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CONSUMER ALLOCATION MODELS: CHOICE OF FUNCTIONAL FORM

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by Anton P. Barten

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CONSUMER ALLOCATION MODELS : CHOICE OF FUNCTIONAL FORM

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ABSTRACT

The functional form of consumer allocation models should be able to satisfy theoretical properties derived from the theory of consumer demand. The paper sketches four approaches that meet this condition. Of course, also empirical performance matters. Next to naive goodness-of-fit comparison, non-nested hypothesis testing can be employed. The latter technique is applied to a comparison of four versions of differential demand systems : the Rotterdam system, a version of the Almost Ideal Demand (AID) system, the CBS system and the NBR system. These systems are artificially nested in a more general model using scalar weights in contrast to Barten and McAleer (1991) who use matrix weights for this purpose. Annual data over the period 1921-1981 for The Netherlands for four major groups of consumer expenditure have been used for the empirical application. The CBS system dominates the others. As a byproduct of this approach a general form, the Synthetic system, results which is more flexible than the elementary models mentioned.

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1. INTRODUCTION

Consumer allocation systems indicate how the consumer allocates his means over the purchase of various commodities, sometimes also including leisure. These models are usually explicitly based on the microeconomic theory of consumer demand, which supplies quite some useful information for the empirical implementation of such systems.

Allocation problems are close to the foundations of economics which, in the eyes of some, is concerned with the optimal allocation of given means over various alternatives, or, its dual, the minimal use of means to reach a given set of objectives. Allocation models are being formulated not only for consumer demand but also for demand for inputs into production, composition of imports by country of origin, investment portfolio's and agricultural acreage allotment. In all these cases the variables explained by these models are, directly or indirectly, the arguments of an objective function minimized or maximized under one or more constraints. The system reflects the curvature of the objective function and/or that of the constraint(s).

While these allocation systems are based on the economic theory of individual behaviour they are most often applied to aggregate behaviour for a market or for an economy as a whole. In fact, exact aggregation is only possible under rather restrictive conditions. Consistent aggregation is less restrictive but holds only asymptotically. Given that one cannot expect an allocation system to reflect faithfully the structure implied by micro theory, one can still impose this structure, if it is not spontaneously reproduced, on the estimation of the system in order to save on degrees of freedom in an interpretable way. Given the usually limited time series, economy of parameters is a must.

After a general discussion of the theoretical foundations of consumer allocation systems, the criteria for choice of a particular functional form and the evaluation of its empirical performance, the paper turns to a class of four differential demand systems which are in a sense similar and therefore easily comparable. The attention is concentrated on their relative empirical performance as a basis of choice. Rather than limiting oneself to a single system one can consider a linear combination of these systems as an alternative which can do more justice to the data than any of the elementary systems.

2. THEORETICAL BACKGROUND

Summaries of the type of consumer theory leading to systems of demand theory can be found in Deaton and Muellbauer (1980b), Barten and Böhm (1982) and Deaton (1986). Here we will limit ourselves to some basic statements.

Under suitable assumptions about the preferences of the consumer these can be represented by a utility function

$$(1) \qquad u(q_1, \ldots, q_n)$$

with as arguments the, usually positive, quantities, q_i, of a finite number of commodities, n. This function is increasing and strictly quasi-concave in the quantities. It is usually taken to be at least twice differentiable. The vector of first-order derivatives, $u_q \equiv [\partial u/\partial q_i]$, is the n-vector of marginal utilities and is positive given the increasing nature of u(q), $q = [q_i]$. The n x n matrix of second-order derivatives, $U = [\partial^2 u/\partial q \partial q']$ is a symmetric matrix. The strict quasi-concave nature of (1) reflects itself in the condition that :

$$(2) \qquad x'Ux \leq 0, \quad \forall x \neq 0, \quad u_q'x = 0$$

The consumer's means m are non-zero but finite. They are used to pay $p_i q_i$, $i = 1, \ldots, n$, for the desired amounts of the commodities, where p_i is the positive unit price of commodity i. These expenditures satisfy the budget equation

(3) $\Sigma_i p_i q_i = m$

The consumer will select the vector q among alternative vectors that satisfy (3) which maximizes his utility (1). Mathematically, this amounts to maximizing the utility function subject to (3). The first-order conditions, next to (3) are

(4) $u_q = \lambda p$

where λ is a (scalar) positive Lagrange multiplier and $p = [p_i]$ is the n-vector of prices. Conditions (4) express the Second Law of Gossen (1854). Together with (3) these are being solved for λ and q. The latter solutions are the (Marshallian) demand functions

(5) $q_i = f_i(m, p_i, \ldots, p_n) \qquad i = 1, \ldots, n$

To guarantee differentiability on the interior of the commodity space *strict* quasi-concavity condition (2) has to be strengthened to

(2a) $x'Ux < 0, \quad \forall x \neq 0, \quad u_q'x = 0$

which is known as *strong* quasi-concavity. Differentiability of the Marshallian demand functions (5) is of interest because most empirical versions are differentiable and because the restrictions theory implies are most conveniently expressed in terms of derivatives of (5) or in terms of elasticities which are derivatives of the logarithmic version of (5).

Indeed write

(6) $d\ln q_i = \eta_i \, d\ln m + \Sigma_j \, \mu_{ij} \, d\ln p_j \qquad i, j = 1, \ldots, n$

for the logarithmic differential of (5). The η_i is the income (budget, means) elasticity of the demand for commodity i and μ_{ij} the price elasticity. These elasticities should satisfy a set of properties - see Frisch (1959) - which involve the budget share w_i, defined as $p_i q_i/m$. Note that (3) implies that $\Sigma_i w_i = 1$. Some of these properties can be conveniently expressed in terms of the Slutsky or *compensated price elasticity*, ϵ_{ij}, defined as :

(7) $$\epsilon_{ij} = \mu_{ij} + \eta_i w_j$$

This elasticity represents the substitution effect of price changes, keeping utility constant.

First, one has the *adding-up conditions*, guaranteeing that explained demand satisfies the budget equation (3) :

(8a) $$\Sigma_i w_i \eta_i = 1$$ (Engel aggregation)

(8b) $$\Sigma_i w_i \mu_{ij} = -w_j$$ (Cournot aggregation)

which together imply, using (7)

(8c) $$\Sigma_i w_i \epsilon_{ij} = 0$$ (Slutsky aggregation)

Next, there are the *homogeneity conditions* reflecting the linear homogeneity of (3) in the p_i and m :

(9a) $$\Sigma_j \mu_{ij} = -\eta_i$$

(9b) $$\Sigma_j \epsilon_{ij} = 0$$

A property of considerable empirical importance is that of Slutsky *symmetry*

(10) $$w_i \epsilon_{ij} = w_j \epsilon_{ji}$$

One next has the *negativity* condition

$$(11) \qquad \Sigma_i \Sigma_j \; x_i w_i \epsilon_{ij} x_j < 0 \qquad\qquad x_i, \; x_j \neq \text{constant}$$

The Slutsky elasticities may reflect particular structures of the preference order or of the utility function. If the preference order can be represented by a utility function which is a sum of n functions in each of the quantities :

$$(12) \qquad \epsilon_{ij} = \varphi\eta_i(\delta_{ij} - \eta_j w_j) \qquad\qquad \text{(complete independence)}$$

with φ being the reciprocal of what Frisch terms "money flexibility" and δ_{ij} a Kronecker delta. It is an extreme parameter reduction corresponding to the very rigid interaction pattern implied by the additivity of utility.

