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EXISTENCE OF NASH EQUILIBRIUM IN MIXED STRATEGIES FOR GAMES WHERE PAYOFFS NEED NOT BE CONTINUOUS IN PURE STRATEGIES*

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1. INTRODUCTION

The purpose here is simply stated: It is to weaken the requirement for existence of Nash equilibrium in mixed strategies that payoff functions be continuous in pure strategy vectors. (This is required, for example, by Glicksburg, 1952.) The weaker requirement adopted here is that each player's payoff is upper semicontinuous in the vector of all pure strategy choices, and continuous in the vector of all other players' strategies, for each own choice. All pure strategy sets are taken to be compact as usual. A minor point is that these strategy sets are taken to be subsets of metric spaces rather than just of Hausdorff spaces, as in Glicksburg.

The following related result should be noted. Dasgupta and Maskin (1986, p. 4, Theorem 2) discuss the existence of Nash equilibrium in *pure* strategies under weaker continuity assumptions than those here. (It is easy to see that requiring payoffs to be continuous in the vector of other players' pure choices implies "graph continuity".) However, they also require payoffs to be quasiconcave in own pure strategies, which is not required here. Suppose one were to attempt to apply the Dasgupta and Maskin result to mixed strategies as in the present context. It is trivial that quasiconcavity in own strategy obtains when mixed strategies are permitted. The difficulty is to show that the weakened continuity properties in pure strategies carry over to precisely analogous statements for mixed strategies. This is essentially the contribution of the present note, although the proof relies directly on Glicksburg (1952).

2. ECONOMIC MOTIVATION

The intention here is to show that it easy to construct economic models in which the weakened continuity requirement is all that can be guaranteed.

Consider indeed Cournot duopoly, given in normal form as

$$G = ((Q_i, \pi_i) \mid i=1, 2)$$

where Q_i, i=1, 2 are compact subsets of R^+ and

$$\pi_i : Q \to R, \text{ for } i = 1, 2, \text{ given } Q = Q_1 \times Q_2$$

are continuous functions. Define sets of mixed strategies as usual by

$$M_i = \{\text{probability measures on Borel subsets of } Q_i\}$$

and expected profits as, say,

$$\tau_i : M_1 \times M_2 \to R, \quad \tau_i(m_1, m_2) = \int \pi_i(q_1, q_2) dm_1(q_1) dm_2(q_2), \quad \text{for } m_i \in M_i, i = 1, 2.$$

It follows from Glicksburg (1952) that a Nash equilibrium in mixed strategies, defined in the usual way, must exist. (Novshek, 1980, however, asserts that a Nash equilibrium in pure strategies, a Cournot equilibrium, that is, need not exist in these circumstances.)

Consider now the following Stackelberg version of this duopoly. Suppose that firm 1 is the leader, moving before firm 2. Firm 2 moves next given knowledge of 1's choice. This game of perfect information has a "subgame perfect equilibrium" in pure strategies. (See Selten, 1975, for the definition of a subgame perfect equilibrium, or SPE, in a finite game. Hellwig, *et al*, 1989, extend this to "continuous action" games.) Indeed the force of subgame perfectness is that 2's choice of quantity, $f_2(q_1)$, say, given any previous choice by firm 1 of q_1 satisfy

$$f_2(q_1) \in R_2(q_1) = \{q_2 | \ \pi_2(q_1, q_2) \geq \pi_2(q_1, q_2') \text{ for all } q_2' \in Q_2\}, \text{ for all } q_1 \in Q_1,$$

where R_2 is nonempty and upper hemicontinuous given that Q_1 and Q_2 are compact and π_2 is continuous. (See Berge, 1963, p. 116.) If 2 breaks ties in 1's favor, so

$$f_2(q_1) \in \arg \max_{q_2 \in R_2(q_1)} \pi_1(q_1, q_2)$$

then firm 1's maximization problem must have a solution. (Hellwig, *et al*, 1989, discuss this issue of tie–breaking for general games with perfect information.) This completes the specification of a Stackelberg equilibrium in pure strategies.

