Multi-item inventory systems with joint ordering and transportation decisions

van Eijs, M.J.G.

Publication date:
1991

TILBURG UNIVERSITY
KATHOLIEKE UNIVERSITEIT BRABANT
POSTBOX 90153
5000 LE TILBURG
THE NETHERLANDS

[redacted]

DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

MULTI-ITEM INVENTORY SYSTEMS WITH
JOINT ORDERING AND TRANSPORTATION
DECISIONS

M.J.G. van Eijs

FEW 528

MULTI-ITEM INVENTORY SYSTEMS WITH JOINT ORDERING AND TRANSPORTATION DECISIONS

M.J.G. VAN EIJS
Tilburg University
P.O. 90153 5000 LE Tilburg, The Netherlands

Abstract:
In many practical situations joint determination of ordering and transportation decisions for a family of items may lead to a considerable cost saving. In this paper we consider a multi-item inventory system with two possibe transportation options for shipping the goods from "oversea" to the central warehouse. These options are a "Full Container Load" (FCL) and a "Less Container Load" (LCL). Orders for the family of items are all triggered by individual periodic (R,S)-strategies. Economics of scale exist because of reduced freight rates when using a FCL instead of a LCL. A FCL can be achieved by enlarging the initial order quantities. A fast and simple algorithm is proposed to decide whether an initial order should be enlarged or not. The heuristic is based on a comparison of the expected saved shipping costs and the expected extra holding costs, caused by an enlargement. Some numerical examples show that the heuristic works quite satisfactorily.

1. Introduction

In many practical situations inventory control and transportation planning are closely related. However, in the main part of inventory management literature, these logistical functions are treated separatedly.

The application area of the system which is described in this paper consists of a distribution centre, which orders various items "oversea". The orders are shipped in containers by boat. Economics of scale exist because of reduced freight rates when using a "Full Container Load" (FCL) instead of a "Less Container Load" (LCL). Full (or close to full) containers can be achieved by coordinating orders of different items. At ordering epochs, one has to decide which transportation option is used. The ordering quantities depend on this transportation decision, wheras the transportation decision depends on the ordering decision from the inventory control planning.

The determination of the optimal strategy of such complex multi-item problems is quite intractable. Therefore, attention is restricted to special classes of strategies, which are simple to implement in practice. A simple ordering control rule is the so-called (R,S_i)-strategy. Under this type of strategy the inventory position of a particular item i is raised up to the order-up-to-level S_i every R periods. The (R,S_i)-strategy is used as a basic strategy in this research. At a review time, a simple heuristic is used to decide (i) whether the normal order has to be enlarged (to achieve economics of scale in the shipping costs) and (ii) which transportation option has to be used.

In the literature most coordinated replenishment systems focus on reducing fixed ordering costs. For a detailed overview of these systems we refer to Aksoy and Erenguc (1988) or Goyal and Satir (1989). Miltenburg (see e.g. (1987)) and Van der Duyn Schouten et al. (1991) investigate classes of coordinated replenishment strategies which also account for discount opportunities. As far as we know, the coordinated replenishment problem, which has been described above has not been investigated until now. Related (deterministic) models with integration of inventory control and transportation planning are discussed in Anily and Federgruen (1990) and Bregman et al. (1990).

This paper is structured as follows. Section 2 gives a detailed description of the problem. In section 3, a heuristic approach is presented to compute the expected saved shipping costs when the normal replenishment is enlarged. The problem of determining the expected extra holding costs, caused by such an action is adressed in section 4. A fast and simple algorithm for the joint ordering- and transportation problem is given in section 5. This algorithm is modified in section 6 to handle the dependency between two subsequent decisions. Section 7 deals with the validation of the method. This paper ends with some concluding remarks in section 8.

2. Description of the model

In this paper we consider a family of N items which are stocked at a single central warehouse. The family of items is ordered from a single supplier "oversea". Inventories are periodically reviewed. Every period the central warehouse may place an order for one or more of the items. This order arrives L time units later (in practice it often occurs that the lead time is longer when a LCL is used instead of a FCL; this fact is neglected in our model).

Demands for item i in subsequent periods are independent identically distributed random variables with expectation μ_i and variance σ_i^2. The demand processes for the various items are supposed to be independent of each other. Excess demands are backordered.

The objective is to minimize the total long run average cost per unit time subject to a given service level constraint. The relevant cost factors are the holding and the shipping costs. Ordering costs and purchasing costs are not explicitly included in the model. We assume that each item is ordered in every replenishment period and hence fixed ordering costs are not affected by the decisions. Furthermore, when no discounts are available, the long run average purchase cost will be the same under different strategies.

If the inventory on hand of item i is H_i, then the holding cost of item i is charged at a rate h_iH_i per unit time. Two options are available to ship the items from "oversea" to the central warehouse. The first option is to use a FCL (Full Container Load). In this case a fixed shipping cost F is charged, regardless of which items are included and regardless of how much of the items is shipped. The capacity of the container is restricted to K m^3. The second option is to ship the items with a LCL (Less Container Load). Now, the costs are entirely variable: c_L dollars are incurred per shipped m^3. The LCL has the same capacity. Economics of scale result from the fact that $K \cdot c_L > F$. It is assumed that one container is enough to ship the required goods.

At each review period one has to decide on the ordering quantities and on the transportation mode (FCL or LCL). We restrict attention to a special class of strategies. The basic strategy is a (R,S_i)-strategy for each item i. The review period R is a common basic period for all items in the family.

Remark 1: Instead of one common review period it is also possible to consider item-dependent review periods R_i. To achieve coordination, the periods R_i are then chosen as some multiple k_i of a base period (e.g. a week). However, in this paper we consider only the case where $k_i = 1 \; \forall_i$. It is simple to adapt the method, to be proposed, for the more general case where the R_i are not equal for all items. □

The parameter S_i is set such that the long run average holding cost is minimized given a certain service level constraint. In literature, there are several procedures to determine the parameter S_i under different assumptions concerning the demand distribution. The parameters are updated periodically (e.g once in half a year, depending on the stability of the input parameters).
At a review time for item i, initial order quantities are obtained by the parameters of the (R,S_i)-strategy: if I_i denotes the inventory position of item i, then the order quantity ,q_i, is given by:

$$q_i = S_i - I_i \tag{1}$$

At a review time, an evaluation is done whether the normal order, denoted by the vector $Q:=(q_1,..,q_N)$ has to be enlarged with $E:=(e_1,..,e_N)$ units to take advantage of the lower charge per m^3 of the FCL. The model, to determine the vector E, can be formulated as follows:

$$\min_{E} \; [\; EHC(E) - SSC(Q,E) \;] \tag{2}$$

s.t.

$$\sum_{i=1}^{N} (\, q_i + e_i \,) \cdot v_i \;\le K \tag{3}$$

$$\sum_{i=1}^{N} (\, q_i + e_i \,) \cdot v_i \;\ge \frac{F}{c_L} \quad \textit{if} \; \exists_i \; \textit{with} \; e_i > 0 \tag{4}$$

$$0 \le e_i \le UB_i \quad \forall_i \tag{5}$$

The objective is given in formula (2). The expected extra holding costs (EHC(E)) and the expected saved shipping costs (SSC(Q,E)) are compared. In section 3 and section 4 methods will be presented to calculate these quantities for given vectors E and Q.
Formula (3) reflects the capacity constraint of the container (recall that it is assumed that one container is enough to ship the quantities Q). v_i denotes the volume of item i in m^3.
Let O denote the vector of order quantities $(o_1,..,o_N)$, then it is clear that a FCL is preferable if

$$\sum_{i=1}^{N} o_i \cdot v_i \geq \frac{F}{c_L} \tag{6}$$

It is also clear that it is only profitable to add extra units to the normal order when the FCL is ultimately used (otherwise there are no economics of scale). Formula (4) ensures that a FCL will be used if the normal order is enlarged. Formula (5) gives an upperbound on the extra order quantity of each item. The upperbounds can be determined in various ways. A specific choice will be given in section 4.

