Evolution of Smart-n Players

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ABSTRACT

To model the evolution of strategic intelligence, players types are drawn from a hierarchy of "smartness" analogous to the levels of iterated rationalizability. If smarter types have demonstrably better survival fitness, then we would have an evolutionary foundation for the standard game-theoretic assumption of super-intelligent players. The results do not support this wishful conjecture.

1.0 Introduction.

The concept of "rationality" is well-defined for a single decision-maker problem and generically predicts a unique decision. However, for multi-decision-maker problems (i.e. games), this standard concept of rationality is not adequate to provide generically unique predictions. The problem of games with multiple Nash equilibria is an obvious example. But even when there is a unique Nash equilibrium, rationality alone (and even common knowledge of rationality) is often insufficient to predict the Nash equilibrium solution [Tan and Werlang, 1988].

In response to this inadequacy, in the last decade or so, the game theory profession has devoted considerable resources to the refinement program - attempting to invent a solution concept that always exists, is unique, and satisfies other esthetic criteria. It is by no means clear that this research program will succeed.

Moreover, in the process, fundamental questions have been raised about what it means to be an "intelligent" player in a game [Binmore, 1987]. With only a handful of exceptions, game theory has taken as an implicit axiom that all players are *super-intelligent*: i.e. possessing omnicient powers beyond those supposed in single decision-maker problems. For example, in many games, all players priors and the selection rule for the solution concept must be common knowledge. The exceptions [e.g. Fudenberg and Maskin, 1986; and Kreps, Milgrom, Roberts, and Wilson, 1982] have found that super-intelligent behavior can be fundamentally different when there is even a small probability of irrational players and is very sensitive to the ad hoc specification of the irrational behavior.

To debate what it means to play "intelligently", we must also give meaning to "unintelligent" play, and then be prepared to demonstrate just how intelligent play is superior to unintelligent play in an environment in which it is possible for some players to be unintelligent. Moreover, we should have a model that justifies the assumptions about unintelligent play that underpin the derived intelligent behavior.

The methods of evolutionary game theory are well-suited for this task.[1] To illustrate, consider a symmetric 2-player game. We can conceive of a large class of behavioral rules including constant-strategy rules as well as sophisticated multi-stage-reasoning rules. We suppose a very large population of potential players and an initial frequency distribution of behavioral types (rules). In the first period, players are randomly matched a large number of times so each behavior type receives its expected payoff against the population distribution. Between the first and second period, each behavioral type reproduces at a growth rate proportional to its payoff, thus generating a new frequency distribution of behavioral types at the beginning of the second period. This process is repeated indefinitely.

The obvious questions include: which types survive and which die out, do the more intelligent types gain in population relative to the less intelligent types, does the frequency distribution converge, and if so to what? Further, we can study the robustness of these properties to the game being played.

Granted we can re-interpret extant evolutionary game theory models evolution *with* intelligence. That is, the growth of a strategy type can be re-interpreted as arising from conscious decisions of intelligent players to switch to better performing strategies (with perhaps lagged and/or noisy information). However,

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[1]For an introduction to the literature, see Friedman, 1991; Hofbauer and Sigmund, 1988; Samuelson, 1988; Samuelson and Zhang, 1991; Selten, 1991; and van Damme, 1987, chapter 9. For an innovative application, see Blume and Easley, 1991.

in this re-interpreted model the intelligent trait itself (the switching behavior) is not subjected to evolutionary selection, and therefore the model tells us nothing about the evolution *of* intelligence. In contrast, the alternative model sketched above and to be developed in this paper is about the evolution *of* intelligence.

1.1 The Specification of Smart Players.

The specification of unintelligent players is obvious. For each strategy, we associate a "dumb" player of that type who will always play that strategy in every situation. The specification of intelligent players on the other hand is far from obvious, and we claim only to have a reasonable beginning model. Henceforth, we will use the term "smart" play rather than intelligent play.

To motivate our approach, consider how a smart player might reason, facing a population of dumb and smart players. Being smart entails doing better than random behavior *given the information available*. Therefore, we must specify the information available to our smart player. We assume that the variables of the game (players, strategies, and payoff matrix) are known. We will also endow our smart players with more information about the population of players, but first it is insightful to consider what a smart player would do if all he/she knew were these game variables. In accord with single decision-maker theory and the expected utility hypothesis, a smart player would have a subjective belief about the other players' strategy choices and choose a best-response. Therefore, a smart player would never choose a strategy that is not a best-response to some distribution over the other players' strategies, and since we have no basis (at this point in our discussion) to restrict the smart player's beliefs, we can impose no further restrictions.[2]

Now suppose that our smart player knows the game variables, the proportion of the population that is dumb and the frequency distribution of dumb players over strategies. This information restricts the possible probability distributions over the other players' strategies. We can define the set of strategies that are first-level rationalizable with respect to this information. But now our smart player knows the proportion of the population that is also smart, and so it is possible to reason further.

Given the information about the dumb players, the payoff matrix can be uniquely modified to represent the payoff to smart players conditional on the fixed known behavior of dumb players. Then, one approach would to assume that smart players play a Nash equilibrium of this modified game [Banerjee and Weibull, 1991]. We reject this approach for several reasons. First, there may be multiple Nash equilibria, in which case selecting any one presupposes a level of coordination that should be explained by the model rather than assumed. Second, even when there is a unique Nash equilibrium, the principles of rationality alone do not compel that solution. Common knowledge (among smart players) about the modified game and the rationality of smart players would lead only to the set of Berheim-Pearce (1984) rationalizable strategies which could be the entire set of strategies.