A less rigid structure is the one of groupwise separability of the preference order and utility function. Here the commodities are organized in non-overlapping groups and the utility function is written as a function of separate utility functions for each group. Then for commodity i, being part of group F, and commodity j, belonging to group G, F $\neq$ G, one has

$$(13) \qquad \epsilon_{ij} = -\varphi_{FG} \; \eta_i \eta_j w_j \qquad\qquad \text{(weak separability)}$$

where $\varphi_{FG} = \varphi_{GF}$ is in common to all interactions between commodities of group F and those of group G. Property (13) reflects *weak* separability. *Strong* separability in groups corresponds to (13) with $\varphi_{FG} = \varphi$, i.e. the same for all group interactions. Clearly, when all groups consist of one good only one has the case of complete independence.

Separability is convenient because it allows one to formulate complete allocation systems for each group separately, given the means to be spent on the group. The allocation of the means over the groups is determined in a higher level model in terms of group characteristics only.

Homothetic preferences express themselves in the property that

(14) $\eta_i = 1, \forall_i$

It represents an extreme in income sensitivity. It implies that budget shares do not vary with income.

A related concept, which has proved its use, is that of the *indirect utility function*

(15) $u^*(m, p_1, \ldots, p_n)$

which is obtained by replacing in (1) the q_i by the optimizing quantities given by (5). Its differential form can be written as

$$(16) \qquad du^* = \Sigma_i (\partial u/\partial q_i)[(\partial f_i/\partial m)dm + \Sigma_j (\partial f_i/\partial p_j)dp_j]$$

$$= \lambda m(\Sigma_i w_i \eta_i \, d\ln m + \Sigma_i \Sigma_j w_i \mu_{ij} \, d\ln p_j)$$

$$= \lambda m(d\ln m - \Sigma_j w_j \, d\ln p_j)$$

where use has been made of the Gossen conditions (4) and adding-up conditions (8a) and (8b). It appears that λ, the Lagrange multiplier in (4), is $\partial u^*/\partial m$, i.e. the marginal utility of the budget. From (16) one can derive demand functions by way of the rule of Roy (1942) :

$$(17) \qquad q_i = -(m/p_i)\left(\frac{\partial u^*}{\partial \ln p_i} \Big/ \frac{\partial u^*}{\partial \ln m}\right)$$

Note that in (16) $d\ln m - \Sigma_j w_j \, d\ln p_j$ can be seen as a kind of real income change. $\Sigma_j w_j \, d\ln p_j$ is a (change in a) price index which is used to deflate m. Constant real income means no change in utility.

Another way to look at this concept of real income is to start from the logarithmic differential of budget equation (3) :

(18) $\quad d\ln m = \Sigma_j w_j \; d\ln p_j + \Sigma_j w_j \; d\ln q_j$

and to write

(19) $\quad \Sigma_j w_j \; d\ln q_j = d\ln m - \Sigma_j w_j \; d\ln p_j$

where the left-hand side variable is a (change in a) quantity index corresponding to the (change in) real income on the right-hand side. In the sequel we will use the notation

(20) $\quad d\ln Q = \Sigma_j w_j \; d\ln q_j \;, \quad d\ln P = \Sigma_j w_j \; d\ln p_j$

to indicate these quantity and price indexes. With (18) one then has :

(21) $\quad d\ln m = d\ln P + d\ln Q$

Another concept of practical use is the *expenditure function* which can be derived from (15) by expressing m in terms of u and p :

(22) $\quad e(u, p_1, \ldots, p_n).$

It gives the minimum expenditure needed to reach utility level u given prices $p_1, \ldots, p_n$. From (16) it follows that its differential form can be written as :

(23) $\quad d\ln e = [1/(\lambda m)]du + \Sigma_j w_j \; d\ln p_j ,$

which serves as the basis for the Shephard (1953) formula :

(24) $\quad w_i = \partial\ln e/\partial\ln p_j , \qquad\qquad i = 1, \ldots, n$

yielding demand equations of the type

$$(25) \qquad q_i = h_i(u, p_1, \ldots, p_n), \qquad i = 1, \ldots, n,$$

also known as Hicksian demand equations. Replacing u in (25) by $u^*(m, p_1, \ldots, p_n)$ one is back to the Marshallian demand equations (5).

Another way to provide the link between the two types of demand equations is to start from (6), use (7) and (16) to obtain

$$(26) \qquad d\ln q_i = [1/(\lambda m)]\eta_i du + \Sigma_j \epsilon_{ij}\ d\ln p_j$$

which is the logarithmic differential version of (25). From this expression the nature of ϵ_{ij} as utility constant price elasticities is clear.

Quite a bit more can be said about the theoretical foundations of applied demand analysis. For the purpose at hand the present summary may suffice.

3. APPROACHES TO FUNCTIONAL SPECIFICATION

In econometrics the ideal specification should be consistent with theory, easy to estimate and fit the data, which includes good prediction performance. Practice falls short of the ideal and usually a reasonable compromise has to be struck between the three requirements.

In the case of the formulation of demand allocation systems "theory" consists of the properties of the demand equations outlined in the previous section. Even though those properties are derived for the single consumer one would like them to be present for the average or aggregate agent as well.

There are basically four approaches to arrive at demand equations satisfying the properties in question. The first one starts off from a functionally specified, increasing and strongly quasi-concave utility function (1), which is maximized subject to budget constraint (3). The first-order conditions are solved for the quantities as a function of prices and income. The parameters of the utility function are the constants of the demand equations. The best-known example of this approach is the Linear Expenditure System (LES) - for a survey see Deaton (1975). The underlying utility function can be written as

$$(27) \qquad u = \Sigma_i \beta_i \ \ln(q_i - \gamma_i) \quad , \quad \Sigma_j \beta_j = 1 \quad , \quad \gamma_i < q_i$$

The resulting demand equations are :

$$(28) \qquad q_i = \gamma_i + (\beta_i / p_i)(m - \Sigma_j p_j \gamma_j)$$

The additive nature of (27) clearly reflects the assumption of complete independence of the preference order. Empirically, (28) is rather restrictive. It is also not so easy to estimate. In (28) the γ_j appear in all equations in a nonlinear combination with the β_i, while the requirement that the γ_i are smaller than the smallest observable q_i value is not so easily met by the data. The first, not ideal, application of the LES is by Stone (1954). It took until Parks (1971) and Solari (1971) before it could be adequately estimated.

More generally, it appears that starting from a well-specified utility function one cannot obtain empirically interesting demand functions.

The second approach starts off from a functionally specified indirect utility function (15) and applies Roy's rule (17) to arrive at estimable demand functions. An example is the Indirect Translog Utility Function of Christensen *et al.* (1975) :

$$(29) \qquad u^* = \alpha + \Sigma_i \beta_i \ \ln(p_i / m) + ½ \ \Sigma_i \Sigma_j \beta_{ij} \ \ln(p_i / m) \ \ln(p_j / m)$$

with $\Sigma_i \beta_i = -1$ and $\beta_{ij} = \beta_{ji}$ resulting in

(30) $$w_i = \frac{\beta_i + \Sigma_j \beta_{ij} \ln(p_j/m)}{-1 + \Sigma_k \Sigma_j \beta_{kj} \ln(p_k/m)}$$

The system is nonlinear in its parameters and not so easy to estimate. Moreover it is impossible to satisfy the condition that u* is monotone increasing in m, or decreasing in the general price level for all possible values of the prices and m, which can be quite awkward in prediction and simulation exercises. Moreover, negative predictions of the value shares cannot be excluded.

