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ABSTRACT

Aaron (1966) compared the capital reserve (CR) and the pay-as-you-go (PAYG) social insurance systems in a special model with two overlapping generations and many cohorts. He found that PAYG yields higher welfare than does CR if and only if the output growth rate is greater than the interest rate. The paper reconsiders this result by relaxing several homogeneity assumptions and deriving optimal quasi-stationary paths from a homogeneous utility function (a specific homothetic function). A further specification with constant relative risk aversion or equivalently, with constant elasticity of substitution shows that "everything goes". In particular, PAYG may be preferred to CR even if the output growth rate is smaller (rather than greater) than the interest rate for youthful (mature) profiles with high (low) risk aversion. This revision calls into question the conventional explanation of the recent crisis of PAYG Social Security system.

1. INTRODUCTION[1]

Since the publication of the seminal paper by Samuelson (1958) economists, dealing with social insurance, have devoted much attention to the comparison of the **pay-as-you-go** (PAYG) system to the **capital reserve** (CR) system. (At the end of the paper a list of abbreviations can be found.) In an influential quasi-stationary model (where all growth rates are constant), assuming two overlapping generations and many cohorts Aaron (1966) found that PAYG is better than CR (in a sense to be defined below) if and only if the output growth rate is greater than the interest rate: Aaron's condition. (A modern treatment is to be found in Blanchard and Fisher (1989, Section 3.2) under the heading of dynamic inefficiency.) This result is widely used to explain the recent crisis of PAYG Social Security system: the growth rates are sinking while the (real) interest rate is rising, reversing the earlier order of PAYG and CR systems (e.g. Verbon (1988) and Peters (1991)).

The present paper reconsiders Aaron's study and its follow-up. For the record we list the following intercohort **homogeneity** assumptions used by Aaron: (D_0) there is no death risk, (K_0) there are no child (student) cohorts, (W_0)-(C_0) cohort wage and consumption profiles are flat. We

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1 This research was financed by the Dutch National Science Foundation (NWO), the Center for Economic Research at Tilburg University and was complemented by the financial support of the Hungarian Science Foundation (OTKA). In April 1992 Eduardo Siandra and I started to work on two joint papers, but in March 1993 we agreed to divide the papers between us and write two one-author papers. As a result, this paper owes much to E. Siandra. I express my intellectual debt to M. Augusztinovics and thanks to E. Dierker, F. X. Hof, P. Kop Jansen, B. Martos, Gy. Molnár, W. Peters, H. Verbon and an anonymous referee for their comments on earlier versions. None of them is responsible for the content of the paper.

shall replace them with assumptions allowing for intercohort **heterogeneity:** (D) there are death risks, (K) child (student) cohorts either have their own consumption or the corresponding consumption is included in the parents' consumption, (W)-(C) general wage and consumption profiles. We retain, however, a basic assumption of Aaron, namely, the profiles are time-invariant. (With a slight abuse of terminology under a cohort consumption or wage we mean per capita rather than total consumption or wage of the given cohort. Profile and path refer to cross-sectional and longitudinal description, respectively.)

Our assumptions are quite customary in certain segments of life cycle modelling. Critically developing Modigliani and Brumberg (1954), already Tobin (1967) introduced a similar framework. In addition to the seemingly technical assumptions D, W and C, he initiated the striking assumption K.

"The Modigliani-Brumberg life cycle model does not allow for children. Presumably, their consumption is assumed to be squeezed from parents' consumption. In this case, however, it is implausible to assume that parents spread their consumption evenly over adult life, independently of the number of mouth to feed. A mechanical amendment to the model, which continues to treat individuals as individuals and ignore their grouping into households, would simply tack the childhood years without income on the beginning of the life cycle. Each person would then be consuming from birth in anticipation of his future adult earnings. He would accumulate debt during his childhood years and he would have to use his earnings to pay back the debt as well as provide for his second childhood" (p. 247).

In his illustrative calculations Tobin introduced households as well, but we stay with his "mechanical amendment". Using Tobin's framework, Arthur and McNicoll (1978, p. 242) criticized Samuelson (1975) for analyzing life cycle problems without taking into account child-dependency.[2] Augusztinovics (1989), (1992) also applied this framework in her synthesis.

Overlooking this approach, two-generation, two-cohort models with scalar consumption still dominate the field of pension systems. We quote an economist, who analyzed excellently the connection between child dependency and pension systems in another paper, but he claimed the following: "For a theoretical analysis it is sufficient to model an overlapping generations framework with two generations only, the old and the young." The usual explanation is as follows: "Balasko et al. (1980) present a simple procedure for redefining periods and generations that converts a model in which consumers live for any finite number of periods into one where they live for only two", (Peters 1991, p. 158 and ftn. 1)[3]. Unfortunately, many users of this theorem forget about the second part of the theorem: in the redefined model the number of the goods increases in proportion to the original number of cohorts (Reichlin (1992) also makes this point).

At this point we make a semantic remark: We prefer the neutral word **cohort** to the more colorful expression **generation**. In fact, we shall consider many (up to a hundred) age groups living together, while in the original meaning of the word only three or four generations may overlap.

Having exposed the role of assumption K, we turn to the issue of assumption C and optimization. We assume that

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2 In his last sentence Samuelson (1975, p. 337) at least acknowledged the existence of the problem, but added: "childhood dependency is intrinsically less costly relative to old-age dependency".

3 It should be noted that in footnote 15 below his statement Peters (1991, p. 169) also remarks that "a more realistic analysis should include an OLG framework which deals with more than two generations..." (cf. Auerbach and Kotlikoff, 1987).

the members of the society have common tastes, and our welfare criterion will be the lifetime utility of a representative agent. A utility function is maximized under CR and PAYG budget constraints one after the other. We say that **PAYG is better than CR** (w.r.t. the utility function) if the first maximum is greater than the second.

Note that Aaron assumed rather than derived assumption C_0 from any optimization framework. However, it can be shown that his result can be deduced from the constrained maximization of the Leontief-Rawls utility function. (This is not the case with the very general model of Verbon (1988, Appendix 7A), where the assumption of common contribution rate for PAYG and CR excludes optimization.) Other researchers applied Cobb-Douglas utility functions (e.g. Verbon, 1988) or **ACRRA** (Additive Constant Relative Risk Aversion) utility functions (Blachard and Fisher, 1989) or general ones (Arthur and McNicoll, 1978).

Unfortunately, the use of general homothetic utility functions are undermined by the presence of death risks. Thus in this paper first we shall consider **homogeneous** utility functions (special homothetic functions), then specify them as ACRRA. Homogeneous functions are not only more general than ACRRA functions, but do not require parametrization. Note that the family ACRRA, i.e. CES[4] contains both the Leontief-Rawls and the Cobb-Douglas utility functions.

In an optimization framework Aaron's condition is transformed by the indirect utility function: the value of the function at the output growth factor is greater than the corresponding value at the interest factor. (Factor is equal to rate plus unity.) Since this function is generally not increasing for all positive values, there may be

4 Obviously an ACRRA function is a non-linear but monotone transformation of a CES (Constant Elasticity of Substitution) function. We shall use both terminologies.

intervals where PAYG is worse than CR though Aaron's condition holds. In contrast, in the two-cohort scalar model the indirect utility function is increasing, independently of the utility function.[5]

In addition to the assumptions D, K, W, C and the macroparameters, the ranking generally also depends on the chosen utility function. The coefficient of relative risk aversion, or equivalently, the elasticity plays a decisive role.[6] For example, for utility functions yielding C_0, the indirect utility functions, as functions of the interest factor, are locally concave and convex functions around the growth rate for lower and higher elasticities, respectively.

The paper is closely related to Augusztinovics (1992). Our frequent references to her work only serve as acknowledgment of priority but its prior knowledge is not requested. In a related paper Siandra (1993), considering three cohorts, no death risks and homothetic utility functions, obtained similar results.

Table 1 displays the different sets of assumptions made by foregoing authors. We only underline the most important three dimensions: (i) realism, (ii) generality of the utility function and (iii) globality of the analysis. The present paper combines the advantages of all the three directions: a rather general utility function is analyzed globally in a realistic framework.

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5 The situation is quite similar to that arising during the capital controversy when neoclassical economists overlooked complications (like reswitching) appearing when vintages in capital are distinguished.

6 This fact contradicts the ambivalence of numerous writers (e.g. Tobin, Arthur and McNicoll and Augusztinovics) as to optimization.

Table 1

Finally, we remind the reader to an implicit assumption: the growth rates of population and productivity, the real interest rate and the activity rate are not only time-invariant but independent of the ruling social insurance system, too. We also confined the analysis to the (unique) quasi-stationary paths, neglecting stability problems studied by Gale (1973) and others. A couple of researchers do not accept this approach (referred to as the case of a small open economy), but we have found it a useful first approximation.[7]

The structure of the paper is as follows. Section 2 presents the model of optimal social insurance systems. In Sections 3 and 4 PAYG and CR are compared for homogeneous utility functions, and the more special ACRRA utility functions, respectively. Sections 5 and 6 contain special local and global results, respectively. Section 7 draws the conclusions. Proofs are relegated to the Appendix. Sacrificing simplicity, we have tried to obtain sharp results, and display their complex relations to the literature. Following Tobin, we aim at numerical and graphical illustrations as well.

2. OPTIMAL SOCIAL INSURANCE SYSTEMS

In this section we shall consider an optimization model of the social insurance systems.

