound renewal process, and lead times of the suppliers are independent and identically mixed Erlang distributed random variables. The compound renewal process is a powerful tool to model real life demand processes, whereas with mixtures of Erlang distribution we can model a wide variety of lead time distributions.

Regarding the literature, most papers on order splitting consider constant demand models or consider at most two suppliers. In that sense these models are special cases of the model presented in this paper. Secondly, we focus on performance measures that are often used in practice, such as the fill rate and the fraction of the time the physical stock is positive. Since stockouts may occur more than once in the same replenishment cycle, we do not think that the often used non-stock out probability (the so-called P_1 service measure) is suitable in a multiple sourcing environment.

The paper is organized as follows. In section 2 the model assumptions are discussed and expressions for the performance measures are derived. Section 3 deals with some computational aspects of the derived performance measures and we present an algorithm to actually calculate these measures. In section 4 the algorithm is verified by discrete event simulation, and we will compare our results with results from Chiang and Benton (1994). Finally we summarize our findings in section 5.

2 The model description

In this single echelon inventory model with order splitting we assume that the demand process is a compound renewal process. I.e., the interarrival times of customers can be described by the sequence $\{A_i\}_{i=1}^{\infty}$ of independent and identically distributed (i.i.d.) random variables with a common distribution function F_A, where A_i represents the time between the arrival of the i-th and $(i-1)$-th customer after time epoch 0. The demand sizes of the customers are described by the sequence $\{D_i\}_{i=1}^{\infty}$ of i.i.d. random variables with a common distribution function F_D, where D_i represents the demand size of the i-th customer after time epoch 0. The sequence $\{D_i\}_{i=1}^{\infty}$ is independent of $\{A_i\}_{i=1}^{\infty}$.

Shortages are backordered, and replenishment decisions are based on the inventory position, being defined as the total stock on hand plus on order minus the total stock backordered. The replenishment strategy that is considered is the continuous review (s, Q) policy. I.e., as soon as the inventory position drops below the reorder point s an amount of Q is ordered, such that the inventory position after ordering is between s and $s+Q$. Hence, we implicitly assume that always an amount of exactly Q is ordered. A replenishment order is equally divided among n different suppliers. The suppliers have independent and identically distributed lead times with a common distribution function G. If we rearrange the realisations of the lead times of the n partial deliveries in an increasing order, we get the order statistics. These order statistics are denoted by
$L_{1:n} \leq L_{2:n} \leq \ldots \leq L_{n:n}$, with distribution functions $G_{k:n}$ for $k = 1, \ldots, n$.

It is assumed that deliveries of two successive replenishment orders (each consisting of n partial deliveries) do not cross in time. Thus, the last partial delivery of a replen-

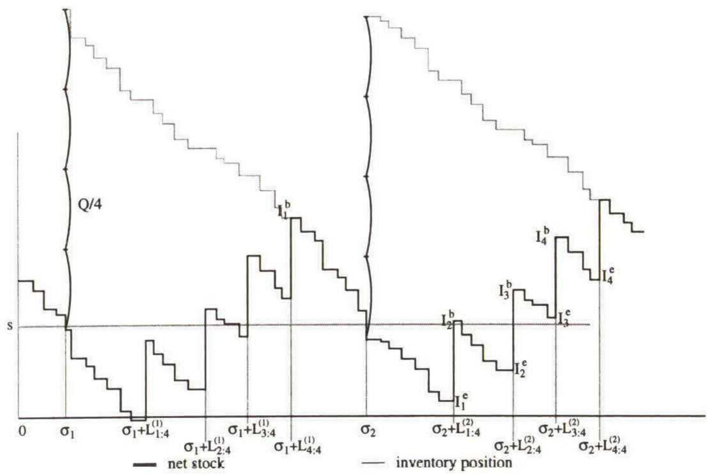

Figure 1: Evolution of the net stock and inventory position during a replenishment cycle for $n = 4$.

ishment order arrives before any partial delivery of a subsequent replenishment order. A replenishment cycle is defined as the time period between two successive **last** arrivals of partial deliveries of two successive replenishment orders. The k-th sub-replenishment cycle of an arbitrary replenishment cycle is defined as the time period between the arrival of the $(k-1)$-th partial delivery and the k-th partial delivery ($k \in \{2, \ldots, n\}$). The first sub-cycle is defined as the time period between the arrival of last partial delivery of the replenishment cycle which preceded the arbitrary replenishment cycle and the arrival of the first partial delivery of the tagged replenishment cycle.

