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Publication date:
1989

Citation for published version (APA):
Rankin, N. (1989). *Monetary and Fiscal Policy in a "Hartian" Model of Imperfect Competition*. (CentER Discussion Paper; Vol. 1989-26). CentER.

CentER for Economic Research

Discussion paper

[redacted]

TILBURG UNIVERSITY

No. 8926

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June, 1989

MONETARY AND FISCAL POLICY IN A 'HARTIAN' MODEL OF IMPERFECT COMPETITION*

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Revised, July 1988

* I am very grateful to Huw Dixon for stimulating my interest in this subject and for comments, and also to Simon Wren-Lewis; errors are of course only mine.

Abstract

Hart's "Keynesian features" model of imperfect competition is interpreted as a model of temporary equilibrium with money, and generalised in three ways: to allow non-zero-elastic expectations of future prices, CES production technology, and government spending. When an unemployment equilibrium exists, an increase in the money supply always raises output, unless expectations are unit-elastic. An ultra-Keynesian result in which the price falls and output increases more than proportionally to money is possible. A rise in government spending lowers output if money is neutral, but otherwise always raises it for some money supply range, if spending is set in cash terms.

JEL classification no. 023

Keywords: imperfect competition, monetary and fiscal policy, expectations

1. Introduction

Hart's (1982) article was the first of a number of recent explorations of the extent to which Keynesian results may be derived from a general equilibrium model of imperfect competition. Other authors to examine this question have included Snower (1983), Mankiw (1985), Akerlof and Yellen (1985), Dixon (1986), Benassy (1987) and Blanchard and Kiyotaki (1987), to name but a few. In this paper, we return to reconsider Hart's original model, and to extend it in several directions. As a first step, we argue that it is most usefully viewed as a model of temporary equilibrium with money. We then generalise the model in three ways: first, by allowing the elasticity of future price expectations to take any possible value, rather than restricting it to zero. Second, instead of assuming firms' technology to be Cobb-Douglas, we generalise it to be CES, permitting the elasticity of substitution to deviate from unity. Third, we introduce government spending as an additional component of aggregate demand, enabling an analysis of fiscal as well as of monetary policy.

The reinterpreted and extended model generates a number of new and interesting results. First, for certain ranges of the parameter space, an unemployment equilibrium exists in which an increase in the money supply will increase output and employment, for *any* value of the elasticity of price expectations not equal to unity. Hart's original result of this sort was (implicitly) derived only for zero-elastic expectations. What deserves comment here is that the outcome is independent of whether the elasticity is above or below unity: all that matters is that it should not be equal to it. Second, it is possible to obtain an unemployment equilibrium which is "ultra-Keynesian", in the sense that when the money supply is increased, output rises more than proportionally to it, and the price level actually falls. Third, when government spending is introduced, it will affect output even when monetary policy does not, but in these circumstances the effect is negative. Given monetary effectiveness, we show, fourthly, that fiscal policy will always also have a positive effect on output for some range of the money supply, provided government spending is set in nominal terms; but if it is indexed to the price level, the conditions for a positive effect become more stringent. These third and fourth results illustrate the conflicting "elasticity" and "fixprice" effects which in general flow from an increase in government spending in this type of model.

Some results related, but not identical, to some of these, have been obtained in two papers which are unpublished at the time of writing. Jacobsen and Schultz (1987) find that, in a model in which the wage is determined by a Nash bargain between employers and unions, an elasticity of price expectations other than unity also gives rise to monetary policy effectiveness. Wren-Lewis (1985) is the first to point out that government spending will always affect output under imperfect competition, even when monetary policy does not, unless very restrictive conditions hold. As a result, he observes, imperfectly competitive models do not possess a "natural rate" of unemployment or output.

The body of the paper falls into three sections. The following one describes the structure of the model, beginning with the microeconomic decisions of individual agents. Section 3 applies this model to the study of the effectiveness of monetary policy. Section 4 introduces government spending, and thence examines fiscal policy. The technical issues of the second-order conditions and the "stability" of equilibrium are considered in an appendix.

2. The Structure of the Model

In Hart's model, imperfect competition takes the form of Cournot-Nash quantity-setting oligopolists, in both the goods and labour markets. We shall simplify this by postulating perfect competition in the goods market. It will be seen that this changes nothing essential in the model's properties, yet makes easier the analysis of these. Throughout, we assume that households have a fixed endowment of time, but obtain no utility of leisure, so implying that at any positive wage their competitive supply of labour would be equal to the exogenous endowment. Therefore if there were perfect competition in the labour market, output would be exogenously determined by the labour supply. With imperfect competition, unions may wish to restrict sales of labour, causing an equilibrium with unemployment to exist. Whether this is the case, or whether equilibrium will still be at full employment, will be seen to depend on various conditions which are the subject of the analysis. Naturally, we shall focus most attention on the cases where unemployment does exist, in order to ask whether and how, in these circumstances, monetary and fiscal policy are effective.

In any imperfectly competitive general equilibrium model in which agents maximise against the "true" demand curves which they face, there are many potential channels of strategic interdependence. Hart limited these by the assumption of a particular micro-market structure, of which we here adopt a simpler form, made possible by the absence of monopoly power in the goods market. Let there be a fixed number of separate locations, each with a market for goods and labour, in which the firms at the location respectively sell and buy. Households who work at one location are assumed to buy goods at a different one, meaning that the local unions do not affect the prices at which their members consume, even though they are able to influence the local goods price. Further, assume that profits of firms at the location are distributed only to households who work or buy goods at different locations. This implies that local unions do not influence the profit receipts of their members; and, in combination with the above, that they do not influence the money incomes of households who buy goods at the location. We shall in fact assume that there is complete symmetry amongst locations as concerns numbers of agents and their preferences, endowments and technologies, and shall focus only on equilibria which are symmetric. Hence it will not be necessary to introduce formal notation to distinguish between different locations, or between aggregate and local variables.

Union behaviour

All households are allocated to a trade union, and membership is fixed. The union's objective function is to be derived from the utility functions of its members. Given that there is no utility of leisure, and that profit incomes of members and the prices of goods which they face are exogenous to the union (under the above assumptions), the appropriate maximand is the union's total money wage revenue. This is the maximand also assumed by Hart (except for an extension in which utility of leisure is introduced). Since, below, households will be assumed identical, it is natural to postulate that when the union restricts labour sales, it rations all members equally; though all-or-nothing unemployment with random selection could equivalently be assumed if households were risk-neutral.

At each location, households are divided equally amongst a fixed number, n, of unions. There is a market labour demand function, $\ell = g(W,...)$, whose derivation will be described below. Under the Cournot-Nash assumption the ith union, in choosing its labour sales ℓ_i, takes as given the sales ℓ' of the other n-1 unions. Thus it solves the problem:

$$\text{maximise} \quad W\ell_i \quad \text{subject to} \quad \ell_i + \ell' = g(W,...) \quad \text{and} \quad \ell_i \leq L$$

where L is the available time endowment. The first-order condition for the solution is:

$$\ell_i + W\frac{d\ell}{dW} \leq 0, \quad \ell_i \leq L, \quad \text{with complementary slack}$$

In labour market equilibrium, since all unions are identical we have $\ell_i = \ell/n$, giving:

$$\frac{1}{n} \leq -\frac{d\ell}{dW}\frac{W}{\ell} \equiv -\varepsilon \qquad (1)$$

We shall be most interested in equilibria in which (1) holds with strict equality, signifying the presence of unemployment. However the possibility that such do not exist, and that the equilibrium is at full employment, cannot be ignored. When it holds with equality, (1) simply states that (the absolute value of) the wage-elasticity of labour demand should equal the reciprocal of the number of unions, a constant. It is from analysing this condition in more detail that most of the properties of the model can be derived.

