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Nijman, T.E.; Palm, F.C.

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…SITY
UNIVERSITEIT
BRABANT
POSTBOX 90153
5000 LE TILBURG
THE NETHERLANDS

DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

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Th.E. Nijman
F.C. Palm

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DISAGGREGATE SAMPLING IN ARIMA - MODELS

T.E. Nijman*

F.C. Palm**

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Preliminary version, comments welcome.

ABSTRACT

In this paper, we compare the forecast accuracy of autoregressive - integrated - moving average (ARIMA) models based on data observed with high and low frequency respectively. We discuss how for instance quarterly models can be used to predict one quarter ahead even if only annual data are available and we compare the variance of the prediction error in this case with the variance if quarterly observations were indeed available. Some insight in the expected gain of information is required to decide on whether to collect data with a higher frequency or to use a model based on observations sampled with a low frequency.
Results on the expected information gain are presented for a number of ARIMA-models including models which describe seasonally adjusted series on gross national product (GNP) in the Netherlands.

* Department of Econometrics, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, The Netherlands.

** Department of Economics, University of Limburg, P.O. Box 616, 6200 MD Maastricht, The Netherlands.

1. INTRODUCTION

In recent years there has been an increased tendency towards collecting and analyzing disaggregate data. In the Netherlands for instance, the Central Bureau of Statistics publishes quarterly National Accounts which were until a few years ago only available on an annual basis. In the U.S., many series are nowaday available on a monthly basis.

In this paper, we show how much additional information is contained in the temporally disaggregate data that can be used to improve the forecast performance at the disaggregate level. Throughout the paper, we assume that the observations are measured without error. Knowledge about the expected gain of information is required to decide on whether to collect data with a higher frequency. Moreover, our results can contribute to solving the choice problem of using infrequently sampled data with negligible measurement errors or data at a disaggregate level which generally include larger errors as they are often partly constructed or estimated.

We restrict ourselves to the comparison of the forecast accuracy of correctly specified univariate ARIMA-models based on data observed with a low and high frequency respectively. The implication of temporal aggregation for the model specification and for parameter estimation have been studied by Brewer (1973) and Weiss (1984) for ARMA and ARMAX-models, and by Engle and Liu (1972), Geweke (1978), Mundlak (1971), Teräsvirta (1980), Wei (1978) and Zellner and Montmarquette (1971) among others for regression models. Palm and Nijman (1984) considered the identification and estimation of ARIMA-models for variables that are sampled with longer intervals than the interval of realization. The estimation of the unobserved realizations has been considered in the literature on interpolation and distribution of time series (see e.g. Chow and Lin (1971), Fernandez (1981), Harvey and Pierse (1984), Nijman (1985), Nijman and Palm (1986) and Litterman (1983)). The loss of information due to contemporaneous aggregation has been analyzed by Kohn (1982), Lütkepohl (1984,a,b), Rose (1977) and Tiao and Guttman (1983). Most closely related to our work are the attempts by e.g. Ahsamullah and Wei (1984), Amemiya and Wu (1972) and Lütkepohl (1986) to quantify the effect of temporal aggregation on the forecast error variance for the disaggregated time series. Ahsamullah and Wei (1984) and Amemiya and Wu (1972) consider flow variables that are generated by known stationary ARMA(1,1) and AR(1) models respectively, whereas Lütkepohl (1986) uses large sample theory and Monte Carlo methods to analyze stock variables that are generated by vector ARIMA-processes with unknown coefficients.

In this paper, we are concerned with predicting disaggregate times series given that the realizations are sampled with a lower frequency. We show how for instance quarterly models can be used to predict one quarter ahead even if only annual data are available and we compare the variance of the prediction error in this case with the prediction error variance when the process is observed each quarter.
The plan of the paper is as follows. In section 2, we present some analytical results on the reduction in the variance of the prediction error due to increasing the frequency of sampling to become identical to that of the realization of the variables. The classical Wiener-Kolmogorov filtering theory is used to derive these results. For cases which are not analytically tractable, numerical results have been obtained using the Kalman filter. Whereas in section 2, the sample is assumed to be sufficiently large to neglect specification and estimation errors of the ARIMA-models considered, the impact of parameter estimation on the forecast accuracy is treated in section 3. In section 4, we analyze quarterly data on the seasonally adjusted gross national product (GNP) series in the Netherlands which has been recently constructed at the Netherlands Central Bank (see De Nederlandsche Bank, 1986). Using results of the previous sections, we show by how much the prediction error variance of quarterly GNP is reduced through the availability of past quarterly observations on this series and we examine whether a further disaggregation to monthly data is desirable. Finally, section 5 contains concluding remarks.

2. ARIMA-MODELS WITH KNOWN PARAMETERS

As an example, assume that the time series Y_t, t = 1,2...T is generated by the univariate AR(1)-model

$$Y_t = \rho\, Y_{t-1} + \epsilon_t, \quad \epsilon_t \sim IN(0,\sigma^2). \tag{1}$$

If Y_t is a stock variable observed every m-th period (m > 1), the sample will consist of the values of Y_t for $t \in T_m = \{m, 2m, \ldots [T/m]m\}$, where $[T/m]$ is the largest integer smaller than or equal to T/m. If Y_t is a flow variable, $\bar{Y}_t = \sum_{i=0}^{m-1} Y_{t-i}$, $t \in T_m$, will be observed. Occasionally, we assume that observations on an infinite past are available in which case we use the notation $T_m(-\infty)$ to denote the set $T_m(-\infty)=\{-\infty,\ldots,m,2m\ldots[T/m]m\}$.
If Y_t is a stock variable, the minimum mean square error (MMSE) predictor of Y_{T+k} (k > 0) is simply

$$E(Y_{T+k} \mid Y_t,\ t \in T_m) = \rho^{k+r}\, Y_{T-r}, \tag{2}$$

where r is the number of periods between T and the last low frequency observation, $r = T - [T/m]m$. The variance of the prediction error is

$$\text{var}\,\{Y_{T+k} - E[Y_{T+k} \mid Y_t,\ t \in T_m]\} = \sigma_\epsilon^2\,(1 - \rho^{2(k+r)})(1 - \rho^2)^{-1},$$
$$= \nu^m_{k,r}, \tag{3}$$

which could be compared with the variance corresponding to $m = 1$. A measure of the loss of forecast accuracy due to temporal aggregation is the reduction in percentage points of the variance due to observing the variable at shorter time intervals

$$\gamma^m_{k,r} = 100\,(\nu^m_{k,r} - \nu^1_{k,o})(\nu^m_{k,r})^{-1}. \tag{4}$$

If Y_t is a stock variable generated by model (1), expression (4) specializes to

$$\gamma^m_{k,r} = 100\,(\rho^{2k} - \rho^{2(k+r)})(1 - \rho^{2(k+r)})^{-1}, \tag{5}$$

which implies that the potential gain of information is purely caused by the possibility that Y_t might have been observed after period $[T/m]m$. In table 1, upperbounds $\gamma^m_{k,m-1}$ on the information gain are given as a function of ρ, k and m.

