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ABSTRACT

The exact likelihood functions for a sequence of job search models are analyzed. The optimality condition implied by the dynamic programming framework is fully imposed. When unemployment duration and reemployment wage data are both available, using the optimality condition allows identification of an offer arrival probability separately from an offer acceptance probability. The estimation problem is nonstandard. The shape of the likelihood function in finite samples is considered, along with asymptotic properties of the maximum likelihood estimator.

Estimation of dynamic programming models is an important new tool for empirical economics. As computational capability increases dramatically - and perhaps as the mathematical level of our profession increases, although somewhat less dramatically - dynamic programming methods once used only by theorists in carefully contrived environments are being used in data analysis. Recent applications include Pakes (1986) on the decision to hold or release a patent, Rust (1987b) on replacement investment, and Rust (1987a) on retirement. An early area of application of dynamic programming models is job search (Kiefer and Neumann (1979) - see Devine and Kiefer (1990) for a discussion of empirical search models). The search model is a natural application because the dynamic programming involved is fairly simple and the theoretical model leads naturally to a likelihood function. Unfortunately, the likelihood function can be difficult to evaluate. Most applications have either imposed some approximation to the optimality condition implied by the d.p. setup (as in Kiefer and Neumann - a linear approximation) or have ignored it altogether. This is unsatisfactory if the search approach is to be evaluated seriously. Recent computationally intensive approaches to imposing the optimality condition are Wolpin (1987) and Stern (1987).

In this paper we consider the exact likelihood function for a job search model. It is worth emphasizing that we consider the full econometric implications of the assumed theoretical model. In some ways these implications may be unsatisfactory from a practical point of view. We address this issue in our conclusion. Nevertheless, it is important to understand the full implications of a model and specification if it is to provide a useful guide to interpreting data. Thus we reject the modelling strategy of writing down a model, accepting the implications we like, and ignoring (or failing to investigate thoroughly) those we do not.

To keep things simple, we focus on the transition from unemployment to employment and consider different data configurations -duration data alone, wage data alone and wage data combined with lengths of spells of unemployment. The likelihood function summarizes without loss all the information in the data given the structure we impose. With the likelihood function in hand we can identify directions in which data are more, or less, informative. This permits a discussion of the sort of supplementary information that would be most useful. The likelihood function is essential if a Bayesian analysis is contemplated - that is, if we wish to make meaningful probabilistic statements about parameters. Of course, many non-Bayesians also find some version of the likelihood principle compelling (see Hill (1985-86)). We also give an algorithm for computing maximum likelihood estimates. These provide an efficient classical alternative to the variations on method of moments estimators often suggested. Efficient hypothesis testing using the likelihood ratio or one of its asymptotic equivalents also requires computation of the likelihood function and its maximum. Of course, moment estimators typically require fewer distributional assumptions than maximum likelihood estimators, but since we are heavily exploiting a stylized search model for interpreting the data, why not exploit it a little more in estimating parameters?

This paper lays out issues of specification, computation and estimation without reference to a specific data set. This way, our results can be easily extended to other search applications. A companion paper applies these methods in an empirical analysis of data from the Survey of Income and Program Participation (SIPP). We have in mind data grouped into subsamples which can be regarded as homogeneous, so the theory will be examined for a homogeneous sample. This approach follows Chesher and Lancaster (1983) and Devine (1989). In addition to leading into an analysis of SIPP data, this paper demonstrates the exact calculation of a nonstandard likelihood function, obtains a suitable asymptotic estimation theory and demonstrates the feasibility of estimating a dynamic programming model.

On the substantive side, we show that full exploitation of the model's assumed structure allows identification of an offer arrival probability and an acceptance probability. Most theoretical search models have focussed on the offer acceptance decision of workers - the determination of the reservation wage. Recent empirical work suggests that variation in obtaining offers is as important (or more important) in explaining variations in lengths of spells of unemployment as is variation in reservation wages (Devine and Kiefer (1990)). These two factors - the offer probability and the acceptance probability, interpreted by Mortensen and Neumann (1984) as "choice" and "chance" - are clearly not nonparametrically identified in the data configurations usually available. Typically their product, the per-period probability of reemployment, is identified. We show that these two probabilities are separately identified under suitable distributional assumptions when joint wage and duration data are available. This is due to ruthless exploitation of the optimality condition. Of course, we might expect that identification is "weak" here - i.e. that data are unlikely to sort out these two effects very sharply, relative to determination of other coefficients.

Section 1 describes the theoretical search model. We use a familiar, discrete time time-homogeneous setup with a small generalization to allow stochastic arrival of offers. The model leads naturally to a likelihood function. In Section 2a-2f we consider properties of the likelihood function in a series of simple models. We note the use of wage and duration data jointly and assess the crucial contribution of duration data to identification. We consider maximum likelihood estimation and asymptotics. We then turn in 3 to a more general model based on a Gamma wage offer distribution, examine the likelihood function and treat issues of computation and estimation by Newton-Raphson iteration along a nonlinear ridge on the boundary of the graph of the likelihood function.

1. A Search Model

Unemployed workers are assumed to know some things about the local labor market for workers with their skills - in particular they are assumed to know the distribution of wages across firms. They do not know which firm offers which wage (alternatively, the firms offer wages at random, eg. reflecting different subjective assessments of worker suitability). We assume that offers w are distributed with density $f(w)$, with support $[0, \infty)$ or possibly $[0, w_{max}]$. A worker who accepts a job expects to hold it forever. This assumption is easily relaxed but simplifies the presentation of the model. A worker who is unemployed but looking for work receives a per-period income of y. One interpretation is that y is unemployment benefits net of search costs. The worker uses a per-period discount factor $\beta \in [0,1)$. We are now in a position to obtain the optimal search policy for a worker who seeks to maximize the expected present discounted value of his income stream by writing down the value function $V(w)$, where w is the outstanding wage offer. It is convenient to define state-dependent value functions V^e and V^u corresponding to employment and unemployment.

$$V(w) = \max \{V^e(w), V^u\}$$

$$V^e(w) = \Sigma_{t=0}^{\infty} \beta^t w = \frac{w}{1-\beta}$$

$$V^u = y + \beta EV$$

so $$V(w) = \max \{\frac{w}{1-\beta}, y + \beta EV\} \qquad (1)$$

Since V^e is monotonically increasing in w and V^u does not depend on w, a reservation wage strategy is optimal: let w^r satisfy $V^e(w^r) = V^u$; then accept the first offer greater than w^r. To make the model interesting, we assume that there is positive probability of an offer greater than w^r (otherwise the worker does not enter the market) and that w^r is greater than 0 (otherwise the worker always accepts the first offer). We use indifference between unemployment and employment at the reservation wage to develop an implicit equation for the reservation wage

$$w^r(y,f) = (1-\beta)\ (y + \beta EV) \qquad (2)$$

This is an implicit equation because EV depends on the optimal policy

$$EV = \int_0^\infty V(w)f(w)dw$$

$$= \int_0^{w^r(y,f)} (y + \beta EV)f(w)dw + \int_{w^r(y,f)}^\infty \frac{w}{1-\beta} f(w)\ dw$$

$$= \frac{1}{1-\beta}\left\{(1-\beta)(y+\beta EV)\int_0^{w^r(y,f)} f(w)dw + \int_{w^r(y,f)}^\infty wf(w)dw\right\}$$

$$\text{so } EV = \frac{1}{1-\beta}\left\{w^r(y,f)\int_0^{w^r(y,f)} f(w)dw + \int_{w^r(y,f)}^\infty wf(w)dw\right\} \tag{3}$$

Let $\Pi(y,f) = \int_{w^r(y,f)}^\infty f(w)dw$

and note that

$$\Pi(y,f) = \text{Prob(accept)}$$

Hence

$$EV = \frac{1}{1-\beta}\left\{w^r(y,f)(1-\Pi(y,f)) + \Pi(y,f)E(w|w\geq w^r)\right\} \tag{4}$$

Suppressing arguments

$$EV = \frac{1}{1-\beta}\left\{(1-\Pi)w^r + \Pi E(w|w\geq w^r)\right\} \tag{5}$$

Substituting (5) in (2) yields the implicit equation for the reservation wage. This analysis is standard - see Mortensen (1986) for an exposition and many extensions.

It is empirically useful to provide a model in which the worker does not necessarily receive an offer each period. Suppose an offer is received with per-period probability p. Note the important assumption that the event of obtaining an offer and the value of the offer are independent. This assumption makes sense in the random search framework since the worker does not know which firms offer which wage, and so the worker's assessment of the probability he will get an offer is the same at each firm. In this case, equation (1) remains relevant and (2) is replaced by

$$w^r(y,f,p) = (1-\beta)(y+\beta EV) \tag{2b}$$

with

$$\begin{aligned} EV &= p\int_0^\infty V(w)f(w)dw + (1-p)(y+\beta EV) \\ &= \frac{1}{1-\beta}\left\{p\left[(1-\Pi)w^r + \Pi E(w|w\geq w^r)\right] + (1-p)w^r\right\} \\ &= \frac{1}{1-\beta}\left\{(p-p\Pi+1-p)w^r + p\Pi E(w|w\geq w^r)\right\} \\ &= \frac{1}{1-\beta}\left\{(1-p\Pi)w^r + p\Pi E(w|w\geq w^r)\right\} \end{aligned} \tag{5b}$$

(5b) replaces (5). Here, $\Pi = \Pi(y,f,p) = \text{Prob}(\text{accept}|\text{offer})$, and $\text{Prob}(\text{employment}) = p\Pi = \lambda(y,f,p)$. Equation (5b) has (5) as special case for $p=1$, so general analysis can be done in (5b). We are going to calculate the reservation wage, so it is useful to note that the right-hand side of (5b) is a contraction; (5b) implicitly defines EV as the unique fixed point of this map:

$$EV = \frac{1}{1-\beta}\left\{(1-p\int_{w^r}^{\infty} f dw)w^r + p\int_{w^r}^{\infty} wfdw\right\}$$
$$=\frac{1}{1-\beta}\left\{(1-p\int_{(1-\beta)(y+\beta EV)}^{\infty} fdw)(1-\beta)(y+\beta EV) + p\int_{(1-\beta)(y+\beta EV)}^{\infty} wfdw\right\}$$
$$=(1-p\int_{(1-\beta)(y+\beta EV)}^{\infty} fdw)(y+\beta EV) + \frac{p}{1-\beta}\int_{(1-\beta)(y+\beta EV)}^{\infty} wfdw$$
$$=T(EV) \tag{6}$$

We verify the contraction property by calculating the derivative

$$T'(EV) = p(1-\beta)\beta f(w^r)\frac{w^r}{1-\beta} + \beta(1-p\Pi) - \frac{p}{1-\beta}(1-\beta)\beta w^r f(w^r) \tag{7}$$
$$= \beta(1-p\Pi)$$

so $0 \leq T' \leq \beta < 1$. Numerically, this allows solving for the expected value by iterating the contraction. By (2b), this gives the reservation wage, too. This is done at each parameter value when calculating the likelihood function and related quantities in Sections 2 and 3. The approach is similar to Rust (1987b), who studies optimal replacement of bus engines.