Consumer behaviour

As noted above, households are taken to be identical, enabling us to work with the concept of a single representative. In Hart's model, the household is assumed to obtain utility from purchases of firms' output, which is taken to be a homogeneous good, and from a non-produced good, the supply of which is fixed. It is natural to wish to interpret the non-produced good as "money", an interpretation which Hart declines to make. The most familiar argument in monetary theory (see, e.g., Grandmont (1983)), for including an intrinsically worthless commodity such as money in the utility function, is that in an economy where money is the only asset, its presence reflects the demand for saving and thus for future consumption. To give an unoriginal illustration, suppose the household faces the very simple intertemporal problem:

$$\text{maximise} \quad u(c,c^e) \quad \text{subject to} \quad M_0 + Y = Pc + M, \quad M = P^e c^e$$

where Y = money income, M_0 (M) = money balances at the start (end) of the first period, c = consumption, P = its price, and ^{e} indicates an expected second-period value. Substituting out c^e as M/P^e, the problem may be re-expressed as:

$$\text{maximise} \quad u(c,M/P^e) \quad \text{subject to} \quad [M_0+Y]/P = c + [P^e/P]M/P^e \qquad (2)$$

From this we see that for Hart's non-produced good to be interpreted as money, it is necessary for P^e, the household's subjective expectation of next period's price, to be an exogenous constant which therefore can be assimilated as part of the definition of u(.), leaving utility effectively a function only of (c,M). Hart's reluctance to accept this interpretation may thus be viewed as an unwillingness to assume exogenous, or zero-elastic, expectations. Remarks in his conclusion suggest that he sees the only plausible assumption as one of unit-elastic expectations, such that $P^e = \psi P$ for some constant, ψ. It is clear that this is equivalent to making utility a function only of (c,M/P), which would require that the non-produced good be deflated by the price level if it is to be interpreted as money. Hart suggests that this would cause money to be neutral, a speculation which below will be shown to be correct.

Here, we shall explicitly take the non-produced good to be money, so allowing the model to contribute to the extremely important debate on the effectiveness of monetary policy. However, in order to avoid constraining price expectations to be exogenous, we adopt the more general assumption widely found in monetary theory (see again Grandmont, op. cit.), that expected prices are some arbitrary subjective function of current prices. For convenience, we take a constant-elasticity parameterisation, $P^e = \psi P^{\gamma}$, where γ is the elasticity of expectations. Such an approach avoids constraining γ to equal zero — though a value of zero generates a model which is formally equivalent to Hart's, and can still be considered as a special case — but also involves a rejection of the view that the only plausible and worthwhile assumption is $\gamma = 1$. At the same time it allows $\gamma = 1$ to be used as a benchmark, so that the effect of small deviations from this, both above and below, can be considered.

It is clear from direct consideration of the consumer's problem as expressed in (2), that consumption may in general be expressed as a function of $([M_0+Y]/P,\ P^e/P)$. Given that P^e is a function of P, then so is the intertemporal relative price, P^e/P. Thus P influences c through two channels: through a real lifetime wealth, or real balance, effect; and through a relative price, or intertemporal substitution, effect. It is worth noting two special cases in which the latter is zero: one is when price expectations are unit-elastic, which locks the relative price at the value ψ; and the other is when current consumption is neither a gross complement nor a gross substitute for future consumption, which occurs most notably when the utility function is Cobb-Douglas. If we assume, as does Hart, that preferences are homothetic, then in both cases consumption is simply equal to a constant times real wealth.

Below it will be found that a key factor in the model is the behaviour of the price elasticity of consumption demand, $\varepsilon_C \equiv [\partial c/\partial P][c/P]$, calculated at constant M_0+Y. It is easy to see that, given homotheticity, ε_C takes the constant value -1 in the two cases just mentioned. In the more general case, whether ε_C lies below or above -1 depends on the combination of two factors: whether γ is greater or less than one (which determines whether

P^e/P rises or falls with P); and whether current and future consumption are gross substitutes or gross complements. In general, ε_C is itself a function of (M_0+Y,P). If preferences are homothetic, as Hart assumes, we obtain the useful simplification that ε_C depends only on P, as may easily be verified.

To illustrate the above remarks, and to provide a fully-specified model of consumer behaviour for use below, let it now be assumed that the utility function has the CES form:

$$u = [c^{\rho} + \delta[M/P^e]^{\rho}]^{1/\rho}, \qquad \delta > 0, \ \rho < 1$$

This, it may be recalled, is homothetic and contains the Cobb-Douglas function as the special case where $\rho = 0$. $\rho > 0$ implies gross substitutability, and $\rho < 0$ gross complementarity, between the two arguments. Incorporating the expectations hypothesis $P^e = \psi P^{\gamma}$, and maximising this subject to the budget constraint, we derive the consumption function:

$$c = \frac{M_0+Y}{P}\frac{1}{1+h(P)} \qquad \text{where} \qquad h(P) \equiv \delta^{1/[1-\rho]}\psi^{\rho/[\rho-1]}P^{\rho[1-\gamma]/[1-\rho]} \tag{3}$$

From this,

$$\varepsilon_C = -1 - \frac{\rho[1-\gamma]}{1-\rho}\frac{h(P)}{1+h(P)} \tag{4}$$

Since h(P) is always non-negative, we can see that whether $\varepsilon_C >$ or < -1 depends on the sign of $\rho[1-\gamma]$. Gross substitutability plus inelastic expectations, or gross complementarity plus elastic expectations, yield an absolute value of ε_C greater than one, while the reverse combinations give a value less than one.

Firm behaviour

There are many firms at each location. Each behaves as a perfect competitor in both the goods and the labour markets, and is endowed with an identical strictly concave production function, $y = f(\ell)$. The representative firm's profit-maximising employment level is then determined from the familiar condition $f'(\ell) = W/P$, giving the strictly decreasing labour demand and goods supply functions:

$$\ell = \ell^d(W/P), \quad y = y^s(W/P)$$

Hart assumes that $f(\ell)$ takes the Cobb-Douglas form, $y = B\ell^\mu$ $(0 < \mu < 1)$. We generalise this to CES, so assuming:

$$y = A[\alpha k^\sigma + \beta\ell^\sigma]^{\pi/\sigma}, \qquad \alpha+\beta = 1,\ \sigma < 1,\ 0 < \pi < 1$$

Although k is here a constant and is — formally — superfluous, it has the natural interpretation of being the (fixed) capital stock, and so is retained for the sake of familiarity. It may also be used to examine the effect of supply shocks. $1/[1-\sigma]$ is the elasticity of substitution between labour and capital. $\alpha+\beta = 1$ ensures that y remains finite as we let σ tend to zero, yielding the Cobb-Douglas function $Ak^{\pi\alpha}\ell^{\pi\beta}$.