Based on these remarks, suppose smart players believe that all other smart players reason as they do, and therefore smart players would never choose a strategy that is not Berheim-Pearce (BP) rationalizable in the modified game. We must also specify how smart players choose from among multiple rationalizable

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[2]Note that since the population contains dumb players, the iterative elimination of never-best-responses is not justifiable.

strategies. A simple, natural (but ad hoc) answer would be to assume a fixed (e.g. uniform) distribution over the rationalizable strategies. It is immediately apparent that this model of smart players is inappropriately limited, because if this fixed random selection rule were known, then a "really smart" player would anticipate that behavior and instead play a best-response to the predicted distribution. Hence, this initial model of smart players is woefully inadequate.

Our approach is to model a hierarchy of evermore thoughful and informed players who reason analogously to the iterative levels of BP rationalizability. At the first level, $smart_1$ players do not assume anything about the non-dumb players, and consequently, $smart_1$ players confine their choices to the "first-level rationalizable" strategies: those that are best-responses to some probability distribution over other players strategies conditional on the information about dumb players. If there is a unique first-level rationalizable strategy, then $smart_1$ players choose that strategy. Otherwise, $smart_1$ players choose among the first-level rationalizable strategies using a (to be specified) deterministic rule.

At the second level, we define $smart_2$ players whose information consists of the information of $smart_1$ players plus the proportion of the population who are $smart_1$ players and the probability distribution over $smart_1$ players' strategy choices. [Equivalently, we could assume that $smart_2$ players know the decision rule of $smart_1$ players and can accurately predict the choices.] We assume that $smart_2$ players assume that the other players are at least as smart as $smart_1$ players and never choose a strategy that is not in the first-level rationalizable set. Consequently, $smart_2$ players confine their choices to the "second-level rationalizable" strategies: those strategies that are are best-responses to some probability distribution over other players' strategies conditional on the information about dumb and $smart_1$ players. If there is a unique second-level rationalizable strategy, then $smart_2$ players choose that strategy. Otherwise, $smart_2$ players choose among the second-level rationalizable strategies using a (to be specified) deterministic rule.

We can continue recursively defining $smart_n$ players for all $n \geq 2$. There are two senses in which a $smart_n$ player is smarter than a $smart_{n-1}$ player. First, the $smart_n$ player reasons that no $smart_{n-1}$ player will choose a strategy that is not (n-1)-level rationalizable. Second, and more importantly, a $smart_n$ player anticipates the response of all less smart players. Nonetheless, the $smart_n$ player is not smart enough to anticipate the behavior of other $smart_n$ players. To fully capture the notion of a "transcendentally smart" player requires an infinite hierarchy and a $smart_\infty$ player. [If this were not true, then the previously discussed problems in game theory would not have arisen.]

We turn now to the difficult issue of how to specify a $smart_n$ player's choice over the n-level rationalizable strategies conditional on his information. Instead of an arbitrary randomization, we opt for a specification that is responsive to evolutionary forces. Suppose that in addition to *primary* preferences over the consequences of the game each $smart_n$ player is endowed with a *secondary* strict transitive preference ordering over the strategies. This secondary preference is superfluous when there is a unique rationalizable strategy. However, when there are multiple rationalizable strategies, this secondary preference is applied to the set of rationalizable strategies. Each distinct secondary preference order distinguishes a smart player's type, and there is an initial distribution of smart_n players by type.[3] Thus, the proportion of smart_n players who choose a particular rationalizable strategy is the proportion of smart_n players whose secondary preference (restricted to the n-level rationalizable set) is for that strategy. In this model, evolutionary dynamics can operate on the (secondary preference) types of smart_n players, thereby making the population of smart_n players more adaptable than they would be with a fixed decision rule.

In section 2, we formalize this model and derive several results about the structure. In section 3, we specify the evolutionary dynamics. In section 4, we present our results. Section 5 concludes, and all proofs are relegated to an Appendix.

2.0 The Formal Structure of the Model.

Let $G \equiv (A,\pi)$ be a symmetric finite 2-player game, where A is a finite set of actions, and π is the payoff matrix. Let $\mathcal{M}(X)$ denote the set of probability measures on a finite set X. Let $\mathcal{P}$ denote the set of all strict transitive orderings of A.

The population of players consists of dumb players and smart_n players for $n \geq 1$. Let y_{0a} denote the proportion of the whole population that consists of dumb players who always choose $a \in A$. Similarly, let y_{nk} denote the proportion of the whole population that consists of smart_n players with secondary preference $k \in \mathcal{P}$. Further, let $y_0 \equiv (y_a)_{a\in A}$, $y_n \equiv (y_{nk})_{k\in\mathcal{P}}$, and $y \equiv (y_j)_{j\geq 0}$. By definition, $\Sigma_{j\geq 0}\, y_j = 1$, and the state variable y completely describes the player

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[3]In biology, species evolve mainly by acquiring new genes that are added to the DNA stock. This analogy suggests that we could think of this secondary preference as being inherited from a primordial pool of dumb players.

population.

We also want to know the distribution by smartness category. The proportion of the population consisting of dumb players is $s_0 = \Sigma_{a\in A} y_{0a}$, and for $n \geq 1$, the proportion of the population consisting of smart_n players is $s_n = \Sigma_{k\in\mathcal{P}} y_{nk}$. Note that $\Sigma_{j\geq 0} s_j = 1$.

Given $s_0 > 0$, the subpopulation of dumb player types is distributed among the strategies according to $f_0 = y_0/s_0 \in \mathcal{M}(A)$, with f_{0a} denoting the proportion of dumb players who always play strategy $a \in A$. Similarly, given $s_n > 0$, the subpopulation of smart_n players is distributed among the $\mathcal{P}$ orderings according to $f_n = y_n/s_n \in \mathcal{M}(\mathcal{P})$, with f_{nk} denoting the proportion of smart_n players with secondary preference type k. Later we will add a time index to these vectors.

2.1 Player Behavior.

It is first convenient to introduce some notation-saving definitions. Let β: $\mathcal{M}(A) \to A$ be the pure-strategy best-response correspondence. For each $b \in B \subseteq A$, let $P(b,B) = \{k \in \mathcal{P} \mid k \text{ ranks } b \text{ highest in } B\}$. In other words, P(b,B) is the set of secondary preference types that rank strategy b highest among the strategies in B. Note that $\{P(b,B),\ b \in B\}$ is a partition of $\mathcal{P}$.