Finally, we note that the existence of the problem is based on the assumption that under a (R,S_i)-strategy, without opportunities to enlarge the normal order, a LCL will be used, in general (otherwise the problem is "how to fill a FCL" instead of "which transportation mode has to be used (FCL or LCL), taking account of opportunities of economics of scale when using a FCL".

Remark 2: An additional effect of enlarging the order quantities is an improvement of the service. However, these effects are not taken into account explicitly in the optimization. □

3. Determination of the expected saved shipping costs, SSC(Q,E)

In this section we analyse the saved shipping costs in the long run when the normal order quantities Q are enlarged to Q+E.

Let V(O) denote the volume (in m^3) of an order $O:=(o_1,..,o_N)$:

$$V(O) = \sum_{i=1}^{N} o_i \cdot v_i \tag{7}$$

As mentioned earlier, a LCL is used when $V(O) \cdot c_L < F$, whereas a FCL is used otherwise.

If the normal order Q is enlarged to Q+E (such that a FCL will be used, as in formula (4)) then the shipping cost per unit will decrease. The problem, however, is to determine the saved expenses due to the extra ordered units. These units would have been ordered at the following review, and the shipping cost of these units depend on the value of V(Q) at that time. In the calculation of SSC(Q,E) it is assumed that these units would have been shipped in a LCL at the following review period. By this assumption the cost of shipping the quantities E at the next review period equals $V(E) \cdot c_L$. The cost of shipping only order Q depends on the value of V(Q), whereas F dollars are charged to ship the order quantities Q+E. Hence, SSC(Q,E) is given by:

$$\begin{aligned} SSC(Q,E) &= [\ V(Q+E)\cdot c_L - F\ ] && \text{if } V(Q) < \frac{F}{c_L} \ \wedge\ \exists_i\, e_i > 0 \\ &= [\ V(E)\cdot c_L\ ] && \text{if } V(Q) \geq \frac{F}{c_L} \ \wedge\ \exists_i\, e_i > 0 \\ &= 0 && \text{if } e_i = 0 \ \forall_i \end{aligned} \tag{8}$$

4. Determination of the expected extra holding costs, EHC(E)

In this section, we derive an approximate expression for the expected total extra holding costs of an extra order E. Since the extra holding costs of item j do not depend on the extra order quantity of any other item i, we focus on a particular item i. At time 0 a number of $q_i + e_i$ units of item i is ordered. This order arrives L periods later. The next order is placed at time R. The normal order quantity for item i at R is e_i units less compared to the case where no extra units would have been ordered at time 0. The arrival of the next order is at time R+L. A consequence of the use of the (R,S_i) replenishment rule is that the order quantity q_i is negative at time R in case the demand during [0,R] is less than e_i. In the model, this situation can be avoided by specifying an upperbound on the enlargement e_i. Let W_i denote the demand of item i during [0,R], then we require that the probability of having a demand W_i less than or equal to e_i is smaller than a given small number B:

$$P\ (\ W_i \leq e_i\ )\ \leq B \tag{9}$$

This restriction imposes an upperbound on e_i. Let $F_{0,R}(\bullet)$ denote the distribution function of W_i, and let [x] denote the integer part of x, then the upperbound UB_i is given by:

$$UB_i = [\ F_{0,R}^{-1}(B)\ ] \tag{10}$$

There are several (practical) justifications for this upperbound. Firstly, if e_i is very large, then the resulting ordering decision yields large deviations from the basic (R,S_i)-strategy. Secondly, a large e_i leads to very high inventory levels, which can be dangerous in case of perishable or obsolescent items. Finally, the upperbounds are an extra incentive to coordinate the orders of several items (economics of scale are not achieved by ordering very much of one single item, but by ordering a little bit more of several items).

The effect of an extra unit, ordered at time 0, disappears when:
- the unit is demanded by a customer, or
- the unit takes the place of another unit which would have been delivered at time R+L if no extra order was placed at time 0.

We conclude that with high probability the inventory effect of an extra unit ordered at time 0 has disappeared ultimately at R+L. So, noting that the effect starts at L, and choosing factor B small enough in formula (9), it is easy to see that the length of the effect of an extra ordered unit is at most R periods.

Denote the demand for item i during period [0,L] by U_i and the demand during period [L,R+L] by V_i. To gain a deeper insight into the effect on the holding costs of an extra order, we distinguish six situations (see also figure 1):

1. $U_i \geq S_i + e_i$, V_i arbitrary ;
2. $S_i \leq U_i < S_i + e_i$, $V_i > S_i + e_i - U_i$;
3. $S_i \leq U_i < S_i + e_i$, $V_i \leq S_i + e_i - U_i$;
4. $U_i < S_i$, $V_i > S_i + e_i - U_i$;
5. $U_i < S_i$, $S_i - U_i < V_i \leq S_i + e_i - U_i$;
6. $U_i < S_i$, $V_i \leq S_i - U_i$.

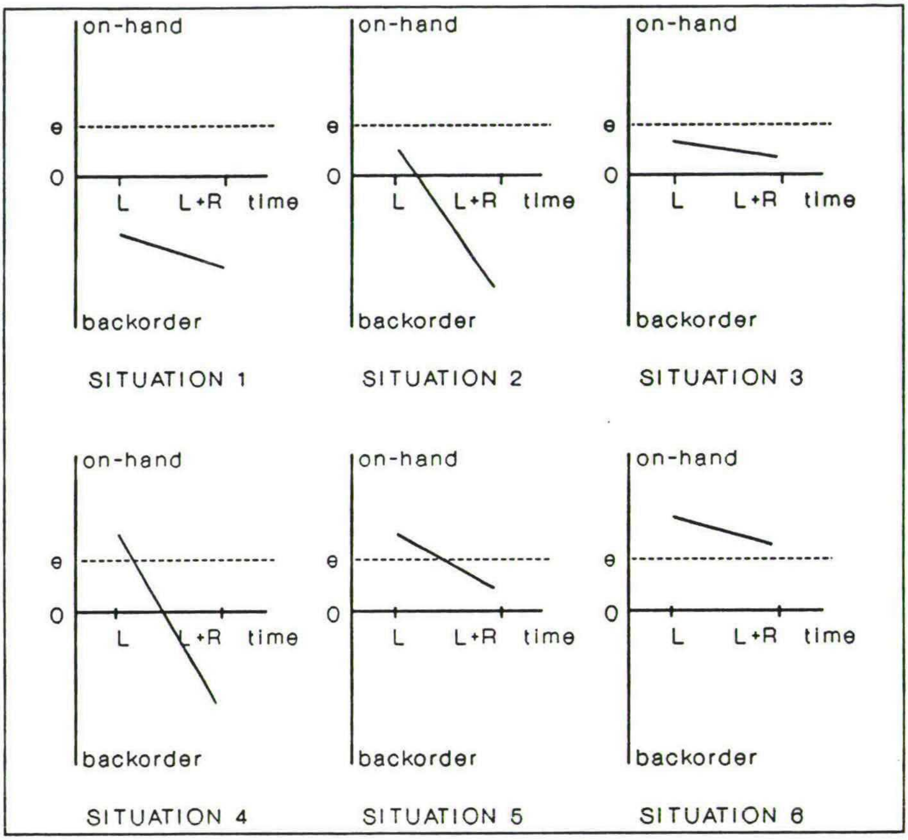

FIGURE 1
Six possible realisations of the net-inventory position

In situation 1, all extra ordered items disappear in the backorders at time L. The inventory cost effect of the extra order is zero. In situation 2 and 4, all extra ordered items have been demanded at time R+L, but there is an effect on the

holding cost in the beginning of the period [L,R+L]. In situation 3 and 5 some extra ordered items are demanded somewhere between L and R+L, whereas the other items are stocked until R+L. If factor B is chosen small enough, then these items take the place of other items which would have been arrived in the system at R+L. Hence, these items are extra in stock during R periods. The same holds for all extra ordered items in situation 6.
It is simple to obtain expressions for the expected extra holding costs in situation 1 and 6. However, the other situations are less straightforward to analyse. We approximate the expected extra holding costs in these situations by using a linear interpolation between $S_i+e_i-U_i$ at time L and $S_i+e_i-U_i-V_i$ at time R+L.