In the model of firms' behaviour, particular significance attaches to the real-wage elasticities of labour demand and goods supply, $\varepsilon_L \equiv [d\ell^d/dw][w/\ell]$ and $\varepsilon_S \equiv [dy^s/dw][w/y]$ $(w \equiv W/P)$. (Note that the former turns out to equal $f'/f''\ell$, which is the reciprocal of the elasticity of the marginal product of labour, below denoted as ε_F.) These are in general functions of W/P, and thus, equivalently, of ℓ or y. With CES technology, they may be computed as the following functions of y:

$$\varepsilon_L = \left[\sigma-1 + [\pi-\sigma][1-\alpha k^\sigma[y/A]^{-\sigma/\pi}]\right]^{-1}, \quad \varepsilon_S = \pi\left[\pi-\sigma + [\sigma-1][1-\alpha k^\sigma[y/A]^{-\sigma/\pi}]^{-1}\right]^{-1} \quad (5)$$

Note that when $\sigma = 0$ (Hart's Cobb-Douglas case), we get $\varepsilon_L = 1/[\pi\beta-1]$, $\varepsilon_S = \pi\beta/[\pi\beta-1]$, *i.e.* constants. The very special fix-price result of Hart turns out to depend on the fact that $\varepsilon_L, \varepsilon_S$ are constants, and thus on the assumption of Cobb-Douglas technology. With the generalisation of this to CES, the simplicity of the fix-price result disappears, but other interesting possibilities arise.

General imperfectly competitive equilibrium

Equilibrium at a given location may be found in two stages. First, for any particular money wage, W, in the local labour market, the equilibrium price, P, in the local goods market is determined from the market-clearing condition:

$$y^s(W/P) \;=\; c(M_0+Y,P) \tag{6}$$

This defines P as an implicit function of (W,M_0+Y). Second, the labour demand curve as faced by unions takes account of this dependence of P on W, and so is given by:

$$\ell \;=\; \ell^d(\,W/P(W,M_0+Y)\,) \;\equiv\; g(W,M_0+Y) \tag{7}$$

Although unions recognise that any increase in the wage will raise the local price, and take account of this in calculating the effect on the demand for their labour, note that Y, the money income of consumers at the location, has been assumed to be earned entirely at other locations, and so is taken as given by the local unions. (7) may now be used in combination with (1) to determine equilibrium in the local labour market.

To find the general equilibrium for the whole economy, we appeal to the assumption of symmetry across locations. This implies that Y, the money income received by households at the typical location, must equal the money value of income generated at the typical location. The latter may be represented either by the money value of goods supplied, or of goods demanded, which in equilibrium are the same. This provides us with a second condition, (9), which, in combination with the labour market equilibrium condition (1) (reproduced as (8)), defines the values (W,Y) consistent with a general imperfectly competitive equilibrium with unemployment:

$$1/n \;=\; -\varepsilon(W,M_0+Y) \tag{8}$$

$$Y/P(W,M_0+Y) \;=\; c(M_0+Y,P(W,M_0+Y)) \tag{9}$$

In (8) we observe that ε, being a log-derivative of $g(W,M_0+Y)$, must in general be a function of the same variables.

(8) and (9), and the preceding expressions, have deliberately been written in terms

of unspecified functional forms, to make clear the general concept of equilibrium. It will be the task of the following section to examine whether, with the forms implied by CES preferences and technology, they can be satisfied; and if so to consider whether monetary policy is effective. Existence of an equilibrium with unemployment is not guaranteed. As Hart notes, the conditions for existence in Cournot-Nash oligopoly are in general more stringent than in perfect competition. However, where an unemployment equilibrium does not exist, it will be found that equilibrium occurs at full employment: in such cases the model is merely less interesting, not incoherent.

3. The Effectiveness of Monetary Policy

Before proceeding to the case of CES preferences and technology, the equilibrium conditions (8) and (9) may be analysed in greater depth for the general case. Differentiating the market labour demand function, (7), ε may be expressed as:

$$\varepsilon \equiv \frac{d\ell}{dW}\frac{W}{\ell} = \varepsilon_L\left[1-\frac{\partial P}{\partial W}\frac{W}{P}\right] \tag{10}$$

where $\partial P/\partial W$ is from $P(W,M_0+Y)$, implicitly defined by the goods market equilibrium condition, (6). Differentiating (6),

$$\frac{\partial P}{\partial W}\frac{W}{P} = \varepsilon_S/[\varepsilon_S+\varepsilon_C] \tag{11}$$

Thus,

$$\varepsilon = \varepsilon_L\varepsilon_C/[\varepsilon_S+\varepsilon_C] \tag{12}$$

ε hence decomposes into three, more basic, elasticities. Of these, ε_L and ε_S depend on firms' technology, and ε_C on households' preferences.

We may now use (12) in the equilibrium condition (8), where, noting that $\varepsilon_L = 1/\varepsilon_F$ and rearranging, we obtain:

$$-\varepsilon_C = \varepsilon_S\varepsilon_F/[n + \varepsilon_F] \tag{13}$$

In the right-hand expression (henceforth denoted as z), ε_S and ε_F we know in general to be functions of y. In the left-hand expression, ε_C we know to be a function of P, and moreover of P alone, if preferences are homothetic. A second relationship between P and y is provided by the equilibrium condition (9). Note that this simply defines the macroeconomic aggregate demand curve. Assuming preferences are homothetic, it may be rewritten as:

$$y = \alpha(P)[M_0/P + y]$$

where $\alpha(P)$ is the wealth-independent marginal and average propensity to consume. Thus,

$$y = \frac{M_0}{P}\frac{\alpha(P)}{1-\alpha(P)} \qquad \text{or, inverted,} \qquad P = D(y/M_0)$$

Substituting this into (13), the conditions for an unemployment equilibrium are conveniently reduced to the following single equation in y:

$$-\varepsilon_C(D(y/M_0)) = \varepsilon_S(y)\varepsilon_F(y)/[n + \varepsilon_F(y)] \equiv z(y) \tag{14}$$

By graphing the two sides of (14), the questions of existence and comparative statics may be studied.

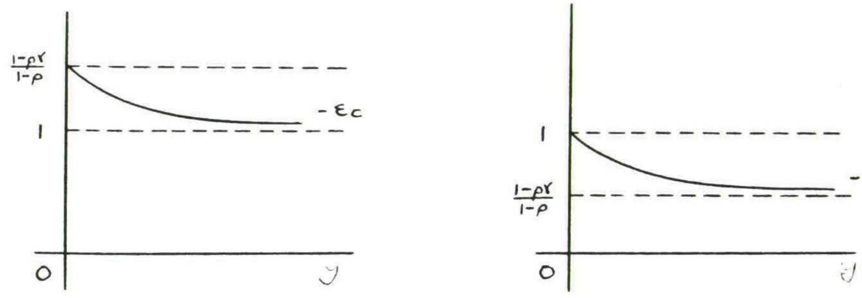
Consider first the graph of $-\varepsilon_C$ against y, utilising now the CES parameterisation of preferences introduced above. ε_C as a function of P was given by (4). From (3), the inverse aggregate demand function is readily found as:

$$P = [y/M_0]^{[\rho-1]/[1-\rho\gamma]}\delta^{1/[\rho\gamma-1]}\psi^{\rho/[1-\rho\gamma]} \tag{15}$$

This is decreasing, as would be expected, provided $\rho\gamma < 1$, a restriction which will henceforth be assumed to hold. Substituting it into (4), $-\varepsilon_C$ as a function of y is:

$$-\varepsilon_C \;=\; 1 + \frac{\rho[1-\gamma]}{1-\rho}\Big[1 + [y/M_0]^{\rho[1-\gamma]/[1-\rho\gamma]}\delta^{1/[\rho\gamma-1]}\psi^{\rho/[1-\rho\gamma]}\Big]^{-1} \tag{16}$$

This is pictured in Fig. 1:



$\rho[1-\gamma] > 0$ $\qquad\qquad$ $\rho[1-\gamma] < 0$

Fig. 1

Only in the case $\rho[1-\gamma] = 0$ (caused either by unit-elastic expectations, or by Cobb-Douglas preferences) does the graph not have a negative slope: in such a case (as noted above) $-\varepsilon_C$ takes the constant value of unity. An increase in M_0, as can be seen from (16), results in a proportionate increase in y at any value of $-\varepsilon_C$.