A dumb player simply plays his strategy type, so the relative distribution of dumb play is f_0. For later notational convenience, we define $\mu_0 = f_0$.

Letting $R_0 = A$, we can recursively define the set of n-level rationalizable strategies conditional on $\{(s_j,\mu_j),\ j=0,\ldots,n-1\}$ for $n \geq 1$:

$$R_n = \beta(Q_n), \text{ where } Q_n = \Sigma_{j<n} s_j\mu_j + (1 - \Sigma_{j<n} s_j)\mathcal{M}(R_{n-1}) . \tag{1}$$

We next construct the relative distribution of smart_n play: $\mu_n = \{\mu_{na},\ a \in A\}$

$$\mu_{na} = \begin{cases} 0 \text{ for all } a \notin R_n, \text{ and} \\ \sum_{k \in P(a,R_n)} f_{nk} \quad \text{otherwise.} \end{cases} \tag{2}$$

Note that if R_n is the singleton {a*}, then $\mu_{na*} = 1$. Furthermore, μ_n is a recursive function of y, so eq(1-2) define $\mu(y) = \{\mu_j\}_{j\geq 0}$.

The aggregate distribution of strategy choices for the whole population is

$$p(y) = \Sigma_j \; s_j \mu_j(y) \; . \tag{3}$$

2.2 Properties of the n-Level Rationalizable Sets.

It is easy to see that $R_n \subseteq R_{n-1}$ for all $n \geq 1$, and hence, $\{R_n, n \geq 1\}$ is a non-increasing sequence of nested sets. Furthermore, if $s_j = 0$ for all $j = 0, \ldots, m$, for m sufficiently large, then R_m is the set of BP rationalizable strategies.

Figure 1 illustrates the construction of the Q_n and R_n sets. First, partition the simplex $\mathcal{M}(A)$ into the pure-strategy best-response regions. Next, locate f_0, and construct Q_1 as a $(1-s_0)$ scaling of $\mathcal{M}(A)$ with f_0 as the common point. Then, R_1 consists of the associated best responses that intersect Q_1: {1,2,3} in Figure 1. By construction, $q_1 = s_0\mu_0 + s_1\mu_1$ lies in Q_1. Then, Q_2 is a $(1-s_0-s_1)$ scaling of $\mathcal{M}(R_1)$ with q_1 as the common point. Then, R_2 consists of the associated best responses that intersect Q_2: {3} in Figure 1. Hence, $R_n =$ {3} for all $n \geq 2$. Consequently, p lies on the straight line connecting q_1 and the {3}-vertex.

Observe that if $s_0 > 0$ and f_0 is an interior distribution, then R_1 cannot contain any weakly dominated strategies. Thus, smart_n players will never choose a weakly dominated strategy given an interior distribution of dumb players.

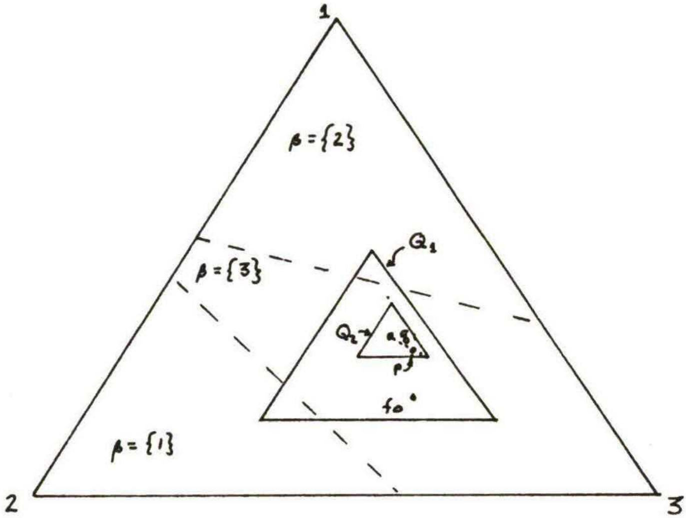

Figure 1.

As a correspondence from $\{(s_j,\mu_j),\ j=0,\ldots,n-1\}$ to A, it is easy to see that for all $n \geq 1$, R_n is upper hemi-continuous. However, when we consider R_n as a correspondence from y to A, upper hemi-continuity does not necessarily hold for $n > 1$, because the distribution of smart$_{n-1}$ play, μ_{n-1}, can change discontinuously. These potential discontinuities are serious problems for continuous-time dynamical systems. Therefore, we will confine our attention to discrete-time dynamics.

3.0 <u>The Evolutionary Dynamics</u>.

To represent the strategy choice of each type of smart$_n$ player, let $\sigma(n,k) \equiv \{\sigma \in \mathcal{M}(A) \mid \sigma_a = 1$ iff a is most preferred relative to R_n by smart$_n$ type k$\}$. By interpreting smart$_0$ as the dumb players and letting the k index range over A instead of $\mathcal{P}$, we also let $\sigma(0,k)$ denote the strategy choice of the dumb players. For $n \geq 1$, note that $\sigma(n,k)$ depends on R_n, which in turn depends on the distribution of strategies of all less smart types $\{(s_j,\mu_j),\ j=0,\ldots,n-1\}$, which is a deterministic function of y.

Given aggregate play p, then πp is the vector of expected payoffs to each strategy when matched with an opponent randomly drawn from the population of players. The expected payoff averaged over the population is $p \cdot \pi p$. The expected payoff to a smart$_n$ player of type k is $\sigma(n,k) \cdot \pi p$.