We are interested in the following long-run performance measures:

$P_2(s, Q, n)$ the fill rate (the fraction of demand directly delivered from stock);

$P_3(s, Q, n)$ the fraction of the time the physical stock is positive.

The renewal reward theorem (see, for example, Tijms (1994)) enables us to derive expressions for the long-run performance measures by deriving the related performance measures derived for an arbitrary replenishment cycle.

Let zero be an arbitrary moment in time, and denote the j-th ordering epoch after zero by σ_j. Let $D(t_1, t_2)$ be the total demand during $(t_1, t_2]$, U_j the undershoot under s at σ_j. $L_{k:n}^{(j)}$ denotes the lead time of the k-th partial delivery in the j-th replenishment cycle after zero. Consider the second replenishment cycle after zero, that is interval $(\sigma_1 + L_{4:4}^{(1)}, \sigma_2 + L_{4:4}^{(2)})$ in Figure 1. Define the net stock as the stock on hand minus the total stock backordered. Then we denote for $k \in \{1, 2, \ldots, n\}$, I_k^b as the net stock at the beginning of the k-th sub-cycle in the second replenishment cycle after zero (just after the partial delivery arrived), and I_k^e as the net stock at the end of the k-th sub-cycle in the second replenishment cycle (just before the partial delivery arrives). Then it can be seen that (see Figure 1):

$$
\begin{aligned}
I_1^b &= s - U_1 + Q - D(\sigma_1, \sigma_1 + L_{n:n}^{(1)}); \\
I_1^e &= s - U_2 - D(\sigma_2, \sigma_2 + L_{1:n}^{(2)}); \\
I_k^b &= s - U_2 + \tfrac{k-1}{n} Q - D(\sigma_2, \sigma_2 + L_{k-1:n}^{(2)}), \quad k \in \{2, 3, \ldots, n\}; \\
I_k^e &= s - U_2 + \tfrac{k-1}{n} Q - D(\sigma_2, \sigma_2 + L_{k:n}^{(2)}), \quad k \in \{2, 3, \ldots, n\}.
\end{aligned}
$$

Since the demand process is a compound renewal process and the lead times are i.i.d., it can be seen that $U_1 \stackrel{d}{=} U_2 \stackrel{d}{=} U$, and $D(\sigma_1, \sigma_1 + L_{n:n}^{(1)}) \stackrel{d}{=} D(\sigma_2, \sigma_2 + L_{n:n}^{(2)})$, where $\stackrel{d}{=}$ denotes equality in distribution. Hence,

$$
I_1^b \stackrel{d}{=} s - U_2 + Q - D(\sigma_2, \sigma_2 + L_{n:n}^{(2)}).
$$

For ease of notation we will suppress the indices 2 in σ_2, U_2 and $L_{k:n}^{(2)}$.

First we will derive an expression for $P_2(s, Q, n)$. The expected shortage during the k-th sub-cycle (denoted by $\beta_k(s, Q, n)$) is given by the expected shortage at the end of the sub-cycle minus the shortage at the beginning (the last term for avoiding double counting), hence

$$
\beta_k(s, Q, n) = \mathbb{E}(-I_k^e)^+ - \mathbb{E}(-I_k^b)^+ \tag{1}
$$

where $x^+ = \max\{0, x\}$.

Since the demand and lead time process are stationary it can be shown that the total demand during a complete replenishment cycle is equal to Q. Hence,

$$
P_2(s, Q, n) = 1 - \frac{1}{Q} \sum_{k=1}^{n} \beta_k(s, Q, n). \tag{2}
$$

By using renewal theory we can derive an expression for $P_3(s, Q, n)$ (see Appendix 1), namely

$$
\begin{aligned}
P_3(s, Q, n) &= \frac{1}{2}(c_A^2 - 1) \mathbb{E} D \sum_{k=1}^{n} \frac{\mathbb{P}(I_k^b < 0) - \mathbb{P}(I_k^e < 0)}{Q} \\
&\quad + \sum_{k=1}^{n} \frac{\mathbb{E}(I_k^b + U)^+ - \mathbb{E}(I_k^e + U)^+}{Q}.
\end{aligned} \tag{3}
$$

Note that $I_k^e + U = s + \frac{k-1}{n}Q - D(\sigma, \sigma + L_{k:n})$. For situations, in which the undershoot is negligible and the demand process is a compound Poisson process it can be seen that $P_2(s, Q, n)$ and $P_3(s, Q, n)$ are equal.