The income elasticity η_i associated with (30) is

(31) $$\eta_i = 1 - (\Sigma_j \beta_{ij}/w_i - \Sigma_k \Sigma_j \beta_{kj})/x$$

with x being the denominator of (30). For the Slutsky elasticity ϵ_{ij}, multiplied by w_i, one has

(32) $$w_i \epsilon_{ij} = (\beta_{ij} - w_i \Sigma_k \beta_{kj} - w_j \Sigma_k \beta_{ik} + w_i w_j \Sigma_k \Sigma_l \beta_{kl})/x$$

which satisfies Slutsky aggregation condition (8c), homogeneity condition (9b) and symmetry condition (10). Controlling the signs of the β_{ij} does not guarantee negativity condition (11), however. Building separability into (30) is a rather complicated affair since it is not a special case of the choice of constants. Still, as is evident from (30) or (32) the system allows for empirically interesting interactions to express themselves in an appropriate value of the relevant β_{ij}. The system becomes homothetic by simply setting $\Sigma_j \beta_{ij} = 0$ for all i. This property can thus either be built into the system, or be tested or both without essentially changing the functional specification.

The third approach bases itself on a specified expenditure function (22). Application of Shephard's lemma (24) results in Hicksian demand equations (25) from which the unobservable utility level is eliminated using the expenditure function to express it in terms of m and p. The best known example of this type of specification is the Almost Ideal Demand (AID) system of Deaton and Muellbauer (1980a). Its expenditure function in logarithmic form reads

$$(33) \qquad \ln e(u,p) = \alpha_0 + \Sigma_i a_i \ln p_i + ½ \Sigma_i \Sigma_j r_{ij} \ln p_i \ln p_j + u \Pi_j p_j^{c_j}$$

with $r_{ij} = r_{ji}$, $\Sigma_i a_i = 1$, $\Sigma_i r_{ij} = 0$, $\Sigma c_j = 0$. Use of Shephard's lemma results in

$$(34) \qquad w_i = a_i + c_i u \Pi_j p_j^{c_j} + \Sigma_j r_{ij} \ln p_j$$

Eliminating u gives

$$(35) \qquad w_i = a_i + c_i (\ln m - \ln P_*) + \Sigma_j r_{ij} \ln p_j$$

with

$$(36) \qquad \ln P_* = \alpha_0 + \Sigma_k a_k \ln p_k + ½ \Sigma_k \Sigma_j r_{kj} \ln p_k \ln p_j$$

A variant of the AID system uses :

$$(37) \qquad \ln P_* = \Sigma_k w_k \ln p_k$$

known as Stone's index. With (37) the budget share equations are linear in the parameters to be estimated, viz., a_i, c_i and the r_{ij}. With (36) for $\ln P_*$ the estimation problem is more complicated.

The income elasticity implied by (35) is

$$(38) \qquad \eta_i = 1 + c_i / w_i$$

Homotheticity clearly corresponds with $c_i = 0$ for all i, which is easy to impose if one sees a reason for it. The implied Slutsky elasticities differ according to the use of (36) or of (37). In the first case, they are, multiplied by w_i :

$$w_i \epsilon_{ij} = r_{ij} + c_i c_j (\ln m - \ln P_*) - w_i \delta_{ij} + w_i w_j \tag{39}$$

In the second case one has

$$w_i \epsilon_{ij} = r_{ij} - w_i \delta_{ij} + w_i w_j \tag{40}$$

It is easy to check that the adding-up, homogeneity and symmetry conditions are satisfied. The negativity condition is less easily reproduced by the system. It cannot be controlled by restricting the value of the estimated parameters and cannot be guaranteed for all p and m combinations. The qualification "Almost" in the name of the system is due to this shortcoming. Another problem is the fact that separability structures are not nested in the general specification. However, as is clear from (35) specific interactions between commodities can be captured easily. In simulations values of w_i outside the (0, 1) interval cannot be excluded. In fact, a basic weakness of (35) is that variations in w_i, which in principle are limited to the (0, 1) interval, are linked to variations in ln m over the $(-\infty, +\infty)$ interval by the constant c_i. The only value of c_i which does not lead to problems is zero, which implies homothetic preferences. A similar complication occurs for the measurement of the price effects.

We now turn to the fourth approach. Many early empirical demand studies worked with double-logarithmic specifications and constant elasticities. They appear to function well empirically but are less adequate in reflecting the theoretical restrictions outlined in the previous section. As we may note those restrictions, apart from homogeneity condition (9), involve budget shares next to the elasticities. To satisfy those properties constant elasticities require constant budget shares. This is theoretically uninteresting and empirically inacceptable.

Byron (1970) working with a double-logarithmic system imposed the theoretical constraints on the estimation using the sample averages of the budget shares. In this way the restrictions contribute to the statistical efficiency of the estimation. Still, for purposes of simulation, one can expect that the explained parts will not sum to the given total, to mention just one of the possible conflicts.

Theil (1965) also started off from a double-logarithmic specification like (6) with μ_{ij} replaced by ϵ_{ij} using (7) :

$$d\ln q_i = \eta_i(d\ln m - \Sigma_j w_j d\ln p_j) + \Sigma_j \epsilon_{ij} d\ln p_j$$

Multiplying both sides through by w_i results in

$$(41) \qquad w_i\, d\ln q_i = b_i(d\ln m - \Sigma_j w_j d\ln p_j) + \Sigma_j s_{ij} d\ln p_j$$

where $b_i = w_i\eta_i$ and $s_{ij} = w_i\epsilon_{ij}$ are now treated as constants. This choice of constants is known as the *Rotterdam system*. It is quickly verified that Engel and Slutsky aggregation imply

$$(42) \qquad \Sigma_i b_i = 1 \;, \quad \Sigma_i s_{ij} = 0 \;,$$

while the homogeneity condition amounts to

$$(43) \qquad \Sigma_j s_{ij} = 0.$$

Symmetry condition (10) translates into

$$(44) \qquad s_{ij} = s_{ji}.$$

The negativity condition now becomes

$$(45) \qquad \Sigma_i\Sigma_j x_i s_{ij} x_j < 0 \qquad\qquad x_i,\ x_j \neq \text{constant}.$$

All these conditions are in terms of the constants of the system and can thus either be tested or built in. Another attractive property of this choice of parameters is that special preference structures are special cases. For complete independence one has

(46) $\quad s_{ij} = \varphi\, b_i(\delta_{ij} - b_j)$

while weak separability expresses itself as

(47) $\quad s_{ij} = -\, \varphi_{FG} b_i b_j$

when i and j belong to groups F and G, respectively. For strong separability the φ_{FG} is replaced by φ.

Since $\eta_i = b_i/w_i$, homotheticity can only be imposed for $b_i = w_i$ for all i, i.e. making the w_i constant also with respect to price changes.

Replacing in (41) the differentials by first differences and w_i by $\bar{w}_{it} = (w_{it} + w_{it-1})/2$ one has a relatively simply estimable system. The model is general in the sense that each i,j interaction is represented by its own s_{ij}. Note one limitation. Because $\eta_i = b_i/w_i$ the sign of η_i is determined by that of b_i. A commodity is estimated to be inferior ($b_i < 0$, $\eta_i < 0$) or to be non-inferior ($b_i \geq 0$, $\eta_i \geq 0$). In the latter case it can be a normal good ($b_i \leq w_i$, $\eta_i \leq 1$) or a luxury ($b_i > w_i$, $\eta_i > 1$). It can change from a luxury into a normal good or vice versa depending on the change in w_i. It cannot change from a non-inferior good to an inferior good. An analogous property is present in the AID system. As is clear from (38) the sign of c_i determines whether η_i is larger than one or not. A good is a luxury or a necessity, without the possibility of a change depending on the exogenous variables. If a good is a necessity it can change from a normal good to an inferior good or vice versa. Both the constant b_i of the Rotterdam system and the constant c_i of the AID system appear to be restrictive. Intuitively, one would like a specification which allows a good to go through an (economic) life cycle, beginning as a luxury, becoming normal and finally inferior as available means for consumption increase. With the usual level of aggregation of goods inferiority is rarely observed, however, reducing the practical importance of the limitation of the constant b_i.