Typically, evolutionary models assume that the growth rate of a species type is proportional to its expected payoff. It follows then that the growth rate of the population share of a species type is proportional to the difference between its expected payoff and the population average payoff. We adopt this approach, and assume for all $j \geq 0$ that

$$\frac{y_{jk}(t+1)-y_{jk}(t)}{y_{jk}(t)} = \nu(t)[\sigma(j,k;t)\cdot\pi p(t) - p(t)\cdot\pi p(t)] , \quad (4)$$

where $\nu(t) > 0$ is the adjustment speed parameter, and t denotes the temporal period. The right-hand side of eq(4) is a deterministic function of the state variable y(t), so given an initial condition y(0), eq(4) defines a unique dynamic path. While the adjustment speed parameter has no affect on the direction of the path at any point, it does affect the length of each step. It is well-known that high adjustment speeds can severly destabilize a system because of overshooting. To reduce the overshooting problems, we will be interested in the behavior of eq(4) for low adjustment speeds.

We have adopted a discrete-time dynamic model to avoid the technical problems due to discontinuities on the right-hand side of eq(4). Doing so, guarantees the existence of a well-defined solution path. However, we must wonder whether the technical problems of the continuous-time version might manifest themselves in some other form (such as instabilities) in our discrete-time version. We believe not because we can modify the model slightly so the right-hand side of eq(4) is continuous. To do so, suppose that every $smart_n$ of type k ($n \geq 1$) unknowingly receives slightly distorted information about $\{(s_j,\mu_j),\ j=0,\ldots,n-1\}$. Further, suppose the information received is, say, uniformly distributed over an ε-ball around the true state, and that each player's distortion is independent of all other player's distortions. Then, integrating over this uncertainty, μ_j and hence p would be Lipschitzian continuous functions of y. In addition, the definition of $\sigma(j,k)$ would be modified, and it too would be a Lipschitzian continuous function of y. Thus, we could formulate a continuous-time dynamical system with unique solution paths.

An artifact of an evolutionary dynamic system specified in terms of growth rates is that if $y_{jk}(t_0) = 0$, then $y_{jk}(t) = 0$ for all $t > t_0$. In other words, player types that do not exist or that die out can never re-emerge. Therefore, initial conditions which have $y_{jk}(0) = 0$ for some (j,k) are of limited interest. We henceforth limit attention to "semi-interior" initial conditions of the form: for all $k \in \mathcal{P}$, $y_{jk}(0) > 0$ for all $j < n^*+1$ and $y_{jk}(0) = 0$ for all $j > n^*$, where $n^* \in \{1, 2, \ldots, \infty\}$ is the maximum level of smartness.

4.0 Results.

The first result identifies a set of strategies that will eventually never be played.

Proposition 1. If $a \in A$ is not BP rationalizable, then starting from any semi-interior $y(0)$, $p_a(t) \to 0$.

This result does not depend on convergence of the solution path. An immediate implication of Proposition 1 is that all dumb types associated with non-BP rationalizable strategies die out. On the other hand, Proposition 1 does not imply that smart_n types whose secondary preference ordering ranks the non-rationalizable strategies first die out. Since a smart_n player would never choose a non-rationalizable strategy, the position of these strategies in his secondary preference ordering is irrelevant to the evolutionary dynamics. It is also noteworthy that Proposition 1 applies to the standard case with no smart players. While the presence of smart players is not vital for this result, the smart players do speed up the demise of non-rationalizable play, since $\mu_{na} = 0$

for all $a \notin R_n$.

We consider next the class of games which have a unique BP rationalizable strategy, say a*.

<u>Proposition 2</u>. If G has a unique BP rationalizable strategy, a*, then (i) $p_{a*}(t) \to 1$ and $f_{0a*}(t) \to 1$, and (ii) there is a $\delta > 0$ such that $\delta < s_j(t) < 1-\delta$ for all t and all $j < n*+1$.

In other words, all dumb players except the a* types die out, but the a* types do not die out. Smart players do not die out, but neither do they dominate. Intuitively, the dumb player who happens to be genetically disposed towards a* is just as "fit" as any $smart_n$ player; i.e., *"being right is just as good as being smart"*.

Before deriving more results on the dynamic behavior of $s_j(t)$, it is necessary to derive more characterizations of the dynamic behavior of y(t). It is also interesting to ask whether known results for standard evolutionary dynamics [$s_0(0) = 1$ in our model] hold in our more general model. One of the typical results is that all NE are dynamic rest points and "almost all" dynamic rest points are NE. Since every pure strategy (and hence every y such that p(y) is a pure strategy) is a rest point of the dynamics, we do not have exact equivalence between NE and rest points. However, if a rest point can be reached from an interior initial condition, then it is a NE. A similar result holds in our model.

<u>Proposition 3</u>. (a) If p[y(t)] is a NE, then $dy(t)/dt = 0$. (b) If y(0) is semi-interior and p(t) converges to p*, then p* is a NE.

Notice Proposition 3(b) does not require that $y(t)$ necessarily converge - only that the aggregate play, $p(t)$, converge. This raises the question of the relationship between convergence of $y(t)$ and $p(t)$. Clearly, if $y(t)$ converges to, say, y^*, then $p(t)$ will also converge *provided* that $p(\cdot)$ is continuous at y^*. However, as stated in section 2, $p(\cdot)$ is discontinuous at points where the correspondences R_j are discontinuous. On the other hand, as we argued at the end of section 3, these discontinuities can be easily eliminated by introducing a tiny bit of noise, so for all practical purposes convergence of the state variable y implies convergence of aggregate play p. The converse proposition is not at all immediate, considering that $p(\cdot)$ is a mapping from the state space to a much lower dimensional space.

Proposition 4. If $p[y(t)]$ converges to some p^* and if for all $n \geq 1$ $R_n(t)$ converges to some R_n^*, then $y(t)$ is also convergent.

Because of the potential discontinuities in the rationalizable sets, convergence of aggregate play in the presense of smart_n players may be more unlikely than with only dumb players. To explore this possibility, we examine cases where the evolutionary dynamics of dumb players only is hyperbolically stable.

