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RESEARCH MEMORANDUM

ANALYSIS AND COMPARISON OF TWO STRATEGIES FOR MULTI-ITEM INVENTORY SYSTEMS WITH JOINT REPLENISHMENT COSTS

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Abstract:

Multi-item inventory systems with joint replenishment costs are considered under constant deterministic demand. Two different types of strategies are distinguished: direct grouping strategies and indirect grouping strategies. Different heuristics are reviewed and compared for different strategies. The performance of the strategies is measured as the percentage cost savings of using a joint replenishment strategy instead of an independent strategy. This performance is compared by means of simulation. Regression analysis is used to summarize the output of several simulation experiments.

Keywords: Multi-item inventory systems, heuristics, simulation, regression

1. Introduction

Joint replenishment strategies are used in a multi-item inventory systems. A characteristic of multi-item systems is the existence of some kind of interaction among the items. Joint replenishment strategies are based on interaction of the set-up or order costs. These costs can be subdivided into major and minor costs. Interaction is caused by the fact that the major set-up cost is independent of the number of items in the replenishment. In addition to the major set-up cost, there is a minor set-up cost, charged to each particular item included in the replenishment. Cost savings can be obtained by coordinating the replenishments of several items. The major set-up cost is then shared if two or more items are jointly replenished. In many practical situations it makes sense to coordinate replenishments of individual items. If several items are purchased from the same supplier; fixed order costs can be shared if two or more items are jointly replenished. Joint replenishments may also be attractive if a group of items use the same mode of transport or production facility.

In the case of constant demand, the strategies can be classified into two classes, which will be called "indirect grouping strategies" and "direct grouping strategies". A group is defined as the set of those items that have the same replenishment cycle. The replenishment cycle is the time between two subsequent replenishments of a particular item. Items of the same group are jointly replenished.

Using the indirect grouping strategy, a family replenishment is made at constant intervals. The replenishment cycle of each item (or group) is an integer multiple of this basic cycle interval. The problem is to determine the basic cycle interval and the replenishment frequency of all items simultaneously. A group is then (indirectly) formed by those items that have the same replenishment frequency. In the last two decades several authors have encountered this sort of joint replenishment problem. For extended reviews of joint replenishment inventory strategies we refer to Aksoy and Erenguc (1988) and Goyal and Satir (1989). Another approach, which is not mentioned in the surveys of Aksoy and Erenguc and Goyal and Satir, is the formulation of a direct grouping strategy. Here, the replenishment cycles of the groups are generally not an integer multiple of the shortest (basic) cycle. In this case the problem is to form (directly) a predetermined number of groups in such a way that the total relevant costs of the items in the family are as low as possible.

To the best of our knowledge, a comparison between indirect grouping and direct grouping strategies has never been made. The purpose of our study was twofold: first, to find out whether the direct grouping strategy outperforms the indirect grouping strategy in some situations; secondly to determine the effect of some variables on the performance of joint replenishment strategies; this performance was measured as the percentage cost savings of using a joint replenishment strategy instead of an independent strategy.
Section 2 gives a short review of the literature on the joint replenishment problem, together with a decision which algorithms are to be used for comparing direct grouping and indirect grouping strategies. The experimental design and simulation results are described in section 3. Finally, section 4 gives the conclusions.

2. Literature review

The joint replenishment problem was investigated under a set of assumptions that are the same as for the classical economic order quantity (EOQ) model, except for the joint set-up cost. Due to these assumptions the relevant cost factors are the set-up costs and the carrying inventory cost. We will review the literature on both grouping strategies; see also figure 1 later on.

2.1. Indirect grouping

The decision variables in the indirect grouping model are the basic cycle time (T) and the frequency (number of basic cycles) of ordering for each item (k_i). The objective is to find a combination (T, k_i) such that the total relevant cost (TRC) of the family is as low as possible:

$$TRC = \frac{1}{T} \left(A + \sum_{i=1}^{N} \frac{a_i}{k_i} \right) + \frac{T}{2} \sum_{i=1}^{N} k_i D_i h_i \ , \qquad (1)$$

s.t. $k_i \in \{1,2,3,\ldots\}$,

where N : number of items in the family.

A : major set-up cost.

a_i: minor set-up cost.

D_i: demand per period for item i.

h_i: inventory carrying cost per unit of item i per period.