Next consider the graph of z against y, utilising the CES parameterisation of technology. ε_L and ε_S as functions of y were given by (5). Substituting these into the definition of z and simplifying, gives:

$$z \;=\; \frac{\pi[1-\alpha k^{\sigma}[y/A]^{-\sigma/\pi}]}{n-1+\sigma + [\pi-\sigma][1-\alpha k^{\sigma}[y/A]^{-\sigma/\pi}]} \tag{17}$$

This is drawn for four different ranges of the parameter σ in Fig. 2:

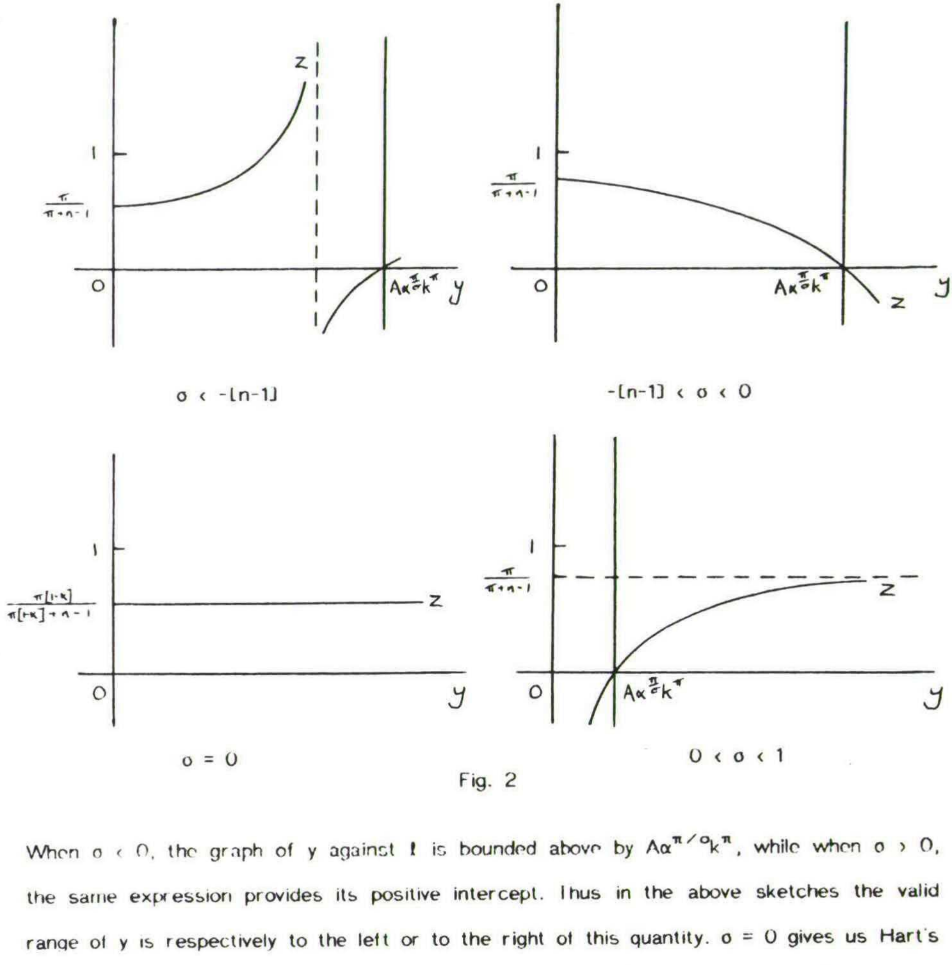


Fig. 2

When $\sigma < 0$, the graph of y against l is bounded above by $A\alpha^{\pi/\sigma}k^{\pi}$, while when $\sigma > 0$, the same expression provides its positive intercept. Thus in the above sketches the valid range of y is respectively to the left or to the right of this quantity. $\sigma = 0$ gives us Hart's case of Cobb-Douglas technology, in which case the graph of z is simply a horizontal line.

Equilibrium may now be found by superimposing the $-\varepsilon_C$ and z curves. The results are clearly sensitive to the parameter values chosen. Take first the closest formal equivalent to Hart's model, in which technology is Cobb-Douglas ($\sigma = 0$) and expectations are zero-elastic ($\gamma = 0$). Since the z-curve is a horizontal line at a value less than unity, it follows that we need $\rho < 0$ to ensure an unemployment equilibrium exists (more specifically, $1/[1-\rho] < \pi[1-\alpha]/[\pi[1-\alpha]+n-1]$). This gives a picture as in Fig. 3:

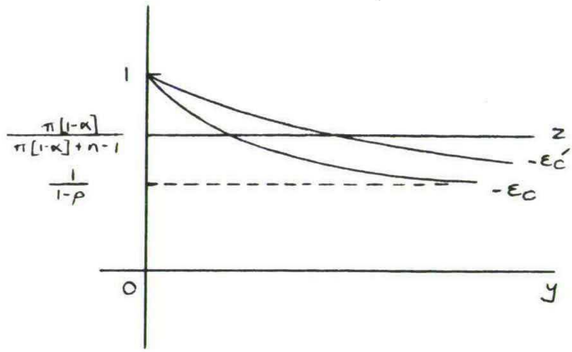


Fig. 3

Here and in the diagrams below, we take it as understood that the value of y at the intersection is less than the "full employment" value implied by the economy's exogenous endowment of labour time, though the latter is not drawn in. This may always be assumed, since both curves are independent of the time endowment. If it is not true, then the equilibrium is simply a full-employment one, in which both monetary and fiscal policy are clearly completely ineffective.

The effect on the $-\varepsilon_C$ curve of a rise in M_0, i.e. of an equal money handout to all households, was explained above. With the z-curve horizontal, this translates into a rise in the equilibrium value of y directly proportional to the rise in M_0, as shown in Fig. 3. Since y/M_0 remains unchanged, we see from the aggregate demand curve, (15) that the price level is also unchanged. This is exactly Hart's "Keynesian" result, except that since M_0 is interpreted not as money but as an endowment of a non-produced good, such a change cannot be presented by him as an example of monetary policy. Our analysis makes it clear that the same result can be generalised to the case of non-zero-elastic expectations, i.e. $\gamma \neq 0$. The necessary and sufficient condition for existence is now $[1-\rho\gamma]/[1-\rho] < \pi[1-\alpha]/[\pi[1-\alpha]+n-1]$. Although it is still true that this is not guaranteed for arbitrary values of γ and ρ, it is clearly achievable for many values of γ other than zero, if ρ takes appropriate values. However, for γ in the range $\left[\pi[1-\alpha]/[\pi[1-\alpha]+n-1,\ 1\right]$, no value of

ρ can secure existence, as may be confirmed by manipulating the above condition. Most notably, this includes the case of unit-elastic expectations, where the $-\varepsilon_C$ curve is horizontal at a height of one. In such cases, the equilibrium must be at full employment, no matter how large the economy's endowment of labour time.