It seems natural to consider the following generalization of both the Cournot simultaneous move game and the Stackelberg sequential move game. Suppose that, prior to either firm's choice, nature chooses state "0", with probability $1-\epsilon$, and state "1" with probability ϵ. Firm 1 next chooses q_1 in ignorance of the state of nature. In state "0", firm 2 then chooses in ignorance of 1's choice, as in the Cournot game. In state "1", however, firm 2 is informed of 1's choice, as in the Stackelberg version of the game. The question is then whether there exists an SPE of this extended game.

Suppose that firm 2 again breaks ties in favor of firm 1 in state "1". Indeed, suppose that firm 2's strategy in state "1" is still denoted by f_2, as in the Stackelberg model and that the best reply correspondence R_2 is also as defined above. It follows that 1's payoff in state "1",

$$\pi_1^L(q_1) \equiv \pi_1(q_1,f_2(q_1)) = \max_{q_2 \in R_2(q_1)} \pi_1(q_1, q_2)$$

say, is upper semicontinuous in q_1. (See Berge, p. 116, Theorem 2.) In addition, 2's payoff in state "1",

$$\pi_2^F(q_1) \equiv \pi_2(q_1,f_2(q_1)) = \max_{q_2 \in Q_2} \pi_2(q_1,q_2),$$

say, is continuous in q_1. (See Berge, 1963, p. 116, "Maximum Theorem".) Hence the overall expected payoffs can then be expressed as, say,

$$\Pi_1(q_1,q_2) = \epsilon\pi_1^L(q_1) + (1-\epsilon)\pi_1(q_1,q_2) \text{ and } \Pi_2(q_1,q_2) = \epsilon\pi_2^F(q_1) + (1-\epsilon)\pi_2(q_1,q_2),$$

where Π_1 is upper semicontinuous in (q_1,q_2) and continuous in q_2, and where Π_2 is continuous in (q_1,q_2). It is then sufficient for existence of an SPE of the original extended game that there exist a Nash equilibrium for this "reduced form" game. Given counterexamples to existence of Cournot equilibrium in pure strategies, it follows that pure strategies will not suffice generally for existence here either. Existence of a Nash equilibrium in *mixed* strategies for the "reduced form" game is ensured by the result of the next section. (Robson, 1990, also makes use of the present result to develop a refinement of Nash equilibrium for two–person nonzero–sum games. This is based on slight uncertainty concerning the order of moves.)

3. THE RESULT

The following equivalent definitions are recalled. (See Berge, 1963, pp. 74–77.)

Definition 1: Upper Semicontinuity

Consider a function

$$U: S \to R$$

where S is a metric space. U is "upper semicontinuous", or "u.s.c." for short, iff any of the following equivalent conditions hold

(a) If $s^n \to s$, $n \to \infty$, then $\forall \delta > 0 \; \exists N \ni n > N$ implies $U(s^n) < U(s) + \delta$

(b) If $s^n \to s$, $n \to \infty$, then $\limsup_n U(s^n) \leq U(s)$.

(c) The upper contour sets $\{s \in S \,|\, U(s) \geq \bar{U}\}$ are *closed* for all $\bar{U} \in R$.

The following class of games is considered.

Definition 2: Game, G

An N–person nonzero sum game is given as

$$G = ((S_i, U_i) \,|\, i = 1,\ldots,N)$$

where S_i are compact metric spaces, and the U_i are functions

$$U_i: S \to R^+, \; i=1,\ldots,N, \text{ where } S = S_1 \times \ldots \times S_N,$$

such that

(i) U_i is u.s.c. on S, i=1,...,N. This implies that each U_i is bounded above (see Berge, p. 76). For convenience, each U_i is assumed nonnegative.

(ii) U_i is continuous in s_{-i}, for all s_i, where s_{-i} is the vector $(s_1,\ldots,s_N)$ with s_i deleted, $i = 1,\ldots,N$.

The following strategy sets are considered.

Definition 3: Mixed Strategies

A mixed strategy m_i for player i is a probability measure on the set of Borel sets of S_i. The set of all such probability measures is denoted M_i, i=1,...,N. The payoff to player i is, say,

$$V_i(m_1,\ldots,m_N) \equiv \int U_i dm, \text{ where } m \equiv m_1 \times \ldots \times m_N,$$

is the unique product measure induced on S by $m_1,...,m_N$. Each U_i is measurable since it is u.s.c., and since it is also bounded, it is integrable.

The sets of mixed strategies are given the following topology.