Generations and cohorts

Following Aaron (1966), Gale (1973), Arthur and McNicoll (1978), Augusztinovics (1989) and (1992), we shall consider

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7 For a combination of classical growth theory and overlapping cohorts models, see Auerbach and Kotlikoff (1987), Blanchard and Fisher (1989) and Peters (1991).

Table 1. Models and assumptions

MODELS	Samuelson	Aaron	Tobin, Arthur McNicoll	Verbon	Auguszti-novics	Siandra, Simonovits
ASSUMPTIONS						
Number of generations	2	2	3	2	3	2 3
Number of cohorts	0/1/2 0/2/3	0/R/D	L/R/D	0/1/2	L/R/D	0/2/3 L/R/D
Death risk	-	-	+	+-	+	- +
Time-variant factors	-	-	-	+	+	-
Productivity increase	-	+	-	+	+	+
Non-flat wage profile	x	-	+	x	+	+
Utility function	G	LR	CD G	CD	LR CD	HT, CES HG, CES
Global analysis	+	+	-	+	+	+

Remarks. Signs + and - mean that the foregoing assumption or its negation is valid, respectively. Sign x refers to a case where the assumption is either automatically satisfied or **not used.** Concerning utility functions, G, HT, HG, LR and CD refer to general, homothetic, homogeneous, Leontief-Rawls, Cobb-Douglas functions, respectively. In the last column the upper signs refer to Siandra (1993), while the lower ones to the present paper.

a model with overlapping **cohorts** rather than generations. A multi-cohort model is a much better description of the economy and is capable to check the validity of results obtained for overlapping generation models.

At time t the population consists of three **age-groups**[8]: children, workers and pensioners. Each age-group consists of several cohorts, L child cohorts, R-L working cohorts and D-R pensioner cohorts, altogether D cohorts numbered as $k=0,1,2,...,D-1$.[9] To reproduce traditional childless populations, we shall not exclude $L=0$. Since we concentrate on pension systems, we shall assume that at least one working and one pensioner cohorts exist: $R\geq 1$, $D\geq 2$. Suppose that in time t B_t "babies" are born, and $N_{k,t+k}=s_kB_t$ persons of them survive age k: $1\geq s_0\geq s_1 ... \geq s_{D-1}>s_D=0$[10]. (The survival probabilities are time-invariant.) Total population N_t is given by

$$N_t=\Sigma_{0\leq k<D}s_kB_{t-k}.$$

We assume that the growth factor of the number of newborns is time-invariant and is equal to b: $B_t=bB_{t-1}$.[11]

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8 Following Augusztinovics (1992), we shall not identify generations with childhood, working age and retirement. For a technical definition, see Section 6.

9 We assume that every member of every working cohort works and no member of any child and pensioner cohort does so. Arthur and McNicoll (1978) and Lee (1980) introduce age-specific participation rates, but in the context of pensions it would raise other questions.

10 In a related model, Blanchard and Fisher (1989, Section 3.3) assumes $D=\infty$ and $s_k=(1-\pi)\pi^k$, $k=0,1,2,...$, $0<\pi<1$. Although life expectancy is finite, $(1/\pi)$, the assumption of unbounded random length of life is quite unrealistic.

11 Of course, in a time-variant demographic system working with cohorts rather than generations, a birth-law $B_t=b_tB_{t-1}$ should be replaced by $B_t=\Sigma_{M\leq k\leq K}\sigma_{k,t}s_{k,t}B_{t-k}$, where σ_k's are the age specific birth coefficients. However, in our time-invariant system, b can be uniquely determined from $1=\Sigma_{M\leq k\leq K}\sigma_k s_k b^{-k}$.

Then the growth factor of the population[12] is also equal to b and

(2.1) $N_t=B_t\Sigma_{0\le k<D}p_k$ where $p_k=s_kb^{-k}$.

Here p_k shows the weight of cohort k in the population, and we shall refer to $\{p_k\}$ as **P profile.**

Having defined the demographic relations, we turn to the economic relations. We assume that the wage share in output is time-invariant. Then per capita (real) average wage w_t grows parallel with per-capita output (productivity), their joint time-invariant growth factor is denoted by g: $w_t=gw_{t-1}$.[13]

It is well-known that wages significantly differ across cohorts (Mincer, 1974). We assume that their profile is time-invariant, i.e.

(2.2) $w_{k,t}=gw_{k,t-1}$, $k=L,..,R-1$.

For convenience, we shall sometimes use the convention that children and pensioners earn zero "wages":

$w_{k,t}=0$, $k=0,..,L-1$ and $k=R,...,D-1$.

We shall denote the per capita (real) **consumption** of cohort m born in time t (in time t+m) by $c_{m,t+m}$.

Utility function

To derive and rank optimal consumption paths for different social insurance systems (represented by different budget constraints), we need a **utility function.** To be precise, we shall denote the variables of this function as $c_0,..,c_{D-1}$,

12 Considering a two-cohort time-variant system, Verbon (1988, p. 58) identifies population growth factor n_t and generation growth factor b_t. Even if we neglect children and cohorts, $n_t=b_{t-1}(b_t+1)/(b_{t-1}+1)$ generally differs from b_t and b_{t-1}. Of course, in a time-invariant system n=b.

13 The neglect of productivity growth (e.g. Samuelson, 1975) might have a strange consequence: it reduces Aaron's condition to "population growth rate is greater than real interest rate", which is much less likely to hold than the original one.

dropping the calendar time. Since the length of the life of our representative agent is a random variable (n), we have to introduce his **conditional utility** $U_n(c_0,...,c_{n-1})$. Then the **expected** (Neumann-Morgenstern) utility function is

$$U(c_0,..,c_{D-1})=\Sigma_{0\le n<D}\pi_n U_n(c_0,...,c_{n-1})$$

where $\pi_n=s_n-s_{n-1}$ is the probability of death at age n.

We assume that all U_n's are **locally insatiable** and strictly concave, thus U is also locally insatiable and strictly concave. To obtain homotheticity of U, we have to assume more than homotheticity of U_n's; namely **homogeneity:** there exist constants σ ($-\infty<\sigma<0$ or $0<\sigma\le 1$) and $\epsilon_0,...,\epsilon_{D-1}>0$ such that for any $\beta>0$ either

(2.3a) $U_n(\beta c_0,..,\beta c_{D-1})=\beta^\sigma U_n(c_0,..,c_{D-1})$, $n=1,...,D$,

or

(2.3b) $U_n(\beta c_0,..,\beta c_{D-1})=U_n(c_0,..,c_{D-1})+\epsilon_n\log(\beta)$, $n=1,...,D$.

Remark. In Section 4 we shall see why we separated the zero-homogeneity case and added the logarithms in (2.3b).

By (2.3), the expected utility function satisfies

(2.4) $$U(\beta c_0,..,\beta c_{D-1})=\beta^\sigma U(c_0,..,c_{D-1})+\Sigma_{0\le n<D}\pi_n\epsilon_n\log(\beta).$$

Hence U is homothetic.

The objective function of a person born in period t is

$$U(c_{0,t},..,c_{D-1,t+D-1})=\Sigma_{0\le n<D}\pi_n U_n(c_{0,t},...,c_{n-1,t+n-1}).$$

CR system (r)

We first consider a capital-reserve (CR) system. Heroically, we assume a perfect annuity market, namely where any baby can sell his stream of future income to an insurance company which pays him a (possibly non-homogeneous) income stream, while he is alive. At the end of his life his expected total net wealth, his **bequest** will be zero.[14] Although a baby cannot write an insurance

14 Blanchard and Fisher (1989, Section 3.1) demonstrate that in a steady state with positive bequest the interest rate is equal to the modified golden rate.

contract, we assume here that his parents manage his assets until he becomes independent (cf. Tobin (1967), Blanchard and Fischer (1989) and Augusztinovics (1992) .

Considering a cohort born in t, we have the following budget constraint (indexed by r):

(2.5r) $N_{0,t}\Sigma_{0\le m<D}s_m r^{-m}c_{m,t+m}\le N_{0,t}\Sigma_{L\le k<R}s_k r^{-k}w_{k,t+k}$.

In fact, the L.H.S. and the R.H.S. of the inequality represent the expected present values of the lifetime consumption and earning taken in time t, respectively.

We shall call a consumption path **optimal CR path** if it maximizes the expected utility function under CR budget constraint (2.5r). Observe that because of concavity, there is a unique optimum, and equality holds in (2.5r).

Taking into account (2.2), (2.5r) can be rewritten as

(2.6r) $\Sigma_{0\le m<D}s_m r^{-m}c_{m,t+m}=g^t\Sigma_{L\le k<R}s_k r^{-k}w_{k,k}$.

Since the utility function is homothetic, the optimal CR consumption profile is time-invariant, or equivalently, the age-controlled consumption vector increases at a rate g-1:

(2.7r) $c_{m,t+m}^{(r)}=gc_{m,t-1+m}^{(r)}$.

Using (2.7r), we shall mostly confine the analysis to t=0 and introduce the notation

(2.8r) $c_m(r)=c_{m,m}^{(r)}$.

Then

(2.9r) $c_{m,t+m}^{(r)}=g^t c_m(r)$.

PAYG system (h)

The study of the pay-as-you-go (PAYG) system is more involved. We have the following **intercohort** (cross-sectional) budget constraint (indexed by h):

(2.5h) $N_{0,t}\Sigma_{0\le m<D}s_m b^{-m}c_{m,t}\le N_{0,t}\Sigma_{L\le k<R}s_k b^{-k}w_{k,t}$.