T : basic cycle time, the time between two successive family replenishments.

k_i: the number of basic cycles between two successive replenishments of item i.

By taking the first derivative of TRC with respect to T and k_i (k_i is then treated as a continuous variable) we can derive the optimal basic cycle time, $T(k_i)$, and the optimal frequency of ordering ,$k_i(T)$. However, T can not be determined without knowing k_i, and vice versa. Several authors have encountered this problem: Brown (1967), Goyal(1973a,1973b,1974a,1974b, 1979,1988), Silver (10), Kapsi and Rosenblatt (1983,1985), etc. Only one of them (Goyal (1974a)) presented an (enumerative) algorithm that gives the global optimum. Although Goyal's approach results in an optimal solution, it may be computationally prohibitive. Therefore, heuristic algorithms were developed. The heuristics may be classified into two classes: iterative algorithms and single iteration algorithms.It is not our intention to give a detailed review of the literature. We refer to the extensive surveys of Aksoy and Erenguc (1988) and Goyal and Satir (1989).

In a simulation study Kapsi and Rosenblatt (1985) compared iterative algorithms due to Brown (1967) and Goyal (1974b), and single iteration algorithms due to Silver (1976), Goyal and Belton (1979) and Kapsi and Rosenblatt (1983). They also suggested a combined approach. This approach uses the single iteration heuristic of Silver (1976) with the modification of Goyal and Belton (1979) as starting point in the iterative algorithm of Goyal (1974b). The heuristic with the smallest average deviation from the optimal solution) was the combined approach, followed by that of Goyal, Brown, Kapsi and Rosenblatt, Goyal and Belton and finally that of Silver. Kapsi and Rosenblatt found that the iterative algorithms, as expected, are more time consuming, but the difference in computation time appeared not to be significant. This was affirmed by our own findings. Consequently, we used the combined heuristic for comparing the direct grouping strategy with the indirect grouping strategy. The algorithm is listed in part one of appendix A.

2.2 Direct grouping strategies

The main difference between indirect grouping and direct grouping strategies is that the replenishment cycles of the groups formed by indirect grouping are a multiple integer of some basic cycle time, whereas this is generally not the case for groups formed by direct grouping. Note that the number of groups is an output variable in indirect grouping, whereas the number of groups is predetermined in direct grouping. Hence, the direct grouping problem is to divide N items into M groups such that the set-up and inventory carrying costs are minimized. The groups must form disjunct sets of the items in the family.

The minimalisation problem is:

$$TRC(S_1,..,S_M) = \sum_{j=1}^{M} \left(\frac{A_j}{T_j} + \tfrac{1}{2} T_j * H_j \right), \tag{2}$$

where M : number of groups to be formed.
S : set of all items in the family.
S_j : set of items in group j.
T_j : replenishment cycle time of group j, the time between two successive replenishments of all items in group j.
A_j : total set-up cost per replenishment of group j.

$$A_j = (A + \sum_{i \in S_j} a_i), \qquad H_j = \sum_{i \in S_j} h_i D_i.$$

The problem of dividing N items into M groups is hard, because there may be numerous combinations. Fortunately, Chakravarty (1981) and Bastian (1986) proved a theorem that they call the "consecutiveness property". The property states that the optimal groups will be consecutive with the ratio $D_i h_i / a_i$. So, when the items are arranged in increasing order with respect to the ratio $D_i h_i / a_i$, the optimal groups can be created from this sequential list.

For example: consider a set of items {1,2,3,4}, which is arranged in increasing order of the ratio $D_i h_i / a_i$ (so, item 1 is the item with the smallest ratio). In this case, the groups S_1={1,2} and S_2={3,4} are consecutive, but S_1={1,3} and S_2={2,4} are not.

Using this ranking scheme, several authors proposed procedures for direct grouping: Page and Paul (1976), Chakravarty (1981,1985), and Bastian (1986). We note that in the orginal papers of Page and Paul and Chakravarty the major set-up cost is not incorporated explicitly. The algorithms of Page and Paul and Chakravarty (1981) can be adjusted easily for the major set-up cost. However, we could not adjust the heuristic of Chakravarty (1976).

Chakravarty (1981) uses dynamic programming to create groups. This algorithm identifies the global optimum of the minimalisation problem in (2). However, computer time increases exponentially with the size of the problem.