Definition 4: Weak Convergence

Suppose $\{m^n\}_{n=1}^{\infty}$ and m belong to M, the set of probability measures on the Borel sets of some compact metric space S. Then "m^n weakly converges to m", written

$$m^n \xrightarrow{w} m \text{ iff } \int f dm^n \longrightarrow \int f dm$$

for all real–valued continuous functions f on S. (See Billingsley, 1968, p. 7.) This topology is consistent with the "Prohorov" metric (Billingsley, pp. 237–238.)

The following three preliminary results are needed to prove the theorem.

Lemma 1: Limit of Product is Product of Limits

Suppose that

$$m_i^n \xrightarrow{w} m_i, \quad n \to \infty$$

where m_i^n, $m_i \in M_i$ for the compact metric spaces S_i, i=1,...,N. It follows that

$$m^n = m_1^n \times ... \times m_N^n \xrightarrow{w} m_1 \times ... \times m_N = m$$

where m^n, and m are the unique product probability measures on the Borel sets of the compact metric space S induced by m_i^n and m_i, i = 1,...,N.

PROOF Consider the firstly the case N = 2. Suppose then

$$f: S_1 \times S_2 \longrightarrow R$$

is any continuous function, which is then clearly bounded and integrable. It must be shown that

$$\int f dm^n \longrightarrow \int f dm$$

Using Fubini's Theorem (Bartle, 1966, pp. 119–120) this is equivalent to showing that

$$\int \left(\int f dm_1^n\right) dm_2^n \longrightarrow \int \left(\int f dm_1\right) dm_2$$

as $n \to \infty$. Define then

$$F^n(s_2) = \int f(s_1,s_2) dm_1^n(s_1) \quad \text{and} \quad F(s_2) = \int f(s_1,s_2) dm_1(s_1)$$

which are continuous functions, given that f is continuous and bounded, using the Lebesgue Dominated Convergence Theorem (Bartle, p. 44). By the weak convergence of m_1^n,

$$F^n(s_2) \longrightarrow F(s_2) \; \forall s_2 \in S_2.$$

Furthermore, $\{F^n\}$ is *equicontinuous*. Indeed, since f is continuous on the compact $S_1 \times S_2$,

$$\forall \delta > 0 \; \exists \; \epsilon > 0 \ni d_2(s_2',s_2) < \epsilon \Rightarrow |f(s_1,s_2') - f(s_1,s_2)| < \delta, \forall \, s_1 \epsilon S_1, \forall \, s_2, s_2' \epsilon S_2$$

where d_2 is the metric on S_2. Hence

$$|F^n(s_2') - F^n(s_2)| \leq \int |f(s_1,s_2') - f(s_1,s_2)| dm_1^n(s_1) < \delta \text{ for all n.}$$

Clearly the $\{F^n\}$ are also pointwise bounded. It follows that

$$F^n(s_2) \longrightarrow F(s_2)$$

uniformly on S_2 (see Rudin, 1964, p. 158). Hence $\forall \delta > 0 \; \exists \; N_1 \ni n > N_1$ implies

$$|F^n(s_2) - F(s_2)| < \frac{\delta}{2}, \quad \forall \, s_2 \in S_2$$

which implies, in turn,

$$\left|\int F^n(s_2) dm_2^n(s_2) - \int F(s_2) dm_2^n(s_2)\right| < \frac{\delta}{2}$$

In addition, weak convergence of m_2^n to m_2 implies:

$$\exists N_2 \ni n > N_2 \text{ implies}$$

$$\left|\int F(s_2) dm_2^n(s_2) - \int F(s_2) dm_2(s_2)\right| < \frac{\delta}{2}$$

Hence $n > \text{Max}\{N_1, N_2\}$ implies the result for $N = 2$:

$$\left|\int F^n dm_2^n - \int F dm_2\right| \leq \left|\int F^n dm_2^n - \int F dm_2^n\right| + \left|\int F dm_2^n - \int F dm_2\right| < \delta.$$

The result for general N now follows by induction, using the associativity of the product operation for measures. (See Friedman, 1982, p. 87.)

Note: A stronger version of the above result for $N = 2$ can be found in Parthasarathy (1967, p. 57, Lemma 1.1). The proof, however, is less direct than that given here.