To replace the cross-sectional social constraint by a longitudinal (intertemporal) individual one, we assume that the consumption profile is time-invariant, or equivalently:

(2.7h) $c_{m,t+m}^{(h)}=gc_{m,t-1+m}^{(h)}$.

Note that while (2.7r) is a result, (2.7h) is an assumption. Furthermore, while (2.7r) refers to an optimum, (2.7h) applies to any feasible path to be considered.

By taking $w_{k,t}=g^{t-k}w_{k,k}$ for $k=L,..,R-1$, $c_{m,t}=g^{-m}c_{m,t+m}$ [(2.7h)] for $m=0,1,...,D-1$ and substituting the newly gained expressions into (2.5h), we obtain

$\Sigma_{0\le m<D}s_m b^{-m}g^{-m}c_{m,t+m}\le\Sigma_{L\le k<R}s_k b^{-k}g^{t-k}w_{k,k}$.

Introduce the **output growth factor**

(2.10) $h=bg$

and simplify the budget constraint:

(2.6h) $\Sigma_{0\le m<D}s_m h^{-m}c_{t+m,m}=g^t\Sigma_{L\le k<R}s_k h^{-k}w_{k,k}$.

The **optimal PAYG path** maximizes the expected utility function under PAYG budget constraint (2.6h). Note that the PAYG optimum is obtained from the CR optimum by replacing r by h. Hence

(2.9h) $c_{m,t+m}^{(h)}=g^t c_m(h)$.

3. COMPARISON OF PAYG AND CR

In this section we start the comparison of the two optimal social insurance systems.

Welfare ranking

As usual, we shall evaluate each insurance system through the **indirect utility function** defined as the value of the utility function at the optimum. Because of homotheticity, we can confine our attention to t=0.

(3.1) $A(x)=U(x,c_0(x),...,c_{D-1}(x))$.

Accordingly, we say that **PAYG is better than CR** with respect to the utility function U if

(3.2) $A(h)>A(r)$.

Aaron's generalized proposition

As was already remarked above, in his pioneering paper Aaron (1966) had not derived the optimal PAYG and CR consumption path from a utility function. Nevertheless, we can attribute him an appropriate utility function and show (see Theorem 5 below) that the corresponding $A(x)$ is an increasing function on the entire interval $(0,\infty)$. Thus (3.2) is equivalent to the so-called **Aaron condition:**

(3.3) $h>r$.

In the remaining part of the paper we shall analyze some conditions under which (3.2) holds or (3.2) and (3.3) are equivalent.

First we give two formulations of Aaron's proposition:

Theorem 1 (Aaron's strong generalized proposition). If the indirect utility function A(x) is increasing on the interval (0,∞), then PAYG is better than CR if and only if h>r holds.

Remarks. 1. This conclusion appears in the literature for the celebrated but somewhat abused two-cohort scalar case (L=0, R=1, D=2). In fact, now the budget sets (2.6r) and (2.6h) collapse to

$$s_0 c_{0,t}^{(x)} + s_1 x^{-1} c_{1,t+1}^{(x)} = g^t s_0 w_{0,0}, \quad x=r,h.$$

It is obvious that the PAYG budget line is higher than the CR budget line if $h>r$. Assuming that $c_{1,t+1}^{(x)}>0$, $A(h)>A(r)$. However, inserting a second worker cohort or a child cohort, the simplicity disappears.

2. In practice we calculate a consumption path from an underlying profile with the help of the growth factor g. The same applies to the demographic parameters $s_k=p_k b^k$. Therefore we often fix b and g, hence h, and vary r. This leads to

Theorem 2 (Aaron's weak generalized proposition). Suppose h is fixed and the indirect utility function satisfies

(3.4) $A(r)<A(h)$ if $r<h$ and $A(r)>A(h)$ if $r>h$.

Then PAYG is better than CR if and only if h>r holds.

Remark. Note that for fixed h, it is conceivable that Aaron's proposition holds but $A(x)$ is only locally increasing around $r=h$. Figure 1 illustrates Theorems 1 and

2: the increasing A curve represents the former; the non-monotone curve, crossing line A=A(h)=1 at the trivial root r=h, corresponds to the latter. (Computer simulation shows that there is no crossing in the interval (0,0.95), which is not shown in the Figure). Note that the non-monotone A curve, also crossing A=1 at a non-trivial root, displays a case where neither the strong nor the weak version holds. (For reasons to be discussed below, transformations of A and r are displayed in Figure 1.)

Figure 1

Youthful and mature profiles

We shall refer to r=h as **the golden root.** In fact, in this case not only the two optimal solutions but also the budget sets coincide. Note that the optimal golden CR path (profile) is the PAYG path (profile).

To clarify the behavior of A(x) at least in the neighborhood of h, we shall introduce Augusztinovics' youthful and mature profiles (cf Gale's Samuelson and classical cases, respectively). First we shall define the **mean age of earning** as average age of workers weighted by the cohorts' shares[15] and wages:

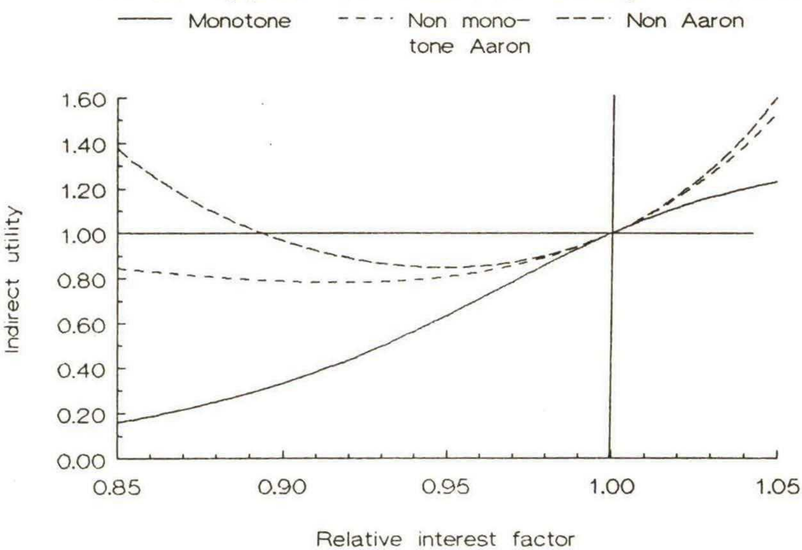
$$(3.5) \qquad \Omega=\frac{\Sigma_{L\le k<R}p_k w_{k,0}k}{\Sigma_{L\le k<R}p_k w_{k,0}}.$$

Now define the **mean age of consuming** as weighted average age of consumers:

$$(3.6) \qquad \Gamma=\frac{\Sigma_{0\le k<D}p_k c_{k,0}k}{\Sigma_{0\le k<D}p_k c_{k,0}}.$$

15 In Augusztinovics (1992) there are no cohort shares, because she neglects individual (per capita) consumption and wages.

Figure. 1
Three types of indirect utility functions



Next we shall call **a PCW profile either youthful or mature or symmetric** if the mean age of earning is higher or lower than or equal to the mean age of consuming, respectively:

(3.7) either $\Omega>\Gamma$ or $\Omega<\Gamma$ or $\Omega=\Gamma$.

Local validity

We shall analyze the validity of Aaron's proposition **locally**, around r=h. We shall prove

Theorem 3 (Local version of Aaron's proposition, cf. Arthur and McNicoll (1978)). Assume that the expected utility function U is differentiable and the optimum is an interior point and r≈h. a) For mature PAYG profiles PAYG is better than CR iff h>r. b) For youthful PAYG profiles PAYG is better than CR iff h<r. c). For symmetric PAYG profiles, if A(r) is concave at h, then PAYG is locally better than CR; if A(r) is convex at h, then PAYG is locally worse than CR.

Remarks. 1. The following observations (cf Augusztinovics, 1992, Proposition 10 and Siandra 1993, Propositions 3 and 4) may illuminate Theorem 3: If a PCW profile, which is optimal under PAYG, is mature, then the expected bequest for any person is positive when h<r and is negative when h>r. Similarly, if a PCW profile, which is optimal under CR, is mature, then the expected social saving for any period is negative when h<r and is positive when h>r. For youthful profiles, both statements are reversed.

2. In practice, profiles are youthful (cf Arthur and McNicoll.) Even an aging population may yield a youthful PAYG profile.

3. Although the Neumann-Morgenstern utility function is determined up to affine transformations, at least in the special case of no-death-risk, the monotone transformations of $U=U_{D-1}$ become legal. Unlike monotonicity, convexity is

not invariant to monotone transformation T. However, for $A'(1)=0$, $[TA]''(1)=T'(1)A''(1)$, i.e. $\mathrm{sgn}[TA]''(1)=\mathrm{sgn}A''(1)$.

Iso-utility roots

Obviously, comparing PAYG and CR, it is very important to know at what interest factors they yield the same indirect utility. By definition, this happens if and only if

(3.8) $A(r)=A(h)$.

Since we are not interested in the trivial root $r=h$, we exclude it, unless it has a multiplicity 2: $A'(h)=0$. Under this restriction, the roots of equation (3.8) will be referred to as **iso-utility roots**.