After analyzing the heuristics of Bastian (1986), Page and Paul (1976) and Chakravarty (1985)), we found that Bastian's heuristic was the best. This simple heuristic starts with N consecutive groups (= the number of items in the family). Each iteration combines two neighbouring groups such that the increase (decrease) of the objective function is minimal (maximal). The procedure terminates when M groups are formed. Bastian proved that this grouping heuristic is optimal when the major set-up cost is zero.

We simulated many inventory situations. These test examples showed that the deviations of Bastian's solution from Chakravarty's optimal solution are very small, even with a high major set-up cost. We also analyzed the computer time needed for both algorithms. As expected, the difference in computer time appeared to be important. Therefore we used Bastian's algorithm, which is outlined in part two of appendix A, for comparing direct grouping and indirect grouping strategies.

Figure 1 summarizes the research papers that were mentioned in this review.

3. Experimental design and simulation results

Several inventory situations with constant demand were simulated to compare the performances of both direct grouping and indirect grouping strategies. Besides an analysis of the differences between these two ways of grouping, the performances of the strategies were compared with the performance of an independent single-item strategy. We used regression analysis to summarize the output of several simulation runs.

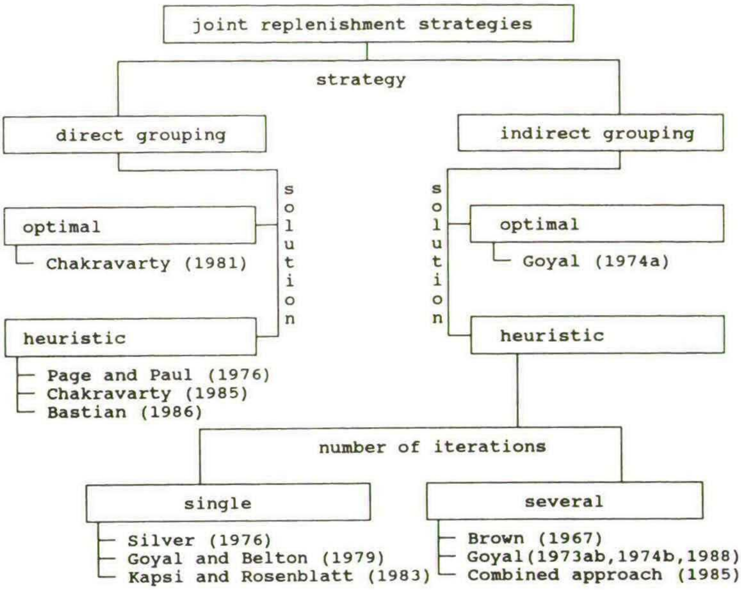

Figure 1. Literature on joint replenishment strategies

Kleijnen (1987) described a hierarchical modeling approach, which can be summarized in the following steps:

1) Determine the response or criterion variable.
2) Determine the independent variables.
3) Construct a regression metamodel (a cause-effect relation between the response variable and the independent variables).
4) Determine the experimental design (the situations that will be simulated).
5) Estimate the regression parameters and validate the metamodel. When the model is not valid step 3 is repeated; otherwise conclusions can be drawn.

Several papers have used simulation to study joint replenishment models. Goyal and Satir [1989, p.11] list some simulation studies. A response variable that often is used is the average cost savings of joint replenishment strategy i expressed as a percentage of the total cost of the independent strategy. This is a dimensionless variable, which we denote by y_i.

$$y_i = 100 \cdot \frac{TRC(EOQ) - TRC_i}{TRC(EOQ)}, \tag{3}$$

where TRC(EOQ) : total cost of the family of items when an independent EOQ strategy is used.
TRC_i : total cost of joint replenishment strategy i.

Kapsi and Rosenblatt (1985) used the deviation from the cost of the optimal joint replenishment strategy. However, this criterion is not adequate for our study since we compared direct and indirect grouping strategies, which are based on a different formulation with different optimal solutions. The response variable (y_i) is not only useful for comparing joint replenishment strategies with independent strategies but also for comparing joint replenishment strategies among themselves.

