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Hoogstrate, A.J.; Osang, T.

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**SAVING, OPENNESS, AND GROWTH**

By André J. Hoogstrate and Thomas Osang

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# Saving, Openness, and Growth

André J. Hoogstrate,<sup>\*†</sup>  
Tilburg University, Tilburg

Thomas Osang<sup>‡</sup>  
Southern Methodist University, Dallas

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<sup>†</sup>Tilburg University, Faculty of Economics and Business Administration, P.O. Box 90153, 5000 LE Tilburg, The Netherlands. Tel: +31-13-466-2031, Fax: +31-13-466-3280, E-mail: a.j.hoogstrate@kub.nl. This research was sponsored by the Economics Research Foundation, which is part of the Netherlands Organization for Scientific Research (NWO).

<sup>‡</sup>Southern Methodist University, Department of Economics, Dallas, TX 75275, Tel: (214) 768-4398, Fax: (214) 768-1821, E-mail: tosang@mail.smu.edu. This research was supported by a grant from the University Research Council of SMU.

## Abstract

Using a country-level panel data set, we investigate whether a country's saving rate matters for the relationship between GDP growth and openness to trade. We first derive a new LM-type statistic to test for the existence of an endogenous threshold separating high from low savings regimes. Once existence is established, estimation of threshold VAR models shows that for countries in the high savings regime, openness has a positive effect on growth, while it has no effect for countries in the low savings regime. Furthermore, there are striking differences in the impulse response functions between the two regimes. In the high savings regime, a positive shock in openness leads to higher GDP growth in all subsequent years, while in the low savings regime, the effect on GDP growth is substantially smaller in size and becomes insignificant after a few periods.

# 1 Introduction

This paper brings together two important parts of the empirical literature on the determinants of economic growth, namely growth and openness to trade on the one hand and growth and saving on the other. By now, there exists a large number of empirical studies on the relationship between growth and various measures of openness to trade (See, for example, Harrison [24], Levine and Renelt [29], Edwards [14], [15], Jorgenson and Ho [26], Balassa [5], [6], and Quah and Rauch [35]). Surprisingly, many of these studies find a rather weak relationship between openness to trade and growth. The estimated coefficient on openness is often statistically insignificant or even has the wrong sign. In Anne Harrison's study, for example, only two of the six measures of openness are significant at the five percent level, while the sign of the trade share in GDP is negative though not significant (Table 6, p. 434). Furthermore, some studies find the estimated coefficient on openness to be sensitive to changes in the econometric model or data set. Levine and Renelt report that "after controlling for the share of investment in GDP we cannot find an independent and robust relationship between any trade ...indicator and growth" (p. 954).

In contrast to the results for the empirical relationship between openness and growth, the empirical literature provides solid evidence on the relationship between growth and saving (See, for example, Maddison [30], Carroll and Weil [11], Bosworth [10], Gupta and Islam [19]). These studies find that countries with higher saving rates have significantly higher growth rates, a result that sensitivity tests show to be fairly robust (Levine and Renelt [29], p. 946)<sup>1</sup>.

Interestingly, none of the studies cited above examines the potentially nonlinear relationship between the GDP growth, openness, and saving. Such a nonlinear link has been the focus of recent theoretical models from the literature on trade and endogenous growth. These models indicate that certain model parameters linked to the consumption and savings behavior of households may play a key role in the interaction between growth and openness (Feenstra [16], Osang and Pereira [31], [32]). Feenstra shows that trade can increase or decrease a country's growth rate depending on the value of

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<sup>1</sup>Levine and Renelt show that the relationship between the investment share in GDP and GDP growth is strong and robust. Since it is well known that investment and saving rates are highly correlated within countries, we interpret their findings as indirect evidence for a robust link between saving and GDP growth.

the *instantaneous* elasticity of substitution, while Osang and Pereira point out that there exists a threshold level of the *intertemporal* elasticity of substitution that separates growth-enhancing from growth-reducing regimes of increased openness to trade<sup>2</sup>. Since the intertemporal elasticity of substitution is a key parameter for saving in the endogenous growth literature, it seems naturally to test the hypothesis that the relationship between openness and growth changes once the saving rate exceeds a certain endogenously determined threshold level. If the hypothesis cannot be rejected by the data, one could argue that the weak link between openness and growth found in the literature may simply be the result of a misspecified model.

In order to test the above hypothesis, we first test for the existence of a threshold saving rate separating high from low saving regimes. We then use a dynamic model and simultaneously estimate the benchmark value of the savings rate and the other parameters of the model using data from 58 countries for the period from 1960 to 1987. The main findings of the paper are as follows. First, using different threshold test statistics we show the existence of an endogenous threshold separating high from low savings regimes. Second, we find that saving rates indeed matter for the link between openness and growth. For countries in the high savings regime, openness to trade has a positive effect on growth, while it has no effect on growth for countries in the low savings regime. Third, there are striking differences in the impulse response functions between the two regimes. In the high savings regime, a positive shock in openness leads to higher GDP growth in all subsequent years, while in the low savings regime the effect on GDP growth is substantially smaller in size and becomes insignificant after a few periods. Interestingly, a positive shock in openness leads to a further decline in savings in the low savings regime.

The paper extends the existing literature in three ways. First, we explicitly estimate the non-linear relationship between saving rates, openness to trade, and growth, while the standard growth literature cited above uses a linear relationship at best. Second, by employing a dynamic model instead of the widely used cross-section analysis, we allow the effect of saving on

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<sup>2</sup>A possible mechanism through which openness affects growth is as follows (see Osang and Pereira [31], [32] for details). International differences in preferences and/or technologies lead to different steady state growth rates across countries. Assuming balanced trade and complete specialization, increasing the volume of trade in both countries (e.g. due to lower trade barriers or changes in consumer preferences) induces changes in the terms of trade. In this situation it is most likely that the country with the weak attitude toward saving will experience an improvement in its terms of trade and, in turn, a decline in output growth, while the country with the strong saving performance will experience the opposite effects.

the relationship between openness and growth to work in the cross section (across countries) as well as the time dimension. This is important since the cross section approach is only justifiable in very specific cases as shown by Harrison [24]. Third, estimating an endogenous threshold in a dynamic model raises some interesting questions concerning the underlying econometric theory since the estimation model is not well defined in the case of a non-existent threshold level. To this regard we introduce a new test statistic which allows us to test for the existence of threshold saving level. The test is based on results derived by Bierens and Ploberger [9] and can be considered as an alternative to tests by Andrews et al. [1] and Andrews and Ploberger [2]. One advantage of the new test is that it is easier to calculate than, for example, the threshold test suggested by Hansen [21], [22].

The remainder of the paper is organized as follows. Section 2 describes the estimated model in detail. We discuss the econometric methodology necessary for estimation and testing of thresholds in panel data sets in section 3. Section 4 describes the data, while section 5 contains the empirical results. Section 6 concludes the paper.

## 2 The Empirical Model

The recent theoretical literature on growth in open economies suggests that GDP growth mainly depends on the following parameters<sup>3</sup>:

- Technology parameters such as total factor productivity,  $A$ , and scale elasticities,  $\alpha$ .
- Taste parameters such as the intertemporal elasticity of substitution,  $\sigma$ , and the discount rate,  $\rho$ .
- Trade policy parameters,  $\tau$ , measuring a country's tariff and non-tariff barriers to trade.

The relation can thus be written as

$$growth = f(A, \alpha, \sigma, \rho, \tau). \quad (1)$$

Unfortunately, in most cases we lack data that directly measure these parameters, especially over time. It is thus common in the literature to approximate these parameters with data that are available both across countries and time.

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<sup>3</sup>See, for example, Lee [27], Rivera-Batiz and Romer [36], and Turnovsky [38].



Taste parameters reflect a country's willingness to postpone current consumption and thus determine the domestic supply of financial capital. Advanced production technologies are intensive in both physical and human capital and correspond to high levels of total factor productivity. Advanced production technologies are thus a major factor behind the domestic demand for financial capital. Given the fact that international capital is not sufficiently mobile (see Gordon and Bovenberg [18] for a recent investigation of this well-known puzzle) it seems reasonable to use the domestic saving rate of a country as a proxy for both sufficient supply and adequate demand in the market for financial capital. Trade policy parameters can be approximated either directly through some index of trade liberalization using country sources on trade barriers (see, for example, Thomas et al [37]) or indirectly through measures such as the blackmarket premium in the currency market or an index measuring price distortions for consumption goods<sup>4</sup>. A widely used indirect measure for trade barriers is the ratio of exports plus imports to GDP. This proxy has the advantage that it is available for many countries for at least three decades. It is also relatively free of different definitions and data collection techniques between countries. Furthermore, Harrison [24] shows that it has the highest correlation coefficient with trade reform compared to all other indirect openness measures (Table 2, p. 429). Its most severe disadvantage is that its value does not depend on trade barriers alone but also on country size or foreign direct investment. Nevertheless, we will use the share of trade in GDP as our proxy for a country's openness to trade. Finally, it is common in the literature to assume that the technology parameters can be incorporated in the function  $f$ . This leaves us with the following estimable model:

$$growth = f(s, o) \tag{2}$$

where  $s$  indicates the saving rate and  $o$  indicates the level of openness. Note that (2) is also a common result in the endogenous growth literature where output growth can be expressed as a function of the (endogenous) savings rate of the economy. Since (2) expresses an equilibrium condition which may or may not hold in reality due to exogenous shocks, it is important to include dynamics in our empirical model. For this reason and because the functional form of  $f$  is unknown we use an unrestricted linear first-order VAR model as an approximation of (2),

$$X_{i,t} = c + A_1 X_{i,t-1} + \varepsilon_{i,t}, \tag{3}$$

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<sup>4</sup>See Harrison [24] for other indirect measures of trade barriers as well as a detailed discussion of measurement problems associated with both direct and indirect proxies of trade barriers.

with  $X_{i,t} = (O_{i,t}, S_{i,t}, \Delta Y_{i,t})'$  where

- $\Delta Y_{i,t}$  denotes GDP growth of country  $i$  at time  $t$ , ( $\Delta Y_{i,t} = \log(Y_{i,t}) - \log(Y_{i,t-1})$ )
- $S_{i,t}$  denotes the gross domestic savings rate of country  $i$  at time  $t$ ,
- $O_{i,t}$  denotes the log ratio of imports plus exports to GDP of country  $i$  at time  $t$ ,

and the disturbance term  $\varepsilon_{i,t}$  is assumed to be independent Gaussian with  $E(\varepsilon_{i,t}) = 0$ , and covariance  $E(\varepsilon_{i,t}\varepsilon'_{i,t}) = \Omega_i$ . We assume that the intercept  $c$  captures the effects of the technology parameters. The assumption that  $c$  is identical across countries is rather restrictive. Unfortunately, at the moment there is no technique available to determine an endogenous threshold in a real panel data context with individual effects. Because we are also interested in the contemporaneous effects (assuming them to be the same for each individual country), we rewrite (3) in the following structural form

$$X_{i,t} = \mu + B_0 X_{i,t} + B_1 X_{i,t-1} + u_{i,t} \quad (4)$$

where  $u_{i,t}$  is again independent Gaussian with zero mean but with diagonal covariance matrix  $E(u_{i,t}u'_{i,t}) = \text{diag}(\sigma_{i,1}^2, \sigma_{i,2}^2, \sigma_{i,3}^2) \equiv \Lambda_i$ , where  $\Omega_i = \Gamma\Lambda_i\Gamma'$  and  $\Gamma$  is lower triangular with ones on the main diagonal. We then have  $\mu = \Gamma^{-1}c$ ,  $B_0 = (I - \Gamma^{-1})$  and  $B_1 = \Gamma^{-1}A_1$ . Observe that  $B_0$  is lower triangular with zeros on the main diagonal<sup>5</sup>.