A more detailed analysis, to understand better the properties of the likelihood function, will require specification of a functional form for the offer distribution. We begin with an exponential wage offer distribution $f(w) = \gamma\exp(-\gamma w)$, $w>0$. Here

$$\Pi = e^{-\gamma w^r} \tag{8}$$

$$E(w|w\geq w^r) = w^r + \frac{1}{\gamma} \tag{9}$$

Substituting in (5b):

$$EV = \frac{1}{1-\beta}\left\{(1-p\Pi)w^r + p\Pi(w^r + \frac{1}{\gamma})\right\}$$

$$=\frac{1}{1-\beta}\left\{ w^r + \frac{p\Pi}{\gamma}\right\} \tag{10}$$

or

$$EV = \frac{1}{1-\beta}\left\{w^r + \frac{p}{\gamma}e^{-\gamma w^r}\right\} . \tag{11}$$

Eliminating w^r

$$EV = T(EV) = \frac{1}{1-\beta}\left\{(1-\beta)(y + \beta EV) + \frac{p}{\gamma}e^{-\gamma(1-\beta)(y+\beta EV)}\right\}$$

$$= y + \beta EV + \frac{p}{(1-\beta)\gamma}e^{-\gamma(1-\beta)(y+\beta EV)}$$

which is a contraction in EV. The alternative implicit equation

$$EV = \frac{y}{1-\beta} + \frac{p}{(1-\beta)^2\gamma} e^{-\gamma(1-\beta)(y + \beta EV)} \tag{12}$$

is used when convenient. Use (2b) for w^r.

The exponential, a one-parameter family, may be restrictive for empirical work. We also consider the Gamma distribution with $f(w) = (\gamma^\alpha/\Gamma(\alpha))w^{\alpha-1}\exp\{-\gamma w\}$, $w>0$, which returns the exponential as a special case when $\alpha=1$.

2. Likelihood functions and ML estimation

In this section we consider the likelihood functions for different data configurations and parametrizations. We begin with a discussion of duration data alone, then turn to wage data and finally to joint wage and duration data. In each subsection we introduce a new concept, building on previous discussions. We do this in order to illustrate the concepts of profile likelihood, asymptotic profile likelihood, singular asymptotic distributions, and the geometric approach separately. We give the likelihood functions, consider maximum likelihood estimation, and give some asymptotic distribution theory for the MLE. Throughout N is the number of individuals, (t_i, w_i) are the duration and accepted wage for the ith individual, N_c is the number of censored spells, $N_e = N - N_c$ is the number of uncensored spells, $T = \Sigma t_i$, $\tilde{T} = T - N_e$ ($\tilde{T}$ has an interpretation as the total number of periods in which workers did not become reemployed), $w_m = \min(w_i)$, and $\bar{w} = \Sigma w_i / N_e$. Of course, wages are observed only for uncensored observations and some of the observed t_i will be censoring times rather than completed durations.

We illustrate our calculations using simulated data. We generated 100 observations on unemployment durations with maximum length 15 periods. Ten observations are censored, so there are 90 wage observations. The underlying parameter values used to generate the data are $\gamma = 1$, $p = 0.3$, $y = -0.66$. The last value was chosen so as to make the acceptance probability $\Pi = 0.5$. For reference, the MLE is (1.026, 0.309, -0.617) and the implied Π is 0.486. The estimated reservation wage is 0.702 and the reservation wage corresponding to the true values of the parameters is 0.690. This data set is used throughout Section 2 except for section 2b which focuses on wage data alone in the estimation of γ and y. In that section data are generated with $\gamma = 1$, $p = 1$, $y = -1$. Here the theoretical reservation wage is 1.35, and the employment probability 0.261. The MLE is (0.962, -1.14); the estimated reservation wage is 1.37. This corresponds to the model with certain offer arrivals each period. This model is discussed in Section 1 and forms the basis for much of the early literature in the search framework.

Throughout, we plot the normed likelihood function $\ell(\theta)/\ell(\hat{\theta})$, where $\hat{\theta}$ is the MLE.

2a. Duration data alone

Spells of unemployment have a geometric distribution with parameter λ. Although the employment probability λ depends on underlying parameters - the offer distribution, the offer probability, unemployment income, etc. - these cannot be separately identified from duration data alone. Nevertheless, duration information is useful in identifying coefficients and improving the efficiency of estimates of the full parameter set in the joint wage-duration case. We set the stage for this discussion by reviewing maximum likelihood estimation of λ. In practice, survey data on durations are typically censored, due to the fixed length of the observation period. Suppose the maximum duration is K periods. Then $P(t_i = k) = (1 - \lambda)^{k-1}\lambda$, $1 \le k \le K$ and $P(\text{censored}) = (1-\lambda)^K$. This is a particularly simple censoring mechanism, but more complicated forms of random censoring could be handled without difficulty. We use subscripts c and e to sum and product operators to indicate ranges (censored or uncensored observations). The likelihood function is

$$\begin{aligned}\ell(\lambda) &= \Pi_c (1-\lambda)^K \, \Pi_e (1-\lambda)^{t_i - 1} \lambda \\ &= (1 - \lambda)^{N_c K + \Sigma_e t_i - N_e} \lambda^{N_e} \\ &= (1 - \lambda)^{\tilde{T}} \lambda^{N_e}\end{aligned}$$

and $\ln \ell(\lambda) = \tilde{T} \ln (1-\lambda) + N_e \ln \lambda$. The score is

$$s(\lambda) = \frac{-\tilde{T}}{1 - \lambda} + \frac{N_e}{\lambda}$$

so the MLE is

$$\hat{\lambda} = \frac{N_e}{N_e + \tilde{T}} = N_e / T.$$

It is straightforward to establish that the MLE is consistent. When we look at the expected contribution of duration data to efficiency we will need the observed information

$$s'(\lambda) = \frac{-\bar{T}}{(1-\lambda)^2} - \frac{N_e}{\lambda^2}$$

The expected information after some reduction is

$$i(\lambda) = -Es'(\lambda) = N \frac{1-(1-\lambda)^K}{\lambda^2(1-\lambda)}$$

The normed MLE $N^{1/2}(\hat{\lambda} - \lambda_0)$ is asymptotically normally distributed with mean zero and variance given by $N/i(\lambda)$. The uncensored case is handled by letting K go to infinity.

2b. <u>Wage data alone - one parameter</u>

The case of wage data alone allows a simple demonstration of the nonstandard nature of the inference problem. We now suppose that we have a sample of accepted wages and that the underlying wage offer distribution is exponential with unknown parameter $\gamma>0$

$$f(w) = \gamma e^{-\gamma w} \qquad w \geq 0$$

so the acceptance probability is

$$\Pi = \int_{w^r}^{\infty} f(w)dw = e^{-\gamma w^r}$$

where $w^r = (1 - \beta)(y + \beta EV)$ and y = unemployment income. Using eq. (11) we find

$$EV = \frac{1}{1-\beta}(w^r(\gamma) + \frac{1}{\gamma} e^{-\gamma w^r(\gamma)}).$$

The density of accepted wages is $f(w)/\Pi$ and we can now write the likelihood in terms of $w^r(\gamma)$ as

$$\ell(\gamma) = \gamma^N e^{-\gamma N(\bar{w}-w^r(\gamma))} 1_{\{w_m \geq w^r(\gamma)\}}.$$

Thus, $t = (\bar{w}, w_m)$ is a sufficient statistic. It is convenient to factor the sufficient statistic as $((\bar{w}-w_m)(w^r)^{-1}(w_m), w_m) = (a, w_m)$. Then a is approximately ancillary in the sense that it is asymptotically distribution constant and in fact becomes distribution constant at a rate faster than $N^{1/2}$.[1] It can be verified generally that the reservation wage is an increasing function of the mean offer

$$(w^r)'(\gamma) < 0 \text{ so } w_m \geq w^r(\gamma) \Leftrightarrow \gamma \geq (w^r)^{-1}(w_m).$$

For $\gamma \geq (w^r)^{-1}(w_m)$ the likelihood is positive and

$$\ln \ell(\gamma) = N\ln \gamma - \gamma N(\bar{w} - w^r(\gamma)).$$

The score is

$$s(\gamma) = \frac{N}{\gamma} - N(\bar{w} - w^r(\gamma)) + \gamma N(w^r)'(\gamma). \qquad (2.1)$$

Now, $\frac{N}{\gamma} > 0$, $\bar{w} \geq w_m \geq w^r(\gamma)$, and $(w^r)'(\gamma) < 0$,

so the sign of the score is ambiguous. It can be demonstrated that there are samples occurring with positive probability for which the likelihood function is maximized in the interior of the set $\{\gamma | w^r(\gamma) \leq w_m\}$. However

$$s(\gamma)/N \xrightarrow[N\to\infty]{} \frac{1}{\gamma} - (w^r(\gamma_0) + \frac{1}{\gamma_0} - w^r(\gamma)) + \gamma(w^r)'(\gamma)$$

$$= \left(\frac{1}{\gamma} - \frac{1}{\gamma_0}\right) + \left(w^r(\gamma) - w^r(\gamma_0)\right) + \gamma(w^r)'(\gamma)$$

For $\gamma \geq \gamma_0$ this expression is nonpositive term by term. In large samples the likelihood is maximized at $\tilde{\gamma} = (w^r)^{-1}(w_m)$. In fact, this occurs quickly. The situation is illustrated in Figure 1.