We did not use the fact that the lead times of the partial deliveries are identically distributed. Hence, (1) and (2) are also valid for non-identically distributed lead times. However, it is well-known that the moments and the distribution function of the order statistics for non-identically distribution random variables are quite complex, see Balakrishnan (1988). For computational convenience we therefore restricted ourselves to identically mixed Erlang distributed lead times for the different suppliers.

3 Computational aspects

A versatile class of distribution functions is the class of mixtures of two Erlang distributions (denoted by ME distributions), i.e.

$$f(x) = \sum_{j=1}^{2} p_j \mu_j^{k_j} \frac{x^{k_j - 1}}{(k_j - 1)!} e^{-\mu_j x}, \quad x \geq 0, \tag{4}$$

where $p_1 \geq 0$, $p_2 \geq 0$, $p_1 + p_2 = 1$, $k_1, k_2 \in \mathbb{N}$.

In Tijms (1994, p.358) formulas are given to fit a ME distribution on a positive random variable based on the first two moments of that variable. When X and Y are ME distributed and $z \in \mathbb{R}$, then closed form expressions for $\mathbb{E}(X - z)^+$, $\mathbb{E}((X - z)^+)^2$, $\mathbb{E}(X - Y)^+$ and $\mathbb{E}((X - Y)^+)^2$ exist.

In the model presented in section 2, we assumed that F_A, F_D and G are known. Expressions (1) and (2) contain the distributions of $D(\sigma, \sigma + L_{k:n})$ $(k = 1, \ldots, n)$ and the distribution of the undershoot U. In general these distribution functions are hard to obtain from F_A, F_D and G. To avoid this problem, we assume that $D(\sigma, \sigma + L_{k:n}) + U$ and $D(\sigma, \sigma + L_{k:n})$ $(k = 1, \ldots, n)$ are ME distributed. So, we only need the first two moments of $D(\sigma, \sigma + L_{k:n}) + U$ and $D(\sigma, \sigma + L_{k:n})$ $(k = 1, \ldots, n)$ to calculate the expressions (1) and (2) for given values of s, Q, and n. Since U is independent of $D(\sigma, \sigma + L_{k:n})$ it is sufficient to find expressions for the moments of U and $D(\sigma, \sigma + L_{k:n})$ separately.

Now we use the fact that the distribution function of the undershoot is approximately equal to the stationary residual lifetime distribution with respect to F_D, when $Q \geq Cond(D)$, (see Tijms (1994, p.14)). For a positive random variable X with finite moments $\mathbb{E}X$, $\mathbb{E}X^2$, and where c_X represents the coefficient of variation of X, $Cond(X)$

is defined as

$$Cond(X) = \begin{cases} \frac{3}{2}c_X^2 \mathbb{E}X & \text{if } c_X^2 > 1; \\ \mathbb{E}X & \text{if } 0.2 < c_X^2 \leq 1; \\ \frac{1}{2c_X}\mathbb{E}X & \text{if } 0 < c_X^2 \leq 0.2. \end{cases} \tag{5}$$

Then using results from renewal theory gives

$$\mathbb{E}U \simeq \frac{\mathbb{E}D^2}{2\mathbb{E}D}, \tag{6}$$

$$\mathbb{E}U^2 \simeq \frac{\mathbb{E}D^3}{3\mathbb{E}D}. \tag{7}$$

The first two moments of $D(\sigma, \sigma + L_{k:n})$ are given by the well-known results

$$\mathbb{E}D(\sigma, \sigma + L_{k:n}) = \mathbb{E}N(\sigma, \sigma + L_{k:n})\mathbb{E}D, \tag{8}$$

$$\mathbb{E}D^2(\sigma, \sigma + L_{k:n}) = \mathbb{E}N(\sigma, \sigma + L_{k:n})\sigma^2(D) + \mathbb{E}N^2(\sigma, \sigma + L_{k:n})(\mathbb{E}D)^2, \tag{9}$$

where $N(\sigma, \sigma + L_{k:n})$ denotes the number of customer arrivals during $(\sigma, \sigma + L_{k:n})$. Recall, that σ is an order epoch. Hence, a customer arrived at epoch σ. Therefore we can use the following asymptotic expressions (see, for example, Cox (1962))