We have no results on the number and location of iso-utility roots for general utility functions. Let us denote the iso-utility roots closest to h from the left or from the right by r_L and r_R, respectively. If r_L or r_R do(es) not exist, then write $r_L=0$ or $r_R=\infty$, respectively.

Combining Theorems 2 and 3 we obtain

Theorem 4 (Iso-utility roots). The local results of Theorem 3 can be extended to the interval $r_L<r<r_R$.

4. ACRRA UTILITY FUNCTIONS: EXPOSITION

Having finished the comparison of PAYG and CR for homogeneous utility functions, we turn to the analysis of a special class. In this way we can sharpen the more general results obtained in the previous section.

Constant relative risk aversion

Additive constant relative risk aversion (ACRRA) utility functions are still quite general and they play an outstanding role in the analysis of life cycle problems. Let σ be a real number, $-\infty\le\sigma<1$, $1-\sigma$ is referred to as the **coefficient of relative risk aversion.** We shall need two series of weights in the utility function. Let $\Phi_0,\Phi_1,\ldots,\Phi_{D-1}$ and $\tau_0,\tau_1,\ldots,\tau_{D-1}$ be positive numbers. Call their double series an **FT path**. Then the conditional utility of a consumption path $c_0,..,c_{n-1}$ is given by

(4.1a) $U_n(c_0,...,c_{n-1})=\Sigma_{0\le m<n}\Phi_m\sigma^{-1}(\tau_m^{-1}c_m)^\sigma$ if $\sigma=0,-\infty$;

(4.1b) $U_n(c_0,...,c_{n-1})=\Sigma_{0\le m<n}\Phi_m\log(c_m)$ if $\sigma=0$.[16]

Note that in (4.1a) $\Phi_m\tau_m^{-\sigma}$ shows the relative weight of utility due to c_m. Note that in (4.1b) Φ_m's are **cumulated discount factors**, if $\Phi_{m+1}\le\Phi_m$ for m=0,...,D-2.

Now we can also define the conditional **no-risk** utility function:

(4.1c) $U_n(c_0,..,c_{n-1})=\min_{0\le m<n}(\tau_m^{-1}c_m)$ if $\sigma=-\infty$.[17]

Remark. Note that (4.1a) and (4.1c) satisfy (2.3a); (4.1b) satisfies (2.3b) with $\epsilon_n=\Sigma_{0\le m<n}\Phi_m$. Hence ACRRA utility functions with joint coefficients are homogeneous. The example of $U_2(c_0,c_1)=c_0+c_1+\min(c_0,c_1)$ shows that there are homogeneous function beyond the ACRRA family.

Observe that formulas (4.1) can be simplified to

(4.2a) $U(c_0..,c_{D-1})=\Sigma_{0\le m<D}s_m\Phi_m\sigma^{-1}(\tau_m^{-1}c_m)^\sigma$ if $\sigma\neq0$, $\sigma\neq-\infty$;

(4.2b) $U(c_0,.,c_{D-1})=\Sigma_{0\le m<D}s_m\Phi_m\log(c_m)$ if $\sigma=0$;

(4.2c) $U(c_0,.,c_{D-1})=\min_{0\le m<D}(\tau_m^{-1}c_m)$ if $\sigma=-\infty$.

We can normalize and unify the three branches of (4.2) as follows [CES utility]:

(4.3) $\Sigma_{0\le m<D}s_m\Phi_m=1$, $Z=U^{1/\sigma}$, i.e. $G=A^{1/\sigma}$

(4.4) $Z(c_0,..,c_{D-1})=[\Sigma_{0\le m<D}s_m\Phi_m(\tau_m^{-1}c_m)^\sigma]^{1/\sigma}$.

As is known, the R.H.S. of (4.4) is not defined for either $\sigma=0$ or $-\infty$, but both limits exist and yield (4.2b) (in fact, less $\Sigma_{0\le m<D}s_m\Phi_m\log(\tau_m)$) and (4.2c), respectively.

We shall frequently use the following transformation of σ:

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16 According to Peters (1987), this is the unique utility function which would be consistent with our framework, if the labor participation rate were also endogeneous and time were continuous.

17 Note that this utility function is not strictly concave but has a unique maximum. Similarly, it is not differentiable, but the corresponding A(x) is smooth.

$$\mu=\frac{\sigma}{\sigma-1} \qquad (-\infty\leq\sigma<1)$$

where $1-\mu$ is **the intertemporal elasticity of substitution.**

Explicit formulas

Here we shall display optimal paths and indirect utility functions for ACRRA functions. To do so we need the following notations:

(4.5) $\delta_m=\Phi_m^{1-\mu}\tau_m^{\mu}$,

(4.6) $W(x)=\Sigma_{L\leq k<R}s_k w_{k,k}x^{-k}$,

(4.7) $C(x)=\Sigma_{0\leq k<D}s_k\delta_k x^{-\mu k}$,

(4.8) $H(x)=\frac{W(x)}{C(x)}$.

Lemma 1. For utility function (4.4) the optimal consumption paths are

(4.9) $c_{m,m}{}^{(x)}=\delta_m x^{(1-\mu)m}H(x)$, $x=r,h$

and the indirect utility function belonging to (4.4) is

(4.10) $G(x)=W(x)C(x)^{-1/\mu}$.

Remark. Although the ACRRA utility functions are concave for the entire parameter interval $\sigma\epsilon(-\infty,1)$, we shall drop the interval $(0,1)$, i.e. we shall exclude $\mu<0$ and maintain δ_m as a logarithmic convex combination of Φ_m and τ_m.

Path versus profile

To simplify exposition, we shall replace intertemporal **paths** with cross-sectional **profiles** in (4.6)-(4.7):

(4.11) $W(x)=\Sigma_{L\leq k<R}s_k b^{-k}w_{k,o}b^k g^k x^{-k}$,

(4.12) $C(x)=\Sigma_{0\leq k<D}s_k(\Phi_k{}^{1-\mu}b^{-\mu k})(\tau_k g^{-k})^{\mu}(b^k g^k x^{-k})^{\mu}$.

To simplify these expressions, in addition to the P profile, we shall introduce the **FT profile**, its **mixture profile**

(4.13) $\Phi_{k,o}=\Phi_k b^k$, $\tau_{k,o}=\tau_k g^{-k}$, $\delta_{k,o}=\Phi_{k,o}{}^{1-\mu}\tau_{k,o}{}^{\mu}$

and **the relative interest factor**

(4.14) $u=\frac{r}{h}$.

Now (4.11)-(4.12) can be rewritten as

(4.15) $W^o(u)=\Sigma_{L\le k<R}p_k w_{k,0}u^{-k}$,

(4.16) $C^o(u)=\Sigma_{0\le k<D}p_k\delta_{k,0}u^{-\mu k}$.

We shall normalize the W, F and T profiles by $\Sigma_{L\le k<R}p_k w_{k,0}=1$, $\Sigma_{0\le k<D}p_k\Phi_{k,o}=1$ and $\Sigma_{0\le k<D}p_k\tau_{k,o}=1$, i.e. $W^o(1)=1$ and $C_\mu{}^o(1)=1$, $\mu=1$ and 0. Introducing $H^o(u)=W^o(u)/C^o(u)$, $H_\mu{}^o(1)=1$ for $\mu=1$ and 0.

Here are the optimal consumption profiles:

(4.17) $c_{m,0}{}^{(u)}=\delta_{m,o}u^{(1-\mu)m}H^o(u)$.

Using profiles, $G^o(u)=W^o(u)C^o(u)^{-1/\mu}$ and the relation 'PAYG is better than CR' can be restated as

(4.18) $G^o(u)<G^o(1)$.

Special cases

We record the optima of two distinguished special utility functions.

a) **Leontief-Rawls**, $\mu=1$:

(4.19) $c_{m,0}{}^{(u)}=\tau_{m,o}H^o(u)$ and $G_1(u)=H(u)$.

b) **Cobb-Douglas**, $\mu=0$, then we can use the fact that the limit of the power means is equal to the geometrical mean, when the exponent converges to zero:

(4.20) $c_{m,0}{}^{(u)}=\Phi_{m,o}u^m H^o(u)$, $G_0(u)=[\pi_{0\le k<D}\tau_k{}^{\Phi k}]\ W(u)u^{\Gamma o}$

where

$\Gamma_0=\Sigma_{0\le m<D}s_m\Phi_m m=\Sigma_{0\le m<D}p_m\Phi_{m,o}m$.

Remark. It can be checked that the optimal solution under $\mu=1$ is the limit of the optima under $\mu_p<1$, $\lim_p\mu_p=1$. The same applies to $\mu=0$.[18]

---

18 Augusztinovics (1992) refers to the solutions occurring in cases a) and b) as the direct and the indirect approach, respectively. Note that the profile is independent of u in the direct approach. At this point we also underline that her notion of path has a special meaning: in our notations it is $\{c_{m,0}u^{-m}\}$, which is independent of u in the indirect approach.

As a benchmark, we shall consider very special profiles studied by Aaron. Under a **flat SCW profile** we mean if cohort survival probability, consumption and wage profiles are flat:

(4.21) $s_k=1$, $k=0,\dots,D-1$,

(4.22) $c_{k,o}=c_{0,o}$, $k=1,\dots,D-1$,

(4.23) $w_{k,o}=w_{0,L}$, $k=L+1,\dots,R-1$.