Without loss of generality, we assume (for the moment) that the unit holding cost (h_i) equals one for all items i. Referring to figure 1, we obtain the following expression for $AHC(e_i)$ (for simplicity, the i subscript on q_i, e_i, and S_i is suppressed):

$$AHC(e_i) =$$

$$\int_{u=S}^{S+e} \int_{v=S+e-u}^{\infty} \frac{(S+e-u)^2 \cdot R}{2 \cdot v} \; dF_{L,R+L}(v)\; dF_{0,L}(u) \qquad (sit.\,2)$$

$$+ \int_{u=S}^{S+e} \int_{v=0}^{S+e-u} \left(R \cdot (S+e-u) - \frac{R \cdot v}{2} \right) dF_{L,R+L}(v)\; dF_{0,L}(u) \qquad (sit.\,3)$$

$$+ \int_{u=0}^{S} \int_{v=S+e-u}^{\infty} \left(\frac{e \cdot (S-u) \cdot R}{v} + \frac{e^2 \cdot R}{2 \cdot v} \right) dF_{L,R+L}(v)\; dF_{0,L}(u) \qquad (sit.\,4)$$

$$+ \int_{u=0}^{S} \int_{v=S-u}^{S+e-u} \left(R \cdot (S+e-u) - \frac{R \cdot v}{2} - \frac{R \cdot (u-S)^2}{2 \cdot v} \right) dF_{L,R+L}(v)\; dF_{0,L}(u) \qquad (sit.\,5)$$

$$+ \int_{u=0}^{S} \int_{v=0}^{S-u} R \cdot e \; dF_{L,R+L}(v)\; dF_{0,L}(u) \qquad (sit.\,6)$$

(11)

where,

$AHC(e_i)$: approximation of the expected extra holding costs of an extra order of e_i units of item i;

$F_{0,L}(\bullet)$: distribution function of demand U_i during [0,L];

$F_{L,R+L}(\bullet)$: distribution function of demand V_i during [L,R+L].

Except for simple demand distributions, such as the uniform distribution, it is not possible to obtain closed form expressions for $AHC(e_i)$. Therefore, we used numerical integration. Several simulation experiments with Mixed Erlang demands showed that the numerical integration method yields very good approximations for the expected extra holding costs. We refer to the appendix for more details on the numerical integration procedure.

It also appeared that $AHC(e_i)$ is quite close to the simple expression $R \cdot e_i \cdot h_i$. When the service criterion requires that at least 95% of demand is satisfied directly from shelf, then $AHC(e_i)/(R \cdot e_i \cdot h_i)$ ranges from 0.96 (for small e_i) to 0.98 (for e_i close to UB_i) (see also table A2 in the appendix). For computational reasons we will use the simple linear expression instead of the more sophisticated formula (11). In this case, the expected total extra holding costs, EHC(E), are simply approximated by:

$$EHC(E) = R \cdot \sum_{i=1}^{N} e_i \cdot h_i \tag{12}$$

Remark 3: Note that EHC(E) is an average value, whereas the realisation of the extra holding costs from an enlargement of the order is a random variable. Thus, the extra holding costs are overestimated (underestimated) if demands (the only source of uncertainty) are higher (lower) than expected. □

5. Algorithm for the evaluation model

In section 2 we have proposed a model to decide whether the vector of normal order quantities Q, which follows from a given (R,S_i)-strategy, has to be enlarged to achieve economics of scale. The evaluation model is given by formula (2) up to (5). In the two foregoing sections we have analysed the expected saved shipping costs and the expected total extra holding costs when a vector of E units is added to the normal replenishment. In this section a fast heuristic algorithm is proposed to solve the optimization problem.

The heuristic is based on an incremental approach. The current solution is improved in every step by adding as much as possible of that item which causes the largest cost decrease.

To formalize the approach, define:

S : set of items which cause a decrease in costs when one unit is added to the replenishment;

Δ_i : incremental cost from adding *one extra unit* of item i to the current order quantity;

S and Δ_i are defined under the condition that a FCL will be used to ship the order. Δ_i can be easily obtained from formula (8) and (12):

$$\Delta_i = R \cdot h_i - c_L \cdot v_i \qquad \textbf{(13)}$$

The algorithm, that evaluates whether the normal order has to be enlarged is outlined below. As mentioned before, the algorithm is used whenever a review time occurs.

Improvement-algorithm

Step 0 : *Feasibility check*

Compute $V(Q+UB)$ from (7);

If $V(Q+UB)<F/c_L$ then go to step 3a, else go to step 1.

Step 1 : *Initialization*

a: Set $e_i = 0\ \forall_i$ and compute $V := V(Q)$ from (7);

b: Compute $\Delta_i\ \forall_i$ from (13);

c: $S := \{\ i \mid \Delta_i < 0 \wedge UB_i > 0 \wedge (\ V + v_i \leq K\ )\ \}$;

d: Go to step 3b if $S = \phi$; otherwise go to step 2.

Step 2 : *Improvement*

a:

$$p := \underset{S}{arg\ \min}\ \Delta_i$$

$$e_p := min\ (\ [\frac{K - V}{v_p}]\ ,\ UB_p\ )$$

where [a] denotes the integer part of a.

$V := V + e_p \cdot v_p$;

$S := S - \{p\}$;

b: For all items $i \in S$: if $V + v_i > K$, then $S := S - \{i\}$;

c: Go to step 2d if $S = \phi$; otherwise go to step 2a.

d: If $V \geq F/c_L$ then go to step 3c; otherwise go to step 3a.

Step 3: *Termination*

a: E:=0; order Q and use a LCL.

b: E:=0; order Q and use a LCL if $V(Q) < F/c_L$, otherwise use a FCL.

c: Compute EHC(E) from (12) and SSC(Q,E) from (8);
If EHC(E) < SSC(Q,E) then order Q+E in a FCL;
If EHC(E) ≥ SSC(Q,E) then order Q in a LCL.

Step 0 checks whether the maximum enlargement is enough to achieve economics of scale. If constraint (4) is violated then a LCL will be advised (see step 3a). In step 1, the vectors E and Δ are initialized, together with the set *S*. If the set *S* is empty, then the order will not be enlarged. The normal order quantities Q will be shipped in a LCL or a FCL, depending on the value of V(Q) (see step 3b).

In step 2a up to step 2c the current order decision is improved by adding as much as possible of item p which causes the largest cost decrease. The maximum enlargement of item p is determined by considering constraints (3) and (5). If the set *S* is empty, it is assumed that the current solution can not be improved anymore.