<u>Theorem 1</u>: Let $\tilde{\gamma} = (w^r)^{-1}(w_m)$ and $\hat{\gamma}$ = the MLE.

A) $\text{plim}N^a (\tilde{\gamma} - \gamma_0) = 0, \quad a \in [0,1)$

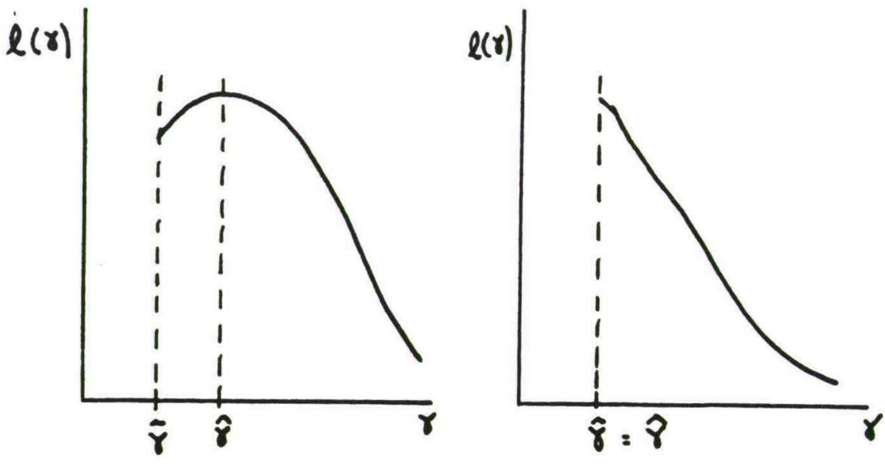
B) $\text{plim}N^a(\hat{\gamma} - \tilde{\gamma}) = 0, \quad a \in [0,1)$

Note that B implies not only that the maximum likelihood estimator is consistent, but that it converges in distribution at a faster rate than $N^{1/2}$.

Proof A: The exact distribution of w_m is $f(w_m) = \gamma N\exp(-\gamma(w_m - w^r)N)$. From this we see that $\text{plim}N^a(w_m - w^r) = 0$ for $a < 1$; in particular plim $N^{1/2}(w_m - w^r) = 0$, a result we will rely on below. Since w^r is a continuous, monotonic function of γ, $\tilde{\gamma}$ is consistent.

Proof B: When $s(\gamma) < 0$ for all $\gamma > \hat{\gamma}$ then $\hat{\gamma} = \tilde{\gamma}$ so it suffices to show that $\Pr(s(\gamma)\, N^{-a} > 0)$ goes to zero as $N \to \infty$. Note that $Es(\gamma)\, N^{-a} = \mu_N = -N^{1-a}c(\gamma)$ where $c(\gamma)$ is a positive function of γ and $Vs(\gamma)N^{-a} = N^{1-2a}\,\gamma_0^{-2}$. These expressions can be obtained by applying a CLT in (2.1) Then

FIGURE 1



A: Small Samples B: Large Samples

Note: $\tilde{\gamma} = (w^*)^{-1}(w_m)$; $\hat{\gamma}$ = MLE

$\Pr(s(\gamma)N^{-a} > 0) = \Pr(s(\gamma)N^{-a} - \mu_N > -\mu_N) \leq \gamma_0^{-2}/(Nc(\gamma)^2)$ by Chebyshev's inequality. Thus $\Pr(S(\gamma)N^{-a} > 0) \to 0$ as $N \to \infty$.

We have illustrated that our estimation problem is nonstandard but tractable. Note that our problem is different from that of estimating a parameter whose true value is at the boundary of the parameter space[2].

2c Wage data - two parameters

In this section we introduce the profile likelihood function in a setting allowing a simple geometric interpretation. We consider the parameter set $(\gamma,y) \in \mathbb{R}^+ \times \mathbb{R}$ (recall that y is unemployment income net of search costs). In this case the reservation wage is given by $w^r(\gamma, y) = (1-\beta)(y+\beta EV)$ and the likelihood function is

$$\ell(\gamma, y) = \gamma^N e^{-\gamma N(\bar{w}-w^r(\gamma, y))} 1_{\{w_m \geq w^r(\gamma, y)\}}$$

Once again the sufficient statistic is $t=(\bar{w}, w_m)$. Here there is no ancillary since the minimal sufficient statistic has the same dimension as the parameter. It is easy to see that w^r is increasing in y - though the dependence is complicated since EV depends on y. Hence, the loglikelihood function is strictly increasing in y and $\ln \ell(\gamma, y) = N \ln \gamma - \gamma N(\bar{w} - w^r(\gamma, y))$ for (γ,y) such that $w^r(\gamma, y) \leq w_m$. The score in y is

$$\frac{d\ln \ell}{dy} = \gamma N \frac{d}{dy} w^r(\gamma, y) > 0$$

so the MLE for y, given γ, can be given explicitly $\hat{y}(\gamma) = (w^r)^{-1}(\gamma, w_m)$.

Upon substitution we obtain the profile likelihood function $\bar{\ell}(\gamma)$. Geometrically, the graph of the positive part of the likelihood function $G_+ = \{ \gamma, y, \ell \mid \ell(\gamma, y) > 0\}$ is a 2-dimensional nonlinear manifold in R^3 (see Figure 2). We have shown that the maximum in ℓ as a function of y lies on ∂G_+, for any γ, and thus that $\bar{G}_+$ the graph of the profile likelihood $\bar{\ell}(\gamma)$ is the projection of ∂G_+ to the (ℓ, γ) plane. Figure 3 shows this profile likelihood function. We would also look at the other profile likelihood $\bar{\ell}(y)$, but note that this is not necessarily the projection of ∂G_+ to the (ℓ, y) plane since the maximum in the ℓ-coordinate of G_+ as γ varies for fixed y need not occur on ∂G_+ (see section 2b). Of course, we know from the geometry that the global maximum in ℓ of G_+ occurs on ∂G_+, so it might make sense to regard the projection of ∂G_+ to (ℓ, y) as an approximate profile likelihood function. This yields a pseudo likelihood function in the sense of Barndorff-Nielsen (1989, p. 30) We do not pursue this here, and return to consideration of $\bar{\ell}(\gamma)$ and $\bar{G}_+$.

The log profile likelihood is

$$\ln\tilde{\ell}\,(\gamma) = N\ln\gamma - \gamma N\,(\bar{w} - w_m)$$

with score

$$\tilde{s}\,(\gamma) = N/\gamma - N(\bar{w} - w_m)$$

and MLE $\hat{\gamma} = 1/(\bar{w} - w_m)$. Since $\text{plim}w_m = w_0^r$ and $\text{plim}\bar{w} = w_0^r + 1/\gamma_0$ it is

FIGURE 2 : NORMED L(GAMMA,Y), PURE WAGE DATA

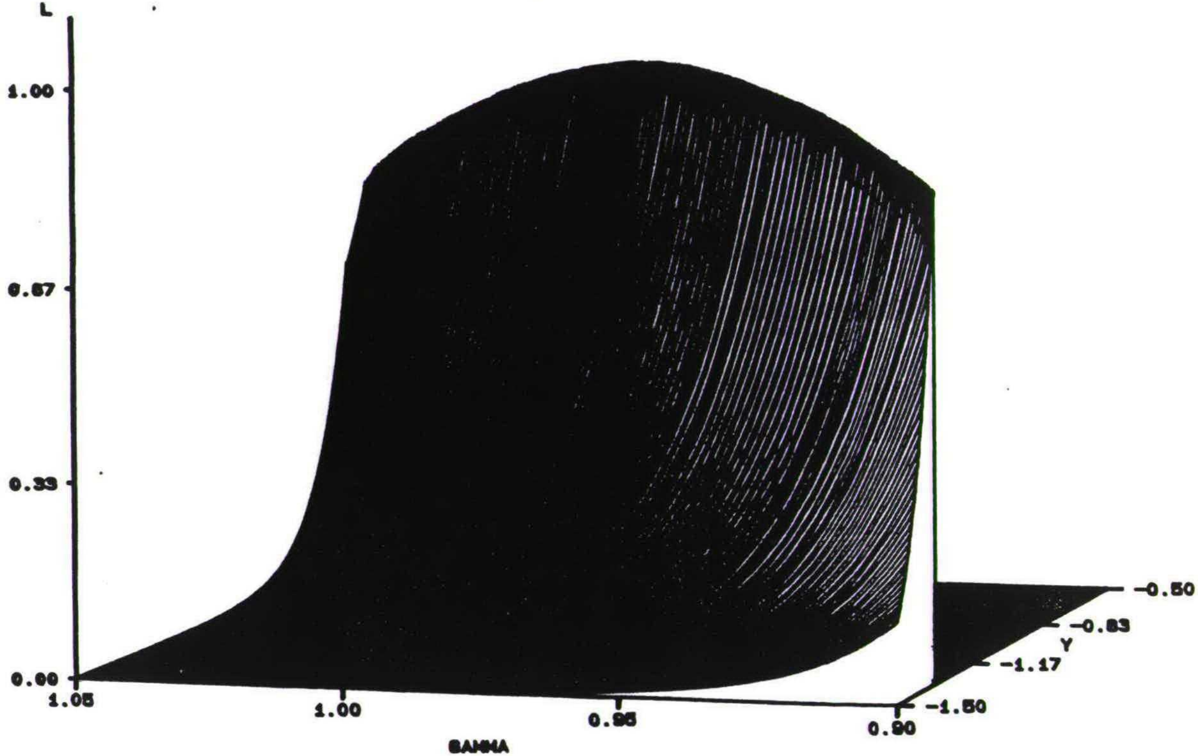
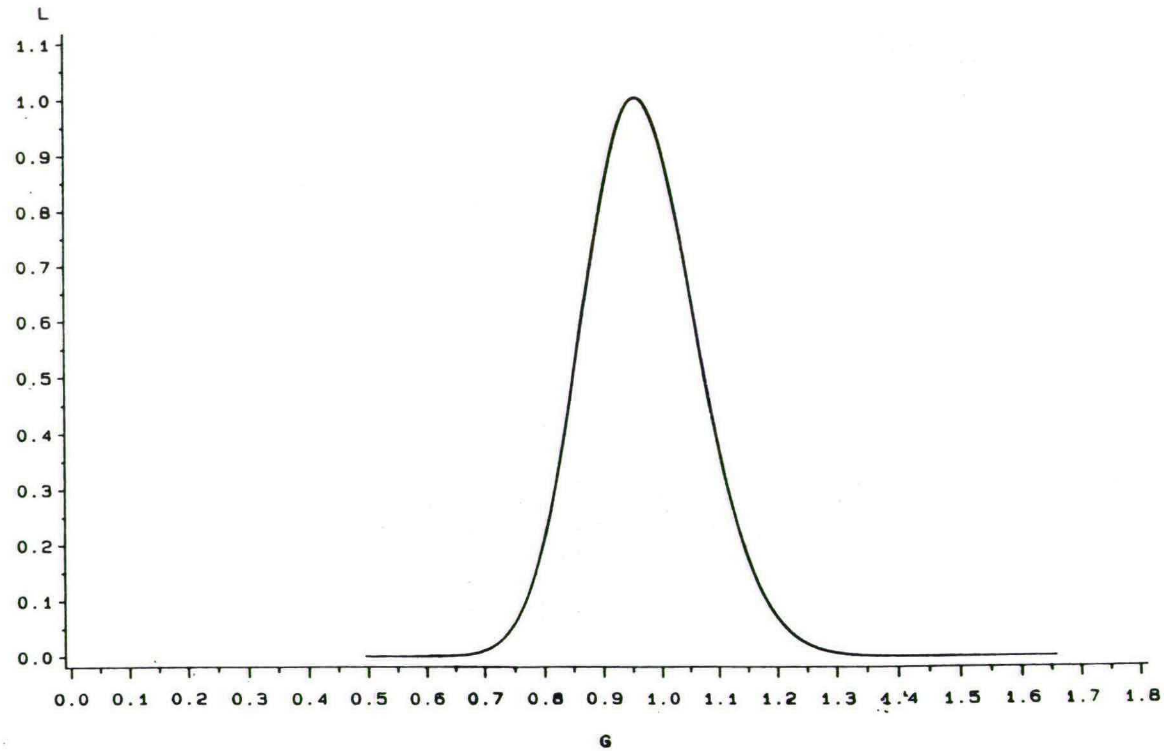