We shall speak of a **flat FT profile** if

(4.24) $\Phi_{k,o}=\Phi_{0,o}$, $k=1,\dots,D-1$,

(4.25) $\tau_{k,o}=\tau_{0,o}$, $k=1,\dots,D-1$,

and of a **flat SFTW profile** if the FT and SW profiles are flat, i.e. (4.21), (4.23)-(4.25) hold. Note that (4.24)-(4.25) imply $\delta_{k,o}=\delta_{0,o}$ for $k=1,\dots,D-1$, regardless of μ.

By (4.17), a flat FT profile implies a flat C profile for any u if $\mu=1$ or for any μ if u=1. Of course, a flat FTW profile results in a flat CW profile under the same conditions.

First we present a basic result for flat SFTW profiles.

Theorem 5 (Aaron's original proposition reformulated). For every childless flat SFTW profile and $\mu=1$ PAYG is better than CR if and only if Aaron's condition h>r holds, or equivalently

(4.26) $u<1$.

Remarks. 1. Aaron (1966) proved an equivalent statement without explicit optimization: For every childless flat SCW profile (3.2) and (3.3) are equivalent.

2. We shall prove a generalization of this Theorem (cf Theorem 7) below.

5. LOCAL ANALYSIS FOR ACRRA

We return to the local analysis started in Section 3, but now confining the attention to special utility functions ACRRA.

Mean age of consuming

According to Theorem 3 local behavior is determined by the difference between mean ages of consuming and of earning. Since the latter is given, we shall study the former, namely, the mean age of consuming at the golden profile $\{c_{m,0}^{(1)}\}$:

$$\Gamma_\mu = \frac{\Sigma_{0\le k<D} p_k \delta_{k,o} k}{\Sigma_{0\le k<D} p_k \delta_{k,o}}.$$

Note that our definition still depends on the partial elasticity of substitution, at least for non-flat FT profiles, and on the P profile.

As an illustration, we shall consider flat PCW profiles, or equivalently, **deterministic and stationary** (b=1) flat PAYG profiles. Evidently

(5.1) $\Omega=(L+R-1)/2$,

(5.2) $\Gamma=(D-1)/2$.

A flat PAYG profile is mature if and only if the retirement period is longer than the childhood period:

$D-R>L$.

Remark. (5.1)-(5.2) highlight a deficiency of two- or three-cohort models: the mean ages of consumption and earning are too low. For example, for D=2 periods (say 60 years) $\Gamma=1/2$ period (15 years), although the correct result would be 1 period (30 years)! This distortion is weaker in multi-cohort models and disappears in Arthur and McNicoll's continuous-time model.

Symmetric elasticity

If Γ_μ depends on μ, we want this dependence to be monotone. First of all we need

Lemma 2. Let α_k, β_k, x_k be non-negative real numbers, $k=0,\dots,D-1$ and let

(5.3) $\{\beta_k/\alpha_k\}$ **and** $\{x_k\}$ **be non-decreasing series.**

Then

(5.4) $$\frac{\Sigma_k\alpha_k x_k}{\Sigma_k\alpha_k}\le\frac{\Sigma_k\beta_k x_k}{\Sigma_k\beta_k}.$$

Remark. If neither $\{\alpha_k/\beta_k\}$ nor $\{x_k\}$ is constant, then strict inequality holds in (5.4).

We shall speak of a **non-decreasing FT profile** if

(5.5) $$\frac{\Phi_{m,o}}{\tau_{m,o}}\le\frac{\Phi_{m+1,o}}{\tau_{m+1,o}},\quad m=0,1,\ldots,D-2.$$

Remarks. 1. Flat FT profiles are non-decreasing.

2. Condition (5.5) can be replaced by a stronger and simpler one:

(5.5′) $\tau_{m+1}\le g\tau_m$ and $\Phi_{m+1}\ge\Phi_m/b$, $m=0,\ldots,D-2$.

Indeed, (4.13) and (5.5′) imply $\tau_{m+1,o}\le\tau_{m,o}$ and $\Phi_{m+1,o}\ge\Phi_{m,o}$, which in turn imply (5.5).

Now we can present

Lemma 3. If the FT profile is non-decreasing [(5.5)], then Γ_μ is a non-increasing function of μ.

For non-decreasing FT profiles with $\Gamma_1<\Omega<\Gamma_0$ we can define the **symmetric elasticity** $1-\mu_o$ by

(5.6) $\Gamma_{\mu o}=\Omega$.

In fact, Γ_μ is a continuous function of μ, thus by Bolzano-theorem, μ_o exists if $\Gamma_1<\Omega<\Gamma_0$. Furthermore, $\mu_o=0-0$ if $\Gamma_0<\Omega$, and $\mu_o=1+0$ if $\Gamma_1>\Omega$.

Combining Theorem 1 and Lemma 3, we obtain

Theorem 6 (Symmetric elasticity). Consider a non-decreasing FT profile. a) For $\mu<\mu_O$ PAYG is locally better than CR iff h>r. b) For $\mu>\mu_O$ PAYG is locally better than CR iff h<r.

Example 1. The simplest non-flat FT profile arises for a flat FT path: $s_k=1$, $\Phi_k=\Phi_0$ and $\tau_k=\tau_0$. Note that $\Phi_k/\tau_k=h\Phi_{k-1}/\tau_{k-1}$, i.e. if the economy grows (h>1), then the FT profile is non-decreasing and (by Lemma 3,) Γ_μ decreases.

As a numerical illustration, let us consider L=15, R=55, D=75, and b=1, h=g=1.03. Numerical calculations yield

μ_o=0.14. Figure 2 displays three curves for μ=0.28; 0.14 and 0. We shall see later that $G_{0.14}$ is a **separatrix**, and curves below and above it resemble G_1 and G_0, respectively.

Figure 2

Critical elasticity[19]

Until now we have struggled with the complications arising from the dependence of Γ on μ. However, for flat PAYG consumption profiles Γ_μ is independent of μ. Thus we can speak of **youthful, mature and symmetric PFTW profiles**, if the FT profile is flat and the resulting PAYG profile is youthful, mature and symmetric, respectively. Note that in practice the C profile is much flatter than the P and W profiles, thus this assumption is acceptable.

Under normal conditions, for every flat FT profile and arbitrary PW profile there exists a real number μ^*, $0<\mu^*<1$, such that around the golden root (u=1) for $\mu>\mu^*$ $G_\mu{}^o$ resembles $G_1{}^o$ (both are concave) and for $\mu<\mu^*$ $G_\mu{}^o$ resembles $G_0{}^o$ (both are convex). We call 1-μ^* **critical elasticity**, and it is implicitly defined by

$G_{\mu^*}{}^{o''}(1)=0$.

Remark. Leaving the realm of symmetric profiles, where sign[TA]'(1)=signA'(1), the invariance to monotone transformations becomes problematic. With Lemma 5 (below) we shall demonstrate that $Z=U^{1/\sigma}$ is the natural transformation, hence G is the appropriate indirect utility function.

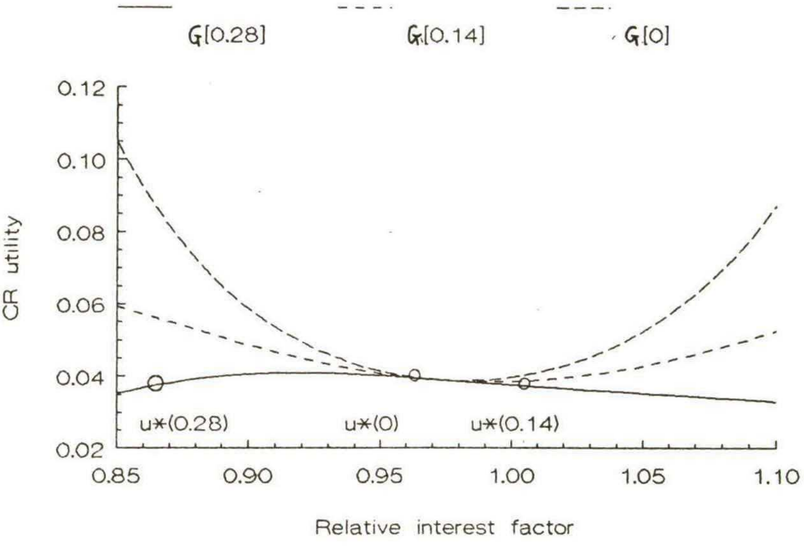
Before embarking on its analysis, we combine Theorem 3 c) and this concept in

Theorem 7 (Symmetric profiles). Consider a symmetric PFTW profile, where FT is flat. (i) If $\mu>\mu^*$, then PAYG is

19 This part may be skipped at first reading and taken up only at the end of the next Section.

Figure 2

Utility, elasticity: non-flat FT profile



locally better than CR; (ii) if $\mu<\mu^*$, then PAYG is locally worse than CR.

The value of the critical elasticity can be determined in terms of the first and second moments of the consumption and wage distributions. We shall need the notations

(5.7) $W_0=\Sigma_k p_k w_{k,0}=1, \quad W_1=\Sigma_k p_k w_{k,0}k, \quad W_2=\Sigma_k p_k w_{k,0}k^2,$

(5.8) $C_0=\delta\Sigma_k p_k=1, \quad C_1=\delta\Sigma_k p_k k, \quad W_2=\delta\Sigma_k p_k k^2.$

Lemma 4 (Critical elasticity). For any flat FT profile and any PW profile the critical elasticity $1-\mu^*$ (if exists) is given by

(5.9) $$\mu^*=12\frac{W_2+W_1-(2W_1-C_1+1)C_1}{D^2-1}.$$

Remarks. 1. For every flat PFTW profile (5.7)-(5.8) reduce to

(5.10) $$W_0=1, \quad W_1=\frac{L+R-1}{2}, \quad W_2=\frac{(R-1)R(2R-1)-(L-1)L(2L-1)}{6(R-L)},$$

(5.11) $$C_0=1, \quad C_1=\frac{D-1}{2}, \quad C_2=\frac{(D-1)(2D-1)}{6}.$$

2. Table 2 displays the dependence of critical elasticity on the triple (L,R,D) for flat PFTW profiles.

Table 2

3. Probably critical elasticity also exists for non-flat FT profiles without symmetric elasticities, but its definition seems to be difficult.