Step 2d checks whether the current solution is feasible with respect to constraint (4). Recall that formula (13) is determined under the condition that (4) holds. In the case that the current enlargement E is not enough to achieve economics of scale, it follows from formula (8) that the expected saved shipping costs are negative. So, a LCL will be advised. Note that step 0 does not guarantee a feasible solution because only items i with a negative value of Δ_i are candidates to enlarge the replenishment.

If a feasible solution Q+E exists, then SSC(Q,E) and EHC(E) are compared in step 3c. Note that SSC(Q,E) is always greater than EHC(E) in case a FCL is already prefered for the normal replenishment. Hence, if $V(Q) \geq F/c_L$, then no evaluation has to be done.

Table 1 illustrates the algorithm with some simple examples.

TABLE 1

Some illustrative examples

R = 2 , L = 1 , F = 240 , c_L = 3 , F/c_L = 80 , K = 100

item	μ_i	σ_i	v_i	h_i	S_i	UB_i	Δ_i
1	10	8	2	1	48	5	-4
2	12	6	1	1	45	11	-1
3	5	2	1	3	18	5	3

item i	q_i	e_i	$q_i + e_i$	Comments
1	18	0	18	After step 2 :
2	20	0	20	E = { 5, 11, 0 }
3	8	0	8	SSC = 15 , EHC = 32
volume	64	0	64	Use a LCL

item i	q_i	e_i	$q_i + e_i$	Comments
1	20	5	25	After step 2 :
2	26	11	37	E = { 5, 11, 0 }
3	12	0	12	SSC = 57 , EHC = 32
volume	78	21	99	Use a FCL

item i	q_i	e_i	$q_i + e_i$	Comments
1	4	0	4	$V(Q+UB) = 71 < F/c_L$
2	25	0	25	
3	12	0	12	
volume	45	0	45	Use a LCL

item i	q_i	e_i	$q_i + e_i$	Comments
1	15	0	15	After step 2 :
2	18	0	18	E = { 5, 11, 0 }
3	8	0	8	$V(Q+E) = 77 < F/c_L$
volume	56	0	56	Use a LCL

item i	q_i	e_i	$q_i + e_i$	Comments
1	24	5	29	After step 2 :
2	25	2	27	E = { 5, 2, 0 }
3	15	0	15	$V(Q) = 88 \geq F/c_L$,
volume	88	12	100	so SSC(Q,E)>EHC(E) Use a FCL

6. An improvement of the algorithm

Until now, we have focused on the effect of one particular decision at a given review time. The improvement-algorithm is used to decide whether to enlarge the normal order or not. However, the inventory planner makes this decision not only once, but he makes a sequence of decisions in time. The algorithm, as presented in section 5, neglects the dependency between two subsequent decisions.

To make this point clear, we consider again the expected saved shipping costs. Recall that the shipping cost per m^3 will decrease if the order is extended from Q to Q+E. Let O_t be a vector of order quantities at time t, and denote the shipping rate per m^3 of such an order by $r(O_t)$, then (noting that the only sensible vectors E_t are those for which $V(Q_t+E_t) \geq F/c_L$):

$$r(O_t) = \frac{F}{V(Q_t + E_t)} \qquad if \;\; O_t = Q_t + E_t$$
$$= \min \left\{ c_L \, , \frac{F}{V(Q_t)} \right\} \quad if \;\; O_t = Q_t \qquad (14)$$

Another formulation of the saved shipping costs is given by formula (8'):

$$SSC(Q_t,E_t) = V(Q_t)\cdot[\,r(Q_t) - r(Q_t+E_t)\,] + V(E_t)\cdot[\,r(O_{t+R}) - r(Q_t+E_t)] \qquad (8')$$

We mention that the saved expenses on the normal order Q_t (first term of formula (8')) are realized at time t. However, the saving on E_t (second term of (8')) will be realized only at time t+R. Recall that formula (8) is derived under the asumption that at the following review time a LCL will be used. In this case $r(O_{t+R}) = c_L$, and hence, formula (8') is the same as formula (8). Because of the assumption mentioned at the end of section 2, this situation usually occurs.

However, if the replenishment is enlarged at time t+R then the shipping rate $r(O_{t+R})$ will change from $r(Q_{t+R})$ to $r(Q_{t+R}+E_{t+R})$ and the realized saving decreases. Consequently, *the outcome of the decision at time t depends on the decision to be made at t+R.*

One possibility to overcome this difficulty is not to allow an enlargement of the order at two subsequent review times. In this case the realization of the saving of an extra order E_t will be forced to be $SSC(Q_t,E_t)$ in formula (8). However, a potential cost saving from a extra order at time t+R will then be neglected.

Therefore, we propose an improvement of the algorithm, as described in section 5. This improvement is based on the observation that the realized saving on E_t decreases if the normal order is also extended at time t+R. Because $r(O_{t+R})$ will decrease from $r(Q_{t+R})$ to $r(Q_{t+R}+E_{t+R})$, the missed saving at time t+R, $MS(Q_{t+R},E_{t+R})$, is equal to:

$$MS_{t+R}(Q_{t+R},E_{t+R}) = V(E_t)\cdot[\,r(Q_{t+R}) - r(Q_{t+R}+E_{t+R})\,] \tag{15}$$

This missed saving has to be included in the decision-making at time t+R. In the algorithm of section 5, the order is enlarged at time t+R if $SSC(Q_{t+R},E_{t+R})>EHC(E_{t+R})$. Now, the order is extended only if the expected saving, $SSC(Q_{t+R},E_{t+R})-EHC(E_{t+R})$, is larger than the missed saving at t+R. Step 3c of the algorithm is corrected as follows (the subscript t is deleted):

Improvement of step 3c

Step 3c': Compute SSC(Q,E) from (8);
Compute EHC(E) from (12);
Compute MS(Q,E) from (15);
If $EHC(E)+MS(Q,E)<SSC(Q,E)$ then order Q+E and use a FCL;
If $EHC(E)+MS(Q,E)\geq SSC(Q,E)$ then order Q and use a FCL or LCL depending on the value of V(Q).

Note that the evaluation now also has to be done if $V(Q)\geq F/c_L$.

Table 2 shows a numerical example, which is based on the same data as the illustrative examples in table 1. We consider the situation where the volume of the extra order at the preceding review time is 20. Note that a FCL will be advised if step 3c is used, because EHC(E) < SSC(Q,E). However, the order will not be enlarged if the missed saving at this review time is explicitly taken into account.

TABLE 2
Illustrative example (continued)

item i	q_i	e_i	$q_i + e_i$	Comments
1	20	0	20	After step 2 :
2	22	0	22	E = { 5, 11, 0 }
3	10	0	10	SSC(Q,E) = 39, EHC(E) = 32
volume	72	0	72	MS(Q,E) = 8.39 Use a LCL

7. Numerical examples

In this section we show how our strategy performs on a set of test problems. Simulation will be used to obtain the long run average expected total cost per unit time. The cost of two different strategies will be compared:

strategy S1 : "normal" (R,S_i)-strategy, with no joint ordering and transportation planning;
strategy S2 : (R,S_i)-strategy, with joint ordering and transportation planning.

Under strategy S1 a LCL is used whenever $V(Q) < F/c_L$ and a FCL otherwise. Changing the "normal" order quantities is not allowed. Strategy S2 uses the algorithm of section 5 (together with the improvement in step 3c, discussed in section 6) to decide whether the normal order should be enlarged or not at a given review time.