We now relax Hart's assumption of Cobb-Douglas technology, turning first to the case where $\sigma < -[n-1]$. This is the case of an elasticity of substitution between labour and capital which falls short of unity by a sufficient margin. Fig. 4 illustrates:

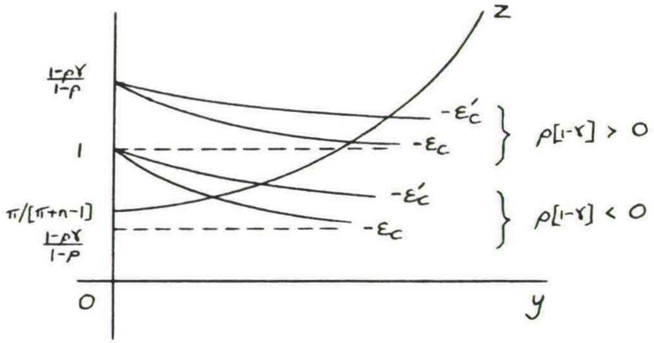


Fig. 4

It is clear that, because the graph of z extends from $\pi/[\pi+n-1]$ (< 1) to $+\infty$ and is upward-sloping, an unemployment equilibrium will exist for all values of (γ,ρ), unlike in the Cobb-Douglas case. This includes, *inter alia*, unit-elastic expectations. A rise in M_0, shifting the $-\varepsilon_C$ curve rightwards as before, clearly still has a positive effect on output. Note especially that this is independent of whether the product $\rho[1-\gamma]$ is positive or negative. Only if it is zero, i.e. only if expectations are unit-elastic (or if preferences are Cobb-Douglas) will monetary policy be ineffective. We encapsulate this as:

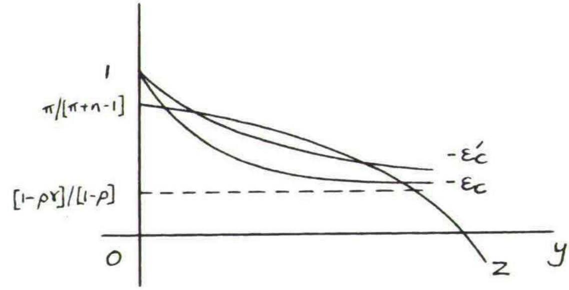
Proposition 1 For CES production technology with sufficiently low substitutability, such that $\sigma < -[n-1]$, a 'Hartian' imperfectly competitive unemployment equilibrium always exists, and an increase in the money supply has a positive effect on output for any value of the elasticity of expectations of future with respect to current prices not equal to one.

That monetary policy is effective for *any* elasticity of expectations not equal to one is a stronger result than before. Under Cobb-Douglas technology, $\gamma = 1$ implies the equilibrium is at full employment and therefore that monetary policy is ineffective; for given values of the other parameters, only when γ sufficiently deviates from one such that it exceeds (if $\rho > 0$) or falls below (if $\rho < 0$) it by an absolute margin of at least $[n-1][1/\rho - 1]/[\pi[1-\alpha]+n-1]$ does unemployment occur and monetary policy become effective.

A difference from Hart's result is that the price level is no longer unaffected. The upward slope of the z-curve dampens the rise in y somewhat, so that the ratio y/M_0 falls rather than remains constant. The aggregate demand curve, (15), indicates that this must cause the price level to rise. In terms of simple macroeconomics, this version of the model therefore resembles an economy with an upward-sloping, rather than a horizontal, aggregate supply curve. (A similar modification is obtained by Hart when he introduces utility of leisure.) The extent to which an increase in nominal demand goes into prices instead of output depends, in part, on the slope of the z-curve, which in turn partly depends on the initial level of output. It can be seen from Fig. 4 that there must be some maximum level of output achievable by monetary expansion, which occurs as the $-\varepsilon_C$ curve tends towards becoming a horizontal line. Whether full employment is achievable by monetary policy thus depends on whether the separately-determined full-employment output level happens to be greater or less than this.

Consider next the effect of ρ in the range $-[n-1] < \rho < 0$. Fig. 5 shows a possible equilibrium in this case:

Fig. 5



It is clear that a necessary condition for existence is $[1-\rho\gamma]/[1-\rho] < \pi/[\pi+n-1]$, a similar condition to that encountered when technology was Cobb-Douglas. Also clear is that this is not yet sufficient. If M_0 is too high, the $-\varepsilon_C$ curve will lie too far to the right to intersect the z-curve; for M_0 sufficiently low, it will be shrunk towards the vertical axis, guaranteeing an intersection.

Since there are two equilibria if there are any, it is natural to ask if they can be distinguished. A natural criterion for this is stability. In the appendix, we consider a possible adjustment mechanism for when the economy is out of equilibrium. This mechanism is not intended as an integrated attempt to extend the model to a dynamic version, but simply as an elementary story, comparable to the tatonnement mechanism used in competitive equilibrium. It is shown that local stability of an equilibrium under this mechanism requires that the z-curve should intersect the $-\varepsilon_C$ curve from below, not above. This then implies that, of the two equilibria in Fig. 5, the one at lower y is stable, and the one at higher y unstable. In looking at comparative statics, we therefore focus on the lower equilibrium, ignoring the upper one.

By comparison with the Cobb-Douglas case of a horizontal z-curve, it can be seen from Fig. 5 that a negative slope implies that y increases *more* than proportionally to a rise in M_0, at the lower equilibrium. Since y/M_0 rises, the aggregate demand curve (15) tells us that the price level now *falls*, rather than remains constant or rises. Thus for this range of σ we have the possibility of an "ultra-Keynesian" result: in terms of simple macroeconomics, it resembles the case of a downward-sloping aggregate supply curve. We summarise this as:

Proposition 2 For CES production technology with intermediate substitutability, such that $-[n-1] < \sigma < 0$, a 'Hartian' imperfectly competitive unemployment equilibrium exists if $[1-\rho\gamma]/[1-\rho] < \pi/[\pi+n-1]$ and if M_0 is sufficiently low, and a rise in M_0 lowers the price level and raises output more than proportionally.

Finally we turn to σ in the range $0 < \sigma < 1$. This is the case of an elasticity of substitution between labour and capital which is greater than unity. Fig. 6 depicts a possible equilibrium:

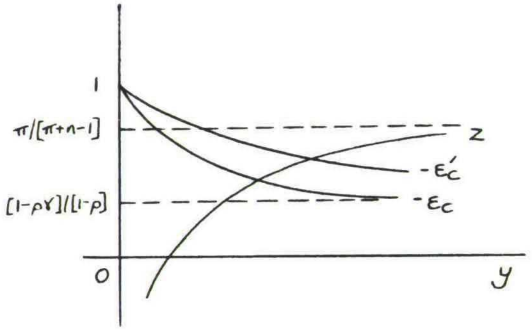


Fig. 6

Existence is once again not guaranteed, depending on the condition $[1-\rho\gamma]/[1-\rho] < \pi/[\pi+n-1]$ as in the preceding instance, though not on the value of M_0. A rise in M_0 raises output and the price level, in a similar way to the case of $\sigma < -[n-1]$.

Throughout this section, we have examined the question of the existence of an unemployment equilibrium in terms of the existence of a solution to equation (14). However, for the range $-[n-1] < \sigma < 0$, the question of the stability of such solutions was also raised. It is easy to confirm, by inspection of the relevant diagrams, that the stability condition that the z-curve should cut the $-\varepsilon_C$-curve from below is satisfied in all the other cases. A further question concerns the satisfaction of the second-order conditions for the union's maximisation problem. This is also examined in the appendix, where it is shown that none of the (stable) equilibria presented above are invalidated by violation of second-order conditions.

4. The Effectiveness of Fiscal Policy

We now introduce government spending, and thus a new component of aggregate demand. This is absent from Hart's original model. Government spending takes the form of purchases of firms' output, distributed evenly across locations. As is most common, any direct effects on households' utility or firms' costs are ignored. Spending is taken to be financed by money issues by the government, whose budget constraint is thus $G = M - M_0$, where G is the nominal value of spending. (Real spending is denoted as g.)