The information gain is substantial only in a situation of short term forecasting when the autoregressive parameter is large in absolute value.

Table 1 : Upperbounds (in percentage points) for the reduction of the prediction error variance $\gamma^m_{k,r}$, when a stock variable is generated by an AR(1)-model.

	ρ = ± 0.8			ρ = ± 0.4		
	m = 2	m = 3	m = 4	m = 2	m = 3	m = 4
k = 1	39	51	57	14	16	16
2	20	29	34	2	3	3
3	12	17	21	0	0	0
12	0	0	0	0	0	0

If Y_t is a flow variable, it is less straightforward to derive MMSE predictors.

Using an ARMA(1,1)model which generates $\bar{Y}_t$, $t \in T_m$, if (1) is valid, Amemiya and Wu (1972) give some results for $k = m$ and $r = 0$. Their results indicate that the variance of the prediction error based on low frequency data exceeds the corresponding figure for the complete sample by less than 10% if $m \leq 4$ and $\rho \leq .6$. To the best of our knowledge, the case of one period ahead forecasts has not been treated in the literature. Filtering theory can be applied to derive one period ahead MMSE predictors even if $\bar{Y}_t$ is observed for $t \in T_m$ only. For reason of simplicity, we consider the case where $m = 2$ and $r = 0$ first. Writing

$$\begin{bmatrix} Y_t \\ \bar{Y}_t \end{bmatrix} = \frac{1}{1 - \rho^2 L^2} \begin{bmatrix} 1 & \rho & \epsilon_t \\ 1 + \rho L^2 & 1 + \rho & \epsilon_{t-1} \end{bmatrix}, \qquad (6)$$

the covariance generating function of $(Y_t, \bar{Y}_t)$ $(t \in T_2)$ is given by

$$g(z) = \frac{\sigma_\epsilon^2}{(1 - \rho^2 z)(1 - \rho^2 z^{-1})} \begin{bmatrix} 1 + \rho^2 & 1 + \rho + \rho^2 + \rho z^{-1} \\ 1 + \rho + \rho^2 + \rho z & 2(1 + \rho + \rho^2) + \rho(z + z^{-1}) \end{bmatrix} (7).$$

Denoting the (i,j)-th element of g by g_{ij}, results from filtering theory (see e.g. Sargent (1979) or Priestley (1981)) imply that

$$E[Y_T \mid \bar{Y}_t, t \in T_m(-\infty)] = \left[\frac{g_{12}(L^2)}{d(L^{-2})}\right]_+ \frac{1}{d(L^2)} \bar{Y}_T = h(L^2)\bar{Y}_t, \qquad (8)$$

where $d(z)$ is defined by $d(z)\, d(z^{-1}) = g_{22}(z)$ and $[\ ]_+$ indicates that only non-negative powers of L between the brackets have to be taken into account.

As $2(1+\rho+\rho^2) + \rho(z^{-1}+z) = \mu(1-\lambda z)(1-\lambda z^{-1})$ with $\mu = 1+\rho+\rho^2+(1+\rho)\sqrt{1+\rho^2}$ and $\lambda = -\rho/\mu$

$$h(z) = \left[\frac{(1+\rho+\rho^2) + \rho z^{-1}}{(1 - \rho^2 z)(1 - \lambda z^{-1})}\right]_+ \frac{1 - \rho^2 z}{(1 - \lambda z)\mu}. \qquad (9)$$

Some straightforward algebra yields that if m = 2 and r = 0, we get

$$E[y_T \mid \bar{Y}_t,\ t \in T_m(-\infty)] = \eta \sum_{i=0}^{\infty} \lambda^i \bar{Y}_{T-im} \tag{10}$$

with

$$\lambda = -\rho(1 + \rho + \rho^2 + (1 + \rho)\sqrt{1 + \rho^2})^{-1} \tag{11}$$

and

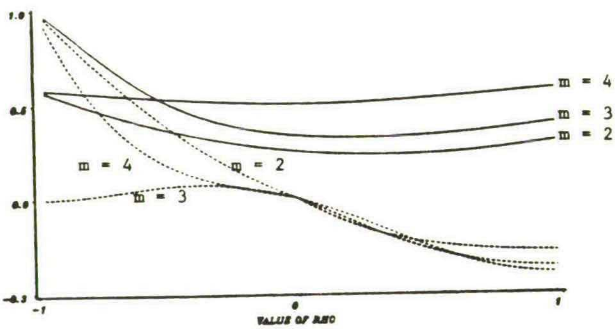
$$\eta = (1 + \rho + \rho^2 + \rho^3)\ (1 + \rho + \rho^2 + (1 + \rho)\sqrt{1 + \rho^2})^{-1} \tag{12}$$

Evidently (10) implies that

$$E[y_{T+k} \mid \bar{Y}_t,\ t \in T_m(-\infty)] = \rho^{k+r}\, \eta \sum_{i=0}^{\infty} \lambda^i \bar{Y}_{T-r-im}, \tag{13}$$

where we drop the assumption that T/m is integer-valued. Truncating the sum in (13) at i = [T/m], these results can be used for example to compute semi-annual forecasts from annual observations on flow variables if Y_t is generated by (1). Equation (13) is also valid for m > 2. Explicit expressions for λ and η if m > 1 are hardly informative. The values of λ and η for m = 2,3,4 and ρ ∈ (-1,1) are given in figure 1. For the derivation of these results see appendix A. Note that when ρ tends to -1 the weight λ tends to 1 if m is even, but to 0 if m is odd.

Figure 1 : Values of lambda and eta for m = 2,3,4 as a function of rho.



From (7) and (10), the covariance generating function of the prediction error of y_T for m = 2 and r = 0 is

$$g^*(z) = [1, -\eta(1-\lambda z)^{-1}]\, g(z) \begin{bmatrix} 1 \\ -\eta(1-\lambda z^{-1})^{-1} \end{bmatrix} \qquad (14)$$

The variance of the prediction error which we denote by ν is the constant term in $g^*(z)$ which after some manipulation can be expressed as

$$\nu = \frac{\sqrt{1+\rho^2}-1}{\rho^2(1+\rho)}\, \sigma_\varepsilon^{2}. \qquad (15)$$

For the variance of the prediction error, we get

$$\bar{\nu}^m_{k,r} = [1-\rho^{2(k+r)}][1-\rho^2]^{-1} + \rho^{2(k+r)}\,\nu, \qquad (16)$$

from which $\bar{\gamma}^m_{k,r}$ can be determined. The magnitudes $\bar{\nu}^m_{k,r}$ and $\bar{\gamma}^m_{k,r}$ are defined as $\nu^m_{k,r}$ and $\gamma^m_{k,r}$ except for the fact that the observations are on fow variables.

Equation (16) is also valid for m ≥ 2 although no simple expression for ν has been obtained. It can be shown that ν tends to infinity if m is even when ρ tends to -1.