$$\mathbb{E}N(\sigma, \sigma + L_{k:n}) \simeq \frac{\mathbb{E}L_{k:n}}{\mathbb{E}A} + \frac{\mathbb{E}A^2}{2\mathbb{E}A} - 1 \tag{10}$$

$$\begin{aligned} \mathbb{E}N^2(\sigma, \sigma + L_{k:n}) \simeq\ & \frac{\mathbb{E}L_{k:n}^2}{(\mathbb{E}A)^2} + \mathbb{E}L_{k:n}\left(\frac{2\mathbb{E}A^2}{(\mathbb{E}A)^3} - \frac{3}{\mathbb{E}A}\right) \\ & + \frac{3(\mathbb{E}A^2)^2}{2(\mathbb{E}A)^4} - \frac{2\mathbb{E}A^3}{3(\mathbb{E}A)^3} - \frac{3\mathbb{E}A^2}{2(\mathbb{E}A)^2} + 1 \end{aligned} \tag{11}$$

These asymptotic relations hold for $k = 1, \ldots, n$ only when $\mathbb{P}(L_{k:n} \leq A)$ is small. In case this probability is larger than say 0.001, we propose a Gamma approximation presented by Smeitink and Dekker (1990) to compute the first two moments of the renewal function.

What remains to compute are the moments of the order statistics $L_{k:n}$. Using an analogous approach as is described in Balakrishnan and Cohen (1991, p.44), enable us to compute $\mathbb{E}L_{k:n}^m$ for $m \in \mathbb{N}$, and $k = 1, \ldots, n$, in case G is ME distributed.

Summarizing, to compute values for the expressions (1) and (2) for given values of s, Q, and n, we have to go through the following three steps

- Compute the moments of the order statistics $\mathbb{E}L_{k:n}^m$ for $m \in \{1, 2\}$ and $k = 1, \ldots, n$.
- Compute the first two moments of U and $D(\sigma, \sigma + L_{k:n})$ for $k = 1, \ldots, n$, by using relations (6) to (11).

Table 1: basic setting parameters for the experiments

n	$\mathbb{E}D$	c_D	$\mathbb{E}A$	c_A	$\mathbb{E}L$	c_L	Q	$P_{3,target}$
1	5	$\frac{1}{2}$	$\mathbb{E}D/5$	$\frac{1}{2}$	5	$\frac{3}{10}$	50	0.50
2	10	1		1	10	$\frac{1}{2}$	100	0.99
3		2		2		1	250	
5								
10								

- Compute $P_2(s, Q, n)$ and $P_3(s, Q, n)$ by fitting ME distributions on $D(\sigma, \sigma + L_{k:n})$ and $D(\sigma, \sigma + L_{k:n}) + U$ $(k = 1, \ldots, n)$, and using relations (1) and (2) respectively.

As been argued in the introduction, the service level contraint can be used to determine the reorder point s for given values of Q and n. The appropriate value of s can be found by solving $P_i(s, Q, n) = P_{i,target}$. Since $0 \leq P_i(s, Q, n) \leq 1$ and is increasing in s, the roots of the $P_i(s, Q, n) = P_{i,target}$ can be found simply by using a local search algorithm (see, for example Press et al.(1992)).

4 Numerical results

In this section we validate the quality of the algorithm described in the previous section for computing $P_2(s, Q, n)$ and $P_3(s, Q, n)$ by discrete event simulation. The numerical experiments are performed for a wide range of parameter values. The input values of the system parameters are given in Table 1. Each of the 3240 experiments consist of 10 sub-runs of 100.000 time units (exclusive 1 initialisation run). The demands, customer arrivals and lead times are drawn from ME distributions. For given values of n, Q and $P_{3,target}$ the reorder point s^* was determined by solving $P_3(s, Q, n) = P_{3,target}$. The output of the simulation experiment are values for the service measures under consideration, denoted by $P_{2,sim}$ and $P_{3,sim}$ respectively, and the fraction of the partial deliveries that crossed any partial deliveries of previous placed replenishment orders, denoted by X_{sim}.

In Table 3 we summarized the results of these experiments. Each line in Table 3 represents 180 simulation experiments, and we calculated the mean absolute deviation of $P_{3,target}$ and $P_{3,sim}$ and the mean absolute deviation of $P_2(s^*, Q, n)$ (denoted by $P_{2,target}$) and $P_{2,sim}$.