Of the four approaches considered here, the first one, starting off from a functionally specified direct utility function, is least attractive because it does not lead to "interesting" demand systems. This also means that the "interesting" systems produced by the three other approaches cannot be integrated to yield a direct utility function. The systems resulting from the second and third approach have by construction an associated indirect utility function and an associated expenditure function. This cannot be taken for granted for systems resulting from the fourth approach. Still, for given levels of real income and a set of prices one can also in that case determine the corresponding minimal nominal expenditure numerically, making welfare comparisons possible.

The systems considered as examples are all regularly used in practice. From a theoretical point of view they have their advantages and disadvantages. Some of them are more easy to estimate than others. To estimation applies, *mutatis mutandis*, the army adage "l'intendance suivra". It is a matter of time and computer power to crack the toughest nuts in that area. What about empirical performance of such systems ? This is the topic of the next section.

4. EMPIRICAL PERFORMANCE

In discussing empirical performance one can distinguish between potential performance for any set of relevant data and actual performance for a particular set of data.

The first type of performance has to a certain extent already been discussed in the previous section because it is very much related to the theoretical properties of the system. We have seen that of the four systems considered as an example, three viz. the Linear Expenditure System, the Indirect Translog system and the Almost Ideal Demand system, cannot maintain proper curvature conditions over the whole (p, m) space. Otherwise said, they break down as demand allocation systems for particular combinations of the exogenous variables of the system. How important this is in actual applications is difficult to say. This defect simply reflects that demand systems are at best a local approximation to the true state of affairs. How local is local ?

A related issue is that of *flexibility* introduced by Diewert (1971). A demand function is considered to be a *flexible functional form* if it is derived from an aggregator function (direct or indirect utility function, expenditure function) which is a second-order approximation to the relevant true one. It means that the demand functions themselves are first-order approximations and that there is some point in the space of exogenous variables where the derivatives of these approximate demand functions are equal to the true ones. The Indirect Translog and the Almost Ideal Demand system are in this sense flexible functional forms, the Linear Expenditure System is not.

What matters is not so much the nature of the aggregator function as well as the nature of the demand functions as a linear approximation, requiring that in each demand function each of the exogenous variables has its own impact. In this sense also the Rotterdam system qualifies as a flexible functional form - see also Deaton (1986).

An unfortunate byproduct of the Diewert flexibility concept is the tendency to formulate the aggregator functions as quadratic forms in the arguments which makes it impossible to guarantee monotonicity over the full range of the exogenous variables, resulting in the type of curvature conflicts mentioned earlier.

A related approach is the one used by Kiefer and MacKinnon (1976). One generates data using one system, adds random terms and estimates other systems. Flexibility expresses itself in the ability to explain the data as well as the system used to generate the data. How does one measure such relative performance ?

This brings us to the issue of *actual* empirical performance where a similar measurement issue prevails. One compares two or more systems using the same set of data on the quantities and the prices. These systems are estimated and their goodness-of-fit is compared. This sounds easier than it is. The usual goodness-of-fit measure is the R^2, the coefficient of determination. These can be calculated for each equation of the system and compared across systems. However, the dependent variables are not necessarily the same for the various systems and then the R^2 is not such a meaningful measure of relative performance. Moreover, one wants to compare systems as a whole and not equation-by-equation.

In the first attempt of empirical comparison of various demand systems Parks (1969) uses the average information inaccuracy concept of Theil (1967). It basically compares the budget shares generated by the estimated system with the actual ones, taking the (weighted) average over the equations and over the sample. A relatively high average information inaccuracy is taken to be an indicator of less satisfactory behaviour. It is not clear to what extent this measure favours models with the budget shares as dependent variables over models with other functions of the quantities like the Linear Expenditure System or the Rotterdam model. The models do not necessarily estimate the same number of parameters. How to account for that difference ? In general, what about the statistical significance of the difference in values for the average information inaccuracy ? In spite of these open questions the average information inaccuracy has been widely used - see for example Klevmarken (1979) and Barten (1989). The comparison of the likelihood values as employed by Deaton (1974) has found little support.

What one clearly needs is a statistical testing procedure allowing one to make statements about the significance of the difference in empirical performance. Some models are special cases of other models, are *nested* within more general specifications. One can, for instance, estimate the Rotterdam model, once with imposing Slutsky symmetry and once without imposing this restriction. The former model is the special case of the unrestricted, more general, second model. The maximum likelihood value of the latter model will be larger than that of the former one. Twice the difference in the logarithms of these values is (asymptotically) distributed as χ^2 with as many degrees of freedom as the number of restrictions involved, if the restricted model is the true one. If dropping the restrictions increases the likelihood significantly, the restrictions are clearly too "restrictive" and are rejected by the data. This type of test has been used already for some time - see, for example, Barten (1969). Similar approaches use the Wald or the Lagrange Multiplier test statistic.

When comparing the empirical performance of alternative systems one model is usually not a special case of another model. The number of coefficients of the two models may be different but one cannot reduce one set to the other by simple manipulation. Otherwise said, there is no restricted version that could act as the natural null hypothesis. Consider the AID model (35) with specification (37) for ln P_* and the Rotterdam model (41). They have the same number of coefficients. They are not nested within each other. One clearly needs a testing procedure for non-nested alternatives.

The first one to apply *non-nested* testing to the comparison of demand systems is Deaton (1978). He applied it to models with the same dependent variables. Such a procedure can be used to compare, for example, the indirect translog system (30) with the AID model (35) which both have the budget share as dependent variable. It is not suitable to compare, say, the AID system with, say, the Rotterdam system.

In Barten and McAleer (1991) an approach is proposed that can deal also with non-nested models with different (vectors of) dependent variables, which are non-linear data transformation of, say, the quantities demanded. The method starts off from a hypothetical general model made up out of a matrix weighted linear combination of two or more basic models. One solves for one of the dependent variables and estimates consistently the transformed matrix weights associated with the other models. Next, one tests whether these matrix weights are significantly different from zero. If this happens to be the case the model for which the dependent variable is on the left-hand side falls short in explaining reality on its own and could usefully employ some of the information contained in the other models in the linear combination. Also here, there is no natural null model. Each model, in turn, can yield the left-hand variables of the estimated system. The alternative models consist then of the other models taken one-by-one or as pairs or as combinations of three, four, ..., models.

Can those linear combinations themselves be considered as a (synthetic) demand allocation system ? There is no problem in letting them satisfy the adding-up and homogeneity conditions but the symmetry conditions cannot be imposed without reducing the matrix weights to scalar matrices. Also the negativity condition cannot be controlled. In the present paper we will consider scalar combinations of systems, which can be used as demand systems in their own right if one is willing to pay the price of not being able to satisfy the negativity condition. The next section presents the models to be compared.

5. A CLASS OF DIFFERENTIAL DEMAND FUNCTIONS

To maintain comparability with the empirical application of Barten and McAleer (1991) attention will be limited to a set of four models that are on the one hand sufficiently different to display differences in empirical performance and are on the other hand sufficiently similar to allow for an interpretation of those differences.

Consider Rotterdam model (41). Use (19) and (20) to rewrite it as

(48) $$w_i \, d\ln q_i = b_i \, d\ln Q + \Sigma_j \, s_{ij} \, d\ln p_j$$

This is one member of the class of four models. Next take AID model (35), bring it into differential form, replace dln P_* by dln P of (20), use (19) and (20) again to write

(49) $$dw_i = c_i \text{ dln } Q + \Sigma_j \, r_{ij} \text{ dln } p_j$$

Note the similarity on the right-hand sides of (48) and (49). The left-hand sides are different, but related. Indeed one can write

(50) $$dw_i = w_i \text{ dln } q_i + w_i \text{ dln } p_i - w_i \text{ dln } m$$

which shows that w_i dln q_i is the quantity component of the change in budget share w_i while w_i dln p_i and $-w_i$ dln m are due to the (exogenous) changes in the price and total means.

