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Abstract

In this paper we consider the Newsvendor Problem. Intuition may lead to the hypothesis that in this stochastic inventory problem a higher demand variability results in larger variances and in higher costs. In a recent paper, Song (1994a) has proved that the intuition is correct for many demand distributions that are commonly used in practice, such as for the Normal distribution function. However, this paper shows that there exist demand distributions for which the intuition is misleading, i.e., for which larger variances occur in combination with lower costs. To characterize these demand distributions we use stochastic dominance relations.

Keywords: Newsvendor problem, demand variability, stochastic dominance.

We consider the traditional single-item single-period Newsvendor Problem with continuous product demand. Let the demand D be randomly distributed with distribution function $F(\cdot)$, finite mean μ and variance σ^2. There is an underage cost p and an overage cost h per unit of product. If Q products are ordered at the beginning of the period, the expected total cost is

$$\begin{aligned} C(Q) &= h\int_0^Q (Q-x)F(dx) + p\int_Q^\infty (x-Q)F(dx) \\ &= p(\mu - Q) + (h+p)\int_0^Q F(x)\,dx. \end{aligned} \tag{1}$$

In the sequel we denote the order quantity for which (1) is minimized by Q^* and we indicate the optimal cost $C(Q^*)$ as the buffer cost. Recently, Song (1994a, 1994b) showed for a particular definition of demand variability that the buffer cost will increase with increasing variability. The key question in this note is whether $C(Q^*)$ will always increase when demand gets more variable.

1 Preliminaries

We consider two inventory systems $i = 1, 2$, with demands D_i and distribution functions F_i. Both systems have underage cost p and overage cost h. We assume that the demands have equal means but different variances.

Condition 1.1

$$\mu_1 = \mu_2 = \mu.$$

Condition 1.2

$$\sigma_1^2 \le \sigma_2^2.$$

One finds in the literature on stochastic dominance relations a family of rules to compare variability between the two demands (Fishburn and Vickson (1978)).

Definition 1

Demand D_2 is more n-variable than demand D_1, denoted by

$$D_1 \ge_n D_2,$$

if

$$H_n(x) \ge 0 \quad \textit{for all} \quad x \ge 0,$$

where

$$\begin{aligned} H_1(x) &= F_2(x) - F_1(x), \\ H_n(x) &= \int_0^x H_{n-1}(t)\,dt \quad (n = 2, 3, \ldots). \end{aligned}$$

In Lemma 1 (below) we show that 2-variability implies higher demand variances in combination with higher buffer costs. The proof is based on the following theorem.

Theorem 1 (Fishburn (1980))
If $D_1 \ge_n D_2$ for some $n \ge 2$ then

$$\left.\begin{aligned} \mu_1 &= \mu_2 \\ \sigma_1^2 &\ne \sigma_2^2 \end{aligned}\right\} \Rightarrow \sigma_1^2 < \sigma_2^2.$$

Lemma 1
Under Condition 1.1,

$$D_1 \ge_2 D_2 \Rightarrow \begin{cases} \sigma_1^2 \le \sigma_2^2 \\ C_1(Q_1^*) \le C_2(Q_2^*) \end{cases}$$

Proof. Apply Theorem 1, the definition of Q_1^* and (1) to obtain that

$$\begin{aligned} C_2(Q_2^*) - C_1(Q_1^*) &\geq C_2(Q_2^*) - C_1(Q_2^*) \\ &= (h+p)\left(\int_0^{Q_2^*} F_2(x)\,dx - \int_0^{Q_2^*} F_1(x)\,dx\right) \\ &= (h+p)H_2(Q_2^*) \geq 0. \qquad \square \end{aligned}$$

Song (1994a) proves that Lemma 1 holds also under alternative stochastic orderings, such as the increasing convex ordering of the demands ($D_2 \geq_{\text{ic}} D_1$), and the cut criterion ordering ($D_2 \geq_{\text{cut}} D_1$). These results follow easily from the observation that these orderings are stronger than 2-variability.