Apart from adding to aggregate demand, the introduction of government spending is important for its effect on the overall price elasticity of demand. Letting $\varepsilon_G \equiv [\partial g/\partial P][P/g]$, and $\varepsilon_D \equiv [\partial y^d/\partial P][P/y^d]$ where $y^d = c + g$, we have $\varepsilon_D = [c/y]\varepsilon_C + [g/y]\varepsilon_G$, i.e. overall price elasticity is a weighted average of the elasticities of the components. For a given level of consumption or of total demand, higher government spending will therefore raise (referring to absolute magnitudes) the overall elasticity if the elasticity of public sector demand exceeds that of private sector demand, and vice versa. The elasticity of public sector demand can be seen to depend on the price-indexation rule adopted by the government. For example, if nominal spending is 100% indexed (i.e., if spending is fixed in real terms), $\varepsilon_G = 0$; or if not indexed at all (i.e. if spending is fixed in cash terms), $\varepsilon_G = -1$. We suppose that spending is in general given by $G = xP^{\varphi}$, where x determines the scale of spending, and φ is an indexation parameter. This gives $\varepsilon_G = \varphi-1$, a constant.

The modification which the introduction of government spending requires to the equilibrium conditions is simply the replacement of ε_C by ε_D in (13), and the addition to demand of $g = xP^{\varphi-1}$ in (9). The expression for ε_D may be written in the form $\varepsilon_D = \varepsilon_C - [xP^{\varphi-1}/y][\varepsilon_C+1-\varphi]$, showing that the difference is the new subtracted term. This introduces a direct dependence of ε_D on (x,y), in addition to the previous dependence on P, of which ε_C is a function. With CES preferences, we have, more specifically:

$$-\varepsilon_D = 1 + \frac{\rho[1-\gamma]}{1-\rho}\frac{h(P)}{1+h(P)} - \frac{xP^{\varphi-1}}{y}\left[\varphi + \frac{\rho[1-\gamma]}{1-\rho}\frac{h(P)}{1+h(P)}\right] \tag{18}$$

When government spending is included in (9), the new reduced form for aggregate demand will show y to be a function of (P, M_0, x). The CES case gives:

$$y = M_0 P^{[1-\rho\gamma]/[\rho-1]} \delta^{1/[\rho-1]} \psi^{\rho/[1-\rho]}$$

$$+ x P^{\varphi-1} + x P^{\rho[1-\gamma]/[\rho-1] - 1+\varphi} \delta^{1/[\rho-1]} \psi^{\rho/[1-\rho]} \qquad (19)$$

To ensure this is downward-sloping, we need $\rho\gamma < 1$ as before, and also $\rho[1-\gamma]/[1-\rho] + 1-\varphi > 0$ (without which the curve becomes forward-bending for sufficiently high P). It is, even so, impossible to write an explicit inverse of (19), comparable to (15). This means P cannot be substituted out of (18) in order to obtain a single equation in y which would be the new version of (17). Instead, we can use (19) to substitute out y from ε_D and from z(y), and so obtain a single equation in P. For the sake of brevity, we do not reproduce the resulting equation here.

The question of the existence of an unemployment equilibrium in the presence of government spending may now be examined by sketching $-\varepsilon_D$ and z as functions of P, for different parameter ranges. Since this is an exercise very similar to the one carried out in Section 3, we shall not lengthen the exposition by repeating it here. Moreover, although comparative statics can be investigated using the diagrams, they do not directly reveal the effect on output, the variable of greatest interest. Hence we merely report a number of findings with regard to existence. For the range $\sigma < -[n-1]$, within which an equilibrium always exists if $x = 0$, too large a value of x, if coupled with a value of φ too close to unity, may cause non-existence. An obvious example of this is where $\varphi = 1$ (spending is fixed in real terms) and x is chosen to exceed the upper bound on production (see above); but problems also occur for less extreme cases. For the range $\sigma > -[n-1]$, within which an equilibrium is not guaranteed to exist if $x = 0$, a positive value of x may bring about existence. Amongst examples of this are several in which an arbitrarily small value of x appears to permit an equilibrium where none existed before. A simple case occurs when spending is fixed in real terms, expectations are unit-elastic and technology is Cobb-

Douglas. The equilibrium value of y then turns out to be in fixed ratio to g, providing emphatically "Keynesian" fiscal policy effects, despite the fact that monetary policy is neutral. Unfortunately this, and other equilibria brought into existence by arbitrarily small x, prove to violate the stability conditions referred to in the previous section, and must therefore be regarded as unsatisfactory.

To examine the effectiveness of fiscal policy where stable equilibria exist, we proceed by differentiation. The effect on the price level, dP/dx, may be computed by differentiating the model in the form of the equation $-\varepsilon_D(P) = z(P)$, whose derivation was described above. Using this in the aggregate demand function then enables the calculation of dy/dx. The expression obtained by this means is given in (20):

$$\frac{dy}{dx} = -\frac{P^{\varphi-1}}{Ph(P)}\left\{\frac{1-\rho\gamma}{\rho-1}\frac{M_0}{Py}\left[\varphi+\frac{\rho[1-\gamma]}{1-\rho}\frac{h(P)}{1+h(P)}\right]+\left[\frac{\rho[1-\gamma]}{1-\rho}\right]^2\frac{h(P)}{1+h(P)}\right\}\Delta^{-1} \tag{20}$$

This has been evaluated at x = 0, being the case which is simplest, and of greatest initial interest. The denominator $\Delta \equiv \partial z/\partial P - \partial(-\varepsilon_D)/\partial P$, and must be negative to satisfy the stability requirement (see the appendix).

From (20) we may immediately derive:

Proposition 3 Government spending will affect output even when monetary policy does not, and in these circumstances its effect will be negative.

To see this, we know that monetary policy is neutral when and only when $\rho[1-\gamma] = 0$, which reduces (20) to $-\varphi[P^{\varphi-1}/Ph(P)]\{[1-\rho\gamma]/[\rho-1]\}[M_0/Py]\Delta^{-1}$. Given existing assumptions, this is unambiguously negative unless $\varphi = 0$, in which case it is zero. The result that, except in a very special case, fiscal policy still affects output in the absence of monetary policy effectiveness in imperfectly competitive models, was first noted by Wren-Lewis (1985). An implication of this, he points out, is that such models do not possess a "natural rate" of output or employment, if the latter is taken to imply independence from both monetary and fiscal policy.

Proposition 3 nevertheless gives no encouragement to a Keynesian view of fiscal policy: rather the reverse. An intuitive explanation of the negative effect starts from the fact that, when monetary policy is neutral, it was seen that private sector demand elasticity ε_C is unity (in absolute magnitude) by virtue of unit-elastic expectations or Cobb-Douglas preferences ($\rho[1-\gamma] = 0$). Public sector demand elasticity ε_G has the (absolute) value $1-\varphi$. Since this is less than ε_C, an increase in x must cause a reduction in the overall demand elasticity ε_D, by the averaging argument given earlier. Any such reduction lowers ε, increasing the monopoly power of trade unions, the effect of which is a cutback in employment and thus output. The fiscal impotence which occurs when $\varphi = 0$ is because in this case ε_C and ε_G are exactly equal, and moreover both exogenous. If negative values of φ were permitted fiscal policy would acquire an expansionary role, but this would imply an unusual indexation rule of *reducing* nominal expenditure in response to a price rise.