To illustrate the situation, in Figure 3 we shall consider a youthful flat PFTW profile and depict three indirect utility curves of low, critical and high elasticities, respectively. We assume that L=20, R=60 and D=73, where $\mu^*=0.336$. We shall choose $\mu_1=1$ and $\mu_2=0$.

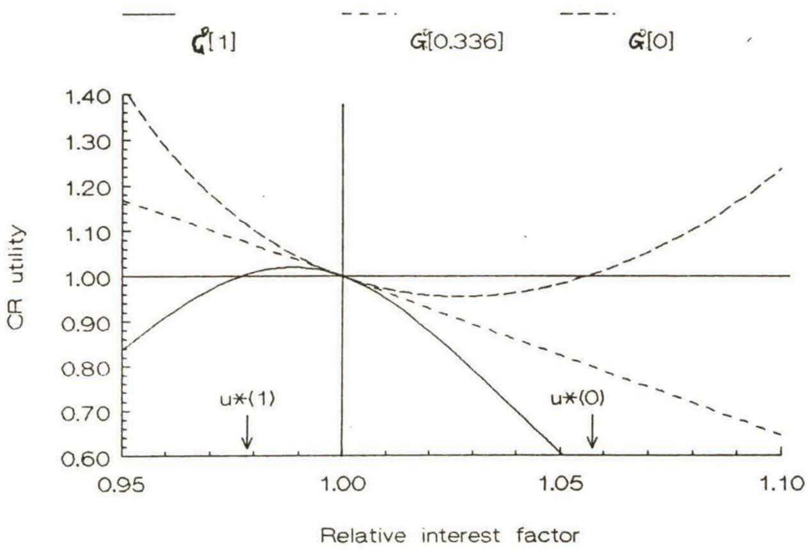
Figure 3

Table 2. Critical elasticities for flat PFTW profiles

Entering the labor force L	Leaving the labor force R	Death D	1-Critical elasticity μ^*
0	40	50	0.736
0	40	60	0.744
10	50	60	0.444
10	50	70	0.375
20	60	70	0.400
20	60	80	0.250

Figure 3

Utility, elasticity: flat FT profile



Continuous time

To avoid absurd results which arise with insufficient time-desaggregation (e.g. for 0/2/3), at this point we introduce **continuous time** modeling.

Let $x=k/D$, $\alpha=L/D$, $\beta=R/D$, $0<\alpha<1/2<\beta<1$, $f(x)=p(x)w_0(x)$ where D is large. Then (5.7)-(5.8) reduce to

(5.7′) $W_0=\int_\alpha^\beta f(x)dx=1, \quad W_1=\int_\alpha^\beta xf(x)dx, \quad W_2=\int_\alpha^\beta x^2f(x)dx$

(5.8′) $C_0=1$, $C_1=1/2$ and $C_2=1/3$.

Substituting (5.7′)-(5.8′) into (5.9), and dropping the negligible terms, we obtain

(5.9′) $\mu^*=12W_2-12W_1+3$.

It can be shown that μ^* lies between 0 and 1.

6. GLOBAL ANALYSIS FOR ACRRA

Having finished the local discussion for ACRRA utility functions, we turn to their global analysis.

To begin with, we reformulate the iso-utility root r^* as $u^*=r^*/h$, which satisfies

(6.1) $G^o(u^*)=G^o(1)$ if $u^*=/=1$ unless $G^{o\prime}(1)=0$.

Generations

To avoid complication in the global analysis, we shall introduce the notions of generations and further special profiles (Augusztinovics, 1992, Section 5).

A set of subsequent cohorts $k=K_1,\dots,K_2$ is called a **generation** if each of its cohorts either saves ($w_{k,t}>c_{k,t}$) or dissaves ($w_{k,t}\le c_{k,t}$) and both cohorts K_1-1 and K_2+1 (if exist at all) make the opposite than $K_1,\dots,K_2$ do.

A CW profile is said to have the **GBR property** if it has the following three generations: **1. gestation** (including children and junior workers), **2. breeder** (including senior workers) and **3. retirement** (including old part-time workers and pensioners).

A CW profile is said to have the **BR property** if it has the following two generations: 1. breeder and 2. retirement (no gestation).

Remark. It can be shown that a CW profile with the BR property always satisfies the so-called cash-constraint: the corresponding wealth is positive for any time except for death.

Leontief-Rawls utility function

To begin with, we shall study the simplest case, that of Leontief-Rawls utility function. Now $G_1^o(u)=H(u)$, i.e. (6.1) is equivalent to $H^o(u^*)=H^o(1)$, which implies **zero bequest** for PAYG systems. Augusztinovics (1992) calls the corresponding roots **singularity roots** and presents several propositions concerning them which we generalize below.

For $\mu=1$, using (4.19) and $H^o(1)=1$, GBR and BR properties can be transferred from PAYG profiles to TW profiles: A PAYG profile has the GBR property if there exist positive integers L^* and R^* ($L \le L^* < R^* \le R$) with the following conditions:

(6.2) $w_{k,0} < \tau_{k,o}$ for $k=0,\dots,L^*$ or $R^*,\dots,D-1$;

(6.3) $w_{k,0} > \tau_{k,o}$ for $k=L^*,\dots,R^*-1$.

A TW profile has the BR property if (6.1)-(6.2) hold for $L^*=0$.

Remarks. 1. In the definitions of GBR and BR, the P profile does not play any role.

2. It is evident that any flat TW profile has the GBR (or BR) property where $L^*=L(=0)$ and $R^*=R$.

3. The celebrated two-cohort case (0/1/2) has the BR property.

The example below will illustrate the meaning of BR profiles in the simplest non-trivial case:

Example 2 (BR TW-profiles in three-cohort models). $L=0$, $R=2$, $D=3$. The TW profile has the BR property if and only if $H^o(\infty)\ge 1$, i.e. $w_{0,0}/\tau_{0,o}\ge 1$.

If the T profile is flat, then $w_{0,0} \geq 1/3$, $w_{1,0}=1-w_{0,0}$. (Since in practice $w_{0,0} \leq w_{1,0}$, the condition is $1/3 \leq w_{0,0} \leq 1/2 \leq w_{1,0} \leq 2/3$.)

However, a childless flat T profile would assign zero consumption to children living with their parents. It is more realistic to assume that $\tau_{0,0}=\tau_{1,0}>\tau_{2,0}$, for example, $\tau_{0,0}=\tau_{1,0}=1.5\tau_{2,0}$, i.e. $\tau_{0,0}=\tau_{1,0}=0.375$ and $\tau_{2,0}=0.25$. Then $w_{0,0} \geq 0.375$ or in a practical setting, $0.375 \leq w_{0,0} \leq 0.5 \leq w_{1,0} \leq 0.625$.

If $w_{0,0}/\tau_{0,0}<1$, then we obtain a GBR TW-profile in a childless population, what Augusztinovics (1992, p. 45) calls trick of congruence.

Specifying Theorem 4 we have

Theorem 8. (cf. Augusztinovics, 1992, Section 5). Leontief-Rawls utility function: a) For every GBR TW-profile PAYG is better than CR if and only if one of the following three alternative conditions holds:

(6.4i) **either** $u<1$ **or** $u>u^*(>1)$ **for a mature profile,**

(6.4ii) **either** $u<u^*(<1)$ **or** $u>1$ **for a youthful profile,**

(6.4iii) $u \neq 1$ **for a symmetric profile.**

b) Every BR TW-profile is mature and (6.4i) holds with $u^*=\infty$.

Remarks. 1. Theorem 8b is a generalization of Theorem 5 from flat childless STW profiles to BR TW-profiles, which can be further extended to μ's close to 1.

2. In his multi-cohort model Verbon (1988, Appendix 7A) assumed **flat contribution rates** which were also independent of the type of social security system. In our notations: $c_{m,0}^{(u)}=(1-\theta)w_{m,0}$, $m=L,\ldots,R-1$. In this framework he found a generalization of Aaron's condition. We only note that his assumption is inconsistent with optimization in general. Indeed, substituting it to (4.17), $\theta=1-\delta_{m,0}u^{(1-\mu)m}H^{o}(u)/w_{m,0}$ is obtained. To have a flat θ, $\delta_{m,0}=\Omega w_{m,0}$ and $\mu=1$, i.e. $\tau_{m,0}=\Omega w_{m,0}$ $(m=L,\ldots,R-1)$ should hold. The resulting $\theta=1-\Omega H^{o}(u)$ still depends on u, i.e. on the type of social security.

Cobb-Douglas utility function

Now we shall consider the other extreme case, the Cobb-Douglas utility function. We shall assume that $\Gamma_0>L$. (This assumption is automatically satisfied for childless profiles and reduces to D-1>2L for flat PF profiles.)