Lemma 2 (Sections 1.3 and 1.5 in Stoyan (1983))
Under Condition 1.1,

$$D_2 \geq_{\text{cut}} D_1 \Rightarrow D_2 \geq_{\text{ic}} D_1 \Rightarrow D_1 \geq_2 D_2.$$

2 The opposite effect

Theorem 1 and the proof of Lemma 1 suggest to investigate n-variability of higher orders ($n \geq 3$) to verify whether the opposite effect may occur, i.e., higher demand variances in combination with lower buffer costs. Indeed, Theorem 2 below states sufficient conditions under which the opposite effect occurs.

Theorem 2 (Sufficiency)
Suppose that Condition 1.1 holds.
(i) If $D_1 \geq_n D_2$ for some $n \geq 3$, then $\sigma_1^2 \leq \sigma_2^2$.
(ii) If $H_2(Q_1^*) < 0$ then $C_1(Q_1^*) > C_2(Q_2^*)$.

Proof. Part *(i)* is Theorem 1. The proof of part *(ii)* is analogous to Lemma 1:

$$\begin{aligned} C_2(Q_2^*) - C_1(Q_1^*) &\leq C_2(Q_1^*) - C_1(Q_1^*) \\ &= (h+p)\left(\int_0^{Q_1^*} F_2(x)\,dx - \int_0^{Q_1^*} F_1(x)\,dx\right) \\ &= (h+p)H_2(Q_1^*) < 0. \qquad \square \end{aligned}$$

In Lemma 3 below we state necessary and sufficient conditions under which the opposite effect occurs. For this purpose we use the relation

$$H_3(\infty) = \int_0^\infty H_2(x)\,dx = \int_0^\infty \int_0^x (F_2(y) - F_1(y))\,dy\,dx = \frac{1}{2}(\sigma_2^2 - \sigma_1^2). \qquad (2)$$

This relation is derived by using the excess equilibrium distribution functions $G_i(x) = \frac{1}{\mu_i}\int_0^x (1 - F_i(y))\,dy$ and Condition 1.1.

Lemma 3 (Necessity and sufficiency)
Under Condition 1.1,

$$\sigma_1^2 \leq \sigma_2^2 \quad and \quad C_1(Q_1^*) \geq C_2(Q_2^*)$$

$$\Leftrightarrow$$

$$H_3(\infty) \geq 0 \quad and \quad H_2(Q_1^*) \leq \frac{p}{h+p}(Q_2^* - Q_1^*) - \int_{Q_1^*}^{Q_2^*} F_2(x)\,dx.$$

Proof. The lemma follows directly from (2) and from

$$\begin{aligned} C_2(Q_2^*) - C_1(Q_1^*) &= p(Q_1^* - Q_2^*) + (h+p)\left(\int_0^{Q_2^*} F_2(x)\,dx - \int_0^{Q_1^*} F_1(x)\,dx\right) \\ &= p(Q_1^* - Q_2^*) + (h+p)\left(H_2(Q_1^*) + \int_{Q_1^*}^{Q_2^*} F_2(x)\,dx\right). \quad \square \end{aligned}$$

3 Examples

Traditional families of demand densities are (truncated) Normal, Lognormal, Beta, Gamma, Weibull and Uniform (Silver & Peterson (1985), Appendix B in Tijms (1994)). When both densities of D_1 and D_2 are taken from one of these families, Condition 1 implies that $D_1 \geq_2 D_2$ (verify the tables in Appendix 1 of Stoyan (1983)). Hence, Lemma 1 applies, and higher demand variability leads to higher buffer cost in these cases.

However, Example 1 below shows that the opposite effect may occur when the demand densities belong to the same family. Example 2 illustrates that Theorem 2 may apply when the demand densities belong to different families.