FIGURE 3 : NORMED PROFILE L(GAMMA), PURE WAGE DATA



clear that $\hat{\gamma}$ is consistent and hence $\hat{y}$ is consistent since $(w^r)^{-1}$ is continuous and $(\hat{\gamma}, w_m)$ is consistent for (γ_0, w_0^r).

The profile information, a measure of the curvature of $\bar{G}_+$, can be calculated

$$\tilde{s}'(\gamma) = -N/\gamma^2$$

$$\bar{i}(\gamma) = -E\tilde{s}'(\gamma) = N/\gamma^2$$

Noting that $\text{plim} N^{1/2}(w_m - w^r) = 0$ we find that

$$N^{1/2}(\hat{\gamma} - \gamma_0) \rightarrow n(0, \gamma_0^2).$$

Of course, y is a parameter as well. The joint MLE is given by

$$\begin{pmatrix} \hat{\gamma} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} 1/(\bar{w}-w_m) \\ (w^r)^{-1}(1/(\bar{w}-w_m), w_m) \end{pmatrix}$$

and

$$N^{1/2} \begin{pmatrix} \hat{\gamma} - \gamma_0 \\ \hat{y} - y_0 \end{pmatrix} \longrightarrow n\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \gamma_0^2 \begin{pmatrix} 1 & \delta \\ \delta & \delta^2 \end{pmatrix} \right)$$

a singular bivariate normal distribution. This singularity arises because of the fact that the MLE is on ∂G_+ - we allow $N^{1/2}$ sampling variation along the boundary but not away from the boundary. Precisely, consider the line tangent to ∂G_+ at the MLE. The area of concentration of the approximating normal distribution is the projection of this line to the (y, γ) plane (This line is given by $\gamma = \hat{\gamma} + k$ and $y = \hat{y} + k\delta$ as k varies). The direction of singularity is perpendicular to ∂G_+ at the MLE.

Here,

$$\delta = \frac{dy}{d\gamma}(\gamma_0) = \frac{d}{d\gamma}(w^r)^{-1}(\gamma,\ w_m)\Big|_{\gamma = \gamma_0}$$

We now use the theory to develop an explicit formula for δ, allowing calculation of asymptotic standard errors. Since

$$w^r(\gamma,\ y) = (1 - \beta)(y+\beta EV(\gamma,y)),$$

$(w^r)^{-1}(\gamma,w_m)$ is the y that solves $w_m = (1-\beta)(y+\beta EV(\gamma,y))$.

But EV depends on y only through $w^r(\gamma,\ y)$, so $(w^r)^{-1}(\gamma,\ w_m)$ is the y that solves $w_m = (1-\beta)(y+\beta EV(\gamma,\ w_m))$, giving

$$(w^r)^{-1}(\gamma,\ w_m) = \frac{w_m}{1-\beta} - \beta EV(\gamma,\ w_m)$$

Recall that from (11)

$$EV(\gamma,\ w_m) = \frac{1}{1-\beta}(w_m + \frac{1}{\gamma}e^{-\gamma w_m})$$

so

$$(w^r)^{-1}(\gamma,\ w_m) = \frac{w_m}{1-\beta} - \beta\frac{1}{1-\beta}\left(w_m + \frac{1}{\gamma}e^{-\gamma w_m}\right)$$

$$= w_m - \frac{\beta e^{-\gamma w_m}}{(1-\beta)\gamma} \ .$$

so

$$\delta = -\frac{\beta}{1-\beta}\left(\frac{-w_m e^{-\gamma w_m}\gamma - e^{-\gamma w_m}}{\gamma^2}\right)\Bigg|_{\gamma = \gamma_0}$$

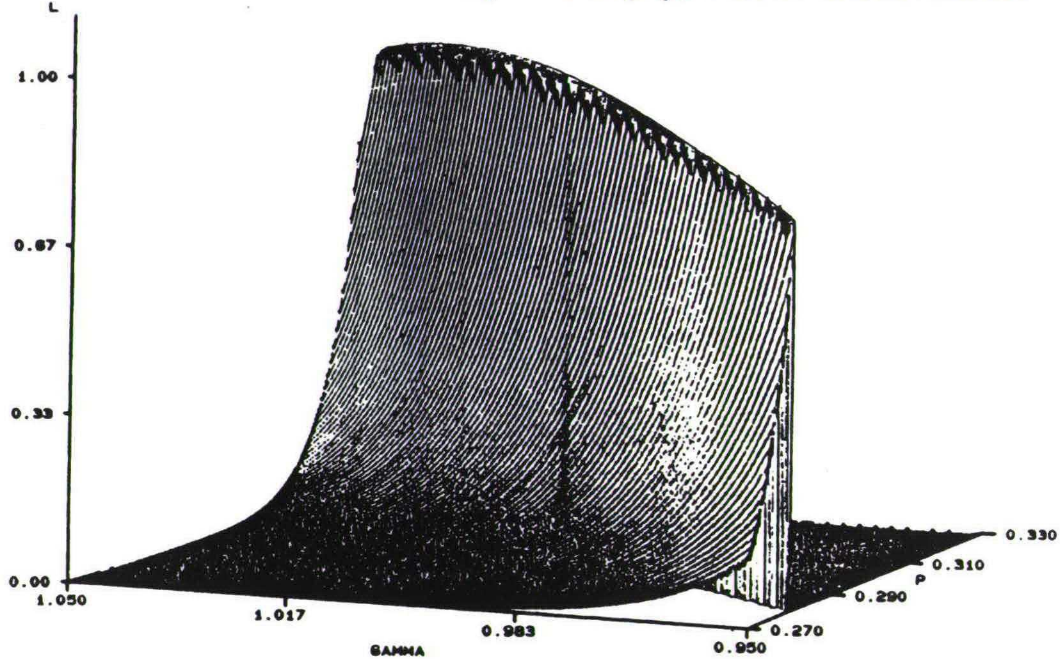
$$= \left(\frac{\beta}{1-\beta}\right)\left(\frac{e^{-\gamma_0 w_m}}{\gamma_0^2}\right)(1 + \gamma_0 w_m) \ .$$

2d. Wage data alone - offer arrival probability

This section demonstrates the identification of (γ, p) and hence (π, p) when y is known. Consider the parameter set $(\gamma, p) \in \mathbb{R}^+ \times [0,1]$, where p denotes the offer arrival probability. Here, the reservation wage is $w^r(\gamma, p) = (1-\beta)(y+\beta EV)$ with EV given by (12). The event of obtaining an offer and the value of the offer are independent. Note that y is treated as known; this is most easily justified when search costs are ignored or not present. In this case, y is interpreted as pure unemployment benefits and may be directly observable. These assumptions allow us to identify p, in addition to γ, from wage data alone. Thus duration data are not needed for identification, even of p and Π separately, but we show in Section 2e below that including durations adds to efficiency.