We consider the following situation :
- the number of items in the family N = 10,
- demands per unit time for item i follow a Mixed-Erlang distribution with mean μ_i and variance σ_i^2,
- the service level requires that at least 95% of demand is satisfied directly from

inventory on hand,

- the order-up-to-level S_i is determined by a solution procedure of De Kok (1990),
- the upperbound UB_i follows from formula (9) with $B=0.05$.

With respect to the review time, the lead time and the capacity of the container, we consider two cases:

case (a): $R=2$, $L=1$, $K=1000$, *case (b)*: $R=1$, $L=2$, $K=700$.

Under strategy S1 the expected average volume that has to be transported is equal to 687 and 344 for case (a) and (b), respectively. Table 3 lists the data for the family of items.

TABLE 3
Data for numerical example

i	μ_i	σ_i	v_i	h_i	case (a): R=2 ,L=1		case (b): R=1, L=2	
					S_i	UB_i	S_i	UB_i
1	10	8	1.0	1.0	48	5	54	0
2	20	10	3.0	2.5	76	19	83	6
3	5	2	1.0	3.0	18	5	19	2
4	12	6	4.0	2.5	45	11	50	4
5	16	10	2.0	2.0	67	12	74	3
6	15	5	4.0	2.0	50	19	54	7
7	8	4	2.0	1.0	30	7	33	2
8	25	10	1.5	1.5	88	29	96	11
9	15	5	2.0	1.0	50	19	54	7
10	45	15	1.0	1.5	151	58	163	23

In the simulation experiments the variable LCL transportation cost (c_L) and the break-even volume (F/c_L), above which a FCL is prefered, are varied over four and three levels, respectively. For each combination of c_L and F/c_L simulation runs are repeated until a 95% confidence interval is obtained with a bandwidth of 1. A single run consists of simulating the multi-item system for 1000 periods. The multi-item inventory system is simulated simultaneously for strategy S1 and S2. So common random numbers (demands for the items) are used for the evaluation of the performance of both strategies. The results are reported in table 4. The simulated average cost per unit time for strategy S1 and S2 are denoted by C1 and C2, respectively.

The percentage cost saving of using strategy S2 instead of strategy S1, is defined by

$$\% c.s. = 100 \cdot \frac{C_1 - C_2}{C_1} \tag{14}$$

TABLE 4
Results for numerical example

case (a): R=2, L=1					case (b): R=1, L=2				
F/c_L	c_L	C_1	C_2	% c.s	F/c_L	c_L	C_1	C_2	% c.s.
700	2	1153.75	1134.17	1.70	400	2	1118.85	1089.72	2.60
800	2	1175.64	1164.19	0.97	500	2	1128.97	1125.08	0.34
900	2	1178.68	1177.71	0.08	600	2	1128.98	1128.91	0.01
700	3	1485.18	1416.86	4.60	400	3	1457.46	1398.66	4.03
800	3	1517.71	1478.35	2.59	500	3	1471.77	1462.08	0.66
900	3	1522.14	1517.72	0.29	600	3	1472.85	1472.54	0.02
700	4	1816.77	1689.44	7.01	400	4	1795.56	1704.11	5.09
800	4	1859.49	1782.46	4.14	500	4	1815.21	1798.02	0.95
900	4	1865.76	1851.98	0.74	600	4	1816.13	1815.57	0.03
700	6	2478.25	2234.10	9.85	400	6	2472.52	2316.82	6.30
800	6	2543.58	2379.70	6.44	500	6	2502.25	2469.85	1.29
900	6	2552.14	2508.29	1.72	600	6	2502.51	2501.31	0.05

It turns out that the percentage cost saving decreases if F/c_L increases while c_L remains the same. This could be expected because the potential cost saving from economics of scale decreases when the difference ($K-F/c_L$) decreases. Table 4 also shows that the percentage cost saving from using strategy S2 instead of S1 increases if c_L increases while F/c_L remains constant. This can be explained by the fact that the proportion of the transportation cost in the total cost increases in case c_L increases and therefore reductions on this cost factor have a larger impact on the total cost.
In comparing case (a) and case (b) we conclude that the observations which are mentioned above hold for both $R>L$ and $R<L$. The percentage cost saving is generaly lower in case (b) since the enlargement opportunities are smaller than in case (a).

We close this section with some additional remarks. Firstly, in section 4 we assumed that the extra holding cost (EHC(E)) from an enlargement E can be approximated by the simple formula (12). Instead of formula (12), we may use a more accurate approximation, which follows from the numerical integration method based on formula (11). The improvement algorithm can be easily

adapted to handle this case. Additional numerical investigations indicate that the perfomance of strategy S2 is not substantially improved by using this more accurate approximation method for EHC(E).
Secondly, these test examples were also runned without the improvement, that is recommended in section 6. The simulation results show that the performance of strategy S2 is improved substantially in some test cases by using step 3c' instead of step 3c.
Thirdly, as mentioned in remark 2, note that not only the total cost per unit time decreases when strategy S2 is used, but that also the service increases for those items i for which the normal order quantities can be enlarged ($UB_i > 0$, $\Delta_i < 0$). In our experiments the service of these items is improved by about 1%.

8. Concluding remarks

In this paper we suggested a simple method to handle the interaction between ordering and transportation decisions, if economics of scale exist because of reduced freight rates when using a FCL instead of a LCL for transportation from "oversea". A FCL is achieved by coordinating the orders of different items.
The periodic review (R,S_i)-strategy is used as a basic strategy for all items i. An improvement-algorithm is proposed which decides to enlarge the normal order or not at a review time. This decision is based on a comparison of the expected saved shipping costs and the expected extra holding costs, if the normal order is extended.

The performance of the strategy was evaluated by simulation. We compared the average long run total cost per unit time of the usual (R,S_i)-strategy with that of the adapted strategy. Numerical results showed that the total cost can be substantially decreased if ordering and transportation planning are integrated. Moreover, the service is increased by enlarging the normal order quantities.

One direction for further research is to apply the proposed method in a multi-echelon environment. Consider a two echelon inventory system consisting of a central warehouse and a number a local warehouses. The items in the system are shipped from an outside supplier "oversea" to the central warehouse,

which allocates the order to the local warehouses. The inventory control planning has not only to take into account the dependencies between the central warehouse and the local warehouses, but also the relationship between inventory planning and transportation planning. It might be worthwhile to investigate whether our method can also be used in this more complex situation.
Another direction of further research is related to the observation that in practice it often occurs that the lead time is shorter when using a FCL instead of a LCL. Additional research is needed to handle this aspect.

Acknowledgement - The author would like to thank Frank van der Duyn Schouten and Ruud Heuts for several fruitful discussions. Further thanks to Jan de Klein, who gave some helpful comments with respect to the numerical integration method which is discussed in the appendix.

References

AKSOY, Y., AND S. ERENGUC, "Multi-item models with co-ordinated replenishments: a survey", *Int. J. Prod. Man.*, 8 (1988), 63-73.

ANILY. S., AND A. FEDERGRUEN, "One warehouse multiple retailer systems with vehicle routing costs", *Management Sci.*, 36 (1990), 92-113.

BREGMAN, R.L., L.P. RITZMAN, AND L.J. KRAJEWKSI, "A heuristic for the control of inventory in a multi-echelon environment with transportation costs and capacity limitations", *J. Opl Res. Soc.*, 9 (1990), 809-820.

DE KOK, A.G., "Hierarchical production planning for consumer goods", *European J. Oper. Res.*, 45 (1990), 55-69.

GOYAL, S.K., AND A.T. SATIR, "Joint replenishment inventory control: deterministic and stochastic models", *European J. Oper. Res.*, 38 (1989), 2-13.