The relevant cost factors in the joint replenishment problem are: a) the major set-up cost A, b) the minor set-up cost a_i, c) the inventory carrying cost of stocking the periodic demand of item i for one period D_ih_i (this factor consists of the demand for item i per period D_i, and the inventory carrying cost per unit per period h_i, which in turn is a constant percentage h of the unit cost of item i v_i: $h_i=hv_i$). Other relevant factors are: d) the number of items in the family N, e) the number of groups to be formed M, and f) the joint replenishment strategy.

Instead of blindly incorporating all these factors in the simulation experiments, these factors were examined, and, after an extensive analysis, it appeared that only two factors must be included in the metamodel. The results are listed below.

* Instead of using D_ih_i and a_i per item, we used the means $\overline{Dh}$ and $\overline{a}$ in our analysis (in the remainder of this study the bar over a and Dh will be deleted).

* Instead of using the major set-up cost (A) and minor set-up cost (a) separately, we used the set-up cost ratio (A/a). It can be shown that a different combination of the major set-up cost (A) and the minor set-up cost (a) with an equal set-up ratio (A/a) yields the same value of the response variable y_i. This is

proven for Bastian's heuristic and the combined heuristic, based on Goyal (1974b), in part one of appendix B.

* It can be shown that an increase of the factor Dh does not affect the response variable y_1, all other things being equal. For that reason, the factor Dh was not used as a separate factor. The proof is also given in appendix 2 (part two).

* A difference between direct grouping and indirect grouping is that the number of groups is an input variable in the first and an output variable in the latter type of strategy. Therefore, Bastian's algorithm was changed a little, so that the number of groups need not be predetermined (see the note in part two of appendix A). In this way the number of groups is not a relevant factor anymore.

* After performing several pilot experiments it appeared that the set-up cost ratio and the number of items are the only factors with a significant impact on the response variable y_1. We also incorporated other factors such as (A+a)/Dh, the variance of Dh, and the variance of the minor set-up cost (a), but these factors were not significant. The response difference between the direct grouping and the indirect grouping strategy seemed to be very small.

Summarizing, two variables were important: the set-up cost ratio (A/a) and the number of items (N). In the remainder of this study we concentrate on these two factors. A graphical analysis of the simulation data of the pilot experiments showed that an increase of the set-up cost ratio or the number of items yields decreasing returns to scale. Therefore a regression metamodel with decreasing returns to scale for the variables A/a and N were specified.

Possible metamodels, with one or more of these characterics are a) a quadratic model, b) a square root model,c) a logarithmic model, and d) a reciprocal model. All these models are linear in the parameters. So we could apply linear regression analysis for estimating the parameter vector β.

An experimental design determines which factor level combinations are simulated. The choice of the experimental design is affected by the metamodel. Since in our case there are only two factors, a full factorial design could be used. The factor A/a was varied over six levels, and the factor N was varied over four levels. The levels are given in table 1.

factor	levels
A/a	1, 2, 4, 8, 12, 16
N	10, 20, 30, 60

Table 1. Factors with corresponding levels

So, there were 24 different combinations. Every combination was used for both joint replenishment strategies. The 24*2 responses (percentage cost savings of both strategies) were generated by simulation.

Given a certain combination, the simulation program generated particular inventory situations: the number of items (N), the major set-up cost (A) and the individual values of a_i and D_ih_i. Individual values of a_i and D_iv_i were randomly generated from a uniform distribution with parameters [1,5] and [1000,9000] respectively. D_ih_i was obtained by multiplying D_iv_i by the given carrying charge h (0.20); the major set-up cost was selected such that A/a was equal to the given value (thus, $A=3 \cdot A/a$). So, we used sampling to generate a situation; once a situation was created, the inventory problem was deterministic. Both direct grouping and indirect grouping were always applied to the same inventory situation. So, the reponses (y_i) of different joint replenishment strategies were based on the same random numbers. Each factor combination was replicated 500 times (a_i and D_iv_i differed, whereas N, A and h were fixed). The performances of the strategies for the given factor combination were then measured by the average cost savings (in %) of the 500 replications.

The simulation output of the 24 factor combinations was summarized by regression analysis. The linear metamodels were estimated with estimated generalised least squares, since common random numbers have been used (remember that both strategies were applied to the same input).

After testing the assumptions for least squares, the four metamodels, mentioned earlier, were estimated. We validated the models with Rao's lack of fit test [18], Kleijnen's cross validation test [19] and interpolation. It seemed that a logarithmic model fits and predicts the simulation data well within the range over which the variables were varied.