The analysis in the present model follows that of Section 2d closely. The likelihood function is the same, with $w^r(\gamma,p)$ replacing $w^r(\gamma, y)$. For $\{\gamma, p \mid w^r(\gamma,p) \le w_m\}$ the likelihood is positive and the score in p is $\partial \ln \ell/\partial p = \gamma N(dw^r(\gamma,p)/dp)$, which is easily seen to be positive, e.g. by differentiating (12). The geometry is the same as in 2c - the graph of the positive part of the likelihood function, G_+, is a 2-dimensional manifold in R^3 (see Figure 4) and the graph of the profile likelihood $\bar{\ell}(\gamma)$ is the projection of the boundary of G_+ to the (ℓ,γ) plane. Given γ, the MLE for p is $\hat{p}(\gamma) = (w^r)^{-1}(\gamma, w_m)$. The profile likelihood is the same as in 2c and as in that section $\hat{\gamma} = 1/(\bar{w} - w_m)$ and $N^{1/2}(\hat{\gamma}-\gamma_0) \rightarrow n(0, \gamma_0^2)$. The parameter γ is identified, clearly, since e.g. the information in the profile likelihood is positive. Substituting back, we obtain $\hat{p} = \hat{p}(\hat{\gamma}) = (w^r)^{-1}(1/(\bar{w} - w_m), w_m)$. p is identified since γ is, and since in any sample, given w_m, $\hat{p}(\gamma)$ is uniquely given. See below for an explicit

FIGURE 4 : NORMED L(GAMMA,P), PURE WAGE DATA



expression. Note that w_m converges to $w_0^r = w^r(\gamma_0, p_0)$ faster than $\bar{w}$ converges to $w_0^r + 1/\gamma_0$. Again, joint consistency of $(\hat{\gamma}, \hat{p})$ follows, and $\hat{p}$ only converges at the same rate as $\hat{\gamma}$. In particular, the asymptotic distribution of the joint MLE is a singular bivariate normal given by

$$N^{1/2}\begin{bmatrix} \hat{\gamma} - \gamma_0 \\ \hat{p} - p_0 \end{bmatrix} \longrightarrow n\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \gamma_0^2 \begin{bmatrix} 1 & \mu \\ \mu & \mu^2 \end{bmatrix}\right).$$

In this case, $\mu = d\hat{p}/d\gamma$. Here the area of concentration is given by $\gamma = \hat{\gamma} + k$ and $p = \hat{p} + k\mu$. See the discussion in 2d. Rewrite the implicit equation $w^r(\gamma, p) =$
$(1 - \beta)(y + \beta EV(\gamma,p))$ as

$$p = \frac{(1 - \beta)}{\beta} (w^r - y)\, \gamma e^{\gamma w^r},$$

using (11). Given γ, the MLE $\hat{p}(\gamma)$ for p is found by substituting w_m for w^r. Hence, differentiation with respect to γ is simple. To get μ, we evaluate at the true values (γ_0, p_0) or equivalently at $(\gamma, w^r) = (\gamma_0, w_0^r)$, yielding

$$\mu = \frac{1-\beta}{\beta} (w_0^r - y)\, e^{\gamma_0 w_0^r}(1 + \gamma_0 w_0^r).$$

For purposes of hypothesis testing, the covariance matrix of the parameter estimates can be estimated by substituting $(\hat{\gamma}, w_m)$ for (γ_0, w_0^r) in the above expressions.

2e Wage and duration data - two parameters

In this section we introduce a pseudo likelihood function (cf. Barndorff-Nielsen (1989)) which approximates the profile likelihood function. We also assess the contribution of duration data to efficiency. In the model of the previous section, suppose we are in addition given data on durations of unemployment spells. Maintaining the notation of Section 2a for durations, we have $T = \Sigma t_i$ and $\tilde{T} = T - N_e = \Sigma_e t_i + KN_c - N_e$, where censoring takes place at duration K. In the formulas below, the uncensored case is obtained by letting K go to infinity. Recall that λ is the reemployment probability, which in the present case can be expressed as $\lambda = p\Pi = pe^{-\gamma w^r(\gamma,p)}$. That is, reemployment occurs if an offer is received, which happens with probability p, and is accepted, which happens with probability Π, given that the offer has been received. The likelihood function for (γ,p) given the total data set $\{(t_i, w_i)\}_i$ is then the product of the likelihood functions from 2a and 2d:

$$\ell(\gamma,p) = (1-\lambda)^{\tilde{T}} \lambda^{N_e} \gamma^{N_e} e^{-\gamma N_e(\bar{w} - w^r(\gamma,p))} 1_{\{w_m \geq w^r(\gamma,\ p)\}}$$

Note that N_e replaces N in the factor corresponding to wage data, since wages only are observed for uncensored spells. The score in p is

$$s_p(\gamma,\ p) = \left[-\frac{\tilde{T}}{1-\lambda} + \frac{N_e}{\lambda} \right] \lambda_p + \left[\gamma N_e \frac{d}{dp} w^r(\gamma,\ p) \right]$$

for $w_m \geq w^r(\gamma,p)$. Here, the first square bracket is the score from the duration model in 2a, and the second square bracket is the score from the wage data model in 2d.

Combining (2b) and (12) of Section 1, we obtain the implicit equation

$$w^r(\gamma,\ p) = y + \frac{\beta p}{(1-\beta)\gamma}\, e^{-\gamma w^r(\gamma,p)}$$

for the reservation wage. From this, it can be shown that

$$\frac{d}{dp} w^r(\gamma,p) = \frac{w^r - y}{p(1+\gamma(w^r-y))}$$

where from above $w^r - y > 0$. Using this in differentiating the above expression for λ, it follows that

$$\lambda_p = \frac{d\lambda}{dp} = \frac{\lambda}{p(1+\gamma(w^r-y))}$$

We now substitute for λ_p and dw^r/dp in the expression for the score in p to get

$$s_p(\gamma,p) = -\frac{\bar{T}\lambda}{(1+\lambda(w^r-y))} + \frac{N_e}{p}$$

$$= \frac{N_e(1+(1-\lambda)\gamma(w^r-y)) - \lambda T}{(1-\lambda)p(1+\gamma(w^r-y))} .$$

From here it is clear that in finite samples, the score in p can be either positive or negative. Especially, it becomes negative when the total sum of durations T is large enough relative to the number of ultimately reemployed workers N_e. It such cases, the MLE $\hat{p}(\gamma)$ of p, given γ, may have to be found in the interior of the interval $(0,(w^r)^{-1}(\gamma,w_m)]$, in which the likelihood function is positive.

Letting G_+ be as usual the graph of the positive part of the likelihood function, our result shows that the finite-sample profile likelihood function cannot in general be graphed by projecting ∂G_+ to the (ℓ,γ) plane. However, this projection does produce a pseudo likelihood function and an asymptotic argument can be given for focussing on this projection. Before turning to this, consider Figure 5. The additional curvature along ∂G_+ relative to Figure 4 is due to the information in the duration data and is verified in calculations given below.

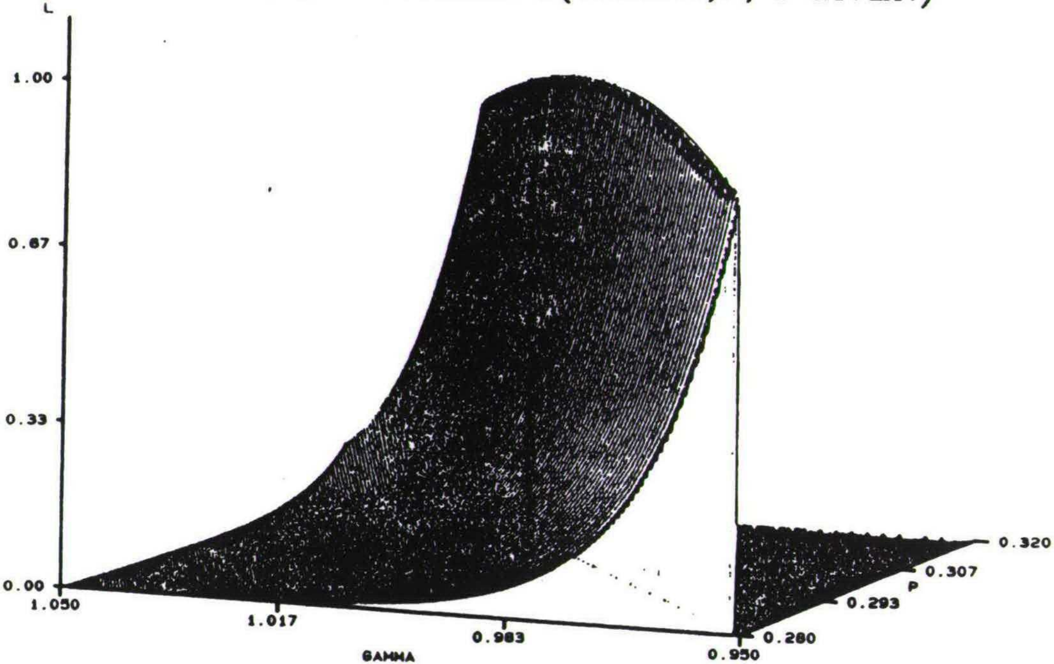
Using the probability limits in Section 2a for N_e/N, N_c/N, and $\Sigma_e t_i/N$ it can be shown that

$$\text{plim}\ \frac{1}{N}s_p = \frac{1-(1-\lambda_0)^K}{\lambda_0 p(1-\lambda)(1+\gamma(w^r-y))}\ (\lambda_0-\lambda + \lambda_0(1-\lambda)(w^r-y)).$$

Hence, asymptotically, the score is positive at least as long as $\lambda_0 > \lambda = \exp(-\gamma w^r(\gamma,p))$. Since $dw^r/dp > 0$ we have available the function $p(\gamma,w) = (w^r)^{-1}(\gamma,w)$, so we can write $\lambda(\gamma,w) = p(\gamma,w)\exp(-\gamma w)$. Clearly, $\lambda(\gamma_0, w^r{}_0) = \lambda^0$. It can be proved that

$$\frac{d\lambda}{d\gamma} = -\ \frac{\lambda y}{1+\gamma(w^r-y)} < 0$$

FIGURE 5 : NORMED L(GAMMA,P; Y GIVEN)



so for $\gamma \geq \gamma_0$, we still satisfy the sufficient condition $\lambda_0 > \lambda$ for the score in p to be asymptotically positive at $(\gamma, p(\gamma, w^r_0))$. Using the expression for dw^r/dp, we have

$$\frac{d\lambda}{dw} = \frac{\partial\lambda}{\partial w} + \frac{\partial\lambda}{\partial p}\,\frac{dp}{dw}$$
$$= -\gamma\lambda + \frac{\lambda}{p}\left(\frac{dw^r}{dp}\right)^{-1}$$
$$= \frac{\lambda}{w^r - y} > 0.$$

so for $\gamma \geq \gamma_0$, the score in p is asymptotically positive for $p \leq p(\gamma, w^r_0)$. This is because decreasing p from $p(\gamma, w^r_0)$ given γ, means decreasing w from w^r_0 and hence decreasing λ from $\lambda(\gamma, p(\gamma, w^r_0)) < \lambda_0$, so the sufficient condition is still satisfied.