Proposition 5. If p^* is a strict NE, then p^* is hyperbolically stable; i.e. given $a^* \equiv \text{supp}(p^*)$, there is a $\delta > 0$ and $\varepsilon > 0$ such that $\|p(y(0))-p^*\| < \varepsilon$ implies $\Delta p_a(t)/p_a(t) < -\delta$ for all $a \neq a^*$ and all t.

Thus, the introduction of smart_n players does not upset the hyperbolic stability of strict NE. On the other hand, Proposition 5 does *not* assert that the basin of convergence is the same. Indeed, because of the possible discontinuities of the $R_n(t)$ sets, when y(0) involves only a small proportion of dumb players, the ε in Proposition 5 could be quite small in comparison to a y(0) which involves predominately dumb players. Smoothing the discontinuities by introducing noise does little to help because there still could be a significant decline in p_{a*} due to the addition of a rationalizable strategy, and that decline might push p outside the basin of convergence.

Since the set of state vectors y that map to a given p* is a manifold, it is too much to hope that any particular point in that manifold is asymptotically stable.

We can now generalize Proposition 2 to cases where G has multiple BP rationalizable strategies.

Proposition 6. If $p(t) \to p^*$, then there is a $\delta > 0$ such that $\delta < s_j(t) < 1-\delta$ for all t and all $j < n^*+1$.

In other words, if the dynamic process converges, then no smart_n nor dumb player has a superior (or inferior) survival fitness. Again, we have the principle that being right is just as good as being smart.

Proposition 6 implies that if we are to find cases for which, say, dumb players die out, we must focus on the non-convergent (often chaotic) cases. Even if we succeed, the victory will be soured by the observation that despite the superior survival fitness of some smart_n players, the aggregate play is non-convergent.

Proposition 7. (a) If $s_0(t) \to 0$, then $s_j(t) \to 0$ for all $j > 0$. (b) Hence, given $n^* < \infty$, there exists a $\delta > 0$ such that $s_0(t) \geq \delta$ infinitely often.

The intuition behind (a) is that because the rationalizable sets R_j are nested, some of the $smart_j$ types are mimicking the $smart_{j+1}$ players, and hence must do as well. Then (b) is an immediate consequence of (a) and the fact that $\Sigma_{j\geq 0}\, s_j = 1$.

Thus, with a finite upper bound on smartness, dumb players will never be driven out. One way to rationalize a finite upper bound n^* would be to assume there are maintenance costs, c_n, for $smart_n$ players, with $c_j > c_{j-1}$ for all $j \geq 1$ and $\lim_{n\to\infty} c_n \gg \max\pi$. Letting $\bar{c}(t) \equiv \Sigma_{j\geq 0} s_j(t)c_j$, we would add $\bar{c}(t) - c_j$ inside the brackets of eq(4). Then, for sufficiently large n, $\Delta y_{nk}(t)/y_{nk}(t) < 0$ for all $k \in \mathcal{P}$; hence, $s_n(t) \to 0$.

Notwithstanding the above remarks, Proposition 7 appears to leave open the possibility that when $n^* = \infty$, mass may escape to infinity: i.e. there may exist an increasing divergent sequence $\{n(t)\}$ such that $\Sigma_{j>n(t)}\, s_j(t) \approx 1$. In other words, a dominant transcendentally smart player would evolve - a result which would vindicate traditional game theory, albeit actual play, p, would not converge.

It also remains an open question whether or not $smart_n$ players can be driven out. The difficulty in trying to prove that a $smart_n$ player will not be driven out (or that mass can escape to infinity) is that the $R_n(t)$ correspondence is discontinuous, and when a new strategy becomes rationalizable, a relatively large proportion of the $smart_n$ players may switch to the newly rationalizable strategy, but it could actually perform worse than average.

5. Discussion.

We have developed a model in which players have varying degrees of smartness, and for which an infinitely smart player is equivalent to the usual super-intelligent player assumed in game theory. We supposed that smartness is subject to evolutionary selection pressures, and asked whether smartness has superior survival fitness. Our findings were negative. Dumb players are never driven out by smart players. Indeed, if there are increasing maintenance costs to smartness, then there is a finite upper bound on surviable smartness.

We have focused our analysis on a fixed (albeit arbitrary) finite, symmetric, two-player game. Given our general definition of smartness, we could be criticised for this focus on a fixed game rather than on a distribution of games in this class. For example, we could consider a set of M×M symmetric games defined by a diverse but finite set of payoff matrices π, and a probability distribution on these games. In each period, players are randomly matched and a payoff matrix is drawn. The evolutionary dynamics would depend on the average performance of all player types over all possible games in this class.

It is reasonable to conjecture that smartness would have superior survival fitness over dumb players in this environment. However, this result would be due to the unreasonable limitations of dumb players. If by "dumb" we mean merely mechanical and independent of information about the current population, then we should permit more complex "genes" that are able to discriminate between the alternative games (just as simple animal species are able to discriminate among various objects in their environment). Given N possible games, and M strategies, there would be M^N possible genetic types. [The secondary preferences of the smart_n players should also be expanded analogously.] The apparent inferiority of dumb players would disappear, and the principle that *being right is just as good as being smart* would seem to hold.

Of course for an infinite set of possible games, we would need an infinity of types, and it may be reasonable to restrict "dumb" players to a finite set of types. However, it is not obvious that these dumb players would be driven out. The dumb players' discrimination abilities could partition the space of payoff matrices in a way that minimizes the consequences of the incomplete information. [For example, for 2×2 games it may suffice to have a three-part partition that recognizes when each of the strategies is strictly dominant.] In general, it may be adequate to have the cardinality of the partition equal to 2^M-1 corresponding to each possible non-empty subset of A.

For the sake of argument, suppose in this infinitely diverse environment of games and finite dumb types, that the dumb types are driven out. Our celebration will be tempered by the observation that after some finite time, the $smart_1$ players will be virtually indistinguishable from the dumb types confined to the rationalizable set R_1, and hence no more fit. Moreover, for any given game, these $smart_1$ players will be indistinquishable from the dumb players of this paper. Therefore, we will not find that "the smarter, the better".