Turning to the more general case where $\rho[1-\gamma] \neq 0$, it is helpful to substitute out $h(P)$ and $M_\bullet/Py$ from (20) using the equilibrium conditions, obtaining an expression in terms of the equilibrium value of z (or equivalently, of $-\varepsilon_D$):

$$\frac{dy}{dx} = \frac{P^{\varphi-1}}{Ph(P)}\frac{1-\rho\gamma}{1-\rho}[1-z]\left[\frac{1-\rho\gamma}{1-\rho}-z\right]^{-1}\left\{1-z+\frac{\rho[1-\gamma]}{1-\rho\gamma}\left[\frac{1-\rho\gamma}{1-\rho}-z\right]-\varphi\right\}\Delta^{-1} \tag{20'}$$

From Figure 1, we know that the equilibrium value of $-\varepsilon_D$ and thus z must lie between the values 1 and $[1-\rho\gamma]/[1-\rho]$, where $[1-\rho\gamma]/[1-\rho] \gtrless 1$ as $\rho[1-\gamma] \gtrless 0$. Thus $\rho[1-\gamma] > 0$ implies $1-z < 0$ and $[1-\rho\gamma]/[1-\rho] - z > 0$, and $\rho[1-\gamma] < 0$ implies the reverse. The product of the two terms is therefore always negative. This shows that the sign of dy/dx in (20') is the same as that of the expression $\{.\}$. In what follows, we label this as Ω.

We may now prove:

Proposition 4 If government spending is fixed in cash terms (i.e. $\varphi = 0$), then whenever monetary policy is effective (i.e. $\rho[1-\gamma] \neq 0$), fiscal policy will also have a positive effect on output for some (and possibly the entire) range of money supply values. If government spending is indexed (i.e. $\varphi > 0$), then in order for the same to be true $\rho[1-\gamma]$ must differ (positively or negatively) from zero by a margin which is strictly increasing in φ.

Proof From the foregoing, the signs of the component terms of Ω are:

$$\Omega \equiv 1-z + \frac{\rho[1-\gamma]}{1-\rho\gamma}\left[\frac{1-\rho\gamma}{1-\rho} - z\right] - \varphi \tag{21}$$

if $\rho[1-\gamma] < 0$	(+)	(−)	(−)
if $\rho[1-\gamma] > 0$	(−)	(+)	(+)

First assume $\varphi = 0$. If $\rho[1-\gamma] < 0$, Ω is unambiguously positive. If $\rho[1-\gamma] > 0$, its sign depends on z. Since it is decreasing in z, it is maximised by minimising z. The minimum possible equilibrium value of z, and thus $-\varepsilon_D$, when $\rho[1-\gamma] > 0$ is $z = 1$ (see Figure 1). This gives:

$$\Omega = \frac{[\rho[1-\gamma]]^2}{[1-\rho\gamma][1-\rho]} - \varphi \tag{22}$$

which is positive at $\varphi = 0$. If z were instead at its maximum of $[1-\rho\gamma]/[1-\rho]$, we would have:

$$\Omega = -\frac{\rho[1-\gamma]}{1-\rho} - \varphi \tag{23}$$

which is negative at $\varphi = 0$. Equilibrium with $\rho[1-\gamma] > 0$ we know requires $o < -[n-1]$. This was depicted in Figure 4. As M_0 tends to zero, the $-\varepsilon_D$ curve shrinks left causing equilibrium z to approach unity; as M_0 tends to infinity it expands right causing z to approach $[1-\rho\gamma]/[1-\rho]$. Therefore Ω is positive for M_0 sufficiently low, negative for M_0 sufficiently high.

Now assume $\varphi > 0$. This clearly makes a positive Ω harder. When $\rho[1-\gamma] < 0$, Ω's sign now depends on z. Ω is decreasing or increasing in z according as $-\rho[1-\gamma]/[1-\rho\gamma] < 1$ or > 1, respectively. Since $-\rho[1-\gamma]/[1-\rho\gamma] = \{-\rho[1-\gamma]/[1-\rho]\}/\{1 + \rho[1-\gamma]/[1-\rho]\}$, this means respectively as $-\rho[1-\gamma]/[1-\rho] <$

$1/2$ or $> 1/2$. If it is less than $1/2$, maximum Ω is where z is minimised, which with $\rho[1-\gamma] < 0$ is at $z = [1-\rho\gamma]/[1-\rho]$, and therefore given by the expression (23). If it is greater than $1/2$, maximum Ω is where z is maximised, which with $\rho[1-\gamma] < 0$ is at $z = 1$, and therefore given by the expression (22). For the possibility of $\Omega > 0$ when $\rho[1-\gamma] < 0$, we therefore need both (22) and (23) to be positive. As seen already, when $\rho[1-\gamma] < 0$, Ω is decreasing in z and maximised at $z = 1$ at a value given by (22), which must thus again be positive for the possibility of $\Omega > 0$. These conditions are necessary, but they are only also sufficient in the case of equilibria where $\sigma < -[n-1]$: in other cases, the extreme values of z used in the argument may lie outside those actually attainable by variation of $M_{\bullet}$, as may be appreciated by reconsidering Figures 3-6.

A link between the expressions in (22) and (23) may be observed by rearranging (22) as $\{\rho[1-\gamma]/[1-\rho]\}^2/\{1+\rho[1-\gamma]/[1-\rho]\} - \varphi$. The conditions for $\Omega > 0$ for some z may then be represented using Figure 7:

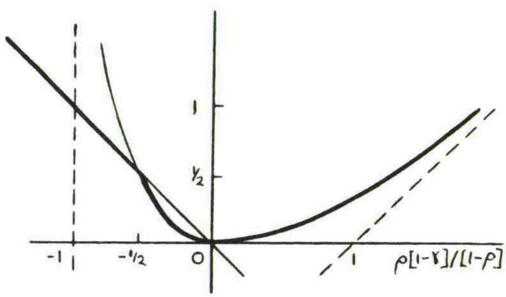


Figure 7

For given φ, $\rho[1-\gamma]/[1-\rho]$ must lie outside the range between the intersection of the heavily-drawn curve and a horizontal line of height φ. This condition is clearly harder to satisfy the greater is φ. Since $\rho[1-\gamma]/[1-\rho]$ cannot be less than -1 (which would violate $\rho\gamma < 1$), at $\varphi = 1$ (spending fixed in real terms) it can only be satisfied for positive values.

An intuitive explanation for the ambiguous impact of fiscal policy which is expressed in Proposition 4 is that a rise in government spending has two conflicting effects. The first, which arises even under monetary neutrality, is the "elasticity" effect dealt with in Proposition 3, whereby higher public spending, to the extent that it has lower price elasticity than private spending, reduces overall price elasticity, so raising the monopoly power of labour and reducing output. The second, which arises from the same source as monetary non-neutrality, i.e. non-unit-elastic expectations, is Hart's "fixprice" effect as studied in Section 3, whereby the endogenous failure of the price level fully to respond to demand shocks channels some of the increase into output. This may be seen more explicitly in (20). The squared term, which is zero if $\rho[1-\gamma] = 0$, represents the fixprice effect, and is always non-negative. The term in $M_{\bullet}/Py$ represents the elasticity effect. The sign of the term in φ which it multiplies is ambiguous, but from (18) it may be seen to determine whether an increase in g/y raises or lowers $-\varepsilon_D$, i.e. to depend on whether $-\varepsilon_G$ > or < $-\varepsilon_C$, respectively. In the latter case it is positive, whence the whole elasticity-effect term is negative, counteracting the fixprice-effect term.