To test the hypothesis that the relation between openness and GDP growth changes if the savings level exceeds a certain level we contrast the benchmark model in (4) with the following threshold VAR model:

$$X_{i,t} = (\underline{\mu} + \underline{B}_0 X_{i,t} + \underline{B}_1 X_{i,t-1})I(S_{i,t-2} \leq \gamma) \quad (5)$$

$$+ (\bar{\mu} + \bar{B}_0 X_{i,t} + \bar{B}_1 X_{i,t-1})I(S_{i,t-2} > \gamma) + w_{i,t}$$

where  $I(S_{i,t-2} > \gamma)$  is an indicator function which equals one when the inequality holds and zero otherwise. The unknown threshold parameter  $\gamma$  indicates at which level of savings the change takes place. The use of  $S_{i,t-2}$  instead of  $S_{i,t}$  in the indicator function allows us to consider the savings level pre-determined. We proceed by introducing tests for the existence of an endogenous threshold as well as recently developed techniques for estimation of (5).

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<sup>5</sup>For further details on the problem of identification, see Lütkepohl [28].

### 3 Econometric considerations

The econometric analysis of the above models requires a number of non-standard techniques which are discussed in detail in this section. In particular, we discuss the following topics: tests for the existence of a threshold, estimation and inference of the threshold VAR model, and nonlinear impulse response analysis.

The econometric analysis of the benchmark VAR model (4) is essentially the analysis of a vector autoregressive model with pooled coefficients. This analysis is similar to that of a large VAR with restrictions on the coefficients of the lag polynomial. Since we assume  $T$  to be large we can apply the standard asymptotic theory on stationary vector autoregressions taking into account the pooling restrictions<sup>6</sup>.

#### 3.1 Testing for the existence of a threshold

To test for the existence of a threshold we choose the following setup. Let the observable variables be denoted by  $(Y_i, X_i)$  for  $i = 1, \dots, N$ .  $X_i$  is a  $(1 \times k_x)$  vector and  $Y_i$  is a scalar variable. Let the following relation hold:

$$Y_i = X_i\beta_1 + Z_i I(q_i > \gamma)\beta_2 + \varepsilon_i \quad (6)$$

where  $Z_i$  is a  $(1 \times k_z)$  vector,  $q_i$  is a one dimensional variable and  $\gamma$  is a scalar which we assume to be contained in a compact subset  $\Gamma$  of  $R$ . We assume that the variables in  $Z_i$  are also contained in  $X_i$ , i.e.,  $Z_i$  is a subvector of  $X_i$  and thus observable as well. Finally, we assume  $\varepsilon_i$  to be a zero mean i.i.d distributed random variable with finite variance  $\sigma^2$ . As noted above,  $I(\cdot)$  denotes an indicator function which determines a possible break between observations satisfying the inequality condition and those not satisfying the condition.

There are several problems which complicate the analysis. First, under the null hypothesis of no threshold (i.e.,  $\underline{\mu} = \bar{\mu}$ ,  $\underline{B}_0 = \bar{B}_0$ , and  $\underline{B}_1 = \bar{B}_1$ ), the threshold parameter  $\gamma$  is not identified, and we therefore cannot apply standard hypothesis testing theory. To solve this issue, we derive a new test statistic appropriate for our case. This new test statistic builds upon and is related to versions of the *Wald*, *LM*, and *LR* statistics introduced by Andrews, Lee and Ploberger [1], Andrews and Ploberger [2] and Hansen [21], [22]. Second, most of the existing test statistics apply to time series and/or cross section data. The test statistic developed in this paper is well suited

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<sup>6</sup>See Lütkepohl [28] or [20] for an introduction to this analysis.

for a panel data set. Third, some of the existing procedures are tedious to calculate due to bootstrapping methods, while others involve the choice of some rather subjective parameters. In contrast, our simple test statistic avoids both problems.

Our test procedure can be derived in two ways, a Wald and an LM version. The LM version of the test is presented below, while the Wald version is derived in Appendix B. To derive the LM based test statistic, we consider estimation of (5) under  $H_0$  in which case the restricted OLS estimator is given by

$$\hat{\beta}_1^r = (X'X)^{-1}X'Y, \quad (7)$$

where  $X = (X'_1, \dots, X'_N)'$  and  $Y = (Y'_1, \dots, Y'_N)'$ . Denoting the residuals of the restricted estimator by  $u^r = Y - X \hat{\beta}_1^r$  and introducing both the functions  $Z_{(\gamma)} = (Z'_1 I(q_1 > \gamma), \dots, Z'_N I(q_N > \gamma))'$  and the matrix  $M = (I - X(X'X)^{-1}X')$  we observe that

$$Z'_{(\gamma)} u^r = Z'_{(\gamma)} (Y - X \hat{\beta}_1^r) = Z'_{(\gamma)} (I - X(X'X)^{-1}X')Y \quad (8)$$

$$= Z'_{(\gamma)} M(X\beta_1 + Z_{(\gamma)}\beta_2 + \varepsilon) \quad (9)$$

$$= Z'_{(\gamma)} M(Z_{(\gamma)}\beta_2 + \varepsilon). \quad (10)$$

This leads to

$$Z'_{(\gamma)} u^r = Z'_{(\gamma)} M \varepsilon \quad \text{under } H_0 \quad (11)$$

$$Z'_{(\gamma)} u^r = Z'_{(\gamma)} M Z_{(\gamma)} \beta_2 + Z'_{(\gamma)} M \varepsilon \quad \text{under } H_1 \quad (12)$$

In Appendix B we show that the normalized stochastic function

$$z_N(\gamma) = \frac{1}{\sqrt{N}} Z'_{(\gamma)} u^r \quad (13)$$

converges, as  $N \rightarrow \infty$ , to a  $k_z$  dimensional Gaussian process  $z(\gamma)$  with covariance kernel  $\sigma(\gamma_1, \gamma_2)$  where

$$\sigma(\gamma_1, \gamma_2) = \lim_{N \rightarrow \infty} \frac{1}{N} \sigma^2 Z'_{(\gamma_1)} M Z_{(\gamma_2)}.$$

Since it is difficult to test whether the stochastic function (13) satisfies the behavior associated with the null hypothesis we use a transformation to summarize its behavior in one test statistic. We introduce the integrals

$$\int_{\Gamma} \mathcal{S}(z_N(\gamma) z'_N(\gamma)) d\gamma, \quad \text{and} \quad \int_{\Gamma} \mathcal{S}(\sigma_N(\gamma, \gamma)) d\gamma \quad (14)$$

where  $\mathcal{S}(\cdot)$  denotes the operator defined as the sum of all elements of its argument and  $\sigma_N(\gamma, \gamma) = \frac{1}{N} \hat{\sigma}^2 Z'_{(\gamma)} M Z_{(\gamma)}$ , where  $\hat{\sigma}^2$  is a consistent estimate of  $\sigma^2$ . The new test statistic, denoted by *BPH*, can now be defined as follows:

$$BPH = \int_{\Gamma} \mathcal{S}(z_N(\gamma) z'_N(\gamma)) d\gamma / \int_{\Gamma} \mathcal{S}(\sigma_N(\gamma, \gamma)) d\gamma. \quad (15)$$

Under the assumptions given in Appendix B, which are essentially the same assumptions as in Hansen [22], we can derive an asymptotic distribution of *BPH*. Since this asymptotic distribution depends on the data through the covariance kernel  $\sigma(\gamma_1, \gamma_2)$  we follow Bierens and Ploberger [9] by using an upper bound to the true asymptotic distribution which is data independent<sup>7</sup>. The following critical regions are taken from Bierens and Ploberger:

$$P(W > 3.23) = 0.10, \quad P(W > 4.26) = 0.05, \quad P(W > 6.81) = 0.01.$$

The main advantage of the BPH test in comparison to the existing tests is its simplicity. In contrast to Hansen's F-test [22] no bootstrapping is needed. Instead we can use the data independent upper bound given in Bierens and Ploberger. Compared to the supremum, average, and exponential LM tests (*supLM*, *aveLM* and *expLM*, respectively) proposed by Andrews [3] and Andrews and Ploberger [2], our test statistic is independent of any nuisance parameters such as the number of regressors or the cut off levels of the threshold region. A disadvantage of our approach is that it can be conservative with the degree of conservatism depending on the problem at hand. Furthermore, it turns out that our test is less powerful than the related LM test statistics mentioned above<sup>8</sup>.

### 3.2 Estimation and inference on the threshold

If the above test statistics lead us to conclude that a threshold exists, we continue by estimating the threshold,  $\gamma$ , as well as the other coefficients of the model, namely  $\beta_1$  and  $\beta_2$ . The estimators for  $\gamma$ ,  $\beta_1$  and  $\beta_2$  are the solution to the following nonlinear least squares problem:

$$(\hat{\gamma}, \hat{\beta}_1, \hat{\beta}_2) = \arg \min_{\gamma \in \Gamma, \beta_1, \beta_2} (Y - X\beta_1 - Z_{(\gamma)}\beta_2)'(Y - X\beta_1 - Z_{(\gamma)}\beta_2).$$

<sup>7</sup>A much more detailed discussion, in particular on the derivation of the upper bound, is given in Appendix B.

<sup>8</sup>Appendix B contains more detailed information about the power of the test as well as a result concerning a family of local alternatives. We also discuss the connection between our and the other LM test statistics.

Finding the global minimum can be achieved in two steps. First, we minimize the sum of squared errors for a fixed  $\gamma$ . Applying OLS gives us an estimate of the variance of the residuals,  $\hat{\sigma}^2(\gamma)$ . Second, we minimize  $\hat{\sigma}^2(\gamma)$  over all  $\gamma \in \Gamma$ . The final estimates are then the OLS coefficients corresponding to the  $\gamma$  which minimizes  $\hat{\sigma}^2(\gamma)$ . Note that when the  $\epsilon_i$  are i.i.d  $N(0, \sigma^2)$ , this estimator is also the MLE.