The values of $\bar{\gamma}^m_{k,r}$ have been presented in table 2 for some illustrative values of ρ and m. Details on their derivation are given in appendix A.
In the worst possible case considered in table 2, the variance of the prediction error approximately doubles if low frequency data are analyzed, but in many cases the loss of information is much smaller. Note that the variance of the prediction error can be smaller if the data are aggregated over three periods than if they are aggregated over two periods.

Table 2 : Upperbounds (in percentage points) for the reduction of the prediction error variance $\bar{\gamma}^{m}_{k,r}$ when a flow variable is generated by an AR(1)-model.

	ρ = .8			ρ = .4		
	m = 2	m = 3	m = 4	m = 2	m = 3	m = 4
k = 1	43	54	59	15	16	16
2	22	32	36	2	3	3
3	13	19	22	0	0	0
12	0	0	0	0	0	0
	ρ = -.8			**ρ = .-4**		
	m = 2	m = 3	m = 4	m = 2	m = 3	m = 4
k = 1	61	57	63	15	16	16
2	38	34	40	3	3	3
3	24	21	25	0	0	0
12	0	0	0	0	0	0

It is well known (see e.g. Palm and Nijman (1984)) that the missing observations problem in an ARIMA (p,d,q) model for flow variables is closely related to that in an ARIMA (p,d+1,q) model for stock variables.
Consider the situation where Y_t is generated by

$$\Delta Y_t = \rho\, \Delta Y_{t-1} + \epsilon_t\ , \quad \epsilon_t \approx NID(0,\ \sigma^2_\epsilon), \tag{17}$$

Assume that Y_t is a stock variable such that $\sum_{i=0}^{m-1} \Delta Y_{t-i} = Y_t - Y_{t-m}$ is observed for $t \in T_m$.

Writing

$$Y_{T+k} = Y_{T-r} + \sum_{i=1}^{k+r} \Delta Y_{T-r+i}, \tag{18}$$

we have

$$E[Y_{T+k} \mid Y_t,\ t \in T_m\ (-\infty)] = Y_{T-r} + \sum_{i=1}^{k+r} \rho^i \eta \sum_{j=0}^{\infty} \lambda^j \Delta_m Y_{T-r-jm} \tag{19}$$

with $\Delta_m = 1-L^m$. The parameters η and λ which have been defined above depend on ρ and m. The variance of the prediction error is now

$$\text{Var}\ \{\sum_{i=1}^{k+r} (\Delta Y_{T-r+i} - \rho^i \Delta Y_{T-r}) \}^2 + (\sum_{i=1}^{k+r} \rho^i)^2 \nu, \tag{20}$$

which yields the upperbounds on $\gamma_{k,r}^{m}$ presented in table 3. The information gain caused by the use of high frequency data is usually much larger than that in table 2. Again the variance of the prediction error of data aggregated over three periods is occasionally smaller than that of aggregates over two periods. Note that the variance of the error of predictions based on the incomplete data is no longer a non-decreasing function of k. Therefore the upperbound no longer coincides with $\gamma_{k,m-1}^{m}$.

For more general ARIMA-models it will usually be cumbersome to derive analytical expressions for the gain in forecast accuracy using e.g. the Wiener-Komolgorov filtering theory. In these cases, the recursive Kalman filter (see e.g. Harvey (1981) or Anderson and Moore (1979)) can be used to evaluate the conditional expectations and the associated variances of the prediction errors numerically in a straightforward way.
The predictive accuracy gain from disaggregate sampling for the ARI(1,1) model (17) and the IMA(1,1) model

$$\Delta Y_t = \epsilon_t + \alpha\ \epsilon_{t-1}\ ,\ \epsilon_t \approx \text{NID}(0,\sigma_\epsilon^2), \tag{21}$$

which are often adequate for describing the dynamics of macroeconomic variables, has been evaluated by Kalman filter methods. Results for the ARI(1,1) model for stock variables were already presented in table 3.

Results for the ARI(1,1) model for flow variables and for the IMA(1,1) model for stock and flow variables are presented in tables 4, 5 and 6 respectively. Form these tables, it becomes evident that if succeeding values of ΔY_t are negatively correlated, the information content of disaggregated data will be much smaller than in the case where the serial correlation of ΔY_t is positive.

Table 3 : Upperbounds (in percentage points) for the reduction of the prediction error variance $\gamma_{k,r}^{m}$ when a stock variable is generated by an ARI(1,1) model.

	ρ = .8			ρ = .4		
	m = 2	m = 3	m = 4	m = 2	m = 3	m = 4
k = 1	79	92	96	67	82	88
2	62	81	88	47	64	73
3	50	70	80	34	51	61
12	14	25	33	9	16	22
	ρ = -.8			ρ = -.4		
	m = 2	m = 3	m = 4	m = 2	m = 3	m = 4
k = 1	58	54	64	29	50	60
2	62	52	67	32	45	55
3	36	36	45	22	35	45
12	15	15	24	8	15	20

Table 4 : Upperbounds (in percentage points) for the reduction of the prediction error variance $\bar{\gamma}^m_{k,r}$ when a flow variable is generated by an ARI(1,1) model.

	ρ = 0.8			ρ = 0.4		
	m = 2	m = 3	m = 4	m = 2	m = 3	m = 4
k = 1	84	94	97	73	86	91
2	69	85	92	53	70	78
3	57	76	85	39	57	67
12	17	30	39	11	20	27
	ρ = -0.8			ρ = -0.4		
	m = 2	m = 3	m = 4	m = 2	m = 3	m = 4
k = 1	26	50	53	31	52	63
2	43	48	59	32	47	58
3	13	32	37	22	38	48
12	7	14	19	1	15	22

Table 5 : Upperbounds (in percentage points) for the reduction of the prediction error variance $\gamma^m_{k,r}$ when a stock variable is generated by an IMA(1,1) model.

	α = 0.8			α = 0.4		
	m = 2	m = 3	m = 4	m = 2	m = 3	m = 4
k = 1	79	88	91	67	80	86
2	47	62	71	41	58	67
3	33	48	58	30	45	55
12	9	16	22	8	15	21
	α = -0.8			α = -0.4		
	m = 2	m = 3	m = 4	m = 2	m = 3	m = 4
k = 1	9	15	20	29	44	54
2	8	15	19	23	37	46
3	8	14	19	19	32	40
12	7	11	15	7	14	19

Table 6 : Upperbounds (in percentage points) for the reduction of the prediction error variance $\gamma^m_{k,r}$ when a flow variable is generated by an IMA(1,1) model.