Table 2: The absolute deviations of the values of P_2 and P_3 computed by the algorithm and simulation.

Q	$P_{3,target}$	c_L	$\lvert P_{3,target} - P_{3,sim}\rvert$	$\lvert P_{2,target} - P_{2,sim}\rvert$	X_{sim}
50	0.50	0.3	0.0143	0.0085	0.4889
50	0.99	0.3	0.0037	0.0047	0.4892
100	0.50	0.3	0.0032	0.0024	0.1446
100	0.99	0.3	0.0022	0.0022	0.1447
250	0.50	0.3	0.0013	0.0011	0.0048
250	0.99	0.3	0.0011	0.0008	0.0047
50	0.50	0.5	0.0331	0.0257	0.5376
50	0.99	0.5	0.0064	0.0084	0.5374
100	0.50	0.5	0.0072	0.0055	0.1987
100	0.99	0.5	0.0039	0.0043	0.1987
250	0.50	0.5	0.0015	0.0016	0.0085
250	0.99	0.5	0.0015	0.0010	0.0085
50	0.50	1.0	0.1307	0.0949	0.5878
50	0.99	1.0	0.0096	0.0118	0.5878
100	0.50	1.0	0.0422	0.0319	0.3094
100	0.99	1.0	0.0085	0.0098	0.3091
250	0.50	1.0	0.0032	0.0031	0.0377
250	0.99	1.0	0.0053	0.0027	0.0377

From these experiments we can conclude the following about the quality of the expressions for the performance measures computed by the proposed algorithm in this section:

- For the situations in which $Q = 100$ or $Q = 250$, the proposed algorithm performs very good in all cases that are considered. Both $|P_{3,target} - P_{3,sim}|$ and $|P_2 - P_{2,sim}|$ are small, even for high coefficients of variation.
- For the situations where $Q = 50$ and $c_L = 1$, we see discrepancies between the target and achieved P_3 service level. The explanation for this deviation is that a large fraction of partial deliveries does cross (up to 58%). When this occurs we need to reconsider the determination of the moments $\mathbb{E}(L_{k:n}^{(2)})^m$, which are now determined by realisations of lead times of partial deliveries from several replenishment cycles.

These results point out that the proposed algorithm performs very well. However we have to be careful in situations where crossing of orders indeed occurs.

In addition to the previous numerical validation we compared our results with results from Chiang and Benton (1994) to check the performance of our algorithm under different model assumptions. Chiang and Benton (1994) considered an (s, Q) inventory model with two suppliers, shifted exponentially distributed Lead times, and normally distributed demand. Chiang and Chiang (1996) and Chiang and Benton (1994) are the only two articles that consider the P_2 service measure. But in both papers the undershoot of the reorder level at ordering epochs is neglected and double-counting is allowed of shortages just before two subsequent partial deliveries. Hence, when there remains a shortage just after the arrival of a partial delivery, this shortage is counted twice. It is easy to see that double-counting can lead to negative service levels. In the computational experiments of Chiang and Benton (1994) they consider normally distributed demand per day with mean 50 units/day and variance 10 units/day, and shifted-exponential lead times with mean 8 and variance equal to 4. To make a fair comparison we used the same first two moments for our models, i.e. $\mathbb{E}A = 1, c_A = 0, \mathbb{E}D = 50$, $c_D = 0.2$, $\mathbb{E}L = 8$, and $c_L = 0.5$. We simulated the model under the conditions of Chiang and Benton, that is with shifted-exponential lead times and normally distributed demand, and compared the results in Table 3.

The examples considered in Chiang and Benton (1994) are for very high service levels and for rather stable demand processes. Then neglecting the undershoot and double-counting have no impact on the calculated reorder level, which is reflected by the good results in Table 3. In spite of the difference in the model assumption and the simulated distribution functions our method did perform very well. In case we simulated lead times

Table 3: A comparison of results from Chiang and Benton (CB) with our model (JK)

Q	$P_{2,target}$		s	$P_{3,sim}$
1350	0.9952	(CB)	531	0.9953 ($\pm$ 0.0009)
		(JK)	521	0.9950 ($\pm$ 0.0009)
1600	0.9954	(CB)	493	0.9956 ($\pm$ 0.0005)
		(JK)	504	0.9960 ($\pm$ 0.0005)
2050	0.9956	(CB)	455	0.9956 ($\pm$ 0.0008)
		(JK)	482	0.9970 ($\pm$ 0.0007)
2850	0.9959	(CB)	422	0.9959 ($\pm$ 0.0008)
		(JK)	456	0.9971 ($\pm$ 0.0007)

from a ME distribution the model presented in this paper performs slightly better than the results of Chiang and Benton.