As is known from the literature (see, for example, Bai [4], Picard [33], and Chan [12]), the estimator for  $\gamma$  has a convergence rate of order  $n$ , which is much faster than the order of convergence ( $\sqrt{n}$ ) for the other parameters of the model. The derivation of the asymptotic distribution of the estimator for  $\gamma$  is rather difficult, in particular when the change between  $\beta_1$  and  $\beta_2$  is considered to be fixed or relatively large. In this case the distance between the two parameters appears in the asymptotic distribution for  $\hat{\gamma}$  which makes inference results almost impossible. However, under the assumption of a local alternative, i.e. a small difference between  $\beta_1$  and  $\beta_2$ , Hansen [22] is able to derive the asymptotic distribution of the Likelihood Ratio test statistic for  $\gamma = \gamma_0$ . He then uses this result to construct a confidence interval for  $\gamma$ . The confidence intervals presented in Table 1 and 4 below are based on his procedure.

### 3.3 Nonlinear impulse responses

Computing impulse response functions for nonlinear dynamic models is more complicated than computing impulse response functions for linear dynamic models for several reasons. One of the complications arises from the fact that in most cases there are no analytical results in the nonlinear case. This means that the impulse responses must be obtained numerically or need to be simulated. Further, it is much harder to present and investigate all the information contained in the impulse response of a nonlinear system. This is due to the fact that the response of a nonlinear system to a shock at time  $t_0$  is path-dependent. The response depends in a nonlinear way on the history of the system, i.e., on the observations before the shock enters the system, and on the disturbances which enter the system between time  $t_0$  and  $t_0 + k$ . Finally, the proportionality of a response to the size of the shock in a linear system does not hold in a nonlinear system<sup>9</sup>.

For the linear VAR we present the traditional impulse responses, including 2 standard-errors confidence bounds, based on 500 drawings from the

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<sup>9</sup>More details on the problems of analyzing nonlinear impulse response can be found, amongst others, in Gallant, Rossi and Tauchen [17] and Potter [34].

distribution of the estimates of the parameters.

For the nonlinear threshold VAR we proceed as follows. We estimate an impulse response function corresponding to the Generalized Impulse Response function proposed by Potter [34] and also analyzed by Balke and Chang [7]:

$$GI(t_0, k, \delta, \Omega_{t_0-1}) = E(X_{t_0+k} | \Omega_{t_0-1}, \varepsilon_{t_0} = \delta) - E(X_{t_0+k} | \Omega_{t_0-1}),$$

where  $\Omega_{t_0-1}$  is the history at time  $t_0$ , and  $\delta$  is the shock given to the system at time  $t_0$ . To obtain this impulse response function we need to integrate out the future shocks  $\varepsilon_{t_0+1}, \dots, \varepsilon_{t_0+k}$ , that is

$$GI(t_0, k, \delta, \Omega_{t_0-1}) = E_{\varepsilon_{t_0+1}, \dots, \varepsilon_{t_0+k}} (E(X_{t_0+k} | \Omega_{t_0-1}, \varepsilon_{t_0} = \delta, \varepsilon_{t_0+1}, \dots, \varepsilon_{t_0+k}) - E(X_{t_0+k} | \Omega_{t_0-1}, \varepsilon_{t_0+1}, \dots, \varepsilon_{t_0+k})).$$

To do this we generate a large number of future zero mean i.i.d. normal shocks  $\varepsilon_{t_0+1}^i, \dots, \varepsilon_{t_0+k}^i$ ,  $i = 1, \dots, R$ , and replace  $GI(t_0, k, \delta, \Omega_{t_0-1})$  by

$$\begin{aligned} \tilde{GI}(t_0, k, \delta, \Omega_{t_0-1}) &= \frac{1}{R} \sum_{i=1}^R E(X_{t_0+k} | \Omega_{t_0-1}, \varepsilon_{t_0} = \delta, \varepsilon_{t_0+1}^i, \dots, \varepsilon_{t_0+k}^i) \\ &\quad - \frac{1}{R} \sum_{i=1}^R E(X_{t_0+k} | \Omega_{t_0-1}, \varepsilon_{t_0+1}^i, \dots, \varepsilon_{t_0+k}^i), \end{aligned}$$

i.e., we average the impulse response function over the future shocks.

Next we notice that  $\tilde{GI}(t_0, k, \delta, \Omega_{t_0-1})$  depends on  $\Omega_{t_0-1}$ . In a threshold model it makes a big difference whether the system is close to the threshold level at  $t_0$ , the time of the shock, or not. Therefore, we consider  $\tilde{GI}(t_0, k, \delta, \Omega_{t_0-1})$  conditional on different histories  $\Omega_{t_0-1}$ . In our case we are especially interested in the behavior of the system for the high and low saving regimes. Therefore, we calculate  $\tilde{GI}(t_0, k, \delta, \Omega_{t_0-1})$  for the following three situations: unconditional on the time we are at time  $t_0$ , conditional on being in the high saving regime at time  $t_0$ , and conditional on being in the low saving regime at time  $t_0$ .

For each of the three situations we generate 100 histories, simulate the model, and leave out the first 500 observations to avoid initial observation problems. For each of the individual histories we calculate the generalized impulse response function based on 100 sets of future shocks ( $R = 100$ ). Finally, we also take into account the uncertainty of the parameter estimates of the system. Therefore, we draw 50 sets of parameters from the asymptotic

distribution of the estimated parameters. For each set of parameters we replicate the above procedure. Therefore, each impulse response function presented in this paper is based on 5000 ( $= 50 \times 100$ ) simulations.

Since the impulse response function of a threshold model can be asymmetric, we investigate both negative and positive unit shocks. The results of our impulse response analysis are presented in Figures 2 to 10 in Appendix A. We present two sets of figures for each of the three classes of histories: we first present the average and the 95% most centered realizations of the impulse response function followed by the average impulse response function together with its two 95% confidence bounds.

## 4 The Data

Per capita GDP, savings and openness are taken from World Bank data [40]. GDP growth is the log difference of real per capita GDP in constant 1987 value of the local currency. The saving rate is the ratio of nominal gross domestic savings to nominal GDP (both in local currency), while openness to trade is the ratio of nominal exports plus imports to nominal GDP (both again in local currency). These data are available for all OECD countries as well as a number of developing countries. Excluding oil exporting countries, non-market economies, as well as countries with a population of less than one million, 58 countries remain in the sample<sup>10</sup>. The sample period ranges from 1960 to 1987<sup>11</sup>.

We use log transforms of both GDP and openness to trade, a procedure that cannot be applied to the saving rate because the rate is negative in some years for some countries. Fortunately, scale problems do not arise since all variables are similarly scaled. Figure 1 in Appendix A displays the empirical distributions of the data. Clearly, the saving rate displays a good deal of variability which is necessary to verify our hypothesis of a changing relation between output growth and openness. Without such variability, testing and

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<sup>10</sup>The countries in our sample are: Algeria, Argentina, Australia, Austria, Bangladesh, Belgium, Bolivia, Brazil, Canada, Chile, Colombia, Costa Rica, Cote d'Ivoire, Denmark, Egypt, El Salvador, Finland, France, Germany, Ghana, Greece, Guatemala, India, Indonesia, Ireland, Israel, Italy, Jamaica, Japan, Kenya, South Korea, Madagascar, Malaysia, Mauritius, Mexico, Morocco, Netherlands, New Zealand, Nigeria, Norway, Pakistan, Panama, Paraguay, Peru, Philippines, Portugal, Senegal, Singapore, Spain, Sri Lanka, Sweden, Switzerland, Thailand, Turkey, United Kingdom, United States, Uruguay, and Venezuela.

<sup>11</sup>The sample period was chosen to allow for a direct comparison between our and earlier results such as Harrison [24] and Levine and Renelt [29].



estimation of a threshold level of saving would be futile.

## 5 Empirical Results

### 5.1 Long-run effects

We first consider the long-run relation between the saving rate and GDP growth. Using time-averaged variables for each country, i.e.,  $\Delta Y_i = \frac{1}{T} \sum_t \Delta Y_{i,t}$ ,  $S_{i,-1} = \frac{1}{T} \sum_t S_{i,t-1}$  where  $T$  denotes the time dimension of the sample, simple OLS yields the following result:

$$\Delta Y_i = \underset{(0.006)}{0.007} + \underset{(0.029)}{0.083} S_{i,-1} \quad R^2 = 0.13, \quad N = 58 \quad (16)$$

The estimated coefficient for savings is positive and significant at the 5% level. Note that this result is based on the assumption of equality across countries. White's test for heteroscedasticity [39] yields a value of 1.972 ( $= N\hat{R}^2$ ) which is less than the critical value at the 5% level of the chi-square distribution with 3 degrees of freedom. We therefore cannot reject homogeneity. Further tests for normality of the residuals lead us to conclude that we cannot reject normality either.

Adding openness to trade to the above estimation model yields the following result:

$$\Delta Y_i = \underset{(0.007)}{0.012} + \underset{(0.029)}{0.083} S_{i,-1} + \underset{(0.003)}{0.005} O_{i,-1} \quad R^2 = 0.16, \quad N = 58 \quad (17)$$

Clearly, the effect of openness is small and insignificant! Again, we cannot reject normality nor homogeneity of the residuals (White's test yields  $N\hat{R}^2 = 2.826$ ).

Next we test for the existence of a threshold. In addition to our own test given in (15) and denoted by *BPH*, we calculate three LM tests (*aveLM*, *expLM*, *supLM*) as well as Hansen's F-test [22] denoted by *HansenF*. Since the regressors are averages over time, the threshold at a certain time would be conditioned on future values of the regressors producing inconsistent estimates. To avoid this problem we use the saving rate of 1960 as the threshold variable. Table 1 contains the values for the five test statistics, their corresponding p-values, as well the threshold estimate and its 95% confidence interval.

Table 1: Threshold Tests and Estimation

	Test Statistic	p-value
<i>BPH</i>	5.44	$0.01 < p < 0.05$
<i>expLM</i>	3.92	$0.05 < p < 0.1$
<i>aveLM</i>	7.05	$0.01 < p < 0.05$
<i>supLM</i>	10.23	$> 0.10$
<i>HansenF</i>	13.98	0.074
	Estimate of threshold	95% conf. interval
$\gamma$	0.1442	(0.049, 0.248)

As explained above, to conduct threshold tests based on the three LM tests as well as Hansen's F-test, we need to cut off a certain fraction,  $\pi$ , both at the top and the bottom of the empirical distribution of the saving rate. Based on 1960 data for the saving rate, this amounts to a search on the interval  $[0.095, 0.276]$  for  $\pi = 15\%$ . With the exception of the *supLM* test, all tests reject the null of no threshold at the 10% significance level. The resulting estimate for the threshold is 0.144. We use this value to split the sample and reestimate the above equation for each subsample. For the low savings countries ( $\gamma \leq 0.144$ ) we find

$$\Delta Y_i = 0.001 + 0.186 S_{i,-1} + 0.005 O_{i,-1} \quad R^2 = 0.38, \quad N = 22$$

$$\begin{matrix} (0.011) & (0.058) & (0.005) \end{matrix} \tag{18}$$

and for the high savings countries ( $\gamma > 0.144$ ) we obtain

$$\Delta Y_i = -0.003 + 0.107 S_{i,-1} - 0.005 O_{i,-1} \quad R^2 = 0.22, \quad N = 36$$

$$\begin{matrix} (0.011) & (0.040) & (0.004) \end{matrix} \tag{19}$$

with a joined  $R^2 = 0.32$ . Interestingly, the sign of the coefficient for openness varies between the two subgroups of countries, but both signs are insignificant. Also, compared to the benchmark model, the estimated coefficient for the average saving rate is larger for both subgroups.