	$\alpha = 0.8$			$\alpha = 0.4$		
	m = 2	m = 3	m = 4	m = 2	m = 3	m = 4
k = 1	82	90	93	72	84	88
2	53	68	76	47	64	72
3	39	55	64	35	51	61
12	12	19	26	11	43	25
	$\alpha = -0.8$			$\alpha = -0.4$		
	m = 2	m = 3	m = 4	m = 2	m = 3	m = 4
k = 1	4	7	12	29	45	55
2	4	7	11	22	38	48
3	4	7	11	19	32	42
12	3	6	8	7	14	20

3. ARIMA MODELS WITH ESTIMATED PARAMETERS

In the preceding section we have assumed that the parameters of the data generating process are known. One could argue that the figures given in section 2 underestimate the efficiency gain in applications where the parameters have to be estimated and can be estimated more accurately if the frequency sampling is increased. In this section we drop the assumption that the parameters are known and we present approximations up to order T^{-1} for the prediction error variances when the parameters have been estimated. Evidently the results in the previous section are valid if T is sufficiently large. The identification and estimation of ARIMA models from temporally aggregated data is discussed in Palm and Nijman (1984). Their results suggest that the efficiency gain of maximum likelihood parameter estimation from increasing the frequency of observation strongly depends on the data generating process. In this section, we analyze how parameter estimation affects the accuracy gain in prediction expected from more frequent observation. As in section 2, we first consider the AR(1)model (1) assuming that Y_t is a stock variable. Straightforward substitution yields that the low freqeuncy data are generated by an AR(1)model as well,

$$Y_t = \psi Y_{t-m} + V_t, \; V_t = IN(0, \sigma_V^2), \tag{22}$$

with $\psi = \rho^m$, $\sigma_\nu^2 = (1-\rho^{2m})(1-\rho^2)^{-1}\ \sigma_\epsilon^2$, V_t and V_s independent if t, s $\subset T_m$.

The parameter ρ is identified in (22) if its sign is known a priori. In appendix A it is shown that if $\rho \neq 0$, we have

$$\sqrt{T}\ (\hat\rho - \rho) \overset{a}{\approx} N(0, \frac{1 - \rho^{2m}}{m\ \rho^{2m-2}}), \tag{23}$$

where $\hat\rho$ is the maximum likelihood (ML) estimator of ρ based on Y_t ($t \subset T_m$).

Substituting the estimate $\hat\rho$ in (2) one obtains

$$Y_{T+k} - \hat\rho^{k+r}\ Y_{T-r} = \sum_{i=0}^{k+r-1} \rho^i\ \epsilon_{T+k-i} + (\hat\rho^{k+r} - \rho^{k+r})\ Y_{T-r}. \tag{24}$$

The mean squared error (MSE) of the second term can be approximated up to order T^{-1} by

$$E\ \{(\rho^{k+r} - \rho^{k+r})Y_{t-r}\}^2 \approx \frac{T}{(k+r)^2\ \rho^{2(k+r-1)}}\ E\{\sqrt{T}\ (\rho - \rho)Y_{T-r}\}^2 \tag{25}$$

In the literature, the (unrealistic) assumption is often made that parameters are estimated from samples independent of the values to be predicted. In that case, we get

$$E(Y_{T+k} - \hat\rho^{k+r}\ Y_{T-r})^2 \approx \frac{1 - \rho^{2(k+r)}}{1 - \rho^2}\ \sigma_\epsilon^2 + d\ \frac{(k+r)^2\rho^{2(k+r-1)}}{T}\ \frac{1-\rho^{2m}}{m\rho^{2m-2}}\ \frac{2\sigma_\epsilon}{1 - \rho^2} \tag{26}$$

with d = 1. If the independence assumption is not made one can bound the covariance between $\sqrt{T}(\hat\rho - \rho)$ and Y_{T-r} using the Cauchy-Schwarz inequality, which implies that the right hand side of (26) is an upperbound for the MSE when d = 3 and a lowerbound when d = -1, if one sample is used both for estimation and prediction. In table 7 we present upperbounds on the relative efficiency $\hat\gamma^m_{k,r}$ of predictors based on high frequency data in case of estimated parameters. For given values of m and k, these upperbounds are the maximum over r of the quotient of the right hand side of (26) evaluated at m,k,r and d = 3 and the same expression evaluated at m = 1,k,r and d = -1.

The reader should compare table 7 with table 1 where the results for the same model with known parameters are presented (or, equivalently, for a very large sample). The information loss caused by temporal aggregation increases if the parameters have to be estimated but the effect is not very substantial in many cases.

Table 7. Upperbounds (in percentage points) for the reduction of the prediction error variance when a stock variable is generated by an AR(1)-model with estimated parameters (T = 100).

	$\rho = \pm.8$			$\rho = \pm.4$		
	m = 2	m = 3	m = 4	m = 2	m = 3	m = 4
k = 1	43	56	62	19	32	69
2	26	36	42	7	21	55
3	19	25	29	2	8	30
12	2	2	3	0	0	0

If a flow variable $\bar{Y}_t$ is generated by the AR(1)model in equation (1), the low frequency data $\bar{Y}_t$, $t \in T_m$, are generated by the ARMA(1,1)model

$$\bar{Y}_t = \psi \bar{Y}_{t-m} + V_t - \lambda V_{t-m}, \qquad (27)$$

where $\psi = \rho^m$, V_t and V_s are independent for $t, s \in T_m$ and $t \neq s$, $v_t \simeq N(0, \sigma_v^2)$ and the parameter λ has been defined in section 2. In appendix A we show that ρ can be identified from the observations Y_t $(t \in T_m)$.
Using the asymptotic distribution of unrestricted efficient estimates of $(\psi, \lambda, \sigma_v^2)$ we moreover show that the maximum likelihood estimator of ρ in the restricted model (27) is asymptotically normally distributed

$$\sqrt{T}\,(\hat{\rho} - \rho) \overset{a}{\simeq} N(0,q) \qquad (28)$$

where q is a function of ρ and σ_ϵ^2 (see appendix A).

If $E[\bar{Y}_{t+k} \mid Y_t \; t \in T_m (-\infty)]$ in equation (13) is expressed as $\sum_{i=0}^{\infty} a_i \; \bar{Y}_{T-r-i,m}$

the MSE of the prediction error in case of parameters estimated from an independent sample can be approximated up to order T^{-1} by

$$E(\bar{Y}_{T+k} - \sum_{i=0}^{\infty} a_i \, Y_{T-r-im})^2 + \frac{dq}{T} \sum_{i,j=0}^{\infty} \frac{\partial a_i}{\partial \rho} \frac{\partial a_j}{\partial \rho} E \, \bar{Y}_{T-im} \bar{Y}_{T-jm}. \qquad (29)$$

with d = 1.

If estimation and prediction are based on the same sample (29) yields upper and lower bounds for the MSE for d=3 and d=-1 respectively, while the results in section 2 (known parameter values) are obtained if d=0. Upperbounds are presented in table 8 which should be compared with table 2. The estimation of parameters affects the conclusions from table 2 only if ρ is highly negative in which case it is very difficult to estimate ρ as becomes evident from table 1 in Palm and Nijman (1984).

In such cases the additional information contained in high frequency data can indeed be very substantial.