MILTENBURG, J.G., "Coordinated control of a family of discount-related items", *Infor*, 25 (1987), 647-671.

TIJMS, H.C., *Stochastic modelling and analysis*, John Wiley & Sons, (1986).

VAN DER DUYN SCHOUTEN, F.A., M.J.G. VAN EIJS, AND R.M.J. HEUTS, "Coordinated replenishment systems with discount opportunities", *Research Memorandum*, FEW 477, Tilburg University, (1991).

Appendix

In this appendix we present the numerical integration method to approximate $AHC(e_i)$ from formula (11). Formula (11) consists of five parts, related to situation 2 up to 6, respectively. For each situation a separate grid will be distinguished. The numerical integration method for situation 2 will now be discussed in detail. After this the results for the other situations are given without extensive discussion. For ease of notation, the subscript i, which refers to item i, is deleted.

Situation 2: $S \leq U < S+e\ ,\ S+e-U < V < \infty$

Let NN1 be the number of integration points at the u-axis. For ease of notation, we define N1:=NN1-1. The integration points $u_0,..,u_{N1}$ are chosen such that

$$S:=u_0 < u_1 < .. < u_{N1} := S+e.$$

If equal interval lengths are used, then $u_j := u_{j-1} + e/N1$ for j:=1,..,N1.
The first NN1 integration points at the v-axis are set such that

$$v_j := S+e-u_{N1-j} \text{ for } j:=0,..,N1 \quad (\text{note that } v_0=0 \text{ and } v_{N1}=e).$$

Define M1 as the number of integration points at the interval $\langle e, v_m]$, where

$$v_m := \{ \min x \mid F_{L,L+R}(x) \geq 0.99 \}. \text{ M1 is set equal to zero if } v_m \leq e.$$

Using equal interval lenghts at $\langle e, v_m]$, it follows (if M1>0) that

$$v_j := v_{j-1} + (v_m - e)/M1 \text{ for } j:=N1+1,..,N1+M1.$$

The distribution function $F_{0,L}(u)$ will be approximated by a piece-wice linear function $F_{0,L}(u)$:

$$\hat{F}_{0,L}(u) = a_j u + b_j \quad for\ u \in [u_j, u_{j+1}] \quad j=0,..,N1-1$$

where the interpolation coefficients a_j and b_j are given by:

$$a_j = \frac{F_{0,L}(u_{j+1}) - F_{0,L}(u_j)}{u_{j+1} - u_j} \quad , \quad b_j = \frac{F_{0,L}(u_j)u_{j+1} - F_{0,L}(u_{j+1})u_j}{u_{j+1} - u_j}$$

On the other hand, the distribution function $F_{L,R+L}(v)$ is approximated by:

$$\hat{F}_{L,R+L}(v) = c_j v + d_j \quad for\ v \in [v_j, v_{j+1}] \quad j=0,..,N1-1+M1$$

where c_j and d_j are given by:

$$c_j = \frac{F_{L,R+L}(v_{j+1}) - F_{L,R+L}(v_j)}{v_{j+1} - v_j} \quad , \quad d_j = \frac{F_{L,R+L}(v_j)v_{j+1} - F_{L,R+L}(v_{j+1})v_j}{v_{j+1} - v_j}$$

Now,

$$\int_{u=S}^{S+e} \int_{v=S+e-u}^{\infty} \frac{(S+e-u)^2 \cdot R}{2 \cdot v} \; dF_{L,R+L}(v) \; dF_{0,L}(u)$$

$$\approx \frac{R}{2} \int_{u=S}^{S+e} \int_{v=S+e-u}^{v_m} \frac{(S+e-u)^2}{v} \; dF_{L,R+L}(v) \; dF_{0,L}(u)$$

$$\approx \frac{R}{2} \Big\{ \sum_{i=0}^{N1-1} a_i \int_{u=u_i}^{u_{i+1}} \Big\{ c_{N1-i-1} \int_{v=S+e-u}^{v_{N1-i}} \frac{(S+e-u)^2}{v} \, dv + \sum_{j=N1-i}^{N1-1+M1} c_j \int_{v_j}^{v_{j+1}} \frac{(S+e-u)^2}{v} \, dv \Big\} du \Big\}$$

$$= \frac{R}{2} \Big\{ \sum_{i=0}^{N1-1} a_i \cdot c_{N1-i-1} \int_{u=u_i}^{u_{i+1}} \int_{v=S+e-u}^{v_{N1-i}} \frac{(S+e-u)^2}{v} \, dv\, du$$

$$+ \sum_{i=0}^{N1-1} a_i \sum_{j=N1-i}^{N1-1+M1} c_j \int_{u=u_i}^{u_{i+1}} \int_{v_j}^{v_{j+1}} \frac{(S+e-u)^2}{v} \, dv \, du \Big\}$$

After some algebraic manupulations, this expression transforms into:

$$\frac{R}{2}\Big\{ -\frac{1}{3}\sum_{i=0}^{NI-2} a_i \cdot c_{NI-i-1} \cdot v_{NI-i-1}^3 \cdot \log\frac{v_{NI-i}}{v_{NI-i-1}}$$

$$+\frac{1}{9}\sum_{i=0}^{NI-1} a_i \cdot c_{NI-i-1} \cdot (v_{NI-i}^3 - v_{NI-i-1}^3)$$

$$+\frac{1}{3}\sum_{i=0}^{NI-1} a_i \cdot (v_{NI-i}^3 - v_{NI-i-1}^3) \cdot \sum_{j=NI-i}^{NI-1+MI} c_j \cdot \log\frac{v_{j+1}}{v_j}\Big\}$$

In the same way, results are obtained for situation 3 up to 6. These results are given below without further explanation.

Situation 3: $S \leq U < S+e\,,\ 0 \leq V \leq S+e-U$

$$\int_{u=S}^{S+e}\int_{v=0}^{S+e-u} \left(R\cdot(S+e-u) - \frac{R\cdot v}{2} \right)\, dF_{L,R+L}(v)\, dF_{0,L}(u)$$

$$\approx \frac{R}{2}\Big\{ \sum_{i=0}^{NI-1} a_i \cdot c_{NI-i-1} \cdot \Big[\frac{1}{2} v_{NI-i}^3 - v_{NI-i}^2 \cdot v_{NI-i-1} + \frac{1}{2} v_{NI-i} \cdot v_{NI-i-1}^2 \Big]$$

$$+ \sum_{i=0}^{NI-2} a_i \cdot \Big[(v_{NI-i}^2 - v_{NI-i-1}^2) \cdot \sum_{j=0}^{NI-i-2} c_j \cdot (v_{j+1} - v_j)$$

$$- \frac{1}{2}(u_{i+1} - u_i) \cdot \sum_{j=0}^{NI-i-2} c_j \cdot (v_{j+1}^2 - v_j^2) \Big] \Big\}$$

where,

NN1 : number of integration points at the interval [S, S+e] at the u-axis;

N1 : NN1-1.