## 5.2 VAR analysis: estimation, identification and testing

First we investigate the stationarity for each of the variables. For this we use the test statistics developed by Im et al. [25]. They develop unit root tests for heterogeneous panels specified by

$$x_{i,t} = (1 - \phi_i)\mu_i + \phi_i x_{i,t-1} + \varepsilon_{i,t}$$

where  $E(\varepsilon_{i,t}) = \sigma_i^2$ . They test the hypothesis  $\phi_i = 1$  for all  $i$  against  $\phi_i < 1$  for all  $i$ . Since we impose that  $\phi_i = \phi$  and  $\mu_i = \mu$  for all  $i$  in our model, we can use this procedure to test for unit roots. Furthermore, since their test is not based on these restrictions it makes our results robust against this type of misspecification. The results, given in Table 2, indicate that we can formally reject the hypothesis of a unit root in GDP growth and the saving rate. For openness, however, we cannot reject the hypothesis of a unit root. This result has some implications for the model proposed in (4) and (5).

Table 2: Tests for unit roots.

Variable	LR <sub>26</sub> (0,0)	p-value
GDP growth	2.85	0.002
saving rate	3.164	<0.001
openness	-0.884	0.199

Since  $O_{i,t}$  is  $I(1)$  and  $\Delta y_{i,t}$  and  $S_{i,t}$  are  $I(0)$  we have to put in place restrictions on the coefficients of openness to obtain a stationary system. Therefore we restrict the coefficients of current and lagged openness to be equal but of opposite sign in all equations including the threshold model given in (5). Imposing this restriction allows us to restate (4) and (5) in terms of  $\Delta O_{i,t} = O_{i,t} - O_{i,t-1}$ :

$$X_{i,t}^* = \mu + B_0 X_{i,t}^* + B_1 X_{i,t-1}^* + u_{it} \quad (20)$$

and

$$X_{i,t}^* = (\underline{\mu} + \underline{B}_0 X_{i,t}^* + \underline{B}_1 X_{i,t-1}^*)I(S_{i,t-2} \leq \gamma) \quad (21)$$

$$+ (\bar{\mu} + \bar{B}_0 X_{i,t}^* + \bar{B}_1 X_{i,t-1}^*)I(S_{i,t-2} > \gamma) + w_{it}$$

where  $X_{i,t}^* = (\Delta O_{it}, S_{i,t}, \Delta Y_{it})'$  and where the following parameter restrictions have been imposed:

$$\underline{B}_0, \underline{B}_0, \bar{B}_0: \begin{bmatrix} 0 & 0 & 0 \\ \cdot & 0 & 0 \\ \cdot & \cdot & 0 \end{bmatrix}, \quad \text{and} \quad \underline{B}_1, \underline{B}_1, \bar{B}_1: \begin{bmatrix} 0 & \cdot & \cdot \\ 0 & \cdot & \cdot \\ 0 & \cdot & \cdot \end{bmatrix}.$$

Table 3: Estimation results: linear VAR.

regressors	Dependent Variable:		
	$\Delta y_{i,t}$	$S_{i,t}$	$\Delta O_{i,t}$
$c$	0.005* (0.002)	0.011* (0.002)	0.008 (0.007)
$S_{i,t}$	0.522* (0.031)	--	--
$\Delta O_{i,t}$	0.033* (0.007)	0.014* (0.005)	--
$\Delta y_{i,t-1}$	0.302* (0.022)	0.066* (0.016)	0.181* (0.062)
$S_{i,t-1}$	-0.466* (0.031)	0.942* (0.007)	0.011 (0.032)
	$R^2 = 0.41$	$R^2 = 0.98$	$R^2 = 0.009$

standard errors in parentheses

\*: significant at 5% level

We present the estimation results of (20) in Table 3. As explained above we can estimate this system as three separate equations with GDP growth, the saving rate, and change in openness as the respective dependent variables (standard errors are given in parentheses).

In the first regression (with GDP growth as the dependent variable) all the explanatory variables are significant at the 5% level. As the next column in Table 3 reveals (with the savings rate as the dependent variable) a change in openness has a significant impact on the savings rate. Therefore, a change in openness has also an indirect impact on GDP growth through its effect on savings. In the third regression (with the change of openness as dependent variable) lagged GDP growth is positive and significant, while the saving rate has no significant impact on the change in openness. The complexity of the relation between openness and growth will become clearer when we present the results of the impulse response analysis in the next section.

To check the validity of this model we perform the Roy-Zellner test for poolability. For the GDP growth and the saving rate regression we obtain F-values of 3.134 (p-value <0.001) and 2.426 (p-value <0.001), respectively, which means that we reject the null hypothesis that all coefficients are the same for each country. This implies that the linear model is misspecified, which is exactly what we would expect if a threshold exists. For the change in openness we do not reject the pooling hypothesis (p-value is equal to 0.4717),

a result which may not be surprising given the low  $R^2$  of this regression.

Next, we test for the existence of a threshold using the same tests as presented in Table 1. In a VAR system as the one we analyze there are two ways to model the breakpoint. A threshold can be modeled separately for each of the three equations. Not surprisingly, we may find a different threshold for each equation in this case. Alternatively, we can restrict the threshold to be the same for all three equations. We pursue both approaches presented in Table 4.

Table 4: Threshold tests and estimation in VAR.

	Threshold Test Statistics			
	Dependent Variable:			Restricted VAR
	$\Delta y_{i,t}$	$S_{i,t}$	$\Delta O_{i,t}$	
<i>BPH</i>	1.62	2.40	3.23*	1.85
<i>explM</i>	17.80**	10.74**	7.41*	34.50**
<i>aveLM</i>	15.78**	12.29**	11.25**	40.00**
<i>supLM</i>	41.92**	27.66*	19.51*	77.66**
<i>HansenF</i>	45.15**	30.73**	21.02*	49.82**
Threshold Estimation				
$\gamma$	0.181	0.240	0.154	0.192
95% conf. inter.	(0.177, 0.204)	(0.229, 0.249)	(0.075, 0.227)	(0.180, 0.247)

\*: significant at 5% level

\*\* : significant at 1% level

The *aveLM*, *explM* and *supLM* tests are based on  $\pi = 0.05$ . The F-test of Hansen is derived under the assumption of maintained homogeneity of the variances of the disturbances among the two groups. First, notice that, as expected (see appendix B), the BPH test suffers from a lack of power as compared to the other tests. Second, the threshold estimates differ across the three equations as expected. In the absence of a reasonable economic interpretation for the different threshold values, we restrict our analysis to the model where the thresholds are restricted to be the same across equations.

Based on the threshold level of 0.192, we estimate the resulting threshold model. The results are presented in Table 5 (standard errors are in parentheses). There are 670 observations in the low savings regime and 838 observations in the high savings regime. Furthermore, while 9 countries are in the low saving regime at every point in time, 15 countries are always in the high saving regime. All other countries switch regimes at some point(s) during the sample period.

Table 5: Estimation results: threshold model.

	$S_{i,t-2} \leq 0.19, N = 670$			$S_{i,t-2} > 0.19, N = 838$		
	Dependent Variable:					
regressors	$\Delta y_{i,t}$	$S_{i,t}$	$\Delta O_{i,t}$	$\Delta y_{i,t}$	$S_{i,t}$	$\Delta O_{i,t}$
$c$	0.005 (0.004)	0.011* (0.003)	-0.019 (0.012)	0.000 (0.004)	0.013* (0.003)	-0.016 (0.014)
$S_{i,t}$	0.333* (0.041)	--	--	0.704* (0.046)	--	--
$\Delta O_{i,t}$	0.0027 (0.009)	-0.006 (0.008)	--	0.033* (0.011)	0.031* (0.007)	--
$\Delta y_{i,t-1}$	0.368* (0.035)	0.050 (0.027)	-0.073 (0.103)	0.253* (0.030)	0.072* (0.020)	0.301* (0.104)
$S_{i,t-1}$	-0.278* (0.045)	0.937* (0.018)	0.242* (0.082)	-0.622* (0.047)	0.931* (0.014)	-0.061 (0.092)
	$R^2 = 0.33$	$R^2 = 0.93$	$R^2 = 0.02$	$R^2 = 0.44$	$R^2 = 0.98$	$R^2 = 0.024$

standard errors in parentheses

\*: significant at 5% level

The most important result is that a change in openness has a positive and significant impact on saving in the high saving regime, while it is negative though not significant in the low saving regime. In addition, both current and lagged savings have larger impact on GDP growth in the high saving than in the low saving regime. Finally, lagged GDP growth has a positive and significant impact on a change in openness in the high saving regime, while its impact is negative though not significant in the low saving regime. In contrast, lagged savings is positive and significant in the low and negative and insignificant in the high saving regime.

### 5.3 VAR analysis: Impulse responses

The impulse response analysis is based on the coefficient estimates presented in Table 3 for the linear model and in Table 5 for the nonlinear threshold model.

Observe that the threshold tests presented in the previous section indicate the existence of a threshold even if only one of the coefficients of  $\mu$ ,  $B_0$  or  $B_1$  changes significantly between the two regimes. Clearly, in this case one could obtain efficiency gains by restricting all other coefficients to be the same. Since we do not restrict the coefficients in this way, a potential efficiency loss

is possible, not only for the coefficient estimates but for the impulse response functions as well. However, any potential efficiency loss does not affect the validity of our analysis or the possible outcomes.

We start by analyzing the linear impulse responses to unit shocks. It is worth noting that in our model the innovations are already orthogonal. The impulse responses for the linear VAR from Table 3 are given in Figure 2. Observe that there are significant responses of GDP growth and savings to a change in openness. The response in GDP growth, however, is small and during a short time even negative. As expected, a shock in savings produces a larger reaction of output growth than a shock in openness. This confirms the theoretical notion that the saving level is a more important determinant of output growth than openness.

We now analyze the nonlinear impulse responses to a unit shock. Figures 3 to 5 present the average nonlinear impulse responses as well as the 95% most centered realizations to a unit shock in openness depending on the starting level of savings. Figure 3 gives the results unconditional whether the starting level of savings is above or below the threshold, Figure 4 conditional on being above, and Figure 5 conditional on being below the threshold.

In each figure, the first row gives the responses to a positive unit shock, while the second row traces the effects of a negative unit shock. The average impulse response with 2 standard deviation confidence bounds, based on the same simulation, are given in the Figures 6 to 8.