The likelihood function is only positive for $p \leq (w^r)^{-1}(\gamma, w_m) = p(\gamma, w_m)$. Asymptotically, $w_m = w^r_0$, so we have shown that for $\gamma \geq \gamma_0$, the likelihood function is asymptotically increasing in p where it is positive. Hence, a natural estimator for p, given γ, is

$$\tilde{p}(\gamma) = (w^r)^{-1}(\gamma, w_m),$$

and for $\gamma \geq \gamma_0$, $\tilde{p}(\gamma)$ is asymptotically equivalent to the MLE $\hat{p}(\gamma)$. If $\tilde{\gamma}$ is an estimator that is asymptotically equivalent to the MLE (γ) of γ, then for the global estimates $(\tilde{\gamma}, \tilde{p}) = (\tilde{\gamma}, \tilde{p}(\tilde{\gamma}))$ and $(\hat{\gamma}, \hat{p}) = (\hat{\gamma}, \hat{p}(\hat{\gamma}))$ we have that they are asymptotically equivalent.

We now substitute $\tilde{p}(\gamma)$ for p in the likelihood function to obtain a pseudo likelihood we call the <u>asymptotic profile likelihood function</u>.

$$\tilde{\ell}(\gamma) = (1 - \lambda(\gamma))^{\tilde{T}} \lambda(\gamma)^{N_e} \gamma^{N_e} e^{-\gamma N_e(\bar{w} - w_m)}$$

whose graph $\tilde{G}_+$ is the projection of ∂G_+ to the (ℓ, γ) plane, and whose domain is $\{\gamma | 0 \leq (w^r)^{-1}(\gamma, w_m) \leq 1\}$. Here,

$$\lambda(\gamma) = \lambda(\gamma, \tilde{p}(\gamma)) = \tilde{p}(\gamma)\, e^{-\gamma w_m} .$$

The score is

$$\tilde{s}(\gamma) = \left[- \frac{\tilde{T}}{1-\lambda} + \frac{N_e}{\lambda} \right] \lambda' + \left[\frac{N_e}{\gamma} - N_e(\bar{w} - w_m) \right].$$

The estimator $\tilde{\gamma}$ found by setting $\tilde{s}$ equal to 0 is consistent, since $\text{plim}\ \tilde{s}(\gamma_0) = 0$. To see the latter property, note that it holds for each square bracket separately: in the duration model of Section 2a, $\hat{\lambda}$ found by setting the first bracket equal to zero was consistent, so the probability limit at γ_0 is 0; similarly for the second bracket, which determined the consistent estimator for γ in Section 2b through 2d. Now, $\tilde{p} = p(\tilde{\gamma}) = (w^r)^{-1}(\tilde{\gamma}, w_m)$ is automatically consistent for p, since w_m converges at a faster rate than $\tilde{\gamma}$, and again we get a singular bivariate normal asymptotic distribution:

$$N^{1/2} \begin{pmatrix} \tilde{\gamma} - \gamma_0 \\ \tilde{p} - p_0 \end{pmatrix} \longrightarrow n \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, V(\gamma_0) \begin{pmatrix} 1 & \mu \\ \mu & \mu^2 \end{pmatrix} \right)$$

with $\mu = dp/d\tilde{\gamma}$ from Section 2d. However, in the presence of duration data, $V(\gamma_0)$ replaces γ_0^2 in the variance expression. Even though γ and p were jointly identified already in the pure wage data model, efficiency is improved by including duration data. This is so because $V(\gamma_0) < \gamma_0^2$, or equivalently $V(\gamma_0)^{-1} > 1/\gamma_0^2$. It can be proved that

$$V(\gamma_0)^{-1} = (1-(1-\lambda_0)^K) \left[\frac{(\lambda'_0)^2}{\lambda_0^2(1 - \lambda_0)} + \frac{1}{\gamma_0^2} \right]$$

where $\lambda_0 = p_0 \exp(-\gamma_0 w_0^r)$ and

$$\lambda'_0 = \frac{1-\beta}{\beta} (w_0^r - y)(1 + \gamma_0 w^r{}_0) - \lambda_0 w_0^r.$$

For variance comparison, it is convenient to consider uncensored samples only. Let K go to infinity in the above expression and note that $V(\gamma_0)^{-1}$ equals $1/\gamma_0^2$ plus a term that is always positive, so the claimed gain in efficiency by including duration data follows. This analysis confirms our impression from the figures that the curvature in ∂G_+ is greater when duration data are included. Again, $(\bar{\gamma}, \bar{p}, w_m)$ can be substituted for $(\gamma_0, p_0, w^r{}_0)$ in the asymptotic distribution to estimate standard errors and facilitate general hypothesis testing.

2f Wage and duration data - three parameters

Obviously, in the case of joint wage and duration data, models with certain offer arrivals can be set up and parameterized by γ or (γ, y) corresponding to Sections 2b and 2c, respectively. Note that this is the simplification of $p = 1$ of the model in 2e. The analysis of these sections is easily combined to yield the results for these alternative models for joint data. In 2e, however, we focused on parameterization by (γ, p) to show that even though these parameters were identified in pure wage data, Section 2d, efficiency was improved by including duration data. However, the score in p was only positive asymptotically. Hence, the efficient estimators $(\hat{\gamma}, \hat{p})$ given in 2e may not be exact maximum likelihood estimators in finite samples, even though their consistency properties and asymptotic distributions are as indicated.

In the present section we develop the exact MLE for (γ, y, p) in the model obtained by freeing up $y \in \mathbb{R}$ in the model of section 2e. The likelihood function is as in 2e, with $w^r(\gamma, y, p)$ replacing $w^r(\gamma, p)$. The graph of the positive likelihood function $G_+ = \{(\gamma, y, p, \ell) | \ell(\gamma, y, p) > 0\}$ is a random 3-dimensional nonlinear manifold in $\mathbb{R}^4$. The score in y for $w_m \geq w^r(\gamma, y, p)$ is

$$s_y(\gamma, y, p) = \left[- \frac{\tilde{T}}{1-\lambda} + \frac{N_e}{\lambda} \right] \lambda_y + \left[\gamma N_e \frac{d}{dy} w^r(\gamma, y, p) \right]$$

Now insert

$$\lambda_y = \frac{d}{dy}\lambda = \frac{d}{dy} pe^{-\gamma w^r(\gamma, y, p)} = -p\gamma e^{-\gamma w^r(\gamma, y, p)} \frac{d}{dy} w^r(\gamma, y, p)$$

$$= - \gamma\lambda \frac{d}{dy} w^r(\gamma, y, p)$$

to obtain the reduction to

$$s_y(\gamma, y, p) = \frac{\tilde{T}\gamma\lambda}{1-\gamma} \frac{d}{dy} w^r(\gamma, y, p) > 0$$

without invoking asymptotic arguments. Hence, given (γ, p), the MLE for y is $(w^r)^{-1}(\gamma, w_m, p)$. This can be expressed explicitly as

$$\hat{y}(\gamma, p) = w_m - \frac{\beta p e^{-\gamma w_m}}{(1-\beta)\ \gamma}$$

parallel to the approach in Section 2c where $p = 1$. Geometrically, the graph of the exact profile likelihood is obtained by projecting ∂G_+, a 2-dimensional submanifold, to the 3-dimensional space with coordinates (γ, p, ℓ). Let $\lambda(\gamma, p) = p \exp(-\gamma w_m)$. The profile likelihood function is always positive, with logarithm

$$\ln\tilde{\ell}\ (\gamma, p) = \tilde{T}\ \ln(1 - \lambda(\gamma, p)) + N_e \ln\lambda(\gamma, p) + N_e \ln\gamma$$
$$- \gamma N_e (\bar{w} - w_m).$$

The scores are

$$\tilde{s}_\gamma(\gamma, p) = \left[-\frac{\tilde{T}}{1-\lambda} + \frac{N_e}{\lambda} \right] \lambda_\gamma + \left[\frac{N_e}{\gamma} - N_e (\bar{w} - w_m) \right]$$

and

$$\tilde{s}_p(\gamma, p) = \left[-\frac{\tilde{T}}{1-\lambda} + \frac{N_e}{\lambda} \right] \lambda_p$$

with $\lambda_\gamma = -w_m \lambda$, $\lambda_p = \Pi = \exp(-\gamma w_m)$. Note that $(\hat{\gamma}, \hat{p})$ obtained by setting the scores equal to zero are consistent, using a mixture of pure duration data and pure wage data arguments as in the previous section. However, in the present section it can be proved that the explicit solution to the profile likelihood equations is

$$\begin{pmatrix} \hat{\gamma} \\ \hat{p} \end{pmatrix} = \begin{pmatrix} 1/(\bar{w} - w_m) \\ N_e / (\tilde{T} e^{-w_m / (\bar{w} - w_m)}) \end{pmatrix}.$$

Now, the MLE for y is given as $\hat{y} = \hat{y}(\hat{\gamma}, \hat{p})$ using the above expression. The profile information in this case is

$$\tilde{i}(\gamma, p) = N \frac{1-(1-\lambda)^K}{1-\lambda} \begin{bmatrix} (w_0^r)^2 + \frac{1-\lambda}{\gamma^2} & -\frac{w_0^r}{p} \\ -\frac{w_0^r}{p} & \frac{1}{p^2} \end{bmatrix}$$

Here, $\lambda = p\exp(-\gamma w_0^r)$. Note that w_0^r has replaced $w^r(\gamma, y, p)$ everywhere. This can be interpreted as a consequence of the rapid convergence of w_m to w_0^r, and is a further indication of the non-standard properties of the present likelihood problem.