Thus, our model does not provide an evolutionary foundation for the usual assumption in game theory that all players are super-intelligent, and it seems unlikely that any other model will satisfactorily meet this goal. Future research should develop more realistic models of intelligence subject to evolutionary selection with the goal of developing a theory of "intelligent" play in an evolutionary context.

APPENDIX OF PROOFS

Proof of Proposition 1.

Let BP_n denote the n^{th}-level BP rationalizable set: i.e. the subset of A that survives n rounds of elimination of never-best-responses (or equivalently, since we are focusing on two-player games, strictly dominated strategies).

(1) Take any $\hat{a} \notin BP_1$, so $\mu_{n\hat{a}} = 0$ for all $n \geq 1$, and $p_{\hat{a}} = s_0 f_{0\hat{a}}$. Since $\hat{a}$ is not first-level rationalizable, $\exists$ $q \in \mathcal{M}(A)$ such that $q \cdot \pi p > e_{\hat{a}} \cdot \pi p$ for all p $\in \mathcal{M}(A)$. Let $\varepsilon = \min_p\{(q-e_{\hat{a}}) \cdot \pi p\} > 0$. Define

$$V(t) = f_{0\hat{a}}(t) \Big/ \left\{ \prod_{a \in A} [f_{0a}(t)]^{q_a} \right\} ,$$

Note that $V > 0$ iff $f_{0\hat{a}} > 0$. Define $\Delta V(t) = V(t+1) - V(t)$. Then,

$$\frac{\Delta V(t)}{V(t)} = \left(\frac{\Pi_{a \in A}(p_a(t))^{q_a}}{\Pi_{a \in A}(p_a(t+1))^{q_a}} \right) \cdot \left(\frac{\Delta f_{0\hat{a}}(t)}{f_{0\hat{a}}(t)} - \sum_{a \in A} q_a \frac{\Delta f_{0a}(t)}{f_{0a}(t)} \right) .$$

Note that, by virtue of the dynamic specification, eq(4), the expression in square brackets is equal to $e_{\hat{a}} \cdot \pi p - q \cdot \pi p < -\varepsilon/2$. Therefore, V(t) is a Liapounov function, so by Liapounov's Direct Method [see, e.g. LaSalle, 1986], $f_{0\hat{a}}(t) \rightarrow 0$.

(2) Next take $\hat{a} \in BP_1 \backslash BP_2$, so $\exists$ $q \in \mathcal{M}(A)$ such that $q \cdot \pi p > e_{\hat{a}} \cdot \pi p$ for all p $\in \mathcal{M}(BP_1)$. By virtue of (1), there exists a t_1 and an $\varepsilon > 0$ such that for all $t > t_1$ and all $p \in \mathcal{M}(R_1(t))$, $(q-e_{\hat{a}}) \cdot \pi p \geq \varepsilon$. Now $p_{\hat{a}} = s_0 f_{0\hat{a}} + s_1 \mu_{1\hat{a}}$. By the same methods used in (1), we can show that $f_{0\hat{a}}(t) \rightarrow 0$. Further, since $f_{0\hat{a}}(t) \rightarrow 0$, for sufficiently large t, $\hat{a} \notin R_1(t)$; hence, $\mu_{1\hat{a}} = 0$.

(4) Repeating these steps for all levels of BP rationalizability, we conclude that if $a \notin \cap_{n \geq 0} BP_n$, then $p_a(t) \rightarrow 0$. Q.E.D.

Proof of Proposition 2.

If G has a unique BP rationalizable strategy a*, then it follows immediately from Proposition 1 that $f_{0a*}(t) \to 1$. To prove the second part, first observe that for all $n \geq 1$, $R_n(t)$ converges to $\{a*\}$ in finite time, say t^*. Thus, for all $t > t^*$, all $smart_n$ types play a* and, therefore, have identical growth rates, which is also the growth rate of y_{0a*}. The population of all these player types remain in constant ratio to each other for all $t > t^*$, and hence none die out. Further, since $s_0 \geq y_{0a*}$, dumb types do not die out either. In other words, for all $j < n*+1$, $\exists\ \delta > 0$ such that $\delta < s_j(t) < 1-\delta$. Q.E.D.

Proof of Proposition 3.

(a) Suppose $p[y(t)]$ is a NE. Then, for all $a \in supp(p(t))$, $(e_a-p)\cdot\pi p = 0$. Thus, $\Delta y_0(t) = 0$, which implies that $R_1(t+1) = R_1(t)$. Then, $\Delta y_1(t) = 0$, and so on. (b) Assume that y(0) is semi-interior and $p(t) \to p*$, and suppose to the contrary that p* is not a NE. In other words, suppose there is a strategy $\hat{a}$ such that $(e_{\hat{a}}-p*)\cdot\pi p* = \delta > 0$. Then, $\exists$ t' such that for all t > t' $(e_{\hat{a}}-p(t))\cdot\pi p(t) \geq \delta/2 > 0$. But then $\Delta p_{\hat{a}}(t)/p_{\hat{a}}(t) \geq \nu\delta/2 > 0$ for all t, which is incompatible with the given convergence of p(t). Therefore, p* is a NE. Q.E.D.

Proof of Proposition 4.

Given that $p[y(t)] \to p*$, let $S \equiv supp(p*)$ and $B \equiv A\backslash S$. Hence, $p_b(t) \to 0$ for all $b \in B$, and $\exists\ \delta > 0$ such that $p_a(t) \geq \delta$ for all $a \in S$.