Finally we consider in this section the case where a stock variable Y_t is generated by the ARI(1,1)model (17). The analogue of equation (27) is

$$\Delta_m Y_t = \psi \, \Delta_m Y_{t-m} + V_t - \lambda \, V_{t-m}, \qquad (30)$$

where the same notation is used. The asymptotic variance of the ML estimate $\hat{\rho}$ is also given in equation (28). Defining the coefficients b_i using (19) by

$E[Y_{T+k} \mid Y_t \; t \in T_m(-\infty)] = Y_{T-r} + \sum_{i=0}^{\infty} b_i \; \Delta_m Y_{T-r-im}$ the analogue of (29) is

$$E[Y_{T+k} - Y_{T-r} - \sum_{i=0}^{\infty} b_i \; \Delta_m Y_{T-r-im}]^2 + \frac{dq}{T} \sum_{i,j=0}^{\infty} \frac{\partial b_i}{\partial \rho} \frac{\partial b_j}{\partial \rho} E \, \Delta_m Y_{T-im} \, \Delta_m Y_{T-jm}. \qquad (31)$$

This equation has been used to derive table 11. Again the conclusions strongly differ from those obtained for known parameter values only if ρ is negative.

Table 8 : Upperbounds (in percentage points) for the reduction of the prediction error variance when a flow variable is generated by an AR(1)model with estimated parameters (T=100).

	$\rho = 0.8$			$\rho = 0.4$		
	m = 2	m = 3	m = 4	m = 2	m = 3	m = 4
k = 1	44	55	60	16	17	17
2	25	33	40	3	3	3
3	16	21	24	1	1	1
12	0	0	0	0	0	0
	$\rho = -0.8$			$\rho = -0.4$		
	m = 2	m = 3	m = 4	m = 2	m = 3	m = 4
k = 1	99	61	98	69	27	50
2	98	40	97	44	11	25
3	97	28	95	19	3	9
12	0	0	0	0	0	3

Table 9 : Upperbounds (in percentage points) for the reduction of the prediction error variance when a stock variable is generated by an ARI(1,)model with estimated parameters (T=100).

	m = 2	m = 3	m = 4	m = 2	m = 3	m = 4
k = 1	80	92	96	68	83	88
2	64	81	89	48	65	74
3	52	71	81	36	52	62
12	17	25	33	9	16	22
	m = 2	m = 3	m = 4	m = 2	m = 3	m = 4
k = 1	98	59	98	67	53	63
2	98	58	97	54	47	59
3	97	41	96	35	38	49
12	15	15	24	8	15	20

4. PREDICTION ACCURACY GAIN FROM DISAGGREGATING THE GNP SERIES FOR THE NETHERLANDS.

In this section, we illustrate how in practice one can determine whether it is worthwhile to increase the frequency of collecting observations on a variable. We consider that quarterly GNP series for the Netherlands that has recently been provided by the Dutch Central Bank (see De Nederlandsche Bank (1986)). Using the results of the previous sections, we show how much the availability of quarterly observations reduces the prediction errror variance of quarterly GNP and whether further disaggregation into monthly data is desirable.
First, we consider seasonally adjusted GNP in millions of guilders in prices of 1980. For the period 1957,I-1984,IV, a Box-Jenkins analysis leads us to select two quarterly models that are both fairly well in agreement with the information in the data. If a month is chosen as the time unit, the two models are

$$\Delta_3 \overline{Y}_t = \underset{(113)}{543} - \underset{(.09)}{0.33}\ \Delta_3\ \overline{Y}_{t-3} + \hat{V}_t \tag{32}$$

and

$$\Delta_3 \overline{Y}_t = \underset{(70)}{408} + \hat{V}_t - \underset{(.09)}{0.35}\ \hat{V}_{t-3} \tag{33}$$

respectively. The parameters have been estimated by maximum likelihood (ML). Standard errors given between parentheses. From tables 4 and 6, it is now obvious that the increase in forecast accuracy due to the availability of quarterly data can be more than 50%. The details are given in table 10. Note that here the increase in efficiency is strictly increasing with r, the number of periods since the last observation. Also, even one is interested in annual forecasts only, quarterly data can be substantially more informative than annual observations.

Table 10 : The reduction (in percentage points) of the prediction error variance of quarterly seasonally adjusted GNP in the Netherlands due to the use of quarterly instead of annual observations (r is the number of periods since the last observation).

Model	r	Number of quarters to be predicted ahead (k)					
		1	2	3	4	8	12
ARI(1,1)	0	21	19	13	11	7	5
"	1	44	39	30	25	15	11
"	2	58	50	41	36	23	17
"	3	66	59	49	44	29	22
IMA(1,1)	0	17	13	10	8	5	4
"	1	39	31	25	22	14	10
"	2	51	43	36	32	21	16
"	3	60	51	44	39	27	21

The quarterly data can also be used to forecast monthly GNP and to estimate the reduction in the variance of monthly prediction errors if monthly data were indeed collected. The first step is to estimate a monthly model from the quarterly data. The monthly IMA(1,1) model

$$Y_t = c + \epsilon_t + \alpha\, \epsilon_{t-1} \qquad (34)$$

implies that $\Delta_3\, \overline{Y}_t = (1 + L + L^2)^2\ \Delta Y_t$ is generated by

$$\Delta_3\, \overline{Y}_t = 9\, c + \sum_{i=0}^{5} \theta_i\ \epsilon_{t-i} \qquad (35)$$

with $\theta_0 = 1$, $\theta_1 = 2 + \alpha$, $\theta_2 = 3 + 2\alpha$, $\theta_3 = 2 + 3\alpha$, $\theta_4 = 1 + 2\alpha$ and $\theta_5 = \alpha$. The model in (35) is a quarterly IMA(1,1) model

$$\Delta_3\, \overline{Y}_t = 9\, c + V_t + \lambda\, V_{t-3}, \qquad (36)$$

where λ is defined by the equality of the first autocorrelation of (35) and (36)

$$\lambda/(1+\lambda^2) = \Sigma_{i=3}^{5} \theta_i\theta_{i-3}/\Sigma_{i=0}^{5}\theta_i^2. \qquad (37)$$

Substituting the ML estimate $\hat{\lambda} = -0.35$ for λ in (37) one gets the ML estimate

$\hat{\alpha} = -0.72$. No other plausible time series model appears to be able to explain the empirical findings in (33) which are almost equivalent to (32).

Monthly forecasts can now be generated from quarterly data using (34). In appendix B, it is shown that

$$E[Y_{t+k} \mid Y_t,\ t \in T_3(-\infty)] = [k + r + 1 - 3\lambda / (1+\lambda)]\ c + \frac{1}{3}(1 + \lambda) \sum_{i=0}^{\infty} (-\lambda)^i\ \bar{y}_{T-r-3i}. \qquad (38)$$

The variance of the prediction error of (38) can be compared with that of the optimal predictor from monthly data

$$E[Y_{T+k} \mid Y_t\ ,\ t \in T_1\ (-\infty)] = [k - \alpha / (1+\alpha)]\ c + + (1 + \alpha) \sum_{i=0}^{\infty} (-\alpha)^i\ Y_{T-i}. \qquad (39)$$

The empirical results are presented in table 11 where it is assumed that α and c are known a priori. Evidently, this table suggests that monthly data on GNP in the Netherlands would hardly contain more information then the existing quarterly series.