5. Conclusions

In all of our extensions of Hart's model, the "Keynesian features which were the particular interest of Hart and which are the focus here, carry over and are in many cases strengthened. In particular, if production technology is sufficiently convex, an unemployment equilibrium will exist for all values of the elasticity of price expectations, not just a restricted range away from unity. In any unemployment equilibrium, a monetary expansion will raise output given only that expectations elasticity is not unity, and regardless of whether it is above or below. These findings together mean that, within this technology range, monetary policy is sure to be effective, except in the chance case that expectations elasticity should equal one. Outside the range of the most convex technology, an unemployment equilibrium is still always possible, and where it occurs monetary policy is always effective, though it now depends on expectations elasticity lying in a particular range away from unity. For no range of technology within the CES parameterisation is full employment automatically guaranteed. In the case of fiscal policy, if government spending is fixed in nominal terms, then whenever monetary policy is effective, a fiscal expansion will also raise output, though under certain conditions a perverse effect may occur for some sub-range of money supply values. If government spending is indexed, either partially or wholly, to the price level, then for an expansionary effect it is necessary for expectations elasticity to lie away from unity by a sufficient margin. These ambiguities result from the fact that the price elasticity of public sector demand is assumed in general to be smaller than that of private sector demand, which generates an "elasticity" effect which is in opposition to Hart's "fixprice" effect.

It is clear that the Keynesian features of the model depend on non-unit-elastic expectations, and there may be those who will argue that the only plausible elasticity is unity. However this would be to reject the whole Hicksian concept of temporary equilibrium, which is a short-run one in which it is recognised that expectations are formed by rule of thumb, so that there can be no powerful reasons for preferring any one value of the elasticity over any other. The strength of the monetary policy result here is that it is not necessary to argue that the elasticity takes some particular value or lies in some particular range in order to get a positive effect, but merely to claim that it will take some arbitrary value which could be either greater or less than one. Moreover it is false to imagine that, if the opportunity of learning were conceded, then only unity would prove to be consistent with "rational" expectations. This depends for example, on the future course of monetary policy: if the current rise in the money supply is known to be part of a permanently higher rate of growth, then the expected inflation rate will change and a unit elasticity would be "irrational". This suggests that 'Hartian' imperfect competition may provide scope for models of monetary effectiveness under rational expectations, opening up a possible avenue for future research.

Appendix: Stability and Second-Order Conditions

Stability

The adjustment mechanism we propose is adapted from early work on the partial equilibrium stability of the Cournot oligopoly model by Theocharis (1960) and other authors as reviewed in Friedman (1977). In view of the fact that this mechanism has been heavily criticised from a game-theory perspective by later writers, it should be stressed that our use of it is not intended as a serious attempt to construct a dynamic model, but simply as a device for discriminating between multiple equilibria, on the same level as the "tatonnement" mechanism used in the theory of perfect competition. All that is necessary is that any conclusions as to stability should not be completely reversed by a more realistic mechanism, which would seem very unlikely.

The solution to the ith union's optimisation problem may be represented as the static reaction function, $\ell_i = r(\ell', M_0+Y)$. For a dynamic story, suppose that in any period t,

$$\ell_{it} = r(\sum_{i \neq j} \ell_{jt-1}, M_0+Y_{t-1})$$

Given the ℓ_{it}'s and thus $\ell_t = \Sigma\ell_{it}$, the wage W_t is determined from the market labour demand function $\ell_t = g(W_t, M_0+Y_t)$. The production function gives $y_t = f(\ell_t)$, and together with the aggregate demand function $P_t = D(y_t/M_0)$, this generates Y_t. By this means, the adjustment path may be mapped out.

Since the concern is with local stability, we linearise about the equilibrium. Denoting deviations from equilibrium values by ^{D}, and partial derivatives of functions, evaluated at the equilibrium, by primes or letter subscripts, the model is reduced to the equations:

$$\ell_{it}^D = r_\ell \sum_{i \neq j} \ell_{jt-1}^D + r_m Y_{t-1}^D$$

$$Y_t^D = [P + yD'/M_0] f' \ell_t^D$$

By manipulating these we may derive the following first-order difference equation in ℓ_t^D:

$$l_t^D = \Big[[n-1]r_l + [P+yD'/M_o]f'nr_m\Big]l_{t-1}^D$$

For stability, we need that the absolute value of the coefficient $[.]$ on l_{t-1}^D should be less than unity.

To relate this condition to the diagram of the $-\varepsilon_D$ and z curves, expressions for r_l and r_m must be computed, as must expressions for the slopes $\partial z/\partial y$, $\partial(-\varepsilon_D)/\partial y$. Lengthy but mechanical calculations then enable us to show that the above coefficient, minus one, may be reformulated as:

$$-ny\Big[\frac{\varepsilon_S}{\varepsilon_S+\varepsilon_D}\Big]^2\Big[\frac{\partial z}{\partial y}-\frac{\partial(-\varepsilon_D)}{\partial y}\Big]\Big[\frac{\partial\varepsilon}{\partial W}\frac{W}{\varepsilon}+\varepsilon+1\Big]^{-1}$$

This must be negative for stability. Satisfaction of the second-order conditions for the union's optimisation problem in fact requires that $[\partial\varepsilon/\partial W][W/\varepsilon]+\varepsilon+1$ should be positive. It can thus be seen that stability requires $\partial z/\partial y > \partial(-\varepsilon_D)/\partial y$, i.e. that the z curve should cut the $-\varepsilon_D$ curve from below.

Second-order conditions

The condition that $[\partial\varepsilon/\partial W][W/\varepsilon]+\varepsilon+1 > 0$ is necessary for a maximum is readily derived from the union's optimisation problem. To examine whether this is satisfied in equilibrium, it is convenient to relate it to $\partial z/\partial y - \partial(-\varepsilon_D)/\partial y$. By obtaining comparable expressions for these two, it may be shown that the critical terms which determine the sign of each, differ by the following quantity:

$$\Lambda \equiv \underset{(?)}{-\frac{\rho[1-\gamma]}{1-\rho}\frac{1}{1+h(P)}\frac{[n-1+\sigma]\sigma F(y)}{\varepsilon_F\varepsilon_S[n+\varepsilon_F]}} + \underset{(+)}{\Big[\frac{n}{n+\varepsilon_F}\Big]^2\frac{n-1}{n}}$$

where $F(y) \equiv \alpha k^{\sigma}[y/A]^{-\sigma/\pi}$. A consequence of this is that if $\partial z/\partial y - \partial(-\varepsilon_D)/\partial y > 0$, then $\Lambda \geq 0$ is sufficient (though not necessary) for $[\partial\varepsilon/\partial W][W/\varepsilon]+\varepsilon+1 > 0$.

The second term in Λ is unambiguously positive. Therefore if $\rho[1-\gamma] = 0$ or $\sigma = 0$, Λ is definitely positive. These cases aside, the three denominators in the first term are all positive, whence sufficient conditions for the first term, and therefore for Λ, to be positive are either (i) $\rho[1-\gamma] < 0$ with $\sigma < -[n-1]$ or $\sigma > 0$, or (ii) $\rho[1-\gamma] > 0$ with $-[n-1] < \sigma < 0$. Reference to Section 3 shows that all remaining equilibria satisfy (i), with the exception of the equilibrium which is the subject of Proposition 2, which satisfies neither. However, this equilibrium only exists for $M_{\bullet}$, and thus y, sufficiently small. With $\sigma < 0$, as y tends to zero, F(y) and thus the first term tend to zero (it may be confirmed that none of the denominators tend to zero), ensuring that $\Lambda > 0$ holds for sufficiently small y.

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