Comparing the nonlinear impulse response functions (Figure 6, first row) with those from the linear model (Figure 2, first column), we notice that the responses of the saving rate and GDP growth to a change in openness are substantially different. In the linear case the effect of the shock quickly disappears, while in the nonlinear case the effect is still significant after 24 periods. Even more important, the effect of a shock of openness on GDP growth becomes negative after a few periods in the linear case but remains positive throughout in the nonlinear case.

Next we analyze the three situations of the nonlinear model. As expected, the magnitude of the unconditional response is between the two conditional responses. Further, we observe clear differences between responses originating from a high saving regime and responses originating from the low saving regime. In the high saving regime there is always a positive effect on savings resulting from a positive shock in openness. This is in contrast to the response initiated from a low saving regime where a positive shock initially has a significant negative impact on the saving rate, while a negative shock has a positive impact on the saving rate. Clearly, this striking asymmetry of responses to positive and negative shocks in the low saving regime cannot

be generated by a linear system. Finally, notice that the response in the high saving regime is larger than the response in the low savings regime. In particular, the response of GDP growth to a positive shock in openness is larger in the high saving regime. Even after 10 years, it is still positive, while in the low saving regime the initially positive response becomes insignificant after 4 years.

To test whether we can observe similar effects for specific countries we calculate the impulse responses for the United States, a notorious low savings country, and Japan, a country known for its high saving rate. The impulse responses are based on the parameter estimates and asymptotic distributions from the model given in Table 5 together with the respective histories of the two countries. For each country we use the last years in the sample, 1985-1987, as the time of the shock. We simulate 100 sets of parameters and then expose the system to a unit shock in openness. The results are shown in Figures 9 and 10.

The figures reveal a number of interesting differences. First, a positive shock in openness has a positive and significant impact on Japan's saving rate, even after 24 periods. In comparison, the same shock has essentially no significant impact on the saving rate in the United States. In addition, a positive shock in openness has a positive impact on GDP growth in Japan for all periods, while the same shock has a much smaller, largely insignificant impact on U.S. growth.

## 6 Summary and Conclusion

In this paper we test a simple hypothesis: does a country's saving rate matter both over time and across countries for the interaction between growth and openness? Using a dynamic model for GDP growth, the saving rate and change in openness to trade, we find that it indeed matters. Countries with high saving rates experience a growth-enhancing effect from an increase in openness, while countries with low saving rates do not experience such an effect. The differences between low and high saving countries are evident from the estimation of the threshold VAR models. In the high saving regime, openness has a positive effect on growth, while the same effect in the low saving regime is negative though insignificant. There are striking differences in the impulse response functions between the two regimes as well. In the high saving regime, a positive shock in openness leads to higher GDP growth in all subsequent years, while in the low saving regime the effect on GDP growth is substantially smaller in size and becomes insignificant after a few



periods.

Our empirical findings have nontrivial policy implications. Clearly, low saving rates are a double curse for a country. On the one hand, a low national saving rate directly diminishes the domestic growth fundamentals of the economy. Furthermore, as our analysis indicates, it also undermines the potential growth effects of increased openness to world trade experienced by high saving countries. Even worse, for some countries a higher volume of trade may even reduce the GDP growth rate which is already relatively low due to the low national savings rate.

In addition to the empirical results, the paper also adds to the theoretical econometric literature by introducing a new simple test for endogenous threshold models. The test has the advantage that it can be calculated efficiently since no bootstrapping is needed. Another advantage is that the test does not involve the choice of any subjective parameters such as the number of regressors or the cut off levels of the threshold region.

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## Appendix A

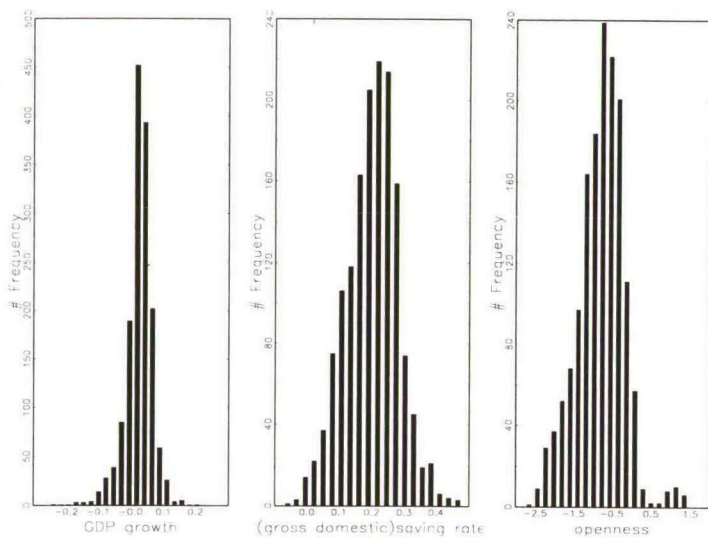


Figure 1: Distribution of the data; frequencies are given on the vertical axis

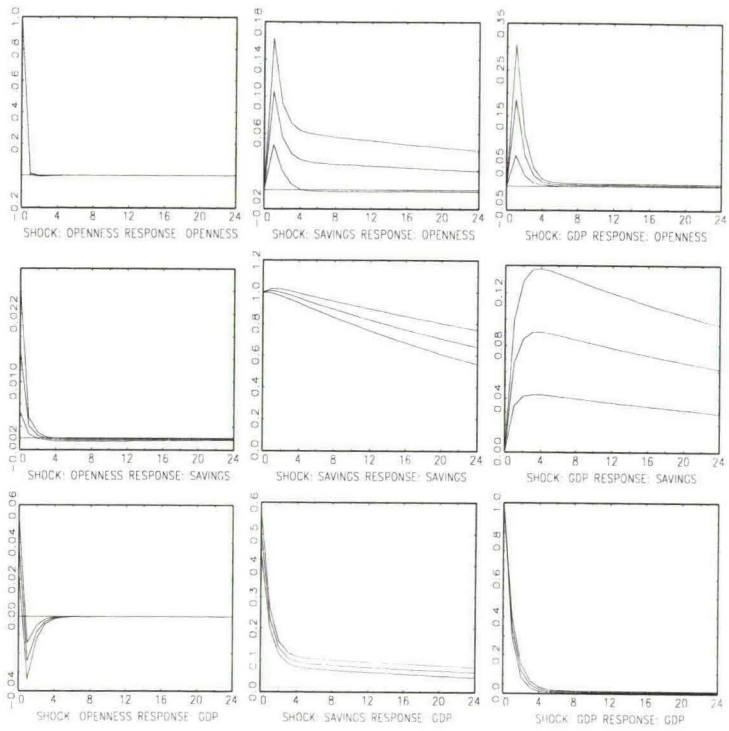


Figure 2: Impulse response functions to unit shocks with 2 standard deviation confidence bounds for the benchmark VAR model.

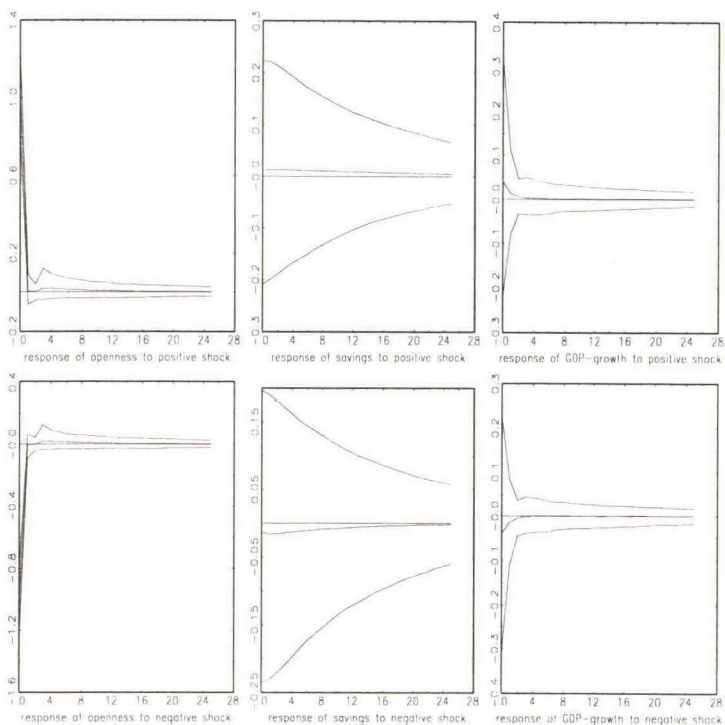


Figure 3: Impulse response functions (average and 95% most centered realizations) to a unit shock in openness for threshold VAR model: initial saving rates unconditional on saving regime



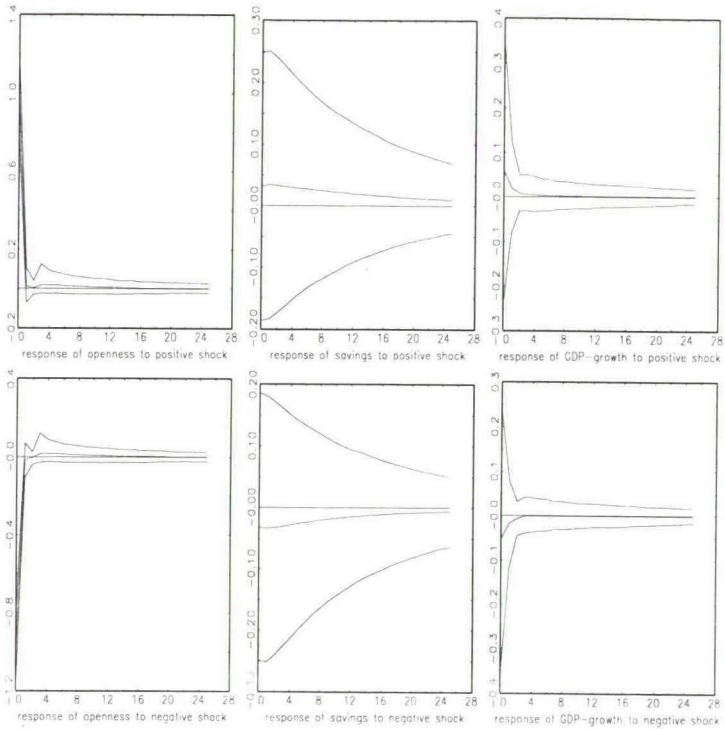


Figure 4: Impulse response functions (average and 95% most centered realizations) to a unit shock in openness for threshold VAR model: initial saving rates conditional on high saving regime