5 Conclusions and future research

In this paper an (s, Q) inventory model is presented with order splitting, where the demand is modelled as a compound renewal process, and lead times of the suppliers are independent and identically distributed random variables. We derived expressions for performance measures which are often used in practice, namely the fill rate, and the fraction of the time that the physical stock is positive.

When shortage cost are hard to specify, a service level constraint can be used to determine the reorder point s. Based only on the first two moments of the underlying demand and lead time process, an algorithm is derived to compute s by solving $P_i(s, Q, n) = P_{i,target}$. The algorithm turns out to perform very good for situations in which the number of order crossings was not too high.

Although the performance measures are derived for non-identically distributed lead times of suppliers, the algorithm is only developed for identically distributed lead times. When a fast algorithm is available for computing the order statistics of non-identically distributed random variables, the key-formulas (1) and (2) can be applied in a similar way.

References

BALAKRISHNAN, N., 1988. Recurrence relations for order statistics from n independent and non-identically distributed random variables. *Annals of Institutional Statistics Mathematics*,**40**, 273-277.

BALAKRISHNAN, N. AND COHEN, A.C., 1991. *Order statistics and inference*, Academic Press, Inc., San Diego.

COX, D.R., 1962. *Renewal Theory*, Methuen, London.

CHIANG, C. AND BENTON, W.C., 1994. Sole sourcing versus dual sourcing under stochastic demands and lead times. *Naval Research Logistics*, **41**, 609-624.

CHIANG, C. AND CHIANG, W.C, 1996. Reducing inventory costs by order splitting in the sole sourcing environment. *Journal of Operational Research Society*, **47**, 446-456.

GUO, Y. AND GANESHAN, R., 1995 Are more suppliers better? *Journal of Operational Research Society*, **46**, 892-895.

JANSSEN, F.B.S.L.P., AND KOK, A.G. DE, 1997 . The optimal number of suppliers in an (s, Q) inventory model with order splitting. *CentER discussion paper 97??*, Tilburg University.

KELLE, P. AND SILVER, E.A. 1990A. Decreasing expected shortages through order splitting. *Engineering Costs and Production Economics*, **19**, 351-357.

KELLE, P. AND SILVER, E.A. 1990B. Safety stock reductions by order splitting. *Naval Research Logistics*, **37**, 752-743.

LAU, H.S. AND LAU, A.H., 1994. Coordinating two suppliers with offsetting lead time and price performance. *Journal of Operations Management*, **11**, 327-337.

LAU, H.S. AND ZHAO, L.G. 1993. Optimal ordering policies with two suppliers when lead times and demand are all stochastic. *European Journal of Operational Research*, **68**, 120-133.

PAN, A.C. AND LIAO, C., 1989. An inventory model under just-in-time purchasing agreements. *Production and Inventory Management Journal*, **30** (1), 49-52.

PRESS, W. H., FLANNERY, B. P., TEUKOLSKY, S. A. AND VETTERLING, W. T., 1992. *Numerical Recipes in C*, Cambridge, England.

SCULLI, D. AND WU, S.Y., 1981. Stock control with two suppliers and normal lead times. *Journal of Operational Research Society*, **32**, 1003-1009.

SILVER, E.A. AND PETERSON, R. 1985. *Decision Systems for Inventory Management and Production Planning.* Wiley, New York.

SMEITINK, E. AND DEKKER, R., 1990. A simple approximation for the renewal function. *IEEE Transactions Reliability*, **R-39**, 71-75.

TIJMS, H.C., 1994. *Stochastic Models: An Algorithmic Approach.* Wiley, Chichester.

ZHAO, L.G. AND LAU, H.S., 1992. Reducing inventory costs and choosing suppliers with order splitting. *Journal of Operational Research Society*, **43**, 1003-1008.