The results are given in table 2 (standard error in parentheses). $\hat{y}_{dg}$ denotes the cost savings (in %) obtained by Bastian's direct grouping algorithm, whereas $\hat{y}_{ig}$ denotes the cost savings (in %) of the combined indirect grouping algorithm, which is mainly based on Goyal (1974b).

$$\hat{y}_{dg} = 6,6588 + \underset{(1.4E-05)}{15,9710} \ln (A/a) + \underset{(2.3E-04)}{5,6209} \ln(N)$$

$$\hat{y}_{ig} = 6,3064 + \underset{(1.6E-05)}{15,7797} \ln (A/a) + \underset{(2.2E-04)}{5,9964} \ln(N)$$

Table 2: Metamodel

No interaction between the variables was included, because this variable was not significant. We used Rao's F-test [18] for linear hypotheses to see if the parameters of the independent variables are equal for both strategies . All coefficients deviated significantly, because the standard errors were virtually zero.

Figures 2 and 3 show that the predicted responses $\hat{y}_b$ and y_g as a function of the cost set-up ratio and the number of items respectively. Over the observed range of table 1 the indirect grouping strategy performed always better than the direct grouping strategy, although the effect is slightly better. So the coefficients differed significantly but not importantly. The estimators of ß show that the better performance of the indirect grouping strategy is due to the effect of the number of items in the family (see table 2).

It is not possible to extrapolate the logarithmic model to the left of the observed range, since for levels of A/a lower than one, the variable ln(A/a) will be negative. Extrapolation to the right of the observed range may result in responses y_i greater than hundred, which is impossible (note that $y_i<100$ because of (3)). So, the metamodel is only adequate for situations within the observed range.

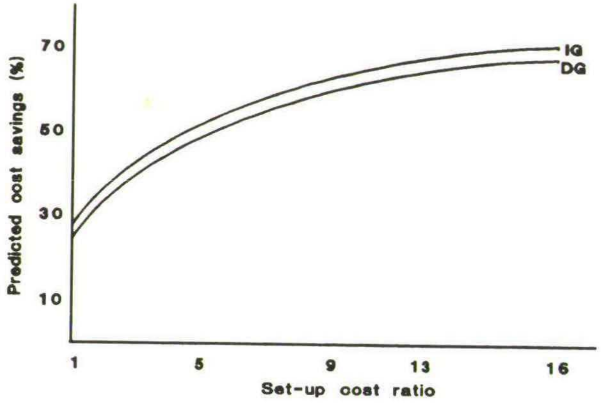

Figure 2. Predicted cost savings of using a direct grouping (DG) strategy (y_{dg}) or an indirect grouping (IG) strategy (y_{ig}) as a function of the set-up cost ratio (N=20)

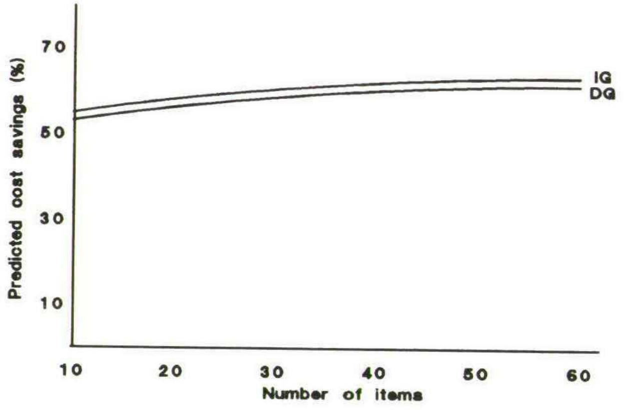

Figure 3. Predicted cost savings of using a direct grouping (DG) strategy (y_{dg}) or an indirect grouping (IG) strategy (y_{ig}) as a function of the number of items N (A/a=8).

Various conditions were simulated with a set-up cost ratio higher than sixteen. Table 3 shows that the responses grow very slowly with an increasing set-up cost ratio when the ratio is higher than twenty-five. If the ratio was higher than seventy-five, the direct grouping and indirect grouping strategy became identical, because only one group is created.