Remark. Note that in the border-line case $\Gamma_0=R-1=L$ $G^o(u)=1$ for every u. The simplest realization of this case is the flat FT profile 1/2/3.

Specifying Theorem 4 again, we have

Theorem 9. (cf. Gale, 1973) Cobb-Douglas utility function: a) For every PFW-profile, satisfying $L<\Gamma_0<R-1$, PAYG is better than CR if and only if one of the two alternative conditions holds:

(6.5i) $u^*<u<1$ **for a mature profile,**

(6.5ii) $1<u<u^*$ **for a youthful profile.**

PAYG is never better than CR for a symmetric profile.

b) Every PFW-profile, satisfying $\Gamma_0\geq R-1>L$, is mature and (6.5i) holds with $u^*=0$.

Remarks. 1. Note the duality between the Leontief-Rawls and the Cobb-Douglas case.

2. Observe that the validity of Aaron's proposition, considering the original childless flat SFTW profiles, depends on the elasticity. While by Theorem 5, Aaron's proposition holds for $\mu=1$; by Theorem 9, this is not the case for $\mu=0$. Indeed, for L=0, $\Gamma_0<R-1$ reduces to (D-1)/2<R-1, which is satisfied for R=3, D=4, and $0<u<u^*$ is not empty.

Because of its importance, we present

Example 3. For a flat PFTW profile 0/3/4 and Cobb-Douglas utility, CR is better than PAYG not only for high but also for sufficiently low relative interest factors.

Numerical calculation yields the iso-utility root $u^*=0.15$ which is around 0.89 per annum, assuming that each cohort consists of 13 year-groups. Note that Figure 1 above depicted almost this situation (0/40/53).

Table 3 displays the iso-utility roots for certain flat PFTW profiles and $\mu=1$ and 0, respectively.

Table 3

General elasticity and flat FT profile

Having analyzed the two extreme cases of $\mu=1$ and 0, we turn to the general case $0\le\mu\le1$. Unfortunately, we have to confine the analysis to flat PAYG consumption profiles (with general PW profiles) already used in connection with critical elasticity.

We first prove

Lemma 5 (Monotonicity). Suppose that FT is a flat profile, u=/=1 and $\mu_2<\mu_1<1$. Then

(6.6) $G_{\mu 1}{}^{\circ}(u)<G_{\mu 2}{}^{\circ}(u)$.

We close this Section with a

Conjecture. For flat FT profiles and reasonable u's, the critical elasticity μ^* divides the interval (0,1) into two parts: any function $G_\mu{}^{\circ}$ with low μ resembles $G_0{}^{\circ}$, while any function with high μ resembles $G_1{}^{\circ}$.

The analysis is confined to a reasonable interval, J, say $0.95\le u\le 1.15$. There are two reasons to do so: (i) In practice the relative interest factors are in J. (ii) $G_\mu{}^{\circ}$ has a jump in $\mu=0$: $G_\mu{}^{\circ}(0)=0$ for $\mu>0$ and $G_0{}^{\circ}(0)=\infty$. Thus for youthful PFTW profiles there are at least two iso-utility roots in the interval (0,1), one in the vicinity of $u_0{}^*$, and the second in $(0,u_0{}^*)$, since $G_\mu{}^{\circ\prime}(u_0{}^*)>0$, and $G_\mu{}^{\circ}(0)=0$.

$G_\mu{}^{\circ}(u)$ may have two iso-utility roots even for μ far from 0, however, we think that between them the indirect utility function hardly changes, i.e. they are **inessential roots**. (For example, for L=9, R=50, D=60 and $\mu=0.4$ there exist two inessential iso-utility roots: 0.91 and 0.93.)

Returning to Figure 3 we can check our conjecture that the critical case separates two worlds. For μ_1 (zero elasticity) PAYG is preferred to CR if u<0.975 or 1<u; for

Table 3. Iso-utility roots as function of L, R and D, for μ=1 and 0.

A G E a t			Iso-utility root for	
Starting to work	Retiring	Death		
L	R	D	μ=1	μ=0
15	55	65	0.976	1.059
15	55	70	1.000	1.000
15	55	75	1.016	0.958
20	60	70	0.961	1.108
20	60	75	0.984	1.059
20	60	80	1.000	1.000
25	65	75	0.949	1.178
25	65	80	0.974	1.108
25	65	85	0.988	1.059

μ_2 (unit elasticity) PAYG is preferred to CR if u<1 or if 1.056≤u.

For additional illustration, Table 4 displays the iso-utility roots lying in the relevant interval, for L=15 and 25, R=55, D=75, for μ=0, 0.1,...,0.9, 1.

Table 4

7. CONCLUSIONS

We have analyzed the status of Aaron's proposition in a rather general framework. It turned out that **logically** the validity of Aaron's proposition is quite limited, since it is based on a number of additional assumptions like a Leontief-Rawls utility function, a BR TW-profile (implying the neglect of children). The **empirical** validity is quite another question. The traditional argument prefers CR to PAYG because of aging population and high relative interest factor and implicitly, low elasticity (Verbon, 1988). In our framework this may be reversed: PAYG is to be preferred to CR because for **youthful** PFTW profiles and high elasticity Aaron's generalized condition may require high rather than low relative interest factor, when the problem of dynamic inefficiency (Blanchard and Fisher, 1989) is ruled out.

Table 4. Iso-utility roots for flat PFTW profiles (R=55 and D=75)

Entering the labor force L	1-elasticity μ	Iso-utility roots u*
15	0.0	0.96
	0.1	0.93
	0.4	1.10
	0.5	1.06
	0.6	1.04
	0.8	1.03
	1.0	1.02
25	0.0	1.08
	0.3	0.92
	0.4	0.95
	0.5	0.96
	0.6	0.97
	0.8	0.98
	1.0	0.98

REFERENCES

AARON, H. J. (1966) "The Social Insurance Paradox", **Canadian Journal of Economics and Political Science 32** 371-374.

ARTHUR, W.B. and MCNICOLL, G. (1978) "Samuelson, Population and Intergenerational Transfers", **International Economic Review 19** 241-246.

AUERBACH, A. J. and KOTLIKOFF, L.J. (1987) **Dynamic Fiscal Policy**, Cambridge, Cambridge University Press.

AUGUSZTINOVICS, M. (1989) "The Costs of Human Life", **Economic Systems Research 1** 5-26.

AUGUSZTINOVICS, M. (1992) "Towards a Theory of Stationary Populations", **manuscript,** Institute of Economics, Budapest.

BALASKO, Y., CASS, D. and SHELL, K. (1980) "Existence of Competitive Equilibrium in a General Overlapping-Generations Model", **Journal of Economic Theory 23** 307-322.

BLANCHARD, O.J. and FISHER, S. (1989) **Lectures on Macroeconomics**, Cambridge, MA, the MTI Press.

GALE, D. (1973) "Pure Exchange Equilibrium of Dynamic Economic Models", **Journal of Economic Theory 6** 12-36.

LEE, R. (1980) "Age Structure, Intergenerational Transfers and Economic Growth: An Overview", **Revue Economique 31** 1129-1156.

MINCER, J. (1974) **Schooling, Experience and Earnings.** New York, NBER.

MODIGLIANI, F. and BRUMBERG, R. (1954) "Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data", **Kurihara, K. K. ed.** Post-Keynesian Economics, New Brunswick, Rutgers University Press, 388-436.

PETERS, W. (1987) "Steady State Growth Paths in a Continuously Overlapping Generations Model", **Zeitschrift für Wirtschafts- u. Socialwissenscheften (ZWS)** 107, 581-594.

PETERS, W. (1991) "Public Pensions in Transition: An Optimal Policy Path", **Journal of Population Economics 4** 155-175.

PÓLYA, GY. and SZEGÖ, G. (1976) **Problems and Theorems in Analysis II**, Berlin, Springer.

REICHLIN, P. (1992) "Endogenous Cycles with Long-Lived Agents", **Journal of Economic Dynamics and Control 16** 243-266.

SAMUELSON, P. A. (1958) "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money", **Journal of Political Economy 66** 467-482.

SAMUELSON, P. A. (1975) "The Optimum Growth Rate for Population", **International Economic Review 16** 531-537.

SIANDRA, E. (1993) "On the Validity of Aaron's Proposition", **manuscript**, Tilburg University.

TOBIN, J. (1967) "Life Cycle Saving and Balanced Growth", **Fellner et al. eds.** Ten Economic Studies in the Tradition of Irving Fisher, New York, Wiley, 231-256.

VERBON, H. (1988) **The Evolution of Public Pension Schemes**, Berlin, Springer Verlag.