Example 1

The nonsymmetric triangular density f is continuous, piecewise linear and characterized by three parameters $0 \leq a < b < c$. The function is nonzero on the interval (a, b) and it attains its maximum at c. Basic algebra yields expressions for the mean, the variance, and the H_n functions. We consider two nonsymmetric triangular densities f_i, $i = 1, 2$. The numerical example below illustrates that the parameters a_i, b_i, c_i can be set such that the conditions of Theorem 2 or Lemma 3 apply.

(a) Let

$$a_1 = 1, b_1 = 2, c_1 = \frac{11}{2}; \qquad a_2 = 0, b_2 = 4, c_2 = \frac{9}{2}, \qquad h = 1, p = 6.$$

It is easily verified that $\mu_1 = \mu_2 = 17/6$, $0.9306 = \sigma_1^2 < \sigma_2^2 = 1.0139$, so that Condition 1 holds. Furthermore, $D_1 \geq_3 D_2$, $Q_1^* = 4$, and $H_2(Q_1^*) = -0.0546$. Hence, Theorem 2 applies. Indeed, $Q_2^* = 3.9279$, $C_1(Q_1^*) = 1.6667 > 1.2883 = C_2(Q_2^*)$.

(b) Let

$$a_1 = 0, b_1 = 4, c_1 = 6; \qquad a_2 = 1, b_2 = 2, c_2 = 7, \qquad h = 5, p = 1.$$

It is easily verified that $\mu_1 = \mu_2 = 10/3$, $28/18 = \sigma_1^2 < \sigma_2^2 = 31/18$, so that again Condition 1 holds. However, there is no $n \geq 1$ for which $D_1 \geq_n D_2$ or $D_2 \geq_n D_1$ (Ridder et al. (1996)). In this case, Lemma 3 applies with $Q_1^* = Q_2^* = 2$:

$$H_3(\infty) = \frac{1}{12} > 0, \quad -\frac{1}{18} = H_2(Q_1^*) < \frac{p}{h+p}(Q_2^* - Q_1^*) - \int_{Q_1^*}^{Q_2^*} F_2(x)\,dx = 0.$$

Example 2

Let D_1 have a Lognormal ($\mu = -0.1, \sigma^2 = 0.2$) density and D_2 a Gamma ($\lambda = 4$, $\alpha = 4$) density, with cost factors $h = 1$, and $p = 24$. Condition 1 holds (with $\mu = 1, 0.2214 = \sigma_1^2 < \sigma_2^2 = 0.25$); $H_3 \geq 0$; $Q_1^* = 1.9797$; $H_2(Q_1^*) = -0.0013$. Hence, Theorem 2 applies. Indeed, $Q_2^* = 2.0214$, $C_1(Q_1^*) = 1.4052 > 1.3712 = C_2(Q_2^*)$.

Remark. Note from the above examples that the optimal ordering quantities Q_i^* may increase as well as decrease with increasing demand variance. In Example 1(a):

$Q_1^* > Q_2^*$, in Example 2: $Q_1^* < Q_2^*$. Note further that the opposite effect may occur when $h < p$ (Example 1(a)), but also when $h > p$ (Example 1(b)). Assuming that the conditions of Theorem 2 hold, these phenomena are explained by the following equivalence.

$$Q_2^* \geq Q_1^* \quad \Leftrightarrow \quad Q_1^* < F_2^{-1}\left(\frac{p}{p+h}\right).$$

4 Conclusion

The conclusion of this paper is that a reduction of the demand uncertainty in stochastic production and inventory systems is economically favorable for most demand distributions that are commonly used in practice. However, for some demand distributions a reduction of the demand uncertainty will *not* result in the desired cost reduction. Whether cost reduction occurs, depends on many factors such as the definition of uncertainty, the structure of the demand distributions, and the ratio between the overage and underage costs.

Our analysis applies to the class of inventory models where the cost function has the form (1). Besides the classical Newsvendor Problem, dynamic inventory systems controlled by a base stock policy belong to this class (D stands for the lead time demand and Q for the base stock level). Furthermore, our analysis carries over easily to discrete demands distributions (see Ridder et al. (1996)).

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