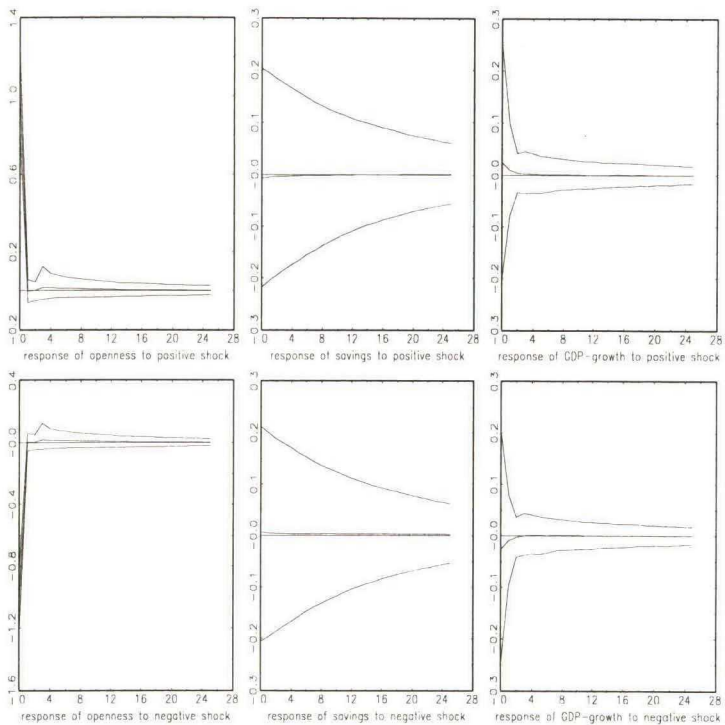


Figure 5: Impulse response functions (average and 95% most centered realizations) to a unit shock in openness for threshold VAR model: initial saving rates conditional on low saving regime

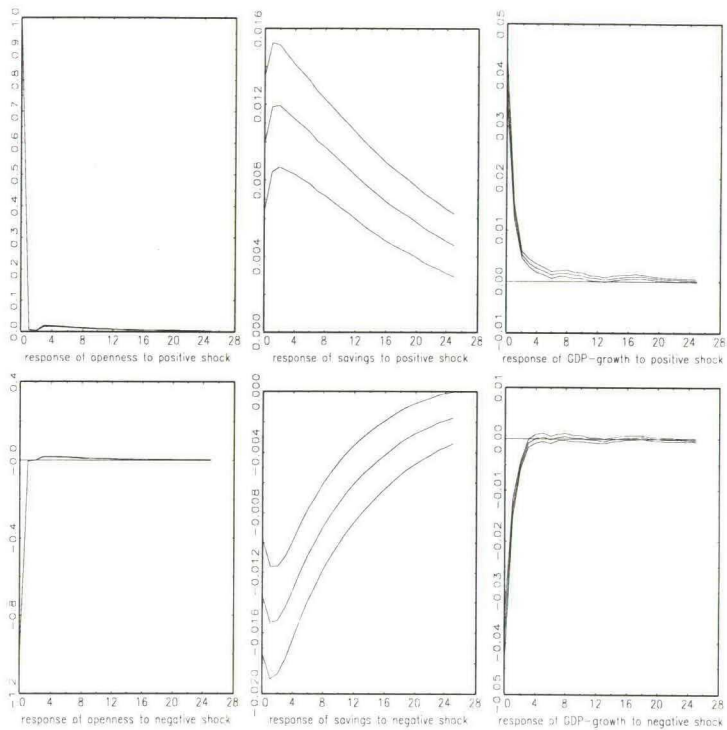


Figure 6: Average impulse response functions to a unit shock in openness with two standard deviation confidence bounds for threshold VAR model: initial saving rates unconditional on saving regime

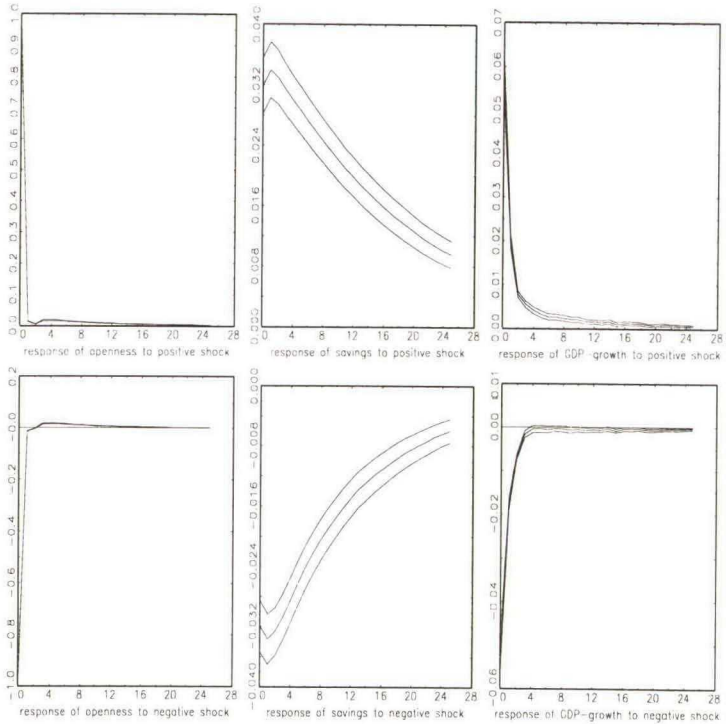


Figure 7: Average impulse response functions to a unit shock in openness with two standard deviation confidence bounds for threshold VAR model: initial saving rates conditional on high saving regime

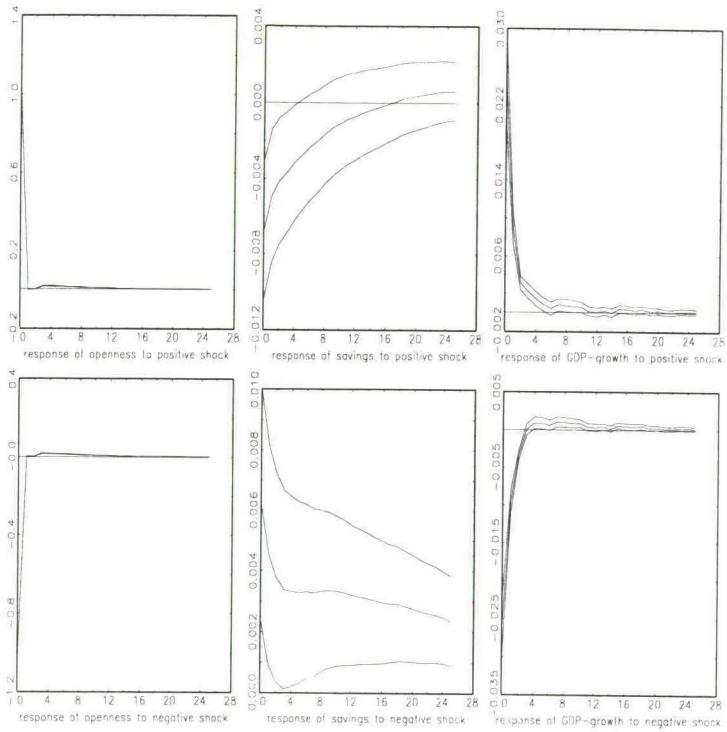


Figure 8: Average impulse response functions to a unit shock in openness with two standard deviation confidence bounds for threshold VAR model: initial saving rates conditional on low saving regime

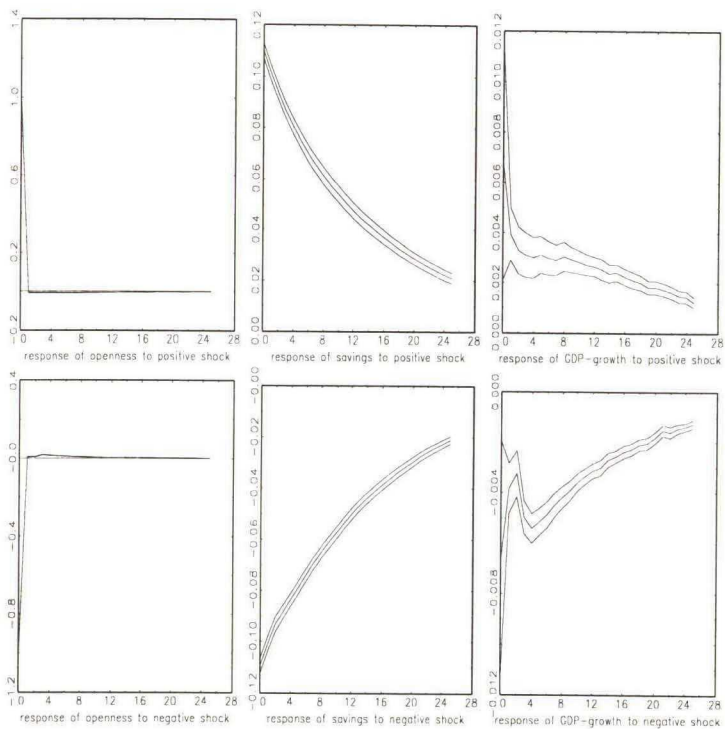


Figure 9: Average impulse response functions to a unit shock in openness with two standard deviation confidence bounds for threshold VAR model: Japan

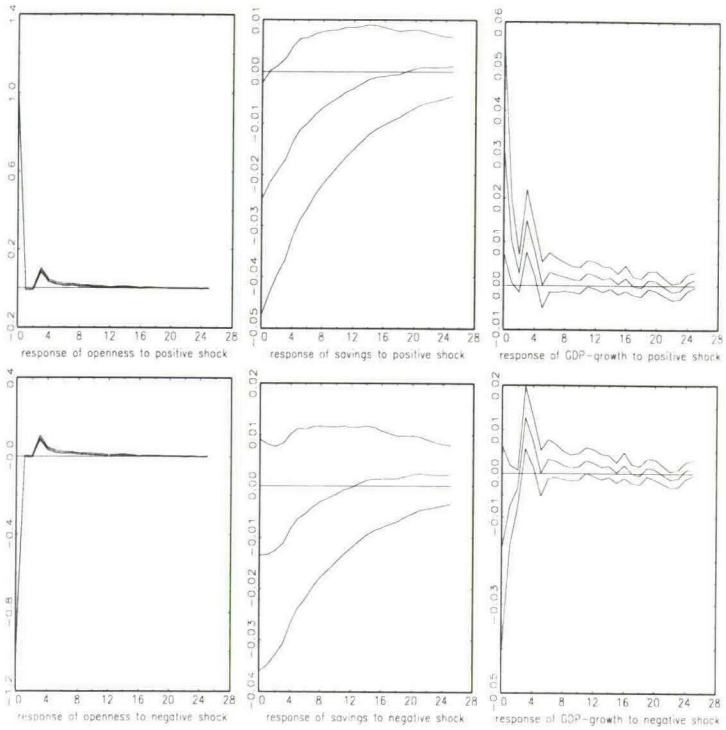


Figure 10: Average impulse response functions to a unit shock in openness with two standard deviation confidence bounds for threshold VAR model: United States

## Appendix B

In section 3.1 we introduced a statistic related to the traditional LM test statistic. Here we derive a test statistic related to the traditional Wald statistic. The Wald statistic is based on the unrestricted parameter estimates of  $(\beta_1, \beta_2)$ ; recall that we test the restriction  $\beta_2 = 0$  against the alternative  $\beta_2 \neq 0$ . For fixed scalar  $\gamma$  the OLS estimator for  $\beta_2$  is given by

$$\hat{\beta}_2(\gamma) = (Z'_{(\gamma)}MZ_{(\gamma)})^{-1}Z'_{(\gamma)}MY, \quad (22)$$

where  $M = (I - X(X'X)^{-1}X')$ . We now define the normalized stochastic function as

$$z_N(\gamma) = \frac{1}{\sqrt{N}}(Z'_{(\gamma)}MZ_{(\gamma)})^{-1}Z'_{(\gamma)}MY. \quad (23)$$

Observe the replacement of  $u^r$ , the residuals based on the restricted estimator of  $\beta_1$  in (13) by  $Y$  in (22). We now have

$$Z'_{(\gamma)}Y = Z'_{(\gamma)}M\varepsilon \quad \text{under } H_0, \quad (24)$$

$$Z'_{(\gamma)}Y = Z'_{(\gamma)}MZ_{(\gamma)}\beta_2 + Z'_{(\gamma)}M\varepsilon \quad \text{under } H_1. \quad (25)$$

Comparing (24) and (25) to (11) and (12) in Section 3.1 reveals that the asymptotic behavior of the test statistic based on the unrestricted estimator is identical to the one based on the restricted estimator. Before we derive this asymptotic distribution we discuss some results concerning the power of the test.