APPENDIX: PROOFS

Proof of Theorem 3. Now the utility function is differentiable and the optimum is an interior point, the method of Lagrange multipliers (where $\alpha(r)$ denotes the multiplier) yields the following D optimality conditions:

$$(A.1)\qquad \Sigma_{n\le m<D}\pi_n\frac{\delta}{\delta c_{m,m}}U_n(c_{0,0},\ldots,c_{n-1,n-1})$$

$$=\alpha(r)r^{-m}\Sigma_{n\le m<D}\pi_n,\qquad \text{for } m=0,\ldots,D-1.$$

Take the total derivative of the indirect utility function according to r:

$$A'(r)=\Sigma_{0\le n<D}\pi_n\Sigma_{0\le m<n}\frac{\delta}{\delta c_{m,m}}U_n(c_{0,0},\ldots,c_{n-1,n-1})c_m'(r).$$

Reversing the order in the double summation

$$A'(r)=\Sigma_{0\le m<D}[\Sigma_{m\le n<D}\pi_n\frac{\delta}{\delta c_{m,m}}U_n(c_{0,0},\ldots,c_{n-1,n-1})]c_m'(r)$$

and making use of (A.1), we obtain

$$A'(r)=\Sigma_{0\le m<D}\alpha(r)s_m r^{-m}c_m'(r).$$

Taking the derivative of the budget constraint (2.6r) at t=0, we obtain

$$\Sigma_{0\le m<D}s_m r^{-m}\{-mr^{-1}[c_m(r)-w_{m,m}]+c_m'(r)\}=0.$$

Combining the last two formulas yields

$$A'(r)=\alpha(r)r^{-1}\Sigma_{0\le m<D}s_m r^{-m}m[c_m(r)-w_{m,m}].$$

Taking into account that $\alpha(r)>0$, (2.5r),

$$A'(h)=\alpha(h)h^{-1}\Sigma_{0\le m<D}p_m m[c_{m,0}^{(h)}-w_{m,0}].$$

Thus $A'(h)>0$ if and only if $\Gamma>\Omega$, etc.

Proof of Lemma 1. a) Specifying the optimality conditions for (4.1a), we obtain

$$\Phi_m\tau_m^{-\sigma}c_{m,m}^{\sigma-1}-\alpha(r)r^{-m}=0,$$

or

$$(A.2)\qquad c_{m,m}^{\sigma-1}=\alpha(r)\Phi_m^{-1}\tau_m^{\sigma}r^{-m}.$$

Taking into account that $\sigma=-\mu/(1-\mu)$, $1-\sigma=1/(1-\mu)$, $1/(1-\sigma)=1-\mu$, (A.2) results in

$$(A.3)\qquad c_m(r)=\alpha(r)^{\mu-1}\delta_m r^{(1-\mu)m}$$

where δ_m is the **weighted geometrical mean** of Φ_m and τ_m. Although for any given μ, $\{\Phi_m\}$ and $\{\tau_m\}$ are replaced by $\{\delta_m\}$, δ_m generally depends on μ.

Substituting (A.3) into (2.6r) yields

$$\text{(A.4)}\qquad \alpha(r)^{\mu-1}=\frac{\Sigma_{L\le k<R}s_k w_{k,k}r^{-k}}{\Sigma_{0\le k<D}s_k\delta_k r^{-\mu k}}.$$

Using the notations (4.6)-(4.8), (A.3)-(A.4) yield (4.9).

b) Inserting the definition of δ_m into (4.9), substituting the new form into (2.4) and separating the common factor H(r) yields the indirect utility function

$$G(r)=H(r)[\Sigma_{0\le m<D}s_m\Phi_m^{1+(1-\mu)\sigma}\tau_m^{-(1-\mu)\sigma}r^{(1-\mu)m\sigma}]^{1/\sigma}.$$

By $-(1-\mu)\sigma=\mu$, the base of the second factor is equal to C(x), i.e. the second factor is equal to $C(x)^{(\mu-1)/\mu}$. Taking into account H=W/C yields (4.10).

Proof of Lemma 2. With rearranging, (5.4) is equivalent to $(\Sigma_k\alpha_k x_k)(\Sigma_j\beta_j)-(\Sigma_k\alpha_k)(\Sigma_j\beta_m x_j)\le 0$. If k=j, then $\alpha_k\beta_j x_k$ appears with plus and minus, cancelling each other. If k=/=j, then $\alpha_k\beta_j x_k-\alpha_k\beta_j x_j$ can be combined with $\alpha_j\beta_k x_j -\alpha_j\beta_k x_k$, their sum is equal to $(\alpha_k\beta_j-\alpha_j\beta_k)(x_k-x_j)$, which is non-positive by assumption (5.3). If neither of the two series is constant, then $(\alpha_{D-1}\beta_0-\alpha_0\beta_{D-1})(x_{D-1}-x_0)>0$, etc.

Proof of Lemma 3. Let $\mu_2<\mu_1$ and apply Lemma 1 with $\alpha_k=p_k\Phi_{k,o}^{1-\mu 1}\tau_{k,o}^{\mu 1}$, $\beta_k=p_k\Phi_{k,o}^{1-\mu 2}\tau_{k,o}^{\mu 2}$, $x_k=k$, it is (5.5) that implies the conditions in (5.3), and by the formula of Γ_μ, (5.4) results in $\Gamma_{\mu 1}\le\Gamma_{\mu 2}$.

Proof of Lemma 4. Calculate $G^{o\prime}(u)$ and $G^{o\prime\prime}(u)$:

$$\text{(A.5)}\qquad G^{o\prime}(u)=C^o(u)^{-1/\mu-1}\{W^{o\prime}(u)C^o(u)-\mu^{-1}W^o(u)C^{o\prime}(u)\},$$

$$\text{(A.6)}\quad G^{o\prime\prime}=C^{o-1/\mu-2}\{W^{o\prime\prime}C^{o2}-2\mu^{-1}W^{o}C^{o}C^{o\prime}+\mu^{-1}(1+\mu^{-1})W^oC^{o\prime 2}-\mu^{-1}W^oC^oC^{o\prime\prime}\}.$$

Substituting (A.6) into $G_{\mu *}^{o\prime\prime}(1)=0$ and using the abbreviations (5.7) and (5.8), we obtain (5.9).

Continuous time. Combining (5.7′) and (5.9′) yields

(A.7) $$\mu^*=12\int_\alpha^\beta (x-1/2)^2 f(x)dx.$$

By (A.7), $\mu^*>0$. To prove $\mu^*<1$, we have to prove that the integral in (A.7) is less than 1/12. Taking into account that cross-sectionally wages increase till the middle of life ($x=1/2$) and then stagnate, while survival probabilities stagnate till $x=1/2$, then start to diminish, we may assume that $f(x)$ increases till 1/2, then decreases. Replacing $f(x)$ by $1/(\beta-\alpha)$, the integral increases, i.e.

$$\mu^*<4(\alpha^2+\alpha\beta+\beta^2)-6(\alpha+\beta)-3.$$

By an elementary calculation, one can show that $\mu^*<1$.

Proof of Theorem 8. Relying on Theorem 4, we have to determine the location of the iso-utility roots. Introducing the notation $D^o(u)=W^o(u)-C^o(u)$, we have $H^o(u)=1$ is equivalent to $D^o(u)=0$. As is evident, the number of changes in signs of coefficients of polynomial $D^o(u)$ is at most two, hence, by Descartes-rule (Pólya and Szegő, 1976), this polynomial has at most two positive roots (cf. Gale, 1973). On the other hand, $D^o(1)=0$, $D^o(0)=w_{D-1,0}-\tau_{D-1,o}<0$ and $D^o(\infty)=w_{0,0}-\tau_{0,o}$ is (a) negative if $L^*>0$ and (b) positive if $L^*=0$. In case (a) D^o and H^o have two roots, in case (b) they have one root. Dropping the golden root, there is one or zero iso-utility root, respectively.

a) GBR: (i) By Theorem 3, if a TW profile is mature, then $H^{o\prime}(1)>0$. Thus $u^*>1$, i.e. PAYG is better than CR if and only if either $u<1$ or $u>u^*$ holds. Cases (ii) and (iii) are similar.

b) BR: $H^o(0)=0$, $H^o(1)=1$, $H^o(u)=1$ has no other root, i.e. $H^{o\prime}(1)>0$, thus by Theorem 3, the TW profile is mature. etc.

Proof of Theorem 9. Similarly to the proof of Theorem 8, first consider

$$G_0^{o\prime}(u)\approx\Sigma_{L\le k<R}(\Gamma_0-k)p_k w_{k,0}u^{\Gamma_0-k-1}.$$

a) If $L<\Gamma_0<R-1$, for small u's u^{Γ_0-R-1} dominates $G_0^{o\prime}(u)$ (with a negative coefficient) and for large u's u^{Γ_0-L-1} dominates $G_0^{o\prime}(u)$ (with a positive coefficient): $G_0^o(u)$ is

U-shaped. In addition to the trivial golden root, equation (6.1) has at least another root. Again relying on Descartes-rule, this polynomial has at most two positive roots, etc.

b) If $\Gamma_0 \geq R-1>L$, then $G_0^{o\prime}(u)>0$, hence G_0^o is increasing, etc.

Proof of Lemma 5. The proof is based on the well-known theorem: the power mean is an increasing function of the exponent μ (Pólya and Szegő, 1976). (4.10) implies $G_\mu^o(u)=W^o(u)/C_\mu^{o1/\mu}$, where the denominator [cf. (4.16)] is the μ-th weighted power mean of $\{u^{-k}\}$ with weights $\{s_k/\Gamma_0\}$. (Note that for a non-flat FT profile, the weights would change with μ.) Thus $C_\mu^{o1/\mu}$ is an increasing function of μ and $G_\mu^o(u)$ is an increasing function of $1-\mu$.

LIST OF ABBREVIATIONS

CR=capital reserve system

PAYG=pay-as-you-go system

ACRRA=additive utility function with constant coefficient relative risk aversion

F=weight path in the utility function

T=weight path in the utility function

S=survival path

C=consumption path

W=wage path

P=population profile

GBR=gestation-breeding-retirement (property)

BR=breeding-retirement (property).

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