Obviously, $y_{0b}(t) \to 0$ for all $b \in B$. Now consider a $smart_j$ player of type k whose secondary preference ordering ranks some $b \in B$ as most preferred. Then, *either* $y_{jk}(t) \to 0$, *or* for t sufficiently large $b \notin R_j(t)$. In the latter case, the fate of these types depends on their preference ordering restricted to S, and they will have the same fate as other types with identical restricted preferences. Thus, without loss of generality, we may assume $S = A$ (or equivalently, reinterpret $\mathcal{P}$ as being restricted to S).

Convergence of p(t) implies that $\Delta p(t) \to 0$, and $\Delta p_a(t)$ consists of four components:

(i) $\Delta y_{0a}(t)$.

(ii) the $smart_j$ types whose secondary most preferred element in both $R_j(t)$ and $R_j(t+1)$ is a: $k \in P(a,R_j(t)) \cap P(a,R_j(t+1)) = P_j^0$.

(iii) the newcomers to strategy a - the $smart_j$ types whose secondary most preferred element in $R_j(t+1)$ is a, but whose most preferred element in $R_j(t)$ was not a: $k \in P(a,R_j(t+1)) \backslash P(a,R_j(t)) = P_j^+$.

(iv) the departers from strategy a - the $smart_j$ types whose secondary most preferred element in $R_j(t)$ was a, but whose most preferred element in $R_j(t+1)$ is not a: $k \in P(a,R_j(t)) \backslash P(a,R_j(t+1)) = P_j^-$.

Now we can write

$$(*) \qquad \Delta p_a(t) = \Delta y_{0a}(t) + \sum_{j \geq 1} \left[\sum_{k \in P_j^0} \Delta y_{jk}(t) + \sum_{k \in P_j^+} y_{jk}(t) - \sum_{k \in P_j^-} y_{jk}(t) \right].$$

The arrival and departures create a troublesome discontinuity in the dynamic process. In principal, the arrivals and departures could be balanced so $\Delta p(t) \to 0$, while y(t) does not converge.

Let $R_j^* = \liminf R_j(t)$. Since the best-response correspondence is upper hemi-continuous R_j^* is non-empty. Further, the nesting of the R_j sets is inherited by the R_j^* sets. Now suppose there is a t^* such that for all $t \geq t^*$, $R_j(t) = R_j^*$ for all j, in which case, P_j^+ and P_j^- are empty and $P_j^0 = P(a,R_j^*)$. Then, since the set of state variables $\{y_{0a}, y_{jk}$ for $k \in P(a,R_j^*)$ and $j < n^*+1\}$ all have identical growth rates, from (*), convergence of p(t) implies that

$\Delta y_{0a}(t) \to 0$ for all a and $\Delta y_{jk}(t) \to$ for all $k \in P(a,R_j^*)$ and $j < n^*+1$. Further, the growth rates of these state variables equal the growth rate of $p_a(t)$, and since $p_a(t)$ converges, so must this set of state variables. Moreover, since $\cup_a P(a,R_j^*) = \mathcal{P}$, all the state variables converge. Q.E.D.

Remark. To see the difficulty in proving a more general result, suppose instead that $\exists\ \hat{a} \in S\backslash R_1^*$ such that $\hat{a} \in R_1(t)$ infinitely often (i.o.). Consider a period t+1 when $\hat{a}$ becomes first-level rationalizable: $\hat{a} \in R_1(t+1)\backslash R_1(t)$. Note that we have $\hat{a} \notin R_j(t)$ for all $j \geq 1$. Then, letting $a = \hat{a}$ in (*), we have for all $j \geq 1$ that $P_j^0 = P_j^- = \emptyset$ and

$$\Delta p_{\hat{a}}(t) = \Delta y_{0\hat{a}}(t) + \sum_{j\geq 1} \sum_{k\in P_j^+} y_{jk}(t) .$$

Since $\Delta p_{\hat{a}}(t) \to 0$, the right-hand side must vanish. Unfortunately, we cannot conlude that $\Delta y_{0\hat{a}}(t)$ and $y_{jk}(t) \to 0$ for all $k \in P_j^+$, because it is conceiveable that $\Delta y_{0\hat{a}}(t)$ is negative just enough to offset the positive newcomers.

Proof of Proposition 5.

Given a strict NE p*, let a* = supp(p*). Strictness implies $\exists\ \delta > 0$ such that for all $a \neq a^*$, $(e_a - e_{a^*})\cdot\pi e_{a^*} \leq -\delta$ for all $a \neq a^*$. Further, there is an $\varepsilon > 0$ such that $\|p-p^*\| < \varepsilon$ implies that $\beta(p) = \{a^*\}$. For any n and y, by construction, $p(y) \in Q_n$. Therefore, $\|p(y)-p^*\| < \varepsilon$ implies $\{a^*\} = \beta(p(y)) \subseteq R_n$.

Consider $Q_1 = y_0 + (1-s_0)\mathcal{M}(A)$. For any $a \neq a^*$, $\min\{q_a \in Q_1\} = y_{0a} < \varepsilon$, $\min\{q_{a^*}\} = y_{0a^*}$, and $\max\{q_{a^*}\} = 1 - \Sigma_{b\neq a^*} y_b > 1-\varepsilon$. In other words, Q_1 is interior to but arbitrarily close to the simplex of A truncated by $q_{a^*} \geq y_{0a^*}$. Further, y_{0a^*} is growing while, for all $a \neq a^*$, y_{0a} is decreasing. Thus, Q_1 is shifting monotonically in the direction of a*. It follows that $R_1(t)$ is non-increasing as long as $\|p-p^*\| < \varepsilon$. Hence, all discontinuities in $R_1(t)$ must be implosions, and since $a^* \in R_1(t)$, all such discontinuities will generate positive jumps in p_{a^*} [referring to (*) of the previous proof], and negative jumps in p_a for the eliminated strategy. Thus, the dynamics of the dumb players and the $smart_1$ players push p(t) towards p*.