*The power of the BPH test:* Consider the model

$$Y = X\beta_1 + Z_{(\gamma)}\beta_2 + u, \quad u \sim N(0, \sigma^2 I)$$

with  $\beta_1$  a  $k_x$  dimensional parameter vector. For fixed  $\gamma$  the LM statistic for the null hypothesis ( $\beta_2 = 0$ ) is given by

$$\begin{aligned} LM(\gamma) &= \frac{(Y - X\tilde{\beta}_1)'Z_{(\gamma)}(Z'_{(\gamma)}MZ_{(\gamma)})^{-1}Z'_{(\gamma)}(Y - X\tilde{\beta}_1)}{Y'MY} \\ &= \frac{u^r'Z_{(\gamma)}(Z'_{(\gamma)}MZ_{(\gamma)})^{-1}Z'_{(\gamma)}u^r}{\hat{\sigma}^2} \end{aligned}$$

where  $\tilde{\beta}_1$  is the estimator based on the restricted estimate, while  $M$  and  $u^r$  are based on  $\tilde{\beta}_1$ . Since  $u^r = MY = Mu$  we find that under  $H_0$

$$LM(\gamma) = \frac{u^r'MZ_{(\gamma)}(Z'_{(\gamma)}MZ_{(\gamma)})^{-1}Z'_{(\gamma)}Mu^r}{\hat{\sigma}^2} \quad (26)$$



$$= \frac{1}{\hat{\sigma}^2} \text{trace}((Z'_{(\gamma)}MZ_{(\gamma)})^{-1}Z'_{(\gamma)}Mu'u'MZ_{(\gamma)})$$

It is now straightforward to explain the relation between our test statistic and the exponential-, average-, and supremum LM statistics. All these test statistics are based on functionals of  $LM(\gamma)$ . In contrast our test makes use of the functional

$$\frac{1}{\hat{\sigma}^2} \mathcal{S}(z_N(\gamma)z'_N(\gamma)) = \frac{1}{\hat{\sigma}^2} \mathcal{S}\left(Z'_{(\gamma)}Mu'u'MZ_{(\gamma)}\right) \quad (27)$$

where  $\hat{\sigma}^2$  is the same as in (26).

Under the null hypothesis the matrix  $(Z'_{(\gamma)}MZ_{(\gamma)})^{-1}Z'_{(\gamma)}Mu'u'MZ_{(\gamma)}$  is distributed as a diagonal matrix with diagonal elements which are i.i.d  $\sigma^2\chi^2$  distributed. This implies that the LM statistic  $LM(\gamma)$  is asymptotically distributed as a standard normal chi-square statistic which is independent of the data. This means that critical values can be tabulated. Clearly  $Z'_{(\gamma)}Mu'u'MZ_{(\gamma)}$  in (27) is asymptotically not diagonal but still depends on the data through  $(Z'_{(\gamma)}MZ_{(\gamma)})^{-1}$ . To take into account the off diagonal elements we introduce the  $\mathcal{S}(\cdot)$  operator.

Since  $\frac{1}{\hat{\sigma}^2} \mathcal{S}(z_N(\gamma)z'_N(\gamma))$  depends on the data,  $\int \frac{1}{\hat{\sigma}^2} \mathcal{S}(z_N(\gamma)z'_N(\gamma)) d\gamma$  does as well. It is shown below that a data independent upper bound of the asymptotic distribution of  $\int \frac{1}{\hat{\sigma}^2} \mathcal{S}(z_N(\gamma)z'_N(\gamma)) d\gamma$  can be given. An additional advantage is that the number of regressors does not affect the asymptotic distribution; this in contrast to the LM statistic which depends on  $k_x$ . Using an upper bound automatically implies that the test statistic will exhibit some conservatism. The degree of conservatism depends on the data.

*Asymptotic distribution of the BPH statistic:* Using the notation from Section 3.1 we again find

$$\begin{aligned} z_N(\gamma) &= \frac{1}{\sqrt{N}} Z'_{(\gamma)} u^r = \frac{1}{\sqrt{N}} Z'_{(\gamma)} M \varepsilon && \text{under } H_0, \\ z_N(\gamma) &= \frac{1}{\sqrt{N}} Z'_{(\gamma)} u^r = \frac{1}{\sqrt{N}} Z'_{(\gamma)} M Z_{(\gamma)} \beta_2 + \frac{1}{\sqrt{N}} Z'_{(\gamma)} M \varepsilon && \text{under } H_1. \end{aligned}$$

Under the assumptions given below  $\frac{1}{\sqrt{N}} Z'_{(\gamma)} M \varepsilon$  converges weakly to a zero mean  $k_z$  dimensional Gaussian process. This means that under  $H_0$  the process  $z_N(\gamma)$  is also a Gaussian process with zero mean. Under  $H_1$ , however, there is an additional term  $\frac{1}{\sqrt{N}} Z'_{(\gamma)} M Z_{(\gamma)} \beta_2$ . Given the assumptions below it can be shown that, for fixed  $\gamma$ ,

$$\lim_{N \rightarrow \infty} \frac{1}{N} Z'_{(\gamma)} M Z_{(\gamma)} = \sigma^2 \Omega_{(\gamma)}.$$

Clearly,  $\frac{1}{\sqrt{N}}Z'_{(\gamma)}MZ_{(\gamma)}\beta_2$  diverges to plus or minus infinity when  $\beta_2 \neq 0$ , a fact that makes our test consistent. In addition, we can consider our test under some local alternative. We may assume that the parameter  $\beta_2$  is given by  $\frac{\tilde{\beta}_2}{\sqrt{N}}$  with  $\tilde{\beta}_2 \neq 0$ , i.e. we assume  $\beta_2$  to be very small initially and to become even smaller as the sample size increases. The first step to derive the asymptotic distribution of the test statistic and its behavior under the local alternatives requires the following theorem.

**Theorem 1** Under  $H_1$ , i.e.  $\beta_2 \neq 0$ , and given the assumptions stated below, we have  $z_N(\gamma) \Rightarrow z(\gamma)$ , where  $z(\gamma)$  is a  $k_z$  dimensional Gaussian process with mean function

$$\eta(\gamma) = p \lim_{N \rightarrow \infty} \frac{1}{\sqrt{N}} Z'_{(\gamma)} M Z_{(\gamma)} \beta_2$$

and variance function

$$\sigma(\gamma_1, \gamma_2) = \lim_{N \rightarrow \infty} \sigma^2 \frac{1}{N} Z'_{(\gamma_1)} M Z_{(\gamma_2)}.$$

Further, by the continuous mapping theorem

$$iz_N \rightarrow iz = \int \mathcal{S}(z_N(\gamma) z'_N(\gamma)) d\gamma \quad \text{in distribution.}$$

**Proof.** We postpone the proof until the end of the appendix. ■

We also need the following two lemmas which are versions of Mercer's Theorem<sup>12</sup>.

**Lemma 2** (Mercer's Theorem) Let  $\Psi(\gamma_1, \gamma_2)$  be a real valued positive semi-definite continuous function on  $\Gamma \times \Gamma$ , where  $\Gamma$  is a compact space, and let  $\mu$  be a probability measure on  $\Gamma$ . The solutions  $\lambda_i$  and  $\psi_i(\bullet)$ ,  $i = 1, 2, 3, \dots$  of the Eigenvalue problem  $\int \Psi(\gamma_1, \gamma_2) \psi_i(\gamma_2) d\mu(\gamma_2) = \lambda_i \psi_i(\gamma_1)$  are real valued and the function  $\Gamma$  has the series representation  $\Psi(\gamma_1, \gamma_2) = \sum_{i=1}^{\infty} \lambda_i \psi_i(\gamma_1) \psi_i(\gamma_2)$ , where the series involved converges uniformly on  $\Gamma \times \Gamma$ .

**Lemma 3** Let the conditions of Lemma 2 be satisfied. The Eigenvalues  $\lambda_i$  are nonnegative and satisfy  $\sum_{i=1}^{\infty} \lambda_i < \infty$ . Moreover, the Eigenfunctions  $\psi_i(\bullet)$  are continuous and can be chosen orthonormal and complete in the space  $C(\Gamma)$  of continuous real functions on  $\Xi$  as well on the space  $L_2(\mu)$  of squared

<sup>12</sup>The two lemmas are also stated in Bierens and Ploberger [9].

integrable functions with respect to  $\mu$ , i.e.  $\int \psi_i(\gamma)\psi_j(\gamma)d\mu(\gamma) = I(i = j)$ , and every function  $\phi$  in  $C(\Gamma)$  or  $L_2(\mu)$  can be written as

$$\phi(\gamma) = \sum_{i=1}^{\infty} g_i \psi_i(\gamma) \quad \text{a.s. } L_2(\mu),$$

with Fourier coefficients

$$g_i = \int \phi(\gamma)\psi_i(\gamma)d\mu(\gamma)$$

satisfying  $\sum_{i=1}^{\infty} g_i^2 < \infty$ .