One can use (50) to show how the coefficients of (48) and (49) are related. Replace w_i dln q_i in (50) by the right-hand side of (48) to obtain

(51) $$dw_i = b_i \text{ dln } Q + \Sigma_j s_{ij} \text{ dln } p_j + w_i \text{ dln } p_i - w_i \text{ dln } m$$

$$= (b_i - w_i)\text{dln } Q + \Sigma_j (s_{ij} + w_i \delta_{ij} - w_i w_j)\text{dln } p_j$$

where (21) and (20) have been used to replace dln m. Comparison with (49) shows equivalence for

(52) $$c_i = b_i - w_i$$

(53) $$r_{ij} = s_{ij} + w_i \delta_{ij} - w_i w_j$$

Note that the w_i are taken to be variable and that taking the b_i and s_{ij} to be constants is essentially different from taking the c_i and r_{ij} to be constant. The two systems are different but comparable.

Keller and van Driel (1985) created a hybrid of the AID and Rotterdam systems by replacing in (48) b_i by $c_i + w_i$ and moving w_i dln Q to the left-hand side. The resulting CBS system reads :

$$(54) \qquad w_i(\mathrm{dln}\, q_i - \mathrm{dln}\, Q) = c_i\, \mathrm{dln}\, Q + \Sigma_j s_{ij}\, \mathrm{dln}\, p_j .$$

This system has the AID income coefficients c_i and the Rotterdam price coefficients s_{ij}. It shares with the two basic models the adding-up, homogeneity and symmetry conditions in terms of the coefficients only. It can also be made to satisfy the negativity condition. Complete independence, weak and strong separability, however, are not special cases of this specification, as is evident from (46) and (47) where the b_i rather than the c_i appear.

Neves (1987) considered another hybrid. He replaced the c_i in the AID system (49) by $b_i - w_i$ to yield the NBR system

$$(55) \qquad dw_i + w_i\, \mathrm{dln}\, Q = b_i\, \mathrm{dln}\, Q + \Sigma_j r_{ij}\, \mathrm{dln}\, p_j$$

It has the Rotterdam income coefficients and the AID price coefficients as constants. It can satisfy the regularity conditions of adding-up, homogeneity and symmetry, but not that of negativity, while special preference structures are also not embedded by this choice of constants.

The right-hand sides of the four systems contain the same variables. The left-hand sides are different. Denote by y_{Ri}, y_{Ci}, y_{Ai}, y_{Ni} the left-hand sides of the Rotterdam system (48), of the CBS system (54), of the AID system (49) and of the NBR system (55), respectively. One then has

$$(56a) \qquad y_{Ci} - y_{Ri} = w_i(\mathrm{dln}\, q_i - \mathrm{dln}\, Q) - w_i\, \mathrm{dln}\, q_i = -w_i\, \mathrm{dln}\, Q$$

$$(56b) \qquad y_{Ai} - y_{Ci} = dw_i - w_i(\mathrm{dln}\, q_i - \mathrm{dln}\, Q) = w_i(\mathrm{dln}\, p_i - \mathrm{dln}\, P)$$

$$(56c) \qquad y_{Ni} - y_{Ai} = dw_i + w_i\, \mathrm{dln}\, Q - dw_i = w_i\, \mathrm{dln}\, Q$$

where use has been made of (50) and of (21). All other pairwise differences can be constructed from these three expressions.

The traditional demand systems take the left-hand side variables as exogeneous, i.e. dln Q, the change in real income and dln p_i, i = 1, ..., n. This reflects the methodological starting point of demand allocation as choosing quantities that maximize utility given the budget m and the prices. It follows from (21) that then also dln Q is exogenous. Both dln Q and dln P involve w_i. Clearly dw_i is endogenous. Assuming intertemporal independence exogenous levels are compatible with endogenous changes. Thus dln Q and dln P, and their finite counterparts, can indeed be taken to be exogenous. This also justifies the treatment of the differences in the left-hand sides of the systems considered as exogenous, because they are defined in terms of exogenous variables only - see (56). This property will be exploited when the systems are being compared and combined. First, however, we will describe the data used for the actual comparison exercise.

6. THE DATA

The dataset used is the same as that of Barten and McAleer (1991). It consists of annual observations of consumer expenditure and corresponding prices for the Netherlands over the period 1921-1981. The original data for 16 groups and services have been aggregated into four major groups, namely Food (FOOD), Pleasure Goods (i.e. confectionery, tobacco, drinks) (PLGD), Durables (DURA) and Remainder (REST).

The complete set of observations consists of four subsets : (i) 1921-1939, taken from Barten (1966); (ii) 1948-1951, an unpublished up-date of the data in Barten (1966) for that period; (iii) 1951-1977, based on data constructed by the Dutch Central Bureau of Statistics (CBS) - see CBS (1982); and (iv) 1977-1981, which also originates from the CBS and is published in Van Driel and Hundepool (1984). The three post-1948 subperiods overlap by one year.

No attempt is made to merge these four subsets into a single set. This is not really necessary. For estimation purposes the models of the preceding section will be written in terms of first differences of the variables. For practical purposes it is simpler to pool the data with estimated dummy variables to absorb the 1939-1948 transition and the 1951 and 1977 shifts. Altogether, the sample consists of 54 observations in first differences.

Given the rather long period, the data display quite some variation. The population doubled. To account for that, per capita expenditures are used. Real income per capita increased by a factor three. Substantial price increases occurred, specifically after 1948. Prices of Durables rose less than the average, those of Remainder, which includes quite some services, increased more than the average. These changes are reflected in variations in the budget shares. Table 1 gives the actual budget shares for the first and last years of the sample and also their average for the whole sample.

Table 1 : Selection Budget Shares

	FOOD	PLGD	DURA	REST
1921	.34	.10	.23	.33
Mean	.25	.09	.25	.40
1981	.13	.06	.22	.58

Engel's law reflects itself nicely in the spectacular decrease in the budget share of FOOD, matched by the substantial increase in the budget share of REST. There appears to be enough variation in the data to allow differences in empricial performance between different functional forms to show up. A first impression of this can be obtained from the estimation experiment of the next section.

7. ESTIMATION OF THE FOUR SYSTEMS

In this section results are given for the estimation of the four systems one-by-one. For the purpose of estimation the differentials have been replaced by finite first differences and the w_i by the moving average, $\bar{w}_{it} = (w_{it} + w_{it-1})/2$. An intercept has been added to represent trends, such as changes in tastes over time. Additive random disturbances together with the data shift dummy variables complete the specification. The disturbances are assumed to have a multivariate normal distribution, independent across observations.

For reasons of comparability, all four systems have been estimated with the homogeneity and symmetry conditions imposed. The negativity condition is not taken into account because the AID and NBR systems cannot be made to respect it. With the type of systems considered here, the adding-up condition is satisfied automatically.

The systems have been estimated by the method of maximum likelihood - see Barten (1969) and Barten and Geyskens (1975) - using the DEMMOD estimation package.

Table 2 gives the point estimates of the b_i or c_i and of the s_{ii} and r_{ii}, together with their asymptotic standard errors. To avoid overburdening the reader with results the intercepts, the s_{ij} or r_{ij} estimates for $i \neq j$, the coefficients of the data shift dummy variables and various performance statistics have not been given. For each system the maximum logarithmic likelihood value (MLV) is also presented.