The matrix in the above expression has determinant $(1-\lambda)/(\gamma p)^2$, clearly non-zero, so we can let $\tilde{V}(\gamma, p) = N\, \tilde{i}(\gamma, y, p)^{-1}$. Explicitly,

$$\tilde{V}(\gamma,p) = \frac{\gamma^2 p^2}{1-(1-\lambda)^K} \begin{bmatrix} \frac{1}{p^2} & \frac{w_0^r}{p} \\ \frac{w_0^r}{p} & \left(w_0^r\right)^2 + \frac{1-\lambda}{\gamma^2} \end{bmatrix}$$

With

$$Dy = \left. \begin{pmatrix} \frac{d\hat{y}}{d\gamma} \\ \frac{d\hat{y}}{dp} \end{pmatrix} \right|_{(\gamma_0,\, p_0,\, w^r{}_0)} = \begin{bmatrix} \lambda_0 (w_0^r + \frac{1}{\gamma_0}) \\ -\frac{\lambda_0}{p_0} \end{bmatrix}$$

where $\lambda_0 = p_0 \exp(-\gamma_0 w_0^r)$ we easily calculate V_o Dy and $(Dy)'V_0$ Dy with $V_0 = V(\gamma_0, p_0)$. We have arrived at the three dimensional, rank two asymptotic distribution of the exact MLE

$$N^{1/2}\begin{pmatrix}\hat{\gamma}-\gamma_0\\ \hat{y}-y_0\\ \hat{p}-p_0\end{pmatrix}\longrightarrow n\left(\begin{pmatrix}0\\0\\0\end{pmatrix},\ \frac{{\gamma_0}^2}{1-(1-\lambda_0)^K}\quad V\right)$$

where V is given by

$$\begin{bmatrix} 1 & \dfrac{\beta\lambda_0}{(1-\beta)\gamma_0^2} & p_0 w_0^r \\[2ex] \dfrac{\beta\lambda_0}{(1-\beta)\gamma_0^2} & \dfrac{\beta^2\lambda_0^2\ (2-\lambda_0)}{(1-\beta)^2\gamma_0^4} & \dfrac{\beta p_0\lambda_0}{(1-\beta)\gamma_0^2}\left(w_0^r - \dfrac{1-\lambda_0}{\gamma_0}\right) \\[2ex] p_0 w_0^r & \dfrac{\beta p_0\lambda_0}{(1-\beta)\ \gamma_0^2}\left(w_0^r - \dfrac{1-\lambda_0}{\gamma_0}\right) & p_0^2\left[\left(w_0^r\right)^2 + \dfrac{1-\lambda_0}{\gamma_0^2}\right] \end{bmatrix}.$$

These matrix calculations and use of ${w^r}_0$ are justified by the rapid convergence of w_m as treated in Section 2b. Here the direction of singularity is given by the line $\gamma = \hat{\gamma} - k\ (w_m + 1/\hat{\gamma})\hat{p}$, $y = \hat{y} + k(1-\beta)\hat{\gamma}/\beta\hat{\Pi}$, $p = \hat{p} + k$ as k varies. The area of concentration of the asymptotic distribution of the MLE is the plane which is the orthogonal complement to this line. As in the previous models, replacing $(\gamma_0, p_0, {w^r}_0)$ by $(\hat{\gamma}, \hat{p}, w_m)$ yields an estimate of the asymptotic variance.

3 Wage and duration data - gamma distribution

As mentioned in Section 1, the exponential distribution may be restrictive for empirical work, being a one parameter family. In this section we consider the Gamma distribution. The parameters of the distribution are $(\gamma, \alpha) \in \mathbb{R}_+^2$, and the exponential is the special case $\alpha=1$. In addition, we consider the parameters $y \in \mathbb{R}$ and $p \in [0, 1]$, unemployment benefit net of search cost and the offer arrival probability, respectively. We study the properties of the resulting four-parameter likelihood function and give a numerical estimation procedure. The density of wage offers is now

$$f(w;\ \gamma,\ \alpha) = \frac{\gamma^{\alpha}}{\Gamma(\alpha)} w^{\alpha-1} e^{-\gamma w}, \qquad w > 0$$

with cumulative distribution function

$$F(w;\ \gamma,\ \alpha) = \int_0^w f(x;\ \gamma,\ \alpha)\ dx, \qquad w > 0$$

The reservation wage depends on all four parameters, and is denoted $w^r(\gamma, \alpha, y, p)$. The corresponding conditional acceptance probability, given an offer has been received, is

$$\Pi(\gamma,\ \alpha,\ y,\ p) = 1 - F\ (w^r(\gamma,\ \alpha,\ y,\ p);\ \gamma,\ \alpha),$$

and we shall often abbreviate notation simply to Π. Thus, the employment probability is written $\lambda = p\Pi$. The likelihood function becomes

$$\ell(\gamma,\ \alpha,\ y,\ p) = (1 - \lambda)^{\bar{T}} \lambda^{N_e} \Pi_e \frac{f(w_i;\ \gamma,\ \alpha)}{\Pi} 1_{(w_m \geq w^r)}$$

where w^r depends on all parameters.

We introduce a compact notation for the parameter vector $\theta = (\gamma, \alpha, y, p) \in \Theta$. The parameter space is

$$\Theta = \mathbb{R}_+^2 \times \mathbb{R} \times [0,1].$$

We consider the graph $G \subseteq \mathbb{R}^5$ of the likelihood function,

$$G = \{(\theta, \ell) \mid \theta \in \Theta\},$$

where from here on pairs on the form (θ, ℓ) should be understood as $(\theta, \ell(\theta))$. We are especially interested in the positive subgraph

$$G_+ = \{(\theta, \ell) \in G \mid \ell > 0\}$$

corresponding to positive data density. Note that G_+ is a four-dimensional, non-linear manifold in $\mathbb{R}^5$.

Let $\overline{\ln w}$ denote $\Sigma_e \ln w_i/N_e$. Here the sufficient statistic is $t=(\bar{w}, w_m, \overline{\ln w}, \bar{T}, N_e)$. For $w_m \geq w^r$, and writing out the density explicitly, we get the log likelihood function

$$\ln \ell(\gamma, \alpha, y, p) = \tilde{T} \ln (1-\lambda) + N_e \ln \lambda$$
$$+N_e\{\alpha \ln\gamma + (\alpha-1)\overline{\ln w} - \gamma\bar{w} - \ln \Gamma(\alpha) - \ln \Pi\}$$

Still for $w_m \geq w^r$, the score in y is

$$s_y = - \frac{\tilde{T}\ \lambda_y}{1 - \lambda}$$

with

$$\lambda_y = - pf(w^r;\ \gamma,\ \alpha)\frac{d}{dy}w^r \qquad < 0$$

so $s_y > 0$. That is, given (γ, α, p), the MLE for y is

$$\hat{y}(\gamma, \alpha, p) = (w^r)^{-1}(\gamma, \alpha, w_m, p)$$
$$= \frac{w_m}{1-\beta} - \beta EV(\gamma, \alpha, w_m, p).$$

As the notation already suggests, the expected value function EV only depends on y through w^r, so the above equation is explicit for y. Using formulas (5b) and (14) of Section 1, we get that $\hat{y}(\gamma, \alpha, p)$ equals

$$\frac{1}{1-\beta}\{(1-\beta(1-p\Pi))w_m - \beta p\alpha(1-F(w_m;\ \gamma,\ \alpha+1))/\gamma\}$$

where from now on Π denotes

$$\Pi(\gamma,\ \alpha) = 1 - F(w_m;\ \gamma,\ \alpha)$$

and hence does not depend on p. Substituting $\hat{y}(\gamma,\ \alpha,\ p)$ in ℓ, we obtain the profile likelihood function $\bar{\ell}$ with parameters $(\gamma,\ \alpha,\ p)$. Note that the graph $\bar{G}_+$ of this function is the projection of ∂G_+, a 3-dimensional submanifold of G_+, to the 4-dimensional space with coordinates $(\gamma,\ \alpha,\ p,\ \ell)$.

The profile likelihood function is always positive, and it's logarithm has the same expression as $\ln\ell$, however keeping in mind the new Π and implied $\lambda = p\Pi$. The score in p is

$$\tilde{s}_p = \left[-\frac{T}{1-\lambda} + \frac{N_e}{\lambda}\right]\Pi$$

where the square bracket is recognized as the score in the pure duration data model of Section 2a. Setting $\tilde{s}_p$ equal to zero hence leads to the MLE for p, given $(\gamma,\ \alpha)$,

$$\hat{p}(\gamma,\ \alpha) = \frac{N_e}{T\Pi(\gamma,\alpha)}\ .$$

Substituting this in the above three-dimensional profile likelihood, we arrive at the new two-dimensional profile likelihood $\tilde{\bar{\ell}}$. The graph $\tilde{\bar{G}}_+$ of $\tilde{\bar{\ell}}$ is a 2-dimensional submanifold of $\bar{G}_+$, although it does not take the simple projection form encountered earlier. Figure 6 illustrates $\tilde{\tilde{G}}_+$. Note that the scale parameter is γ and the shape parameter α.