Situation 4: $0 \leq U < S\,,\ S+e-U < V < \infty$

$$\int_{u=0}^{S}\int_{v=S+e-u}^{\infty} \left(\frac{e\cdot(S-u)\cdot R}{v} + \frac{e^2\cdot R}{2\cdot v} \right)\, dF_{L,R+L}(v)\, dF_{0,L}(u)$$

$$\approx \sum_{i=0}^{N2-1} a_i \cdot c_{N2-i-1} \cdot \Big\{ e \cdot R \cdot [\, v_{N2-i-1} (e - \frac{1}{2} v_{N2-i-1}) \cdot \log \frac{v_{N2-i}}{v_{N2-i-1}} - \frac{1}{4} (v_{N2-i-1}^2 - v_{N2-i}^2) - e \cdot (u_{i+1} - u_i) \,] + \frac{e^2 \cdot R}{2} \cdot [\, -v_{N2-i-1} \cdot \log \frac{v_{N2-i}}{v_{N2-i-1}} + (u_{i+1} - u_i) \,] \Big\}$$

$$+ \sum_{i=0}^{N2-1} a_i \cdot \Big\{ \frac{e \cdot R}{2} \cdot [\, (S - u_i)^2 - (S - u_{i+1})^2 \,] + \frac{e^2 \cdot R}{2} \cdot (u_{i+1} - u_i) \Big\} \cdot \sum_{j=N2-i}^{N2-1+M2} c_j \cdot \log \frac{v_{j+1}}{v_j}$$

where,

NN2 : number of integration points at the interval [0, S] at the u-axis;

N2 : NN2-1;

v_m : $\{ \min x \mid F_{L.L+R}(x) \geq 0.99 \}$;

M2 : number of integration points at the interval $\langle S+e, v_m]$ (M2=0 if $v_m \leq S+e$).

Situation 5: $0 \leq U < S$, $S-U < V \leq S+e-U$

This situation is handled differently from the other situations. The integration points at the u-axis are determined in the same way as before.

Define,

NN3 : number of integration points at the interval [0, S] at the u-axis;

N3 : NN3-1; then

$0 := u_0 < u_1 < .. < u_{N3} := S$, and $u_j := u_{j-1} + S/N3$ for j:=1,..,N3.

However, points at the v-axis are chosen with respect to a point at the u-axis. Let,

MM3 : number of integration points at the v-axis belonging to one particular integration point at the u-axis;

M3 : MM3-1; then

$S-u_i := v_i^{(0)} < v_i^{(1)} < .. < v_i^{(M3)} := S+e-u_i$, and

$v_i^{(j)} := S-u_i + j \cdot e/M3$ for j:=0,..,M3 and i=0,.,N3.

The factor e/M3 will be denoted by δ.

$$\int_{u=0}^{S} \int_{v=S-u}^{S+e-u} \left(R\cdot(S+e-u) - \frac{R\cdot v}{2} - \frac{R\cdot(u-S)^2}{2\cdot v} \right) \, dF_{L,R+L}(v)\, dF_{0,L}(u)$$

$$\approx \sum_{i=0}^{N3-1} a_i \sum_{j=0}^{M3-1} c_{ij} \int_{u=u_i}^{u_{i+1}} \int_{v=S-u+\delta\cdot j}^{S-u+\delta(j+1)} \left(R\cdot(S+e-u) - \frac{R\cdot v}{2} - \frac{R\cdot(u-S)^2}{2\cdot v} \right) dv\, du$$

$$= \sum_{i=0}^{N3-1} a_i \sum_{j=0}^{M3-1} c_{ij} \Big\{ \frac{R\cdot\delta}{2}\cdot\left(v_i^{(M3)\,2} - v_{i+1}^{(M3)\,2} \right)$$

$$- \frac{R}{4}\cdot\left[(2j+1)\cdot\delta^2\cdot(u_{i+1}-u_i) + \delta\cdot\left((S-u_i)^2 - (S-u_{i+1})^2\right) \right]$$

$$- \frac{R}{2}\cdot\left(f(i+1,j) - f(i,j) \right) \Big\}$$

where,

$$f(i,j) := \frac{1}{3}\cdot(u_i - S)^3\cdot\log\frac{v_i^{(j+1)}}{v_i^{(j)}} - \frac{1}{3}\cdot(\delta\cdot(j+1))^3\cdot\log v_i^{(j+1)}$$

$$+ \frac{1}{3}\cdot(\delta\cdot j)^3\cdot\log v_i^{(j)} + \frac{1}{6}\cdot\left[-4\delta(j+1)u_i - 2\delta^2 u_i + 2\delta S u_i - \delta u_i^2 \right]$$

The first and the third term in f(i,j) are equal to zero if i=N3 and j=0 ($v_{N3}^{(0)}=0$).

Situation 6: $0 \le U < S$, $0 \le V \le S\text{-}U$

$$R\cdot e\cdot \int_{u=0}^{S} \int_{v=0}^{S-u} 1 \; dF_{L,R+L}(v)\, dF_{0,L}(u)$$

$$\approx R\cdot e\cdot \Big\{ \frac{1}{2}\cdot \sum_{i=0}^{N4-1} a_i\cdot c_{N4-i-1}\cdot\left(v_{N4-i}^2 - v_{N4-i-1}^2 \right)$$

$$+ \sum_{i=0}^{N4-2} (u_{i+1}-u_i)\cdot \sum_{j=0}^{N4-2-i} c_j\cdot(v_{j+1}-v_j) \Big\}$$

where,

NN4 : number of points at the interval [0,S] at the u-axis; (N4:=NN4-1).

The correctness of these formulas was tested by using a simple uniform distribution function for $F_{L,L+R}(v)$ and $F_{0,L}(u)$. The performance of the numerical approximation is tested for Mixed-Erlang distribution functions. The integral for situation 6 is not approximated, since the distribution function of the demand during [0,R+L], $F_{0,R+L}(\bullet)$, is easy to determine for Mixed-Erlang demands (see e.g. Tijms (1986)). The integral in situation 6, which takes the largest part of AHC(e), is obtained by $R \cdot e \cdot F_{0,R+L}(S)$.

Let μ denote the average demand per unit time and σ^2 the variance of demand per unit time. For different combinations of μ and σ^2, we calculate the order-up-to-level S and the maximum enlargement UB (based on B=0.05). Denote P2 the percentage of the demand which is satisfied directly from shelf. Using randomly generated demands from the Mixed-Erlang distribution with mean μ and variance σ^2, we determine the extra holding cost from an extra order of e units with help of simulation. The simulated extra holding cost of e units, SHC(e), is obtained for e=1 up to UB (if UB>0). The numerical approximations, AHC(e), are then compared with the simulation results. Table A1 lists the mean average percentage deviation, which is defined as:

$$MAD = 100 \cdot \frac{1}{UB} \cdot \sum_{e=1}^{UB} \frac{| AHC(e) - SHC(e) |}{SHC(e)}$$

TABLE A1

MAD for different combinations of μ and σ

		R = 1 , L = 2				R = 2 , L = 1			
			P2				P2		
μ	σ	UB	90%	95%	98%	UB	90%	95%	98%
10	8	0	-	-	-	5	0.67	1.07	0.91
20	10	6	0.39	0.42	0.31	19	0.81	0.67	0.43
5	2	2	1.48	1.40	0.80	5	3.41	2.61	1.49
12	6	4	0.67	0.53	0.55	11	1.07	0.90	0.51
16	10	3	0.74	0.75	0.63	12	0.62	0.83	0.52
15	5	7	1.05	0.74	0.44	19	1.83	1.65	1.06
8	4	2	0.61	0.43	0.31	7	1.54	1.25	0.60

Table A1 shows that the performance of the numerical integration method is quite good. For different (μ,σ) combinations the simulated values differed about 1% or 2% from the approximated values. In all test cases, the simulated value exceeded the value which was determined by numerical integration. An explanation is that formula (11) assumes that the effect of the extra ordered items lasts no longer than one review period.

Numerical investigations indicated also that AHC(e) is quite close to R·e·h for P2≥95%. This suggests to use the simple linear expression instead of the complicated formula (11) to approximate the extra holding cost. Table A2 gives the range over which the ratio AHC(e)/(R·e·h) varies for different situations when P2=95%. It's obvious that the range for P2≥95% is even closer to one.