Table 2 : Selected Point Estimates of the Four Systems

	ROT		CBS		AID		NBR	
	b_i	s_{ii}	c_i	s_{ii}	c_i	r_{ii}	b_i	r_{ii}
FOOD	.142 (.026)	-.105 (.023)	-.121 (.023)	-.093 (.021)	-.125 (.023)	.102 (.020)	.138 (.025)	.091 (.022)
PLGD	.069 (.011)	-.049 (.014)	-.026 (.011)	-.049 (.014)	-.025 (.012)	.034 (.014)	.070 (.012)	.034 (.014)
DURA	.561 (.033)	-.029 (.031)	.302 (.033)	-.030 (.032)	.301 (.035)	.150 (.033)	.560 (.034)	.151 (.032)
REST	.228 (.038)	-.040 (.039)	-.155 (.034)	-.038 (.035)	-.151 (.034)	.192 (.036)	.232 (.038)	.190 (.040)
MLV	662.621		669.442		667.390		661.016	

Economists have built up their intuition around elasticities. To compare the b_i with the c_i and the s_{ii} with the r_{ii} these concepts have been converted into elasticities using the formulae

(57) $\eta_i = b_i/w_i$ $\qquad$ $\eta_i = c_i/w_i + 1$

(58) $\epsilon_{ii} = s_{ii}/w_i$ $\qquad$ $\epsilon_{ii} = r_{ii}/w_i - 1 + w_i$

based on (38), (40) and the definitions of b_i and s_{ij} in (41). The elasticities are variable because the w_i are variable. They have been evaluated for the budget shares of Table 1. In view of the similarities between point estimates Table 3 and 4 only report the income or budget elasticities η_i and the own substitution elasticities ϵ_{ii} for ROT and AID.

Table 3 : Selected Elasticities for the Rotterdam model

	η_i			ϵ_{ii}		
	1921	mean	1981	1921	mean	1981
FOOD	.421	.566	1.060	-.312	-.418	-.784
PLGD	.726	.758	1.095	-.516	-.538	-.778
DURA	2.397	2.209	2.538	-.124	-.114	-.131
REST	.683	.564	.391	-.120	-.099	-.069

Table 4 : Selected Elasticities for the AID model

	η_i			ϵ_{ii}		
	1921	mean	1981	1921	mean	1981
FOOD	.629	.504	.067	-.360	-.344	-.105
PLGD	.737	.721	.603	-.547	-.533	-.397
DURA	2.286	2.188	2.362	-.125	-.156	-.100
REST	.548	.625	.741	-.091	-.121	-.088

It appears from Tables 3 and 4 that all the ϵ_{ii} are negative. There is no conflict with the negativity condition in this respect. In both cases the ϵ_{ii} are rather small : no value less than -.8. For the mean budget share and for 1921 the elasticities are rather similar for both models. This contrasts with the considerable difference in the elasticities for 1981. The η_i values for that year indicate FOOD to be a luxury in the Rotterdam model and almost an inferior good for the AID system. This is due to the rather low budget share for FOOD for 1981. As is evident from (57) a decrease in w_i increases the η_i in the Rotterdam and decreases it in the AID context for negative c_i. Which of these two sets of results are to be believed ? The maximum likelihood values of Table 2 give little support for an answer to this question. CBS appears to do best but is its difference with the others significant ? Note that all models have the same number of parameters estimated, so difference in performance is not a matter of difference in degrees of freedom. The next section is concerned with a more formal test.

8. PAIRWISE COMPARISONS

The models considered in the preceding sections are not nested. They are of the type

$$y_{jt} = X_t \beta_j + u_{jt}$$

where the n-vector y_{jt} represents the j-th nonlinear data transformation of a vector of basic endogenous variables. The X_t is a n x k matrix of exogenous variables and β_j is a vector of coefficients, specific for each system. The n-vector u_{jt} are disturbance terms. Let j = 1,2 and construct the following general model

$$\alpha_1 (y_{1t} - X_t \beta_1) + \alpha_2 (y_{2t} - X_t \beta_2) = v_t \tag{59}$$

No loss of generality is involved by letting $\alpha_1 + \alpha_2 = 1$ or $\alpha_1 = 1 - \alpha_2$. Thus the general model is

$$(1 - \alpha_2) y_{1t} + \alpha_2 y_{2t} = X_t \ ((1 - \alpha_2)\beta_1 + \alpha_2 \beta_2) + v_t$$

or

$$y_{1t} = X_t \ ((1 - \alpha_2)\beta_1 + \alpha_2 \beta_2) + \alpha_2 (y_{1t} - y_{2t}) + v_t \tag{60}$$

As pointed out before, for our set of models $y_{1t} - y_{2t}$ is a vector of exogenous variables. One can thus very simply estimate α_2 in (60) and test whether it is significantly different from zero. If it is not, the second model cannot be missed in the general model; the first model is not able to explain the data adequately.

Rather than letting $\alpha_1 = 1 - \alpha_2$ one can also have $\alpha_2 = 1 - \alpha_1$ and obtain as counterpart of (60)

$$(61) \qquad y_{2t} = X_t(\alpha_1\beta_1 + (1 - \alpha_1)\beta_2) + \alpha_1(y_{2t} - y_{1t}) + v_t$$

If one rejects the null hypothesis $\alpha_1 = 0$ one rejects the second model as a satisfactory explanation of reality.

One may observe that (60) and (61) are identical from the point of view of estimation, with $\alpha_1 = 1 - \alpha_2$ also for the estimates. Otherwise said, one can use (60) to test the null hypotheses of $\alpha_2 = 0$ (no contribution from model 2) and of $\alpha_2 = 1$ (no contribution from model 1).

It can be verified easily that the coefficients of the X_t in (60) satisfy the adding-up conditions and can be made to satisfy the homogeneity and symmetry conditions, but not the negativity condition except when one considers the ROT-CBS pair. For all possible pairs, (60) has been estimated with (only) the homogeneity and symmetry conditions explicitly imposed.

The DEMMOD package does not allow straightforward maximum likelihood estimation of the α_2. The term $\alpha_2(y_{1t} - y_{2t})$ has therefore been moved to the left-hand side of the system. For a given value of α_2 the system was estimated by DEMMOD. Trial and error produced the maximizing value of this coefficient.

These are given in Table 5 together with their asymptotic standard errors. The $\hat{\alpha}_R$, $\hat{\alpha}_C$, $\hat{\alpha}_A$, $\hat{\alpha}_N$ correspond with the α_2 of (60) when the ROT, CBS, AID and NBR systems, respectively, are in the role of the second model. The table also gives the corresponding information for the four single systems and the maximum logarithmic likelihood values for all the cases considered. The ML values of the first four systems are those of Table 2.

Table 5 : Coefficients and ML Values for Single and Pairs of Systems

System(s)	$\hat{\alpha}_R$	$\hat{\alpha}_C$	$\hat{\alpha}_A$	$\hat{\alpha}_N$	MLV
ROT	1	0	0	0	662.621
CBS	0	1	0	0	669.442
AID	0	0	1	0	667.390
NBR	0	0	0	1	661.016
ROT + CBS	-.42 (.34)	1.42 (.34)	0	0	670.197
ROT + AID	.07 (.30)	0	.93 (.30)	0	667.424
ROT + NBR	.96 (.54)	0	0	.04 (.54)	662.623
CBS + AID	0	1.06 (.52)	-.06 (.52)	0	669.447
CBS + NBR	0	1.35 (.32)	0	-.35 (.32)	670.086
AID + NBR	0	0	1.36 (.38)	-.36 (.38)	667.935

The value of $\hat{\alpha}_C$ of 1.42 in the model that combines ROT and CBS is the value of the coefficient of the difference between the vector of dependent variables of ROT and that of CBS with the former being on the left-hand side. Under the null hypothesis of $\alpha_C = 0$, α_R is one. This is the case of the first system (ROT). One can compare the corresponding logarithmic likelihood values. Twice their difference is asymptotically distributed as χ^2 with one degree of freedom. For the same combination, i.e. ROT + CBS one can also test whether $\hat{\alpha}_R = -.42$ is significantly different from zero. Under the null hypothesis of α_R being zero, α_C is one which is the case of the second system (CBS).