Table 11 : The reduction (in percentage points) of the prediction error variance of quarterly seasonally adjusted GNP in the Netherlands due to the use of monthly instead of quarterly series.

Model	r	Number of months to be predicted ahead (k)					
		1	2	3	6	9	12
IMA(1,1)	0	0.8	0.8	0.7	0.6	0.5	0.4
IMA(1,1)	1	8.0	7.4	6.9	5.8	5.0	4.5
IMA(1,1)	2	14.2	13.3	12.5	10.1	8.8	8.2

Now we examine whether as a result of the assumption of known parameters the true predictive accuracy gain from increasing the frequency of data collection is substantially underestimated in table 11. The comparison between tables 3 and 9 suggests that the impact of this assumption might not be negligible, but these tables refer to ARI(1,1) processes for stock variables and not to an IMA(1,1) process for a flow variable as in (34). The relative efficiency of the ML estimator of α in (34) from monthly data compared with that from quarterly data can be found in Palm and Nijman (1984) for various values of α. When the true value of α is -.6, the relative efficiency is only 2.7, which suggests that the results in table 11 where $\alpha = -.72$ should not be too sensitive to the assumption of known parameters. Numerical results on upperbounds for the reduction of the prediction error variance are given in table 12. For the derivation we refer to appendix B. From table 12, it becomes clear that the impact of parameter estimation is not sufficiently large to alter the main conclusions. Nevertheless, the change in the predictive accuracy for longer forecast horizons is probably larger than suggested by table 11.

Table 12. Upperbounds (in percentage points) for the reduction of the prediction error variance of quarterly seasonally adjusted GNP in the Netherlands due to the use of monthly series instead of quarterly information corrected for the effect of parameter estimation.

Model	r	Number of periods ahead (k) to be predicted.					
		1	2	3	6	9	12
IMA(1,1)	0	5.6	6.6	7.8	11.5	15.4	19.2
IMA(1,1)	1	13.4	14.1	14.8	17.4	20.4	23.5
IMA(1,1)	2	20.3	20.6	20.9	22.8	25.1	27.6

Until now, we have considered seasonally adjusted data. The quarterly model for seasonally adjusted GNP implies the annual model

$$\Delta_{12} \bar{Y}_t = 6496 + \nu_t + 0.13\, \nu_{t-12}, \qquad (40)$$

which is also compatible with the quarterly model

$$(1 - 0.90L^3)\Delta_{12} \bar{Y}_t = 472 + (1 - 0.34L^3)\ (1 - 0.66L^{12})\nu_t, \qquad (41)$$

that describes the seasonally unadjusted data constructed by De Nederlandsche Bank (1982) very well.

One might be tempted to think that (41) can be used to obtain quarterly unadjusted forecasts from annual data. However, evidently the seasonal pattern cannot be reconstructed from annual data only. In appendix C, we show that the quarterly forecasts from annual data generated by (41) coincide with those obtained from (33), that is for both models one obtains forecasts of the adjusted series. For policy purposes, these will usually be the most interesting. If forecasts of the seasonals are required, some information on the seasonal pattern will have to be provided.

5. CONCLUDING REMARKS

In this paper, we analyzed the predictive accuracy gain of k-step ahead forecasts from univariate ARIMA models which results from increasing the frequency of sampling. For simple time series models with known parameters, analytical expressions for the information gain were obtained. For more general ARIMA models, this gain was evaluated numerically using the Kalman filter equations. Next, we obtained approximations for the predictive accuracy gain due to more frequent sampling for models with estimated parameters. These results were used to evaluate the additional information content of recently collected quarterly GNP data for the Netherlands and to consider whether it is worthwhile to construct monthly data.

The main conclusions are as follows :

For variables generated by a first order autoregressive model with known parameters, the information gain is substantial only in shortrun forecasting when subsequent realizations are strongly correlated.

We conjecture that this result can be extended to more general stationary processes. Note however that the information gain can be substantially larger if the parameters in the model have to be estimated as suggested by table 8. For variables generated by non-stationary models, the efficiency gain of more frequent sampling can be important in shortrun forecasting but will often be negligible when the forecast horizon becomes large. The results for the GNP series in the Netherlands suggest that the construction of quarterly GNP data has reduced the variance of prediction errors considerably but that further disaggregation into monthly data would hardly yield extra information. Although we limit ourselves to univariate time series models, the results are likely to contain relevant indications for multivariate models as the variances of the prediction errors for univariate and multivariate models have often similar properties. Finally, as many macroeconomic variables can be adequately described by IMA(1,1) processes, the results in this paper can often be applied to decide whether increasing the frequency of observation will be worthwhile.

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APPENDIX A : SOME ANALYTICAL RESULTS ON AR(1)-MODELS.

In this appendix we present analytical results on the prediction of flow variables generated by a first order autoregressive process using lower frequency data. As stressed in section 2 these results are also relevant for the case where a stock variable is generated by an ARI(1,1)-model. First we derive the covariance generating function g(z) of $(Y_t, \bar{Y}_t)$ if $t \in T_m$ by generalizing the result in (7) for m = 2. Define a_i (i = 0,...,2m-2) by

$\sum_{i=0}^{2m-2} a_i L^i = \sum_{i,j=0}^{m-1} \rho^i L^{i+j}$. Then it can be checked that

$$g(z) = \frac{1}{(1-\rho^m z)(1-\rho^m z^{-1})} \begin{bmatrix} b_0 & b_1+b_2 z^{-1} \\ b_1+b_2 z & b_3+b_4(z+z^{-1}) \end{bmatrix}, \qquad (A.1)$$

with $b_0 = \sum_{i=0}^{m-1} \rho^{2i}$, $b_1 = \sum_{i=0}^{m-1} \rho^i a_i$, $b_2 = \sum_{i=0}^{m-2} \rho^i a_{i+m}$, $b_3 = \sum_{i=0}^{2m-2} a_i^2$

and $b_4 = \sum_{i=0}^{m-2} a_{i+m} a_i$. Factorize the numerator of the lower right element as
$b_3 + b_4(z + z^{-1}) = \mu(1 - \lambda z)(1 - \lambda z^{-1})$,

where $\mu = \tfrac{1}{2} b_3 \pm \sqrt{b_3^2 - 4 b_4^2}$ and $\lambda = -b_4 \mu^{-1}$. For the case where m = 2, explicit expressions for μ and λ are given in section 2.The polynomial h(z) in (8) equals

$$h(z) = \left[\frac{b_1+b_2 z^{-1}}{(1-\rho^m z)(1-\rho^m z^{-1})} \; \frac{1-\rho^m z^{-1}}{1-\lambda z^{-1}} \right]_+ \frac{1-\rho^m z}{(1-\lambda z)\mu}$$

$$= \eta(1-\lambda z)^{-1},$$

where $\eta = (b_1 + b_2\rho^m)(1-\rho^m\lambda)^{-1}\mu^{-1}$. From (14) the covariance generating function of the prediction error of Y_t is obtained as

$$g^*(z) = [d_1 + d_2(z + z^{-1})] \,/\, (1-\lambda z)(1-\lambda z^{-1})(1-\rho^m z)(1-\rho^m z^{-1}) \;,$$

with $d_1 = b_0(1+\lambda^2) - 2\eta(b_1-b_2\lambda) + \eta^2 b_3$ and $d_2 = \eta^2 b_4 - b_0\lambda - \eta b_1\lambda - \eta b_2$.