Lemma 2: Inheriting U.S.C.

Consider the u.s.c. function

$$U: S \longrightarrow R^+$$

where S is a compact metric space. Suppose $\{m^n\}_{n=1}^{\infty}$, $m \in M$, the set of probability measures on Borel sets of S, and $m^n \xrightarrow{w} m$. It follows that $\int U dm$ is u.s.c. in m:

$$\limsup_n \int U dm^n \leq \int U dm.$$

Proof Billingsley (1968, pp. 12–13, Theorem 2.1) proves the equivalence of five conditions characterizing weak convergence. Part of his proof yields the required result, although this is not explicit. For completeness, the proof is as follows.

Without loss of generality, it is assumed that

$$0 < U(s) < 1 \quad \forall s \in S.$$

For fixed $k \in N$, define

$$F_i = \{s \mid U(s) \geq \tfrac{i}{k}\} \quad i=0,\ldots,k$$

which are closed precisely because U is u.s.c. (Billingsley assumes U is continuous and uses the following argument on both U and $-U$.)

All the F_i are measurable sets, U is a measurable function, and it follows that

$$\sum_{i=1}^{k} \frac{i-1}{k} m\{s \mid \tfrac{i-1}{k} \leq U(s) < \tfrac{i}{k}\} \leq \int U dm \leq \sum_{i=1}^{k} \frac{i}{k} m\{s \mid \tfrac{i-1}{k} \leq U(s) < \tfrac{i}{k}\}$$

Thus

$$\sum_{i=1}^{k} \frac{i-1}{k}[m(F_{i-1}) - m(F_i)] \leq \int Udm < \sum_{i=1}^{k} \frac{i}{k}[m(F_{i-1}) - m(F_i)].$$

The two sums can be simplified to yield

$$\frac{1}{k}\sum_{i=1}^{k} m(F_i) \leq \int Udm < \frac{1}{k} + \frac{1}{k}\sum_{i=1}^{k} m(F_i).$$

Billingsley shows that

$$m^n \xrightarrow{w} m \text{ iff } \lim\sup_n m^n(F) \leq m(F) \text{ for all closed sets } F.$$

Thus applying the upper bound above for the integral over m^n, and the lower bound for the integral over m,

$$\lim\sup_n \int Udm^n \leq \frac{1}{k} + \frac{1}{k}\sum_{i=1}^{k} m(F_i) \leq \frac{1}{k} + \int Udm$$

and, since k is arbitrary,

$$\lim\sup_n \int Udm^n \leq \int Udm \qquad \text{Q.E.D.}$$

Lemma 3: Inheriting Continuity

Suppose that U_i and V_i, 1,...,N, are as in Definitions 1, 2 and 3. Then $V_i(m_i,m_{-i})$, where m_{-i} denotes the vector $(m_1,\ldots,m_n)$ with m_i deleted, is continuous in m_{-i} in that:

$$m^n_{-i} \xrightarrow{w} m_{-i} \text{ implies } V_i(m_i,m^n_{-i}) \rightarrow V_i(m_i,m_{-i}),\ \forall m_i \epsilon M_i,\ \forall m_{-i},\ m^n_{-i} \in M_{-i}$$

where M_{-i} is the Cartesian product of $M_1,\ldots,M_N$ with M_i omitted and where weak convergence is as in Definition 4.

PROOF Define

$$v_i(m_i,s_{-i}) = \int U_i(s_i,s_{-i})dm_i(s_i)$$

This is a continuous function of s_{-i}, for any $m_i \in M_i$, given $U_i(s_i,s_{-i})$ is continuous in s_{-i} and bounded, by the Lebesgue Dominated Convergence Theorem. Hence Fubini's Theorem implies

$$V_i(m_i, m^n_{-i}) = \int v_i(m_i,s_{-i})d\{\prod_{j\neq i} m^n_j(s_j)\} \longrightarrow V_i(m_i, m_{-i})$$

as $m^n_{-i} \xrightarrow{w} m_{-i}$, using Lemma 1.