For each combination one has two likelihood ratio statistics (LRT). All those statistics are given in Table 6.

Table 6 : LRT Values for Paired Tests

Null Model	Alternative Model			
	ROT	CBS	AID	NBR
ROT	–	15.2	9.6	0.0
CBS	1.5	–	0.0	1.3
AID	0.1	4.1	–	1.1
NBR	3.2	18.1	13.8	

For 1 degree of freedom, the 5 and 1 per cent critical value of the χ^2 distribution are 3.8 and 6.6, respectively. A high value of the test statistic means that the null model needs the alternative model to explain the facts. One can also say that the alternative model rejects the null model as a stand alone model. From Table 6 one can see that at the 1 per cent significance level CBS rejects ROT and NBR and at the 5 per cent level also AID. CBS itself is not being rejected by any of the three other models. The NBR system does not reject any other model and is itself rejected by CBS and AID. It appears to be the weakest of the four. Clearly, CBS is the strongest with AID and ROT in the second and third position. This global picture corresponds with that of McAleer and Barten (1991) for the same data but with a different test set-up. There, however, AID and ROT were virtually at a par.

In interpreting these results one may note that the models performing best have both the AID type income coefficients. They differ in the price coefficients. AID has, of course, AID type price coefficients but CBS has ROT type price coefficients. It may be that AID type income coefficients perform better than the ROT type ones and that the ROT type price coefficients are in this respect better than the AID type counterparts. The next section will treat this issue more explicitly.

9. HIGHER ORDER COMPARISONS

There is no reason to limit the general model to a pair as in (59). One may write more generally :

$$\alpha_R y_{Rt} + \alpha_C y_{Ct} + \alpha_A y_{At} + \alpha_N y_{Nt} = X_t \gamma + v_t \tag{61}$$

where $\gamma = \alpha_R\beta_R + \alpha_C\beta_C + \alpha_A\beta_A + \alpha_N\beta_N$. Again here one may normalize by letting the α_j add up to one. One can eliminate α_R from (61) in this way. The resulting extended version of (60) is

$$y_{Rt} = X_t \gamma + \alpha_C(y_{Rt}-y_{Ct}) + \alpha_A(y_{Rt}-y_{At}) + \alpha_N(y_{Rt}-y_{Nt}) + v_t \tag{62}$$

Setting $\alpha_C = 1$, $\alpha_A = \alpha_N = 0$ one has the CBS system. Analogous specifications hold for the AID and NBR systems. ROT corresponds with all three α's being zero. The first three pairwise combinations of Table 5 of the preceding section correspond with two of the α's in (62) equal to zero and the third one being estimated. The second set of three pairwise combinations estimated a pair of the α's but with their sum restricted to unity.

Conceptually, one can extend the procedure of the preceding section by estimating (62) with two or three degrees of freedom for the α's. There is a problem, however. For the systems considered, relations (56) hold, which imply

$$y_{Rt} - y_{Ct} + y_{At} - y_{Nt} = 0 \tag{63}$$

or

$$(y_{Rt}-y_{Ct}) - (y_{Rt}-y_{At}) + (y_{Rt}-y_{Nt}) = 0 \tag{64}$$

This means that the three additional variables in (62) are perfectly collinear. Unconstrained estimation of the three α's of (62) is not possible.

One can use (64) to rewrite (62) as

$$(65)\qquad y_{Rt} = X_t\gamma + (\alpha_C+\alpha_A)(y_{Rt}-y_{Ct}) + (\alpha_A+\alpha_N)(y_{Rt}-y_{Nt}) + v_t$$

$$= X_t\gamma + \delta_1(y_{Rt}-y_{Ct}) + \delta_2(y_{Rt}-y_{Nt}) + v_t$$

Three observations can be made. First, there are only two degrees of freedom available. Equation (60) of the preceding section can only be extended by one extra variable. Second, the α's cannot be identified from the δ_1 and δ_2. The other way around is clear. One can calculate the δ_1 and δ_2 values corresponding with the $\hat{\alpha}$ of Table 5 - see e.g. Table 7 below. Third, the way (65) is written lets δ_1 be the coefficient associated with the difference between the ROT and the CBS systems, i.e. with the difference in the specification of the income coefficient. The estimate of δ_1 reflects the empirical importance of this difference in parametrization. The δ_2 is associated with the difference between the ROT type and AID type price coefficients, because it is only in that respect that ROT and NBR differ. So also the impact of that difference can be measured separately.

The estimated version of (65) is

$$(66)\qquad y_{Rt} = X_t\hat{\gamma} + \underset{(.34)}{1.43}\,(y_{Rt}-y_{Ct}) - \underset{(.53)}{.12}\,(y_{Rt}-y_{Nt}) + \hat{v}_t$$

The MLV is 670.221. One should realize that $\delta_1 = 1$ corresponds with AID type income coefficients and $\delta_2 = 1$ with AID type price coefficients. Zero's indicate ROT type of coefficients. As (66) shows, the sample favours AID type income coefficients and ROT type price coefficients. This conclusion agrees with the strong performance of CBS, which combines these two types of coefficients, in Table 6.

One can use (66), indicated by SYN, as the alternative model to test the null hypothesis of the models presented earlier. Table 7 summarizes the relevant information about these models.

Table 7 : Coefficients of General Model (65) and Test Statistics

Systems	$\hat{\delta}_1$	$\hat{\delta}_2$	MLV	LRT
ROT	0	0	662.621	15.2**
CBS	1	0	669.442	1.6
AID	1	1	667.390	5.7
NBR	0	1	661.016	18.4**
ROT + CBS	1.42 (.34)	0	670.197	.0
ROT + AID	.93 (.30)	.93 (.30)	667.424	5.6*
ROT + NBR	0	.04 (.54)	662.623	15.2**
CBS + AID	1	-.06 (.52)	669.447	1.5
CBS + NBR	1.35 (.32)	-.35 (.32)	670.086	.3
AID + NBR	1.36 (.38)	1	667.935	4.6*
SYN	1.43 (.34)	-.12 (.53)	670.221	-

* significant at 5 per cent significance level
** significant at 1 per cent significance level.

From Table 7 the artificial nesting procedure is clear. The δ_1 and δ_2 parametrize the differences between the various models. The last line gives the results for the general model of which the preceding systems are special cases. The last column gives the values of the likelihood ratio test statistic for the model in question as the null model and (66) as the alternative model. For the first four models two degrees of freedom are involved. For the other models only one degree of freedom has been used up.

The (asymptotically valid) test results show that SYN rejects ROT and NBR as single models firmly. Accordingly, it also rejects the combination of ROT and NBR. SYN almost rejects AID as a single model. AID combined with ROT or NBR cannot raise the likelihood enough to avoid rejection of these combinations. The most salient feature is the strength of CBS. It is not rejected by SYN. Moreover, none of combinations of CBS with the other models is rejected.

In fact, CBS, standing alone, fits the data not significantly worse than any of the other models or combinations of these models, including those with CBS itself. Still, SYN is a generalization of CBS - and for that matter of the other models as well - which could be considered as a demand system in its own right with two extra degrees of freedom to better adjust to the data. This aspect is further elaborated in the next section.

