The quality of some approximation formulas in a continuous review inventory model

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RESEARCH MEMORANDUM

The quality of some approximation formulas

in a continuous review inventory model

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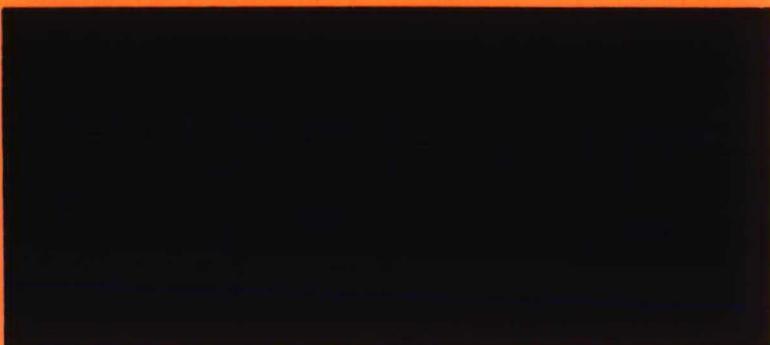
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Abstract

This paper extends an earlier paper by Heuts and v.Lieshout [3] by repre-
senting more extensive information about the quality of some approximation
formulas for the reorder point and the order quantity in a continuous
review inventory model. Schmeiser and Deutsch [5] introduced the so-called
S-D distribution, which we used as lead time demand distribution, as it
has many desirable properties. It will be shown that in all cases consid-
ered the approximation for the order quantity is an underestimation of the
optimal value, while the two approximations for the reorder level are an
overestimation of the optimal value. However, what we are really interes-
ted in are relevant cost differences between the exact solution method in
comparison with approximations. It will be seen that the quality (in terms
of cost differences) of the approximation formulas in comparison with
exact solutions, very much depends on the following cost ratio: shortage
cost p. unit per unit of time in relation to inventory cost p. unit per
unit of time.
The quality of one of the two proposed approximations is better than the
other for many realistic cost structures. This "best" approximation
appears to be a good alternative for the more complex exact solution,
when the shortage cost is high in relation to the inventory cost.

1. Introduction

In this paper an inventