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A Homotopy Approach to the Computation of Economic Equilibria on the Unit Simplex

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Abstract

In the model of a pure exchange economy with $n+1$ commodities, the excess demand is a continuous function from the n-dimensional unit simplex S^n to the $(n+1)$-dimensional Euclidean space R^{n+1}. A zero point of this function is a price vector at which the demand is equal to the supply in the economy. Such a price vector yields an economic equilibrium. In this paper, we propose a homotopy method on the unit simplex to compute such an economic equilibrium. This method has a clear economic interpretation. Along the path of generated prices the excess demand of each commodity is a multiple of the difference between the current and initial prices of that commodity.

Keywords: Pure Exchange Economy, Excess Demand, Economic Equilibria, Homotopy, Simplicial Subdivision, Piecewise Linear Path

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1 Introduction

In a pure exchange economy with $n+1$ commodities and a finite number of consumers the excess demand is a function of the prices of the commodities. Due to the homogeneity of the demand and supply in the prices, these prices can be normalized to lie in the n-dimensional unit simplex, where the prices of the commodities sum up to one. Further, an excess demand function is continuous, satisfies Walras' law and has the property of desirability. Walras' law states that value of the excess demand vector is always zero and desirability implies that excess demand of a commodity is positive whenever its price is zero. A price vector is called an equilibrium price vector for the economy when demand equals supply, i.e. the price vector is a zero point of the excess demand function.

In this paper we introduce a simplicial homotopy algorithm on the unit simplex. Simplicial algorithms were introduced by Scarf in [12] (see also [8], [13] and [14]). A simplicial algorithm subdivides the set on which the problem is defined into simplices and searches by generating a sequence of adjacent simplices for a simplex that contains an approximating solution to the problem. To make an arbitrary restart, Merrill introduced [11] a simplicial homotopy algorithm on the Euclidean space. In such an algorithm the underlying set is imbedded in a set of one dimension higher and in this set a path of solutions is generated leaving from the solution of a trivial problem for an approximating solution to the original problem. Van der Laan and Talman [9] introduced a simplicial variable dimension restart algorithm on the unit simplex (see also [3] and [7] for an overview of these methods). Such an algorithm generates a sequence of adjacent simplices of varying dimension and initiated at an arbitrary point. This sequence contains a piecewise linear path, connecting the starting point with an approximating solution. Instead of using a simplicial algorithm, such a path can also be generated by a continuation method, e.g. see Allgower and Georg [1]. Kamiya introduced such a method to find an equilibrium price vector in the set of price vectors on which the sum of the first n prices is at most equal to one. Kamiya's method generates a path of zero points of some homotopy function on that set, disregarding the excess demand of the last commodity. Because of Walras' law Kamiya's method can not be applied on the unit simplex itself.

By taking the vector of values of the excess demand instead of the excess demand vector itself we are able to introduce a homotopy function on the unit simplex. The path of zeros of this function connects an arbitrarily chosen price vector in the relative interior of the unit simplex with an equilibrium price vector, under some weak regularity condition. Moreover, we give a simplicial algorithm to follow the path piecewise linearly. The price vectors along that path have a simple economic interpretation. For every commodity the value of excess demand is the same multiple of the difference between the current and initial price vectors. So, for commodities in excess demand (supply) the prices are kept larger (smaller) than the initial price value. This property does hold not only for the first n commodities as in Kamiya's method but for all commodities simultaneously.

The paper is organized as follows. The model of a pure exchange economy is described in Section 2. We introduce the homotopy function and the path with an economic interpretation in Section 3. The piecewise linear path is given in Section 4. Finally, we present the steps of the algorithm in Section 5.

2 Model

Suppose there are $n+1$ commodities in a pure exchange economy, each of which is represented with an index i in N_0 given by $N_0 = \{0, 1, \cdots, n\}$. These commodities are traded among m consumers, each of which is represented with an index j in M given by $M = \{1, 2, \cdots, m\}$. Consumer j has an initial endowment of the $n+1$ commodities that is denoted by a vector $w^j \in R_+^{n+1}$, where w_i^j is his endowment of commodity i for $i = 0, 1, \ldots, n$. We assume that $w_i^j > 0$ for all i and j. For a price vector $p \in R_+^{n+1} \setminus \{0\}$, the budget set of consumer j is given by $B^j(p) = \left\{x \in R_+^{n+1} \mid p^\top x \leq p^\top w^j \text{ and } x \leq w\right\}$, where $w = \sum_{j=1}^m w^j$ represents the aggregate endowment of the consumers. The preference ordering of consumer j over all commodity bundles is represented by a utility function $u^j : R_+^{n+1} \to R$. We assume that u^j is continuous, strictly monotone, and strictly quasi-concave. Given a price vector $p \in R_+^{n+1} \setminus \{0\}$, the demand $d^j(p)$ of consumer j at p is determined by his preference ordering and budget constraint and equal to a solution of the nonlinear programming problem given by

$$\max \quad u^j(x)$$

$$\text{subject to} \quad x \in B^j(p).$$

Notice that $d^j(p)$ is uniquely determined for any given price vector p and that $d^j : R_+^{n+1}\backslash\{0\} \to R_+^{n+1}$ is continuous. From the strict monotonicity of u^j, it follows that consumer j will use up all of his income, i.e.

$$p^\top d^j(p) = p^\top w^j,$$

$p \in R_+^{n+1}\backslash \{0\}$. Moreover, $d_i^j(p) = w_i$ whenever $p \in R_+^{n+1}\backslash \{0\}$ and $p_i = 0$. Further, d^j is homogeneous of degree zero, i.e. $d^j(\lambda p) = d^j(p)$ for every $p \in R_+^{n+1}\backslash \{0\}$ and $\lambda > 0$. Given a price vector $p \in R_+^{n+1}\backslash \{0\}$, let $d(p)$ represent the aggregate demand of the consumers, so $d(p) = \sum_{j=1}^m d^j(p)$. Then the function $d : R_+^{n+1}\backslash\{0\} \to R_+^{n+1}$ satisfies the following properties:

1. (continuity) d is continuous on $R_+^{n+1}\backslash \{0\}$,
2. (Walras' law) $p^\top d(p) = p^\top w$ for $p \in R_+^{n+1}\backslash \{0\}$,
3. (relative prices) d is homogeneous of degree zero.
4. (desirability) $d_j(p) > w_j$ when $p_j = 0$.

Because of the homogeneity one can normalize a price vector $p \in R_+^{n+1}\backslash \{0\}$ such that the sum of its components is equal to one. Let S^n be the n-dimensional unit simplex, i.e. $S^n = \left\{p \in R_+^{n+1} \mid \sum_{i=0}^n p_i = 1\right\}$. Given a price vector $p \in S^n$, the excess demand at p is determined by $z(p) = d(p) - w$. The excess demand function $z : S^n \to R^{n+1}$ satisfies the following properties:

1. z is continuous on S^n,

2. $p^\top z(p) = 0$ for $p \in S^n$,

3. $z_i(p) > 0$ when $p_i = 0$.

When the excess demand is zero for every commodity, we call the corresponding price vector an equilibrium price vector.

3 Homotopy

In this section, we introduce the homotopy function on the unit simplex, which underlies the method, and analyse paths of zero points of this function. We assume in this section that the excess demand function $z : S^n \to R^{n+1}$ is continuously differentiable. Let $p * z(p) = (p_0z_0(p), p_1z_1(p), \ldots, p_nz_n(p))^\top$. We take v to be an arbitrary point in $\text{int}(S^n)$, where $\text{int}(S^n)$ is the relative interior of S^n. For $(t,p) \in [0,1] \times S^n$, we define

$$h(t,p) = t(p-v) - (1-t)p * z(p).$$

Let $e = (1,1,\ldots,1)^\top \in R^{n+1}$. Then $e^\top h(t,p) = 0$, $(t,p) \in [0,1] \times S^n$. Let the set S_0 be given by $S_0 = \left\{y \in R^{n+1} \mid e^\top y = 0\right\}$. Then h is a continuously differentiable function from the $(n+1)$-manifold $[0,1] \times S^n$ to the n-manifold S_0 without boundary. We are interested in the zero set of h. Obviously, when $t = 1$, there is only one zero point of h in $\{1\} \times S^n$, given by $(1, v)$.

Lemma 1 *There exists a $0 < \delta$ such that, for every $i \in N_0$, $h_i(t,p) < 0$ whenever $0 < p_i \le \delta$ and $(t,p) \in [0,1] \times S^n$.*

Proof: Notice that $z_i(p) > 0$ and $p_i - v_i < 0$ if $p_i = 0$. Then the lemma follows immediately since $[0,1] \times S^n$ is compact. □

This lemma implies that all zero points of h are contained in $[0,1] \times \text{int}(S^n(\delta))$, where $S^n(\delta)$ is given by $S^n(\delta) = \{p \in S^n \mid \delta \leq p_i,\ 0 \leq i \leq n\}$. We assume that 0 is a regular value of h on $[0,1] \times \text{int}(S^n)$. Then there exists a path of zero points of h starting at $(1, v)$ and terminating at some point $(0, p^*)$ in $\{0\} \times S^n$, which yields an economic equilibrium. We call this path the homotopy path, denoted by P.

An economic interpretation of the homotopy path P from $(1, v)$ to $(0, p^*)$ is as follows. It holds on P that for every point $(t, p) \in P$ with $t < 1$,

$$z_j(p) = \beta \frac{p_j - v_j}{p_j},$$

$j = 0, 1, \ldots, n$, where $\beta = t/(1-t)$. So, in economic terms, along P the positive components of the excess demand correspond to those commodities with prices higher than the initial ones given by v. Similarly, the price of a commodity is lower than the initial one given by v if its excess demand is negative. More precisely, for every commodity the value of the excess demand for it is the same multiple of the difference between the prevailing and initial prices of that commodity, i.e. $p * z(p) = \beta(p - v)$. Notice that along the path β and t need not monotonically decrease to zero.

4 Piecewise Linear Path

In this section we describe how the homotopy path p of zeros of h leaving from $(v, 0)$ to $(p^*, 1)$ can be followed by a simplicial homotopy algorithm. Such an algorithm subdivides the underlying space into simplices and generates a piecewise linear path of zeros of the piecewise linear approximation of h with respect to this triangulation. To do so, let T be a simplicial subdivision of $[0,1] \times \mathrm{aff}(S^n)$ such that all the vertices of simplices of T are contained in $\{0,1\} \times \mathrm{aff}(S^n)$, where $\mathrm{aff}(S^n)$ denotes the affine hull of S^n. We represent a simplex $\sigma \in T$ by $< y^{-1}, y^0, y^1, \ldots, y^n >$, where $y^k = (y^k_{-1}, y^k_0, \ldots, y^k_n)^\top$ is a vertex of σ for $k = -1, 0, \ldots, n$. Given $y = (y_{-1}, y_0, \ldots, y_n)^\top \in [0,1] \times \mathrm{aff}(S^n)$, we define the projection of y on $\mathrm{aff}(S^n)$ by $\mathrm{proj}(y) = (y_0, y_1, \ldots, y_n)^\top$. For a simplex $\sigma \in T$, the diameter of σ is given by

$$\mathrm{dia}(\sigma) = \max_{-1 \leq \iota < \kappa \leq n} \|\mathrm{proj}(y^\iota - y^\kappa)\|.$$

Then the mesh of T is given by $\mathrm{mesh}(T) = \sup_{\sigma \in T} \mathrm{dia}(\sigma)$. We assume $\mathrm{mesh}(T) \leq \delta$. Let $q : \mathrm{aff}(S^n) \to S^n(\delta)$ be the continuous mapping given by

$$q_j(p) = \begin{cases} \delta & \text{if } p_j < \delta, \\ p_j / (\sum_{i \in \{k \mid \delta \leq p_k\}} p_i + \sum_{i \in \{k \mid p_k < \delta\}} \delta) & \text{otherwise,} \end{cases}$$

$j = 0, 1, \ldots, n$. Recall that zero points of h are contained in $[0,1] \times \mathrm{int}(S^n(\delta))$. Thus, if $h(t, q(p)) = 0$ then $p \in S^n(\delta)$. For $(t, p) \in \{0,1\} \times \mathrm{aff}(S^n)$, let

$$f(t,p) = \begin{cases} p - v & \text{if } t = 1, \\ -q(p) * z(q(p)) & \text{otherwise.} \end{cases}$$

We do not require the function z on S^n to be continuously differentiable anymore. Given a vector $(t, p) \in [0,1] \times \mathrm{aff}(S^n)$, when

$$(t, p) \in \sigma = < y^{-1}, y^0, \ldots, y^n >,$$

we define the piecewise linear approximation of h with respect to T by

$$H(t,p) = \sum_{k=-1}^{n} \lambda_k f(y^k),$$

where λ_k, $-1 \leq k \leq n$, satisfy $0 \leq \lambda_k$, $-1 \leq k \leq n$, $\sum_{k=-1}^{n} \lambda_k = 1$, and

$$(t,p) = \sum_{k=-1}^{n} \lambda_k y^k.$$

Clearly, $H : [0,1] \times \text{aff}(S^n) \rightarrow R^{n+1}$ is a piecewise linear and continuous function. Moreover, $H(1,p) = p - v$, and when mesh(T) goes to zero, $H(0,p)$ approaches to $p * z(p)$ if $p \in S^n(\delta)$.

Let θ be a positive number. The set $J(\theta)$ is given by

$$J(\theta) = \left\{(-\epsilon, \epsilon - \epsilon^2, \ldots, \epsilon^{n-1} - \epsilon^n, \epsilon^n)^\top \mid 0 < \epsilon < \theta\right\}.$$

Obviously, any convex set containing $J(\theta)$ for some $\theta > 0$ is of dimension n. Let T^k be the collection of k-dimensional faces of simplices in T for $k = 0, 1, \ldots, n$. A face in T^n is called a facet. Notice that every facet is a face of at most two simplices in T and a face of just one if and only if it lies in $\{0\} \times \text{aff}(S^n)$ or $\{1\} \times \text{aff}(S^n)$.

Definition 1 *The facet $\tau \in T^n$ is complete if $J(\theta) \subset H(\tau)$ for some $\theta > 0$, where $H(\tau) = \{H(y) \mid y \in \tau\}$.*

Clearly, there is only one complete facet contained in $\{1\} \times \text{aff}(S^n)$. We denote it by τ_0.

Lemma 2 *Every simplex in T with a complete facet has precisely two complete facets.*

Proof: Suppose that σ is a simplex in T with a complete facet τ. According to Definition 1, there is a positive number θ such that $J(\theta) \subset H(\tau)$. Thus, the restriction of H on τ, $H|\tau : \tau \to H(\tau)$, is an affine homeomorphism. Therefore, restricted on σ, the piecewise linear function H is an affine transformation with full row rank of n. Let ρ be an arbitrary $(n-1)$-dimensional face of σ. Then it can be seen that $J(\theta) \cap H(\rho)$ is either finite or empty since $H|\tau$ is an affine homeomorphism. Therefore, decreasing θ if necessary, one can find a positive number $\bar{\theta}$ satifying $J(\bar{\theta}) \subset H(\tau)$ and $J(\bar{\theta}) \cap H(\rho) = \emptyset$. Using the fact that $H|\sigma$ is an affine transformation of full row rank of n, one can easily see that for all $0 < \epsilon < \bar{\theta}$ the set

$$\sigma \cap H^{-1}((\epsilon, \epsilon - \epsilon^2, \ldots, \epsilon^{n-1} - \epsilon^n, \epsilon^n)^\top)$$

is a closed line segment with one end point in the interior of τ and another end point in the interior of another facet of σ. The lemma follows. $\square$

Let $C^n(\text{mesh}(T))$ be given by

$$C^n(\text{mesh}(T)) = \left\{ p \in R^{n+1} \mid \sum_{i=0}^{n} p_i = 1,\ -\text{mesh}(T) \le p_i,\ 0 \le i \le n \right\}.$$

Lemma 3 *There are no complete facets outside* $[0,1] \times C^n(mesh(T))$.

Proof: Let τ be an arbitrary facet outside $[0,1] \times C^n(\text{mesh}(T))$. Then, for some j, all the jth components of vertices of τ are nonpositive. Let y be a vertex of τ. If $y = (1,p)$ then $f_j(1,p) = p_j - v_j \le -v_j < 0$. If $y = (0,p)$ then $f_j(0,p) = -q_j(p)z_j(q(p)) < 0$ since $q(p) \in \partial S^n(\delta)$. Thus $0 \notin H(\tau)$. This implies that τ is not complete. $\square$

Using Lemma 2 and Lemma 3, one can readily derive the following result.

Theorem 1 *There exists a unique sequence of adjacent simplices, which starts at the simplex having τ_0 as its facet and ends at a simplex having a complete facet that is contained in $\{0\} \times S^n$.*

The projection of the zero point of H in the complete facet contained in $\{0\} \times \text{aff}(S^n)$ with which the sequence terminates can be considered as an approximating zero point of z. When the accuracy of approximation is not sufficient, to improve it, a triangulation with a smaller mesh size can be employed and the starting point v can be chosen to be the known zero point of H in $\{0\} \times \text{aff}(S^n)$.

5 Algorithm

As a simplicial subdivision to underly the algorithm, we take the U-triangulation of $\text{aff}(S^n)$ introduced in [9]. Let e^k be the kth unit vector of R^{n+2} for $k = -1, 0, \ldots, n$. We take b to be the barycenter of $\{0\} \times S^n$, i.e.

$$b = (0, 1/(n+1), \ldots, 1/(n+1))^\top.$$

Let m be a given positive integer. The vector u^k is given by

$$u^k = (e^{k-1} - b)/m$$

for $k = 1, 2, \ldots, n$. We set $u^0 = e^{-1}$. We take $y = (y_{-1}, y_0, \ldots, y_n)^\top$ to be a vector in $\{0\} \times \text{aff}(S^n)$ satisfying

$$y = b + \sum_{k=0}^{n} a_k u^k$$

for certain integers a_k, $k = 0, 1, \ldots, n$. Let $\pi = (\pi(0), \pi(1), \ldots, \pi(n))$ be a permutation of the elements in the set $\{0, 1, \ldots, n\}$. Then the vectors y^{-1}, y^0, $\ldots$, y^n are given as follows:

$$y^{-1} = y,$$

$$y^k = y^{k-1} + u^{\pi(k)}, \; k = 0, 1, \ldots, n.$$

Clearly, y^{-1}, y^0, $\ldots$, y^n are affinely independent. Therefore, their convex hull is a simplex denoted by $U(y, \pi)$. Let U be the collection of all such simplices. It can be seen that U is a simplicial subdivision of $[0, 1] \times \mathrm{aff}(S^n)$ with $\mathrm{mesh}(U) \leq (n+1)^{3/2}/2m$ (see [10]). We call it the U-triangulation of $[0, 1] \times \mathrm{aff}(S^n)$ with grid size m^{-1}. Clearly, all the vertices of simplices in U are in $\{0, 1\} \times \mathrm{aff}(S^n)$.

We take m to be an arbitrary positive integer such that $\mathrm{mesh}(U) < \delta$. The algorithm to follow the piecewise linear path of zeros of the piecewise linear approximation H of h with respect to the underlying triangulation is as follows.

Initialization: Take v to be an arbitrary point in $\mathrm{int}(S^n)$ and let $\iota = 0$.

Step 0: Let τ_0 be the unique complete facet contained in $\{1\} \times \mathrm{aff}(S^n)$. Notice that τ_0 is a facet containing $(1, v)$ and can be easily obtained since $h(1, p) = p - v$ on $\{1\} \times \mathrm{aff}(S^n)$. Let $\sigma_0 = U(y, \pi)$ be the unique simplex in U with vertices y^{-1}, y^0, $\ldots$, y^n such that τ_0 is its facet. Let $k = 0$. Go to Step 1.

Step 1: Let τ_{k+1} be the unique other complete facet of σ_k. If τ_{k+1} is contained in $\{0\} \times \mathrm{aff}(S^n)$, go to Step 3. Otherwise, let y^ν be the vertex of σ_k, not being a vertex of τ_{k+1}. Go to Step 2.

Step 2: If $\nu = -1$ then set $y = y + u^{\pi(0)}$ and $\pi = (\pi(1), \ldots, \pi(n), \pi(0))$. If $-1 < \nu < n$ then set $\pi = (\pi(0), \ldots, \pi(\nu+1), \pi(\nu), \ldots, \pi(n))$. If $\nu = n$ then set $y = y - u^{\pi(n)}$ and $\pi = (\pi(n), \pi(0), \ldots, \pi(n-1))$. Let $\sigma_{k+1} = U(y, \pi)$ with vertices y^{-1}, y^0, $\ldots$, y^n, so τ_{k+1} is a facet of σ_{k+1}. Let $k = k+1$ and go to Step 1.

Step 3: Let $(0, p^\iota)$ be an arbitrary point in τ_{k+1}. Take m to be a larger integer. Let $v = p^\iota$ and $\iota = \iota + 1$. Go to Step 0.

Using Theorem 1, one obtains the following result.

Theorem 2 *Every cluster point of the sequence $\{p^\iota\}$ generated by the algorithm gives a zero point of z.*

Within a finite number of steps a vector p^ι is generated, for which it holds that $\max_{0 \leq h \leq n} |z_h(p^\iota)| < \epsilon$ for any a priori chosen accuracy level $\epsilon > 0$.

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