The inverse of the denominator of $g^*(z)$ can be written as

$$\sum_{i=-\infty}^{\infty} c_i z^i \text{ with}$$

$$c_i = [(1-\lambda^2)\rho^{m(i+1)} - (1-\rho^{2m})] \,/\, (\rho^m-\lambda)(1-\lambda\rho^m)(1-\lambda^2)(1-\rho^{2m})$$

which yields that the constant term of $g^*(z)$ or the variance of the prediction error is

$$\nu = [(1+\rho^m\lambda)d_1 + 2d_2(\lambda+\rho^m)] \,/\, (1-\lambda\rho^m)(1-\lambda^2)(1-\rho^{2m})$$

Next we consider the asymptotic variances of maximum likelihood estimators based on low frequency data only. In the case where a stock variable is generated by an AR(1)-model, the ML estimator of ρ is simply $\hat{\rho} = \hat{\psi}^{1/m}$ with $\hat{\psi}$ being the ML estimator of ψ in (23) and the sign is chosen according to the a priori identifying information. Therefore if $\rho \neq 0$ the asymptotic variance of $\hat{\rho}$ equals Avar $(\hat{\psi})/(m^2\rho^{2m-2})$ or

$$\sqrt{T}\,(\hat{\rho} - \rho) \overset{a}{\approx} N(0, \frac{1-\rho^{2m}}{m\rho^{2m-2}}).$$

If a flow variable is generated by a first order autoregressive model, the asymptotic variance of $\hat{\theta}_1' = (\hat{\rho}, \hat{\sigma}^2_\epsilon)$ is obtained from that of $\hat{\theta}_2' = (\hat{\psi}, \hat{\lambda}, \hat{\sigma}^2_v)$ using the facts that

$$\sqrt{T}\,(\hat{\theta}_1 - \theta_1) \overset{a}{\approx} N(0,Q)$$

and

$$m^{-1}Q = -E\frac{1}{T}\frac{\partial^2 L}{\partial\theta_1\partial\theta_1'} = \frac{\partial\theta_2'}{\partial\theta_1}\left[-E\frac{1}{T}\frac{\partial^2 L}{\partial\theta_2\partial\theta_2'}\right]\frac{\partial\theta_2}{\partial\theta_1'},$$

where

$$\frac{\partial\theta_2'}{\partial\theta_1} = \begin{bmatrix} m\rho^{m-1} & -\partial\lambda/\partial\rho & \partial\sigma_v^2/\partial\rho \\ 0 & 0 & \partial\sigma_v^2/\partial\sigma_\epsilon^2 \end{bmatrix}$$

and

$$-E\frac{1}{T}\frac{\partial^2 L}{\partial\theta_2\,\partial\theta_2'} \begin{bmatrix} \frac{1}{1-\rho^{2m}} & \frac{1}{1-\rho^m\lambda} & 0 \\ \frac{1}{1-\rho^m\lambda} & \frac{1}{1-\lambda^2} & 0 \\ 0 & 0 & \frac{1}{2}\sigma_v^{-4} \end{bmatrix}.$$

Some straightforward algebra yields that the asymptotic variance of $\sqrt{T}(\hat{\rho} - \rho)$ in (28) is given by

$$q = m\left\{\frac{m^2\,\rho^{2m-2}}{1-\rho^{2m}} - \frac{2m\,\rho^{m-1}\,\partial\lambda/\partial\rho}{1-\rho^m\lambda} + \frac{(\partial\lambda/\partial\rho)^2}{1-\lambda^2}\right\}^{-1}.$$

APPENDIX B : MONTHLY FORECASTING OF GNP FROM QUARTERLY DATA.

We derive the optimal monthly prediction of GNP in the Netherlands based on quarterly data assuming that (34) holds, and we discuss the derivation of the numerical results on the reduction of the prediction error variance that could be obtained if monthly data were collected.

For simplicity we assume that $r = 0$. In order to derive (38) recall that

$$E[\bar{Y}_{T+3} \mid \bar{Y}_t, t \in T_3(-\infty)] = 9c + \bar{Y}_T + \lambda \sum_{i=0}^{\infty} (-\lambda)^i [\Delta \bar{Y}_{T-3i} - 9c]$$

$$= 9(1 - \frac{\lambda}{1+\lambda})c + (1+\lambda) \sum_{i=0}^{\infty} (-\lambda)^i Y_{T-3i} \qquad (B.1)$$

and define the coefficients b and a_i $(i = 0,1,...)$ by

$$E[Y_{T+1} \mid \bar{Y}_t, t \in T_3(-\infty)] = b + \sum_{i=0}^{\infty} a_i Y_{T-3i} \qquad (B.2)$$

As

$$E[Y_{T+k} \mid \bar{Y}_t, t \in T_3(-\infty)] = (k-1)c + E[Y_{T+1} \mid \bar{Y}_t, t \in T_3(-\infty)], \qquad (B.3)$$

a second expression for the expectation in (B.1) can be obtained by adding up the expectations in (B.3) for $k = 1,2$ and 3 respectively which yields

$$E[\bar{Y}_{T+3} \mid \bar{Y}_t, t \in T_3(-\infty)] = 3c + 3b + 3 \sum_{i=0}^{\infty} a_i \bar{Y}_{T-3i} . \qquad (B.4)$$

Comparing (B.1) and (B.4) we obtain $a_i = 1/3(1+\lambda)(-\lambda)^i$ and $b = (2 - 3\lambda/(1+\lambda)c$ which yields (38).