We now apply these two lemmas to our statistic. Let  $\Psi$  in Lemma 2 be

$$\Psi(\gamma_1, \gamma_2) = \mathcal{S}(\sigma(\gamma_1, \gamma_2)).$$

Then the continuity of the Gaussian process  $z(\gamma)$ , and the compactness of  $\Gamma$  imply that  $z(\gamma)$  is squared integrable. Further since the set  $\{\psi_i, i = 1, 2, \dots\}$  of Eigenfunctions is complete we can apply Parseval's identity

$$\begin{aligned} \int \mathcal{S}(z(\gamma)z'(\gamma)) d\mu(\gamma) &= \int \mathcal{S}(z(\gamma))^2 d\mu(\gamma) \\ &= \sum_{i=1}^{\infty} \left( \int \mathcal{S}(z(\gamma)) \psi_i(\gamma) d\mu(\gamma) \right)^2 \end{aligned}$$

Observe that  $\phi(\gamma) = \mathcal{S}(z(\gamma))$ . By the fact that the sum of Gaussian processes is again Gaussian it follows that the Fourier coefficients

$$\int \mathcal{S}(z(\gamma)) \psi_i(\gamma) d\mu(\gamma), \quad i = 1, 2, 3, \dots \quad (28)$$

are Gaussian as well. For the characterization of their joint distribution we only need the covariances and means. The covariances are given by

$$\begin{aligned} &E \left[ \int \mathcal{S}(z(\gamma) - \eta(\gamma)) \psi_i(\gamma) d\mu(\gamma) \int \mathcal{S}(z(\gamma) - \eta(\gamma)) \psi_j(\gamma) d\mu(\gamma) \right] \\ &= \int \int \mathcal{S}(\sigma(\gamma_1, \gamma_2)) \psi_i(\gamma_1) \psi_j(\gamma_2) d\mu(\gamma_1) d\mu(\gamma_2) = \lambda_i I(i = j), \end{aligned}$$

which means that the sequence (28) is independent. Further it is easy to see that the mean of the  $i$ -th element of the sequence (28) is just the  $i$ -th Fourier coefficient of  $\eta(\gamma)$

$$\eta_i = \int \eta(\gamma) \psi_i(\gamma) d\mu(\gamma).$$

Note that the way in which we apply Lemma 2 and 3 is similar to Bierens and Ploberger [9]. From the above it directly follows that under the local alternative  $\beta_2 = \frac{\tilde{\beta}_2}{\sqrt{N}}$  with  $\tilde{\beta}_2 \neq 0$  the following theorem holds.

**Theorem 4** Under  $H_1$ , i.e  $\beta_2 \neq 0$ . we have

$$iz = \int \mathcal{S}(z(\gamma)z'(\gamma)) d\gamma \rightsquigarrow \sum_{j=1}^{\infty} (\eta_j + \epsilon_j \sqrt{\lambda_j})^2$$

where  $\epsilon_j$  i.i.d  $N(0, 1)$  and  $\lambda_j$  and  $\eta_j$  are as described above.

Under the null we have

$$H_0: T_0 = \sum_{j=1}^{\infty} \epsilon_j^2 \lambda_j, \quad (29)$$

where the  $\lambda_j$  depend on  $\Psi$  and are therefore data dependent. This implies that the asymptotic distribution of  $T_0$  also depends on the data. However, using a theorem derived by Bierens and Ploberger we can obtain critical values which are data independent.

**Theorem 5** (Theorem 5 of Bierens and Ploberger) Let  $\epsilon_j$  be i.i.d  $N(0,1)$  and let

$$\bar{W} = \sup_{T \geq 1} \frac{1}{T} \sum_{j=1}^T \epsilon_j^2.$$

For  $\eta > 0$ ,  $P(T_{H_0} > \eta E(T_{H_0})) \leq P(\bar{W} > \eta)$ , where  $T_{H_0}$  is the random variable defined by (29). Consequently, under the null hypothesis of  $\beta_2 = 0$

$$\lim_{N \rightarrow \infty} P\left(iz_{H_0} > \eta \int \mathcal{S}(\sigma_N(\gamma, \gamma)) d\gamma\right) \leq P(\bar{W} > \eta).$$

Bierens and Ploberger also simulate the distribution of  $\bar{W}$ . Using 10,000 replications they obtain the following critical values:

$$P(\bar{W} > 3.23) = 0.10, \quad P(\bar{W} > 4.26) = 0.05, \quad P(\bar{W} > 6.81) = 0.01.$$

This means that we reject the null hypothesis at the 10% significance level if

$$\int \mathcal{S}(z_N(\gamma)z'_N(\gamma)) d\gamma / \int \mathcal{S}(\sigma_N(\gamma, \gamma)) d\gamma > 3.23 \quad (30)$$

To prove Theorem 1 we need the following definitions and assumption.

*Definitions:*

$$\begin{aligned} V_{11} &= E(X_i' X_i) & \Omega_{11} &= E(X_i' X_i \varepsilon_i^2) \\ V_{12}(\gamma) &= E(X_i' Z_i | I(q_i = \gamma)) & \Omega_{12}(\gamma) &= E(X_i' Z_i \varepsilon_i^2 | I(q_i = \gamma)) \\ V_{21}(\gamma) &= E(Z_i' X_i | I(q_i = \gamma)) & \Omega_{21}(\gamma) &= E(Z_i' X_i \varepsilon_i^2 | I(q_i = \gamma)) \\ V_{22}(\gamma) &= E(Z_i' Z_i | I(q_i = \gamma)) & \Omega_{22}(\gamma) &= E(Z_i' Z_i \varepsilon_i^2 | I(q_i = \gamma)) \end{aligned}$$

and

$$V(\gamma) = \begin{bmatrix} V_{11} & V_{12}(\gamma) \\ V_{21}(\gamma) & V_{22}(\gamma) \end{bmatrix}, \quad \Omega(\gamma) = \begin{bmatrix} \Omega_{11} & \Omega_{12}(\gamma) \\ \Omega_{21}(\gamma) & \Omega_{22}(\gamma) \end{bmatrix}$$

We also need the  $s$ -th moment of these conditional variance matrices:

$$\begin{aligned} V_{11}^s &= E(X_i' X_i)^s & \Omega_{11}^s &= E(X_i' X_i \varepsilon_i^2)^s \\ V_{12}^s(\gamma) &= E((X_i' Z_i)^s | I(q_i = \gamma)) & \Omega_{12}^s(\gamma) &= E((X_i' Z_i \varepsilon_i^2)^s | I(q_i = \gamma)) \\ V_{21}^s(\gamma) &= E((Z_i' X_i)^s | I(q_i = \gamma)) & \Omega_{21}^s(\gamma) &= E((Z_i' X_i \varepsilon_i^2)^s | I(q_i = \gamma)) \\ V_{22}^s(\gamma) &= E((Z_i' Z_i)^s | I(q_i = \gamma)) & \Omega_{22}^s(\gamma) &= E((Z_i' Z_i \varepsilon_i^2)^s | I(q_i = \gamma)) \end{aligned}$$

and

$$V^s(\gamma) = \begin{bmatrix} V_{11}^s & V_{12}^s(\gamma) \\ V_{21}^s(\gamma) & V_{22}^s(\gamma) \end{bmatrix}, \quad \Omega^s(\gamma) = \begin{bmatrix} \Omega_{11}^s & \Omega_{12}^s(\gamma) \\ \Omega_{21}^s(\gamma) & \Omega_{22}^s(\gamma) \end{bmatrix}$$

Finally, we denote  $f(q)$  as the probability density of the variable  $q$ .

*Assumptions:*<sup>13</sup>

1. Let the series  $(X_i, q_i, \varepsilon_i)$  be strictly stationary with  $\beta$  mixing coefficients  $\beta_m$  satisfying
$$\beta_m^{(s-1)/2s} = O(m^{-(1+\delta)}).$$
2.  $E(\varepsilon_i | F_{i-1}) = 0$ ;
3.  $E|X_i|^{2s} < \infty$ , and  $E|\varepsilon_i|^{2s} < \infty$ ;
4.  $f(\gamma), V(\gamma), \Omega(\gamma), V^s(\gamma), \Omega^s(\gamma)$  be continuous at  $\gamma = \gamma_0$ ;
5.  $V(\gamma), \Omega(\gamma)$  are positive definite and  $f(\gamma) > 0$ ;
6.  $P(q_i \in \Gamma) < 1$ .

Under these assumptions the following lemma can be stated:

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<sup>13</sup>These assumptions are similar to those made by Hansen [22], [23]. For a discussion, see Hansen [23].

**Lemma 6**  $\frac{1}{\sqrt{N}}Z'_{(\gamma)}\varepsilon \implies G(\gamma)$ , a zero mean Gaussian process with covariance kernel  $\sigma^2V_{22}(\gamma)$ , where  $V_{22}(\gamma)$  is defined as above and  $\gamma = \min(\gamma_1, \gamma_2)$ .

The proof of the lemma follows directly from Theorem 3 of Hansen [22] and the above definitions. We are now ready to prove Theorem 1.

*Proof of Theorem 1:* Define  $X^*_{(\gamma)} = [X \ Z_{(\gamma)}]$ . Then, for fixed  $\gamma$ ,

$$\frac{1}{\sqrt{N}}X^*_{(\gamma)}\varepsilon = \begin{bmatrix} \frac{1}{\sqrt{N}}X'\varepsilon \\ \frac{1}{\sqrt{N}}Z'_{(\gamma)}\varepsilon \end{bmatrix} \rightarrow N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \sigma^2 \begin{bmatrix} V & V_{12}(\gamma) \\ V_{21}(\gamma) & V_{22}(\gamma) \end{bmatrix}\right)$$

and also weak convergence

$$\frac{1}{\sqrt{N}}X^*_{(\gamma)}\varepsilon = \begin{bmatrix} \frac{1}{\sqrt{N}}X'\varepsilon \\ \frac{1}{\sqrt{N}}Z'_{(\gamma)}\varepsilon \end{bmatrix} \implies \begin{bmatrix} N(0, \sigma^2V) \\ G(\gamma) \end{bmatrix},$$

where  $G(\gamma)$  is defined as in Lemma 6. Since

$$\begin{aligned} \frac{1}{\sqrt{N}}Z'_{(\gamma)}M\varepsilon &= \frac{1}{\sqrt{N}}Z'_{(\gamma)}(I - X(X'X)^{-1}X')\varepsilon \\ &= \frac{1}{\sqrt{N}}Z'_{(\gamma)}\varepsilon - \left(\frac{1}{N}Z'_{(\gamma)}X\right)\left(\frac{1}{N}X'X\right)^{-1}\left(\frac{1}{\sqrt{N}}X'\varepsilon\right) \\ &= \left[-\left(\frac{1}{N}Z'_{(\gamma)}X\right)\left(\frac{1}{N}X'X\right)^{-1} \ I\right]\frac{1}{\sqrt{N}}X^*_{(\gamma)}\varepsilon \\ &\implies G^*(\gamma) \end{aligned}$$

where  $G^*(\gamma)$  is a zero mean Gaussian process with covariance kernel

$$\begin{aligned} &-((V_{21})(V_{11})^{-1})(\sigma^2V_{11})((V_{21})(V_{11})^{-1})' + \sigma^2V_{22}(\gamma) \\ &= \sigma^2(-Z'_{(\gamma)}X(X'X)^{-1}X'Z_{(\gamma)} + Z'_{(\gamma)}Z_{(\gamma)}) \\ &= \sigma^2Z'_{(\gamma)}MZ_{(\gamma)}. \end{aligned}$$

The theorem follows. ■

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