Factor combination		Cost savings (%)	
Cost set-up ratio (A/a)	number of items (N)	Direct grouping (Bastian)	Indirect grouping (Goyal)
25.00	20	69.34	69.44
50.00	20	72.73	72.74
75.00	20	73.94	73.94
100.00	20	74.59	74.59
500.00	20	76.25	76.25
1000.00	20	76.49	76.49

Table 3. Simulations with A/a>16 (N=20)

We already mentioned that indirect grouping strategies performed slightly better than direct grouping strategies within the observed range of table 1. Table 4 shows that for very small values of the set-up cost ratio, direct grouping strategies performed better than indirect grouping strategies. With a set-up cost ratio of 0.01, the indirect grouping strategy performs even badder than the independent strategy, because the replenishment cycles of the groups have to be a integer multiple of the basic cycle. In this case the extra carrying cost is higher than the saved major set-up cost. However, in these situation a joint replenishment strategy does not make much sense.

Factor combination		Cost savings (%)	
Cost set-up ratio (A/a)	number of items (N)	Direct grouping (Bastian)	Indirect grouping (Goyal)
0.01	20	0.28	-0.56
0.05	20	1.78	1.33
0.10	20	3.66	3.50
0.25	20	8.87	9.24
0.50	20	15.76	16.56
0.75	20	21.24	22.26

Table 4. Simulations with A/a<1 (N=20)

4. Conclusions

Joint replenishment strategies reduce the set-up costs of a family of related items. In this paper two types inventory strategies were investigated, namely indirect and direct grouping strategies, assuming constant demand.

We presented a short review of the literature on both strategies. In indirect grouping, individual items are replenished at fixed time intervals. The replenishment cycle of an individual item is a integer multiple (k_i) of a basic cycle time (T). We mentioned some algorithms for determining (T,k_i). Obtaining the optimal solution requires much computational effort. Therefore, several heuristics were proposed. These can be classfied into heuristics that require several iterations and heuristics that require only a single iteration. A comparitive simulation was done by Kapsi and Rosenblatt (1985). They recommended a combined approach, based on Goyal (1974b), provided an iterative algorithm is allowed; when only a single iteration is allowed the approach of Kapsi and Rosenblatt was recommended. Kapsi and Rosenblatt found that the iterative procedures, as expected, are more time demanding, but the difference in computation time is not significant. In direct grouping the items are partitioned into a predetermined number of groups with a common order interval for each group. The replenishment cycle of the groups is not a integer multiple of some basic cycle. Since the number of groups is predetermined, the problem is to divide N items into M groups such that the total costs are minimized. Dynamic programming yields the optimal solution, but becomes too expensive for large problems. Bastian's heuristic (1986) seems to be a good alternative. Several test examples were examined. It appeared that the deviations of Bastian's solution from Chakravarty's optimal solution are very small, whereas the difference in computer time seems important. Therefore, Bastian's algorithm is recommended whren a direct grouping strategy is used.

In the literature, simulation is used to compare different algorithms. However, the algorithms that were compared in these studies were all based on the indirect grouping strategy. The direct grouping strategies are not mentioned at all. To the best of our knowledge, a comparison between direct grouping and indirect grouping strategies has never been made.

In section three we presented a simulation study to analyse the differences of both indirect grouping and direct grouping strategies with respect to some factors that were expected to be important. The performance of the strategies was measured as the percentage cost savings of using a joint replenishment strategy relative to an independent strategy.

Instead of blindly incorporating all factors of the joint replenishment problem in the experiments, these factors were examined. We concluded that only two factors had be included in the metamodel, namely: (i) the ratio of the major set-up cost (A) to the mean minor set-up cost (a), and (ii) the number of items in the family (N).

A full factorial design was used with six levels of (A/a) (ranging between the values 1 and 16) and four levels of N (ranging between the values 10 and 60). 500 deterministic inventory situations were generated for each factor combination (A/a,N). The response for that factor combination was measured by the cost savings (in %) of joint replenishment strategy averaged over the 500 replications. For the direct grouping strategy we used the algorithm of Bastian; for the indirect grouping strategy we used the combined approach, mainly based on Goyal.

Regression analysis was used to model the input-output behaviour of the simulation experiments. A logarithmic model fitted and predicted the experimental data well within the range over which the variables were varied. We did also some extra simulation experiments outside the observed range.