10. A SYNTHETIC SYSTEM

One can consider (65) as a demand system. This becomes somewhat more clear if one realizes that

$$(y_{Rt} - y_{Ct})_i = \bar{w}_{it}\ \Delta \ln Q_t$$

$$(y_{Rt} - y_{Nt})_i = (y_{Ct} - y_{At})_i = - \bar{w}_{it}\ (\Delta \ln p_{it} - \Delta \ln P_t)$$

where (56) and (63) are employed. Use of these relations results in the typical demand equation

$$(67) \qquad \bar{w}_{it}\ \Delta \ln q_{it} - \delta_1\ \bar{w}_{it}\ \Delta \ln Q_t + \delta_2\ \bar{w}_{it}\ (\Delta \ln p_{it} - \Delta \ln P_t)$$

$$= \text{constant}_i + d_i\ \Delta \ln Q_t + \Sigma_j\ e_{ij}\ \Delta \ln p_{jt} + v_{it}$$

with the right-hand side, apart from v_{it}, being the explicit form of the i-th element of $X_t\gamma$. Here, the *adding-up conditions* for the d_i and the e_{ij} are

$$(68) \qquad \Sigma_i d_i = 1 - \delta_1 \qquad\qquad \Sigma_i e_{ij} = 0$$

Since the expression on the left-hand side of (67) is homogeneous of degree zero in the prices, one should have on the right-hand side the *homogeneity condition* :

(69) $\quad \Sigma_j e_{ij} = 0$

A further inspection of the properties of the coefficients can be conveniently done with the following variant of (67)

(70) $\quad \bar{w}_{it} \, \Delta \ln q_{it} = \text{constant}_i + (d_i + \delta_1 \bar{w}_{it}) \, \Delta \ln Q_t$

$$+ \Sigma_j \, (e_{ij} - \delta_2 \, \bar{w}_{it} \, (\delta_{ij} - \bar{w}_{jt})) \, \Delta \ln p_{jt} + v_{it}$$

It is then easy to see that the income or budget elasticity η_i is

(71) $\quad \eta_i = (d_i + \delta_1 \bar{w}_{it}) \,/\, \bar{w}_{it} = d_i \,/\, \bar{w}_{it} + \delta_1$

and the Slutsky substitution elasticity ϵ_{ij} is given by

(72) $\quad \epsilon_{ij} = (e_{ij} - \delta_2 \bar{w}_{it}(\delta_{ij} - \bar{w}_{jt})) \,/\, \bar{w}_{it} = e_{ij} \,/\, \bar{w}_{it} - \delta_2(\delta_{ij} - \bar{w}_{jt})$

Clearly, symmetry of $\bar{w}_{it}\epsilon_{ij}$ in i and j requires symmetry of e_{ij}. The *symmetry condition* can be imposed in the same way as in the original elementary models. The negativity condition can only be guaranteed when $\delta_2 = 0$.

As the preceding discussion has made clear the elementary models specify δ_1 and δ_2 to be either zero or one. Expressions (71) and (72) offer somewhat more flexibility. It is one step on the way towards loosening the strait jacket of functional rigidity of the systems approach which is the butt of the Johansen (1981) complaint. As one can see from (71), with negative d_i and positive δ_1 a good can be a luxury for high values of $\bar{w}_{it}$ and an inferior good for sufficiently low values of $\bar{w}_{it}$. The reverse is true for positive d_i and negative δ_1. If d_i and δ_1 are both positive the elasticity is on the range $(d_i + \delta_1, \infty)$ which is positive. If they are both negative the range is $(-\infty, d_i + \delta_1)$ and thus negative, i.e. good i is an inferior good. For ϵ_{ij} one has a similar type of adaptability : the sign of ϵ_{ij} is in part dependent on the value taken by the variable $\bar{w}_{jt}$. With changing budget shares a pair of goods can turn from (Hicksian) complements into (Hicksian) substitutes. Of course, a negative ϵ_{ii} can turn into a positive one, which is the undesirable aspect of flexibility.

Table 8 presents the point estimates of the income and own price coefficients of the synthetic system along the same lines as Table 2 for the elementary systems. The d_i are somewhat more negative or less positive than the c_i of Table 2. The ϵ_{ii} are also more negative than the s_{ii} of Table 2. The standard errors are roughly the same as those of that table. These standard erros are conditional ones. They are obtained for δ_1 and δ_2 fixed at their estimated values.

Table 8 : Selected Point Estimates of the Synthetic System

	d_i	e_{ii}
FOOD	-.234 (.023)	-.113 (.021)
PLGD	-.068 (.011)	-.059 (.014)
DURA	.191 (.033)	-.053 (.032)
REST	-.320 (.032)	-.064 (.034)
MLV	670.221	

Of course the d_i and e_{ij} are conceptually different from the b_i and s_{ij} and the c_i and r_{ij}, respectively. To compare their estimates they have been converted into elasticities using (71) and (72) and the value of the budget shares of Table 1. Like in Tables 3 and 4, Table 9 limits itself to the income or budget elasticities and the own price substitution elasticities.

Table 9 : Selected Elasticities for the Synthetic System

	η_i			ϵ_{ii}		
	1921	Mean	1981	1921	Mean	1981
FOOD	.736	.498	-.316	-.256	-.360	-.739
PLGD	.714	.683	.351	-.512	-.539	-.824
DURA	2.246	2.182	2.294	-.135	-.119	-.146
REST	.472	.638	.881	-.112	-.087	-.060

Comparing the elasticities of Table 9 with those of Tables 3 and 4 one notices a somewhat greater variability across the data points used for evaluation. The estimation of δ_1 and δ_2 has indeed resulted in a greater flexibility. The negative budget elasticity for FOOD for 1981 is a consequence of this. It is somewhat difficult to fully accept it as realistic. Given the limitations of the Synthetic System, however, this outcome maximizes the likelihood, is the most realistic. To avoid it a further increase in flexibility may be needed.

11. CONCLUDING REMARKS

Demand systems allocate a given total budget over a set of commodities taking into account the effects of price variation. They are a tool in the hands of the demand analyst to describe and predict empirical consumer behaviour, for a whole economy or for individual consumption units. They are based on the microeconomic theory of individual consumer behaviour, which supplies various restrictions on the allocation system. These are not sufficient to determine the functional form. Four alternative approaches to the functional specification have been sketched, with well-known demand systems as illustrations. A particular choice of constants can be judged from a theoretical point of view, but clearly empirical considerations are of no less importance. Comparing alternative specifications empirically can be and has been done using some goodness-of-fit criteria. Still, the statistical interpetation of that approach is not clear. Statistical test procedures have to take into account that mostly the models compared are not nested within each other.

One approach is the variable addition method of McAleer (1983) which was extended to combination of vector valued functions by Barten and McAleer (1991) and applied to a comparison of a set of four related demand systems, namely the Rotterdam system, the Almost Ideal Demand system, the CBS system and the NBR system, using data for The Netherlands 1921-1981. Barten and McAleer studied linear combinations of these sytems using matrices to weight the various members. This has the advantage of allowing for detailed interactions. The linear combinations are, however, themselves not typical demand systems because they cannot be made to satisfy Slutsky symmetry.

This is different when one restricts oneself to linear combinations with scalar weights. These combinations can be taken as demand systems in their own right. This is the approach taken in the present paper, where the same set of models and the same set of data as in the Barten-McAleer paper has been used. Given the nature of the dependent variables of the four systems considered, the test basically reduces to assessing the extra explanatory power of vectors of exogenous variables. The Likelihood Ratio Test statistic can be used for this purposes. As it turns out the CBS model performs best.

One of the alternatives in the testing set-up is a combination of all four systems. This Synthetic System nests not only the four elementary systems but also all combinations of two or three of those systems.

It is a demand system in its own right. It has two degrees of freedom more than the elementary systems to adjust to the data and is therefore somewhat more flexible. For the sample used, however, it performs not significantly better than the CBS system on its own.

The fact that the Synthetic System does not outperform the CBS system in a particular sample does not mean that is a useless generalization. Other samples may yield other findings. The increased flexibility is an advantage on its own. As a representation of a class of differential demand functions it can, moreover, serve also a theoretical purpose.

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