Appendix 1: Proof of relation (2)

Define $T(x)$ as the expected total time that the physical stock is positive, in case the physical stock level on epoch 0 equals x ($x \geq 0$), there are no outstanding replenishment orders, and time epoch 0 is an arrival moment of a customer. Let M be the renewal function associated with F_D, then by definition

$$T(x) = \mathbb{E}A \; M(x) \tag{A.1.1}$$

Analogously, we define $\tilde{T}(x)$ as the expected total time that the physical stock is positive, in case the physical stock level on epoch 0 equals x ($x \geq 0$), there are no outstanding replenishment orders, and time epoch 0 is an arbitrary moment in time. Let $\tilde{A}$ be the arrival time of the first customer after zero. By conditioning on the first arriving customer after time epoch 0, results into

$$\tilde{T}(x) = (\mathbb{E}\tilde{A} - \mathbb{E}A) + \mathbb{E}A\, M(x) \tag{A.1.2}$$

Lemma A.1.1.
Let M be the renewal function associated with F_D, and let U the equilibrium excess distribution of D, then

$$(M * U)(x) = \frac{x}{\mathbb{E}D} \tag{A.1.3}$$

Proof:
Let $\tilde{F}_D(y)$ be the Laplace transform of F_D, thus $\tilde{F}_D(y) = \int\limits_0^\infty e^{-yx} dF_D(x)$. Since $\tilde{U}(y) = (1 - \tilde{F}_D(y))/(y\mathbb{E}D)$ and $\tilde{M}(y) = 1/(1 - \tilde{F}_D(y))$, it follows that the Laplace transform of the convolution equals $1/(y\mathbb{E}D)$. Hence, taking the inverse Laplace transform of $1/(y\mathbb{E}X)$ yields $(M * U)(x) = x/\mathbb{E}D$. □

Now, consider the k-th sub-cycle ($k \in \{1, \ldots, n\}$). The expected physical stock at the beginning of the k-th sub-cycle (just after the replenishment arrived) is equal $(I_k^b)^+$, whereas the expected physical stock at the end of the k-th sub-cycle (just before the replenishment arrives) is equal to $(I_k^e)^+$. Then it is easy to see that the the expected time that the physical stock is positive during the k-th sub-cycle is given by $\mathbb{E}\tilde{T}((I_k^b)^+) - \mathbb{E}\tilde{T}((I_k^e)^+)$. By using relation (A.1.2), Lemma A.1.1., and by conditioning on I_k^b, we find

$$\begin{aligned}
\mathbb{E}\tilde{T}((I_k^b)^+) &= \int\limits_0^{s+\frac{k-1}{n}Q} \tilde{T}(s + \frac{k-1}{n}Q - x) dF_{D(\sigma,\sigma+L_{k-1:n})+U}(x) \\
&= (\mathbb{E}\tilde{A} - \mathbb{E}A) \int\limits_0^{s+\frac{k-1}{n}Q} dF_{D(\sigma,\sigma+L_{k-1:n})+U}(x) \\
&\quad + \mathbb{E}A \int\limits_0^{s+\frac{k-1}{n}Q} \int\limits_0^{s+\frac{k-1}{n}Q-x} d(M * U)(y) dF_{D(\sigma,\sigma+L_{k-1:n})}(x) \\
&= (\mathbb{E}\tilde{A} - \mathbb{E}A)\mathbb{P}(I_k^b < 0) + \frac{\mathbb{E}A\mathbb{E}((I_k^b + U)^+)}{\mathbb{E}D},
\end{aligned}$$

and for $\mathbb{E}\tilde{T}((I_k^b)^+)$ an analogue expression can be derived. Finally using that the length of a replenishment cycle equals $\frac{Q\mathbb{E}A}{\mathbb{E}D}$, and summing up the expected time the net stock is positive during the n sub-cycles, yields

$$\begin{aligned}
P_3(s, Q, n) &= \frac{\mathbb{E}D}{Q\mathbb{E}A} \sum_{k=1}^n \left(\mathbb{E}\tilde{T}((I_k^b)^+) - \mathbb{E}\tilde{T}((I_k^e)^+) \right) \\
&= \frac{1}{2}(c_A^2 - 1)\mathbb{E}D \sum_{k=1}^n \frac{\mathbb{P}(I_k^b < 0) - \mathbb{P}(I_k^e < 0)}{Q} \\
&\quad + \sum_{k=1}^n \frac{\mathbb{E}(I_k^b + U)^+ - \mathbb{E}(I_k^e + U)^+}{Q} \qquad \text{(A.1.4)}
\end{aligned}$$

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