The prediction error of $E[Y_{T+k} \mid \bar{Y}_t, t \in T_3(-\infty)]$ is

$$\epsilon_{T+k} + (1+\alpha) \sum_{i=1}^{k-1} \epsilon_{T+i} + \alpha\epsilon_T + Y_T - 1/3\, \bar{Y}_T - 1/3\, \lambda \sum_{i=0}^{\infty} (-\lambda)^i \Delta_3 \bar{Y}_{T-3i} , \qquad (B.5)$$

which can be rewritten as

$$\epsilon_{T+k} + (1+\alpha) \sum_{i=1}^{k-1} \epsilon_{T+i} + \sum_{i=0}^{\infty} \alpha_i \epsilon_{T-i}$$

with

$$\sum_{i=0}^{\infty} d_i L^i = (\frac{2}{3} + \alpha) + (\frac{1}{3} + \frac{2}{3}\alpha) L + \frac{1}{3}\alpha L^2 - \frac{1}{3}(1+L+L^2)^2 (1+\alpha L)\sum_{i=0}^{\infty}(-\lambda L^3)^i \qquad (B.6)$$

Therefore, the variance of the prediction error if α and c are known is

$$\nu^3_{k,0} = \sigma^2_\epsilon [1 + (k-1)(1+\alpha)^2] + \sigma^2_\epsilon \sum^{\infty}_{i=0} d^2_i . \qquad (B.7)$$

Because the variance of the prediction error would be the first term of (B.7) if monthly data were used, equation (B.7) can be easily used to determine the additional information content of the monthly data in table 11.

Extending the results in section 3 to the case where the predictor depends on a consistently estimated (nx1) vector of parameter θ, the variance of this predictor $\hat{Y}_{T+k}(\hat{\theta})$ can be approximated up to order T^{-1} by

$$E[Y_{T+k} - \hat{Y}_{T+k}(\hat{\theta})]^2 + \frac{1}{T}\sum^{n}_{i,j=1} E \frac{\partial \hat{Y}_{T+k}(\theta)}{\partial\theta_i} \frac{\partial \hat{Y}_{T+k}(\theta)}{\partial\theta_j} \sqrt{T}(\hat{\theta}_i - \theta_i)\sqrt{T}(\hat{\theta}_j - \theta_j). \qquad (B.7)$$

When $\sqrt{T}(\hat{\theta}-\theta) \overset{d}{\approx} N(0,V)$, the Cauchy-Schwarz inequality can be used to obtain an upperbound for (B.7)

$$E[Y_{T+k} - \hat{Y}_{T+k}(\hat{\theta})]^2 + \frac{1}{T}\sum^{n}_{i,j=1} E \frac{\partial \hat{Y}_{T+k}(\theta)}{\partial\theta_i} \frac{\partial \hat{Y}_{T+k}(\theta)}{\partial\theta_j} V_{ij}$$
$$+ \frac{2}{T}\sum^{n}_{i,j=1} [E(\frac{\partial \hat{Y}_{T+k}(\theta)}{\partial\theta_i})^2 E(\frac{\partial Y_{T+k}(\theta)}{\partial\theta_j})^2 V_{ii}V_{jj}]^{1/2} \qquad (B.8)$$

In the present case $\theta' = (c,\lambda)$ and $V = \text{diag}(3\sigma^2_v/81, 3(1-\lambda^2))$. Moreover

$$\frac{\partial}{\partial\lambda}\hat{Y}_{T+k}(\theta) = -\frac{1}{3}\sum_{i=0}^{\infty}(-\lambda)^{i}\, v_{t-i}$$ so that

$$E\left[\frac{\partial}{\partial\lambda}\hat{Y}_{T+k}(\theta)\right]^{2} = \frac{\sigma_v^2}{9(1-\lambda^2)}$$. Substitution in (B.8) finally yields that

$$\bar{\nu}^{3u}_{k,0} = \bar{\nu}^{3}_{k,0} + \left(k+1-\frac{3\lambda}{1+\lambda}\right)^{2}\frac{\sigma_v^2}{9T} + \frac{\sigma_v^2}{T} + 2\left|k+1-\frac{3\lambda}{1+\lambda}\right|\frac{\sigma_v^2}{9T} \qquad (B.9)$$

is an upperbound for the variance of the prediction error of monthly forecasts based on quarterly data which is accurate to order T^{-1}. Along the same lines a lowerbound for the prediction error variance for monthly data can be obtained as

$$\nu^{1}_{k,0} - \left(k-\frac{\alpha}{1+\alpha}\right)^{2}\frac{\sigma_\varepsilon^2}{T} - \frac{\sigma_\varepsilon^2}{T} - \left|k-\frac{\alpha}{1+\alpha}\right|\frac{\sigma_\varepsilon^2}{T}, \qquad (B.10)$$

but evidently $\nu^{1}_{k,0}$ is a larger lowerbound. The figures which are given in table 12 are computed as

$$100\ .\ (\bar{\nu}^{3u}_{k,r} - \nu^{1}_{k,0})\ /\ \bar{\nu}^{3u}_{k,r} \qquad (B.11)$$

with $\bar{\nu}^{3u}_{k,r} = \bar{\nu}^{3u}_{k+r,0}$.

APPENDIX C : QUARTERLY FORECASTS OF GNP FROM ANNUAL DATA USING THE MODEL FOR SEASONALLY UNADJUSTED DATA.

In this appendix we show that if only annual data are available, models (33) and (41) yield identical quarterly forecasts. For simplicity we restrict ourselves to the case where $k = 0$. Generalizations are straightforward. Ignoring the constant term, equation (41) can be written as

$$(1-\rho L)(1-\psi L^4)Y_t = (1+\alpha L)(1+\theta L^4)\epsilon_t \tag{C.1}$$

with $\theta = -\rho^4$. Equation (C.1) implies that

$$(1-\psi L^4)Y_t = \sum_{i=0}^{4} a_i \epsilon_{t-i} \tag{C.2}$$

and

$$(1-\psi L^4)\,\bar{Y}_{t-r} = \sum_{i=0}^{10} b_i\,\epsilon_{t-i}\,, \qquad r = 0,1,2,3, \tag{C.3}$$

where $\sum_{i=0}^{4} a_i L^i = (1 + \rho L + \rho^2 L^2 + \rho^3 L^3)(1+\alpha L)$ and $\sum_{i=0}^{10} b_i L^i =$

$L^r(1 + L + L^2 + L^3) \sum_{i=0}^{4} a_i L^i$. Using a slight generalization of equation (8) this implies

$$E[Y_T \mid \bar{Y}_t\ (t \in T_m(-\infty))] = \sum_{i=0}^{\infty} h_i \bar{Y}_{t-r-4i}\,, \tag{C.4}$$

where

$$h(z) = \left[\frac{A_0 + A_1 z^{-1} + A_2 z^{-2}}{(1-\psi z)\ (1+\lambda z^{-1})}\right]_+ \frac{1-\psi z}{\mu(1+\lambda z)}\,, \tag{C.5}$$

λ is the annual MA(1) parameter and the weights A_0, A_1, A_2 and μ depend on the coefficients a_i and c_i. Rewriting (C.5), one obtains

$$h(z) = \left\{ \frac{A_0 + A_1 z^{-1} + A_2 z^{-2}}{1+\lambda\psi} \left[\frac{1}{1+\lambda z^{-1}} + 1 \quad \frac{1}{1-\psi z} - 1 \right]_+ \right\} \frac{1-\psi z}{\mu(1+\lambda z)} \qquad \text{(C.6)}$$

$$= A_3(1+\lambda z)^{-1}$$

which is also obtained if (33) is assumed to be the data generating process (compare appendix B).

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