The simulation yielded the following conclusions:

a) Over the observed range of the experiments the indirect grouping strategy always outperform the direct grouping strategy did. However, the differences between the responses were very small. The better performance of the indirect grouping strategy is due to the effect of the number of items in the family.

b) The cost savings increase only slightly when the ratio becomes greater than fifty. If the ratio is higher than seventy-five the direct grouping and indirect grouping strategy are identical, because only one group is created.

c) Simulation showed that for very small values of the set-up cost ratio, direct grouping strategies outperform indirect grouping strategies. However, in this situation a joint replenishment strategy does not make much sense.

d) A joint replenishment strategy yields high percentage cost savings, when the cost set-up ratio exceeds a half.

So when it makes sense to replenish items jointly, we recommend an indirect grouping strategy, since

1) the indirect grouping strategies outperform the direct grouping strategies slightly;
2) the indirect grouping algorithms need less computer time than the direct grouping algorithms do.

Appendices

A.1. The combined heuristic for indirect grouping.

Step 1: Determination of the starting point with Silver's single iteration algorithm (1976) (with the modification of Goyal and Belton (1979)):

1^a: Determine the item with the maximum value of $D_i h_i/(A+a_i)$ and define this item as item r (reference-item).

1^b: Determine the integer value k_i=L from

$$L(L-1) < \frac{(a_i/D_i h_i)}{(A+a_r)/(D_r h_r)} \leq L(L+1). \qquad (A.1)$$

Step 2: Use the integer values, obtained in step 1^b, as starting point in the iterative heuristic of Goyal (1974b).

2^a: Determine the replenishment frequencies k_i=L from

$$L(L-1) \leq \frac{B_i}{A_i} * \frac{a_i}{D_i h_i} \leq L(L+1), \qquad (A.2)$$

where :

$$A_i = (A + \sum_{j=1}^{N} a_j/k_j) - a_i/k_i, \qquad (A.3)$$

and,

$$B_i = \sum_{j=1}^{N} k_j D_j h_j - k_i D_i h_i. \qquad (A.4)$$

Repeat this iteration until all the integer values of k_i remain unchanged in two successive iterations.

2^b: Determine the basic cycle (T) from

$$T^2(k_i) = \frac{2 (A + \sum_i a_i/k_i)}{\sum_i k_i D_i h_i} \qquad (A.5)$$

A.2. Bastian's algorithm for direct grouping.

Step 1: Rank the items in ascending order of $D_i h_i / a_i$.

Step 2: Create N groups with $S_j = \{j\}$, $A_j := a_j + A$, $H_j := D_j h_j$.
Determine μ_j, the marginal cost of combining group j and j+1, for j=1 to N-1:

$$\mu_j = [2*(A_j + A_{j+1} - A)*(H_j + H_{j+1})]^{½} - [2*A_j*H_j]^{½} - [2*A_{j+1}*H_{j+1}]^{½} \quad (A.6)$$

AI (number of created groups) is N.

Step 3: Repeat the following procedure until the number of created groups (AI) is equal to M:
Determine $k = \min_j \mu_j$.
Combine groups k and k+1: $S_k := \{S_k + S_{k+1}\}$, $A_k := A_k + A_{k+1} - A$, $H_k := H_k + H_{k+1}$.
Rank the groups (j=1,..,AI-1).
Determine μ_k en μ_{k-1} (the other μ_j have already been calculated).
AI:=AI-1.

Step 4: Determine the replenishment cycles for each group j:

$$T_j = (2*A_j/H_j)^{½} \quad (A.7)$$

Note: In our experiments we adjusted this algorithm a little. Instead of repeating step 3 until the number of groups AI is equal to M, we repeated the procedure in step 3 until $\mu_j > 0$ for all groups. In this case the objective function can not decrease when combining any two neighbouring groups. In our experiments, however, we restricted the number of groups formed to less than ten.

B.1. A proof for:"a different combination of major set-up cost (A) and minor set-up cost (a) with an equal set-up ratio A/a yields the same value of the response variable y_1".

Assume the following situation:

combination	major set-up cost	minor set-up cost
1	A	a[i] (i=1,..N)
2	t·A	t·a[i](i=1,..,N)

We want to prove that $y_{i1}=y_{i2}$, where y_{ij} denotes the cost savings for strategy i and combination j. y_{ij} is defined in section 3 as: $y_{ij} = 100*(TRC_{eoq,j}-TRC_{ij})/TRC_{eoq,j}$.