Next, consider $Q_2 = y_0 + s_1\mu_1 + (1-s_0-s_1)\mathcal{M}(R_1)$. For any $a \neq a^*$, $\min\{q_a \in Q_2\} = y_{0a} + \Sigma_{k\in P(a,R_1)}\, y_{1k} < \varepsilon$, $\min\{q_{a^*}\} = y_{0a^*} + \Sigma_{k\in P(a^*,R_1)}\, y_{1k}$, and $\max\{q_{a^*}\} > 1-\varepsilon$. In other words, Q_2 is interior to but arbitarily close to the simplex of A truncated by $\min\{q_{a^*}\}$. Further, the latter quantity is growing while, for all $a \neq a^*$, $\min\{q_{0a}\}$ is decreasing. Thus, Q_2 is shifting monotonically in the direction of a*. It follows that $R_2(t)$ is non-increasing as long as $\|p-p^*\| < \varepsilon$. Hence, all discontinuities in $R_2(t)$ must be implosions, and since $a^* \in R_2(t)$, all such discontinuities will generate positive jumps in p_{a^*}, and negative jumps in p_a for the eliminated strategy. Thus, the dynamics of the $smart_2$ players also push p(t) towards p*.

Iterating this argument for all n, the $R_n(t)$ sets are non-increasing in t, so for all $a \neq a^*$, $\Delta p_a(t)/p_a(t) < -\delta$. Q.E.D.

Remark: The proof of Proposition 5 can be adapted to prove that if a* is a "robust" NE (i.e. a* is the unique perfect best response to itself, Okada, 1983), then a* is asymptotically stable.

Proof of Proposition 6.

Given $p(t) \to p^*$, since the best-response correspondence is upper hemi-continuous, $\exists\, a^* \in \beta(p^*)\cap\beta(p(t)) \subseteq R_j(t)$ for all $j \geq 1$ and t sufficiently large. Therefore, all player types who rank a* highest, do not die out and grow at the same strictly positive rate as y_{0a*}. Therefore, $\exists\ \delta > 0$ such that $s_0(t) \geq y_{0a*}(t) \geq \delta$ for t sufficiently large. For $n \geq 1$, let k* denote the secondary preference type that ranks a* highest. Then, similarly, $\exists\ \delta > 0$ such that $s_n(t) \geq y_{0k*}(t) \geq \delta$ for t sufficiently large. Q.E.D.

Proof of Proposition 7.

(a) Suppose $s_0(t) \to 0$. Then, observe that, for sufficiently large t, $R_1(t)$ differs from the first-level BP rationalizable set only by deleting all strategies (if any) that are never perfect best-responses to some $g \in \mathcal{M}(A)$; call this set PBP_1. For each $a \in PBP_1$ and each $k \in P(a,PBP_1)$, y_{1k} and y_{0a} have the same growth rate; hence, together with Proposition 1, $s_0(t) \to 0$ implies $s_1(t) \to 0$.

Next, for sufficiently large t, $R_2(t)$ differs from the second-level BP rationalizable set only by deleting all strategies (if any) that are never perfect best-responses to some $g \in \mathcal{M}(PBP_1)$; call this set PBP_2. For each $a \in PBP_2$ and each $k \in P(a,PBP_2)$, y_{2k} and y_{0a} have the same growth rate; hence, together with Proposition 1, $s_0(t) \to 0$ implies $s_1(t) \to 0$. Therefore, by induction, $s_0(t) \to 0$ implies $s_j(t) \to 0$ for all $j \geq 1$.

(b) Given $n* < \infty$ and $\Sigma_{j\geq 0}\ s_j(t) = 1$ for all t, we cannot have $s_0(t) \to 0$, so there must be a $\delta > 0$ such that $s_0(t) > \delta$ infinitely often.

REFERENCES

Banerjee, A. and Weibull, J.W., (1991), "Evolutionary Selection and Rational Behavior," Princeton and Stockholm, mimeo.

Bernheim, B.D. (1984), "Rationalizable Strategic Behavior," Econometrica, 52, 1007-1028.

Binmore, K., (1987), "Modeling Rational Players, I and II," Economics and Philosophy, 3 (179-214) and 4 (9-55).

Blume, L. and Easley, D., (1991), "Economic Natural Selection and Adaptive Behavior," mimeo.

Damme, E. van, (1987), Stability and Perfection of Nash Equilibrium, chp 9, Springer-Verlag.

Friedman, D., (1991), "Evolutionary Games in Economics," Econometrica, 59, 637-666.

Fudenberg, D., and Maskin, E., (1986), "The Folk Theorem in Repeated Games with Discounting and Incomplete Information," Econometrica, 54, 533-554.

Hofbauer, J. and Sigmund, K., (1988), The Theory of Evolution and Dynamical Systems, Cambrige.

Kreps, D., Milgrom, P., Roberts, J., and Wilson, R., (1982) "Rational Cooperation in Finitely Repeated Prisoners' Dilemma," J. of Econ. Theory, 27, 245-252.

LaSalle, J.P., (1986), The Stability and Control of Discrete Processes, Springer-Verlag.

Okada, A., (1983), "Robustness of Equilibrium Points in Strategic Games," Tokyo Center for Game Theory, Department of Information Sciences, Tokyo Institute of Technology, Japan.

Pearce, D., (1984), "Rationalizable Strategic Behavior and the Problem of Perfection," Econometrica, 52, 1029-1050.

Samuelson, L. (1988), "Evolutionary Foundations of Solution Concepts for Finite, Two-Player Normal-Form Games," in Vardi, M.Y. (ed) Theoretical Aspects of Reasoning About Knowledge, Morgan Kaufmann.

________, and Zhang, J., (1991), "Evolutionary Stability in Asymmetric Games," CentER Discussion Paper No. 9132, Tilburg University, The Netherlands.

Selten, R., (1991), "Evolution, Learning, and Economic Behavior," Games and Economic Behavior, 3 3-24.

Tan, T., and Werlang, S.R.C., (1988), "The Bayesian Foundations of Solution Concepts of Games," J. of Econ. Theory, 45, 370-391.

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