The expression for $\ln\tilde{\tilde{\ell}}$ is still that of $\ln\ell$, but λ is now fixed at $N_e/\tilde{T}$, and Π denotes $\Pi(\gamma, \alpha)$. Hence, the scores in the remaining parameters, γ and α, are

$$\tilde{\tilde{s}}_\gamma = N_e\left[\frac{\alpha}{\gamma} - \bar{w} - \frac{\Pi_\gamma}{\Pi}\right], \quad \tilde{\tilde{s}}_\alpha = N_e\left[\ln\gamma + \overline{\ln w} - \Psi(\alpha) - \frac{\Pi_\alpha}{\Pi}\right],$$

where $\Psi(\cdot)$ is the digamma function. It can be shown[3] using $(x/\alpha)f(x;\ \gamma, \alpha) = F(x;\ \gamma, \alpha) - F(x;\ \gamma, \alpha+1)$ that

$$\Pi_\gamma = -(w_m/\gamma)\ f(w_m;\ \gamma, \alpha)$$
$$\Pi_\alpha = \Pi(\ln\gamma - \Psi(\alpha) + E(\ln w|w > w_m))$$

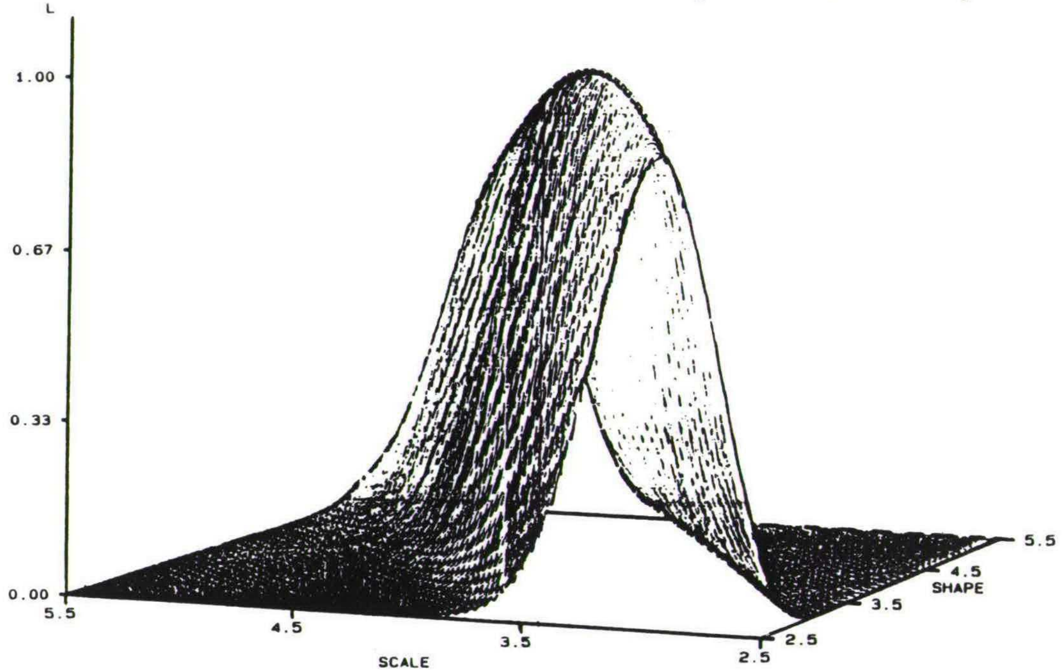
so the expression for $\tilde{\tilde{s}}_\alpha$ reduces to

$$\tilde{\tilde{s}}_\alpha = N_e(\overline{\ln w} - E(\ln w|w > w_m)).$$

Here, the expectation has to be computed numerically in applications. Upon setting the scores $\tilde{\tilde{s}}$ equal to zero, simultaneous solution in (γ, α) is necessary. Note that this is possible for the full parameter space, whereas the untruncated case requires $\alpha > 1$; see Johnson and Kotz (1970, p. 185). We now describe an iterative procedure, analogous to Newton-Raphson on $\tilde{\tilde{G}}_+$.

Along the manifold $\tilde{\tilde{G}}_+$ the consecutive parameter estimates are $(\gamma_k, \alpha_k, (\hat{y}, \hat{p})(\gamma_k, \alpha_k))$ with $(\hat{y}, \hat{p})(\cdot)$ given above, and

FIGURE 6 : NORMED PROFILE L(SCALE,SHAPE)



$$\begin{pmatrix}\gamma_{k+1}\\ \alpha_{k+1}\end{pmatrix} = \begin{pmatrix}\gamma_k\\ \alpha_k\end{pmatrix} + t_k H(\gamma_k, \alpha_k)^{-1}\, \tilde{\tilde{S}}(\gamma_k, \alpha_k).$$

In each iteration, the scalar t_k is determined by line search. $\tilde{\tilde{S}}_k(\gamma_k, \alpha_k)$ is $(\tilde{\tilde{S}}_\gamma, \tilde{\tilde{S}}_\alpha)'$ from above, evaluated at (γ_k, α_k). The 2x2-matrix $H(\cdot)$ is minus the observed profile information, the matrix $d\tilde{\tilde{S}}'/d\mu$ of second derivatives of $\ln\tilde{\tilde{\ell}}$, here denoting (γ, α) by μ, or it can be taken to be one of the asymptotic equivalents. Of these, we focus on

$$H(\gamma, \alpha) = \frac{1}{N_e}\Sigma_e\, \tilde{\tilde{s}}_i \tilde{\tilde{s}}_i',$$

namely, the sample average of outer products of profile scores per observation, $\tilde{\tilde{s}}_i$. Note carefully that the analysis at this stage is carried out using uncensored observations only, and from these only the wage information, the estimation of (y, p) having exhausted all information in the duration data. Writing

$$f_m = f(w_m; \gamma, \alpha)$$

H is easily computed using

$$\tilde{\tilde{s}}_{i\gamma} = \frac{\alpha}{\gamma} - w_i + \frac{w_m f_m}{\gamma\Pi}$$

$$\tilde{\tilde{s}}_{i\alpha} = \ln w_i - E(\ln w | w > w_m)$$

where $\Pi = 1 - F(w_m; \gamma, \alpha)$. We estimate the 3x3 full rank variance-covariance matrix of $(\hat{\gamma}, \hat{\alpha}, \hat{p})$ by

$$\bar{V} = (\frac{1}{N}\Sigma \bar{s}_i \bar{s}_i)^{-1}$$

in parallel with the approach in the estimation procedure above. The score $\tilde{s}_i$ for the ith worker is evaluated at the MLE. Note that the entire sample now is used. The entries of $\tilde{s}_i$ are, firstly

$$\tilde{s}_{ip} = \left[-\frac{t_i - 1(i)}{1-\lambda} + \frac{1(i)}{\lambda} \right] \Pi,$$

the i'th contribution to $\tilde{s}_p$ above, with $1(i) = 1$ iff the i'th worker was ultimately employed, and secondly $(\tilde{s}_{i\gamma}, \tilde{s}_{i\alpha})$ which can be calculated using

$$\tilde{s}_{i\gamma} = \tilde{s}_{ip} \frac{p\Pi_\gamma}{\Pi} + 1(i)\, \approx{s}_{i\gamma},$$

$$\tilde{s}_{i\alpha} = \tilde{s}_{ip} \frac{p\Pi_\alpha}{\Pi} + 1(i)\, \approx{s}_{i\alpha}$$

The expression for the gradient Dy of the map $(\gamma, \alpha, p) \to \hat{y}(\gamma, \alpha, p)$ above is complicated and is left out here, but we note that $\sqrt{N}(\hat{\gamma}-\gamma_0, \hat{\alpha}-\alpha_0, \hat{p}-p_0, \hat{y}-y_0)$ has a rank three (singular) normal asymptotic distribution, with mean zero and variance-covariance matrix consistently estimated by

$$V = \begin{bmatrix} \tilde{V} & \tilde{V}(Dy) \\ (Dy)'\tilde{V} & (Dy)'\tilde{V}(Dy) \end{bmatrix}$$

The rapid convergence of w_m discussed in Section 2b still holds in the Gamma case. Indeed, it is the justification of the above calculation of the asymptotic distribution of the MLE, where we can ignore the distribution of w_m.

4. Conclusion

We have examined the full econometric implications of our theoretical job-search model. The likelihood function is tractable and maximum likelihood estimation is feasible. The asymptotic distribution theory is slightly nonstandard but also tractable. Rather than treating the model as merely a suggestive prelude to analysis, we have explored its implications thoroughly. The model implies that information is accumulated rapidly in one dimension. Although this is theoretically appealing and is a strict implication of the specification, practical considerations suggest that the estimates might be more than usually sensitive to specification error or measurement error. Of course, addressing the question of the effect of these errors requires more modelling, but intuition suggests that estimates based on averages rather than on order statistics may be more satisfactory. With our current analysis in hand, it is straightforward to explore generalizations of the specification based, for example, on introducing heterogeneity in mean offers (through γ) or in search costs (through y). Some generalizations lead to regular estimation problems; some do not. This is an area of active current research.[4]

The fact that sufficient statistics are available in our model suggests that a Bayesian analysis involving natural conjugate updating might be feasible. This is true in simple cases, although the problem of assessment remains difficult (the conjugate prior requires a complicated reparametrization). Bayesian analysis of this model is also an area of current research.

Finally, we are proceeding with an application to data from the SIPP files. The best way to assess the usefulness of an econometric specification and estimation procedure is through application. We will report these empirical results separately.

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FOOTNOTES

1. A statistic is distribution constant if its marginal distribution does not depend on the parameter being estimated. (cf. Barndorff-Nielsen (1989)).

2. Heckman and Singer (1984) discuss general issues of estimation in a broad class of models including search models. They note the nonstandard nature of the estimation problem but do not pursue the implications of imposing the optimality condition.

3. Proof:
$$
\begin{aligned}
F(x;\ \gamma,\ \alpha) &= \int_0^x (\gamma^\alpha/\Gamma(\alpha)) w^{\alpha-1} e^{-\gamma w} dw \\
&= (\gamma^\alpha/\Gamma(\alpha))(w^\alpha/\alpha) e^{-\gamma w} + \int_0^x (\gamma^{\alpha+1}/\Gamma(\alpha))(w^\alpha/\alpha) e^{-\gamma w} dw \\
&= (x/\alpha)(\gamma^\alpha/\Gamma(\alpha)) x^{\alpha-1} e^{-\gamma x} + \int_0^x (\gamma^{\alpha+1}/\Gamma(\alpha+1)) w^\alpha e^{-\gamma w} dw \\
&= (x/\alpha) f(x;\gamma,\alpha) + F(x;\gamma,\alpha+1)
\end{aligned}
$$

4. A related issue arises in the literature on estimation of frontier production functions (see Greene (1980)). We expect that our methods could be used to provide a full treatment of the estimation problem in that setting as well.

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