The total cost of the independent strategy is $\Sigma_i[(A+a_i)D_ih_i\;]^{½}$.
So, $TRC_{eoq,2} = \;\Sigma_i\;[t(A+a_i)D_ih_i\;]^{½}\;=\sqrt{t}\cdot TRC_{eoq,1}$. Hence, $y_{i2}=y_{i1}$ when $TRC_{i2}=\sqrt{t}\cdot TRC_{i1}$.

<u>Proof for Bastian's direct grouping algorithm</u>

Step 1a: The ranking scheme of combination 2 is the same as that of combination 1, since the ratio D_ih_i/a_i is multiplied by a constant factor for all i.
Step 1b: Hence, the groups of the combinations 1 and 2 are the same in the first iteration. The only difference is that $(A_j)_2=t\cdot(A_j)_1$, where $(A_j)_2$ is the value A_j of group j for input combination 2. It is simple to derive that $(\mu_j)_2=\sqrt{t}\cdot(\mu_j)_1$ for all groups j.
Step 2: The group with the minimal value of μ_j is the same for both combinations; so the groups of combinations 1 and 2 remain the same after the first iteration.

The total cost $(TRC_{b.})$ is $\Sigma_j\;[\;2\cdot A_j\cdot H_j\;]^{½}$. Using $(A_j)_2=t\cdot(A_j)_1$, it is obvious that $TRC_{b2}=\sqrt{t}\cdot TRC_{b1}$ (b denotes Bastian's algorithm), Q.E.D.

<u>Proof for Goyal's indirect grouping algorithm (combined heuristic)</u>

It will be shown that $TRC_{g2}=\sqrt{t}TRC_{g1}$ (g denotes Goyal's combined algorithm) in the same way we did for Bastian's algorithm:

Step 1a: The reference item of combination 2 is the same as that of combination 1, since the ratio $D_ih_i/(A+a_i)$ is multiplied by a constant factor t for all items i.
Step 1b: In the first computation of the replenishment frequencies, $(k_i)_1$ and $(k_i)_2$ are the same as for combination 2 the factor t appears both in the numerator and the denumerator.
Step 2: It follows that $(B_i)_2=(B_i)_1$ and $(A_i)_2=t\cdot(A_i)_1$; so $(k_i)_2=(k_i)_1$ in all iterations.

The total cost is $TRC_{g.} = [\;(A + \sum_i \frac{a_i}{k_i})\cdot\sum_i\; k_itD_ih_i\;]^{½}$;

so $TRC_{g2}=\sqrt{t}TRC_{g1}$, Q.E.D.

B.2. Proof for: "an increase of factor Dh for all items with a factor t does not affect the response variable y_i"/

Assume the following situation:

combination	factor Dh	
1	D[i]h[i]	(i=1,..,N)
2	t·D[i]h[i]	(i=1,..,N)

The proof runs along similar lines as for result b. The total cost of the independent strategy is: Σ_i [$(A+a_i)D_ih_i$]$^{1/2}$; so $TRC_{eoq,2}=\surd t \cdot TRC_{eoq,1}$.

Since y_{ij} is defined as $100\cdot(TRC_{eoq,j}-TRC_{ij})/TRC_{eoq,j}$ we have again to prove that $TRC_{i2}=\surd t \cdot TRC_{i1}$.

Proof for Bastian's direct grouping algorithm

Analogue to result b it can be shown that:
step 1a): the ranking scheme of combination 2 is the same as that of combination 1; step 1b): in the first iteration the groups S_j of combination 1 and 2 are the same with $(H_j)_2=t\cdot(H_j)_1$ and $(A_j)_2=(A_j)_1$; $(\mu_j)_2=\surd t\cdot(\mu_j)_1$ for all groups j ; step 2): the groups of combinations 1 and 2 remain the same after the first iteration, so $TRC_{b2}=\surd t\cdot TRC_{b1}$.

Proof for Goyal's indirect grouping algorithm

step 1a): the reference-item of combination 2 is the same as that of combination 1; step 1b): $(k_i)_1=(k_i)$ the first computation of the replenishment frequencies, step 2): $(A_i)_2=(A_i)_1$ and $(B_i)_2=t\cdot(B_i)_1$; so $(k_i)_2=(k_i)_1$ in all iterations; so $TRC_{g2}=\surd t\cdot TRC_{g1}$.

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