Finally, a Nash equilibrium is defined in the obvious fashion:

Definition 5: Nash Equilibrium

A Nash equilibrium in mixed strategies for the game described in Definitions 1, 2, and 3 is

$$(m^*_1,\ldots,m^*_N) \in M_1 \times\ldots\times M_N$$

such that

$$V_i(m^*_i, m^*_{-i}) \geq V_i(m_i, m^*_{-i}) \;\; \forall m_i \in M_i, \text{ for } i = 1,\ldots,N.$$

The main result is then

Theorem 1: Existence of NE

Any game G, as in Definitions 1, 2, and 3 has a Nash equilibrium as in Definition 5.

PROOF Define the best–reply correspondences

$$R_i\colon\; M_{-i} \longrightarrow M_i$$

$$R_i(m_{-i}) = \{m_i \in M_i \,|\, V_i(m_i, m_{-i}) \geq V_i(m_i', m_{-i}) \;\; \forall m_i' \in M_i\}, \, i = 1,\ldots,N$$

In order to apply the generalization of Kakutani's theorem due to Glicksburg (1952, p. 171, "Theorem"), note the following:

(i) $C(S_i)$, the continuous functions on the compact metric space, S_i, is a Banach space.

(ii) $C(S_i)^*$, the linear functionals on $C(S_i)$ is a convex Hausdorff linear topological space.

(iii) M_i, the probability measures on Borel sets of S_i, is a convex compact subset of $C(S_i)^*$.

Note that (i), (ii) and (iii) are as in Glicksburg. He refers to the "ω^* topology" that is, the "weak* topology" but this is identical to the "topology of weak convergence" as defined by Billingsley. (1968, p. 236.)

(iv) Each R_i is convex valued, given the use of mixed strategies and expected payoffs. Thus, for example, if $t \in (0,1)$ $m_i, m_i' \in R_i(m_{-i})$ then

$$tm_i + (1-t)m_i' \in R_i(m_{-i}).$$

Also, R_i is nonempty. This follows immediately from Lemma 2, since u.s.c.functions attain a maximum on a compact set. (See, for example, Berge, p. 76, Theorem 2.)

(v) The heart of the matter is to show that each R_i has a *closed* graph. That is, suppose $m_{-i}^n \xrightarrow{w} m_{-i}$, $m_i^n \in R_i(m_{-i}^n) \xrightarrow{w} m_i$. (Since M_i are metrizable, i=1,2, they are first countable and the usual sequences characterize the topology. See Dugundji, 1966, pp. 186–187, pp. 217–218. There is a more general characterization in Glicksburg.) It must then be shown that $m_i \in R_i(m_{-i})$. Suppose not. Then there exists $m_i' \in M_i$ such that

$$V_i(m_i', m_{-i}) - V_i(m_i, m_{-i}) = 3\Delta > 0.$$

Now Lemmas 1 and 2 imply that

$$m^n = m_i^n \times m_{-i}^n \xrightarrow{w} m_i \times m_{-i} = m$$

and there exists an N_1 such that $n > N_1$ implies

$$V_i(m_i^n, m_{-i}^n) < V_i(m_i, m_{-i}) + \Delta$$

Also, Lemma 3 implies there is an N_2 such that

$$n > N_2 \text{ implies } |V_i(m_i', m_{-i}^n) - V_i(m_i', m_{-i})| < \Delta.$$

Hence $n > \text{Max}\{N_1, N_2\}$ implies

$$V_i(m_i', m_{-i}^n) - V_i(m_i^n, m_{-i}^n) =$$

$$[V_i(m_i', m_{-i}^n) - V_i(m_i', m_{-i})] + [V_i(m_i', m_{-i}) - V_i(m_i, m_{-i})] + [V_i(m_i, m_{-i}) - V_i(m_i^n, m_{-i}^n)]$$

$$> -\Delta + 3\Delta - \Delta = \Delta > 0$$

This contradicts

$$m_i^n \in R_i(m_{-i}^n). \qquad \text{Q.E.D.}$$

Remark It might be that the continuity assumptions here can be further relaxed. Note, indeed, that Berge (1963, pp. 210–211) presents a result due to Sion of an existence theorem for a two–person zero sum game for which each player's payoff is u.s.c. (and quasiconcave) in his own choice and l.s.c. (and quasiconvex) in the other. The present result imposes stronger conditions on V_1 and V_2 considered as functions of (m_1,m_2), given that $V_2 = -V_1$; indeed they must then be continuous in (m_1,m_2).

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