TABLE A2

range of AHC(e)/(R·e·h) for different combinations of μ and σ

		R = 1 , L = 2		R = 2 , L = 1	
			P2		P2
μ	σ	UB	95%	UB	95%
10	8	0	-	5	0.968 - 0.971
20	10	6	0.966 - 0.971	19	0.962 - 0.977
5	2	2	0.960 - 0.962	5	0.950 - 0.966
12	6	4	0.967 - 0.971	11	0.959 - 0.971
16	10	3	0.969 - 0.971	12	0.965 - 0.975
15	5	7	0.962 - 0.968	19	0.957 - 0.965
8	4	2	0.965 - 0.966	7	0.959 - 0.965

IN 1990 REEDS VERSCHENEN

419 Bertrand Melenberg, Rob Alessie
A method to construct moments in the multi-good life cycle consumption model

420 J. Kriens
On the differentiability of the set of efficient (μ,σ^2) combinations in the Markowitz portfolio selection method

421 Steffen Jørgensen, Peter M. Kort
Optimal dynamic investment policies under concave-convex adjustment costs

422 J.P.C. Blanc
Cyclic polling systems: limited service versus Bernoulli schedules

423 M.H.C. Paardekooper
Parallel normreducing transformations for the algebraic eigenvalue problem

424 Hans Gremmen
On the political (ir)relevance of classical customs union theory

425 Ed Nijssen
Marketingstrategie in Machtsperspectief

426 Jack P.C. Kleijnen
Regression Metamodels for Simulation with Common Random Numbers: Comparison of Techniques

427 Harry H. Tigelaar
The correlation structure of stationary bilinear processes

428 Drs. C.H. Veld en Drs. A.H.F. Verboven
De waardering van aandelenwarrants en langlopende call-opties

429 Theo van de Klundert en Anton B. van Schaik
Liquidity Constraints and the Keynesian Corridor

430 Gert Nieuwenhuis
Central limit theorems for sequences with m(n)-dependent main part

431 Hans J. Gremmen
Macro-Economic Implications of Profit Optimizing Investment Behaviour

432 J.M. Schumacher
System-Theoretic Trends in Econometrics

433 Peter M. Kort, Paul M.J.J. van Loon, Mikulás Luptacik
Optimal Dynamic Environmental Policies of a Profit Maximizing Firm

434 Raymond Gradus
Optimal Dynamic Profit Taxation: The Derivation of Feedback Stackelberg Equilibria

IN 1991 REEDS VERSCHENEN

466 Prof.Dr. Th.C.M.J. van de Klundert - Prof.Dr. A.B.T.M. van Schaik
Economische groei in Nederland in een internationaal perspectief

467 Dr. Sylvester C.W. Eijffinger
The convergence of monetary policy - Germany and France as an example

468 E. Nijssen
Strategisch gedrag, planning en prestatie. Een inductieve studie binnen de computerbranche

469 Anne van den Nouweland, Peter Borm, Guillermo Owen and Stef Tijs
Cost allocation and communication

470 Drs. J. Grazell en Drs. C.H. Veld
Motieven voor de uitgifte van converteerbare obligatieleningen en warrant-obligatieleningen: een agency-theoretische benadering

471 P.C. van Batenburg, J. Kriens, W.M. Lammerts van Bueren and R.H. Veenstra
Audit Assurance Model and Bayesian Discovery Sampling

472 Marcel Kerkhofs
Identification and Estimation of Household Production Models

473 Robert P. Gilles, Guillermo Owen, René van den Brink
Games with Permission Structures: The Conjunctive Approach

474 Jack P.C. Kleijnen
Sensitivity Analysis of Simulation Experiments: Tutorial on Regression Analysis and Statistical Design

475 C.P.M. van Hoesel
An $O(nlogn)$ algorithm for the two-machine flow shop problem with controllable machine speeds

476 Stephan G. Vanneste
A Markov Model for Opportunity Maintenance

477 F.A. van der Duyn Schouten, M.J.G. van Eijs, R.M.J. Heuts
Coordinated replenishment systems with discount opportunities

478 A. van den Nouweland, J. Potters, S. Tijs and J. Zarzuelo
Cores and related solution concepts for multi-choice games

479 Drs. C.H. Veld
Warrant pricing: a review of theoretical and empirical research

480 E. Nijssen
De Miles and Snow-typologie: Een exploratieve studie in de meubelbranche

481 Harry G. Barkema
Are managers indeed motivated by their bonuses?

497 L.W.G. Strijbosch and R.M.J. Heuts
Investigating several alternatives for estimating the compound lead time demand in an (s,Q) inventory model

498 Bert Bettonvil and Jack P.C. Kleijnen
Identifying the important factors in simulation models with many factors

499 Drs. H.C.A. Roest, Drs. F.L. Tijssen
Beheersing van het kwaliteitsperceptieproces bij diensten door middel van keurmerken

500 B.B. van der Genugten
Density of the F-statistic in the linear model with arbitrarily normal distributed errors

501 Harry Barkema and Sytse Douma
The direction, mode and location of corporate expansions

502 Gert Nieuwenhuis
Bridging the gap between a stationary point process and its Palm distribution

503 Chris Veld
Motives for the use of equity-warrants by Dutch companies

504 Pieter K. Jagersma
Een etiologie van horizontale internationale ondernemingsexpansie

505 B. Kaper
On M-functions and their application to input-output models

506 A.B.T.M. van Schaik
Produktiviteit en Arbeidsparticipatie

507 Peter Borm, Anne van den Nouweland and Stef Tijs
Cooperation and communication restrictions: a survey

508 Willy Spanjers, Robert P. Gilles, Pieter H.M. Ruys
Hierarchical trade and downstream information

509 Martijn P. Tummers
The Effect of Systematic Misperception of Income on the Subjective Poverty Line

510 A.G. de Kok
Basics of Inventory Management: Part 1
Renewal theoretic background

511 J.P.C. Blanc, F.A. van der Duyn Schouten, B. Pourbabai
Optimizing flow rates in a queueing network with side constraints

512 R. Peeters
On Coloring j-Unit Sphere Graphs

513 Drs. J. Dagevos, Drs. L. Oerlemans, Dr. F. Boekema
Regional economic policy, economic technological innovation and networks

514 Erwin van der Krabben
Het functioneren van stedelijke onroerend-goed-markten in Nederland - een theoretisch kader

515 Drs. E. Schaling
European central bank independence and inflation persistence

516 Peter M. Kort
Optimal abatement policies within a stochastic dynamic model of the firm

517 Pim Adang
Expenditure versus consumption in the multi-good life cycle consumption model

518 Pim Adang
Large, infrequent consumption in the multi-good life cycle consumption model

519 Raymond Gradus, Sjak Smulders
Pollution and Endogenous Growth

520 Raymond Gradus en Hugo Keuzenkamp
Arbeidsongeschiktheid, subjectief ziektegevoel en collectief belang

521 A.G. de Kok
Basics of inventory management: Part 2
The (R,S)-model

522 A.G. de Kok
Basics of inventory management: Part 3
The (b,Q)-model

523 A.G. de Kok
Basics of inventory management: Part 4
The (s,S)-model

524 A.G. de Kok
Basics of inventory management: Part 5
The (R,b,Q)-model

525 A.G. de Kok
Basics of inventory management: Part 6
The (R,s,S)-model

526 Rob de Groof and Martin van Tuijl
Financial integration and fiscal policy in interdependent two-sector economies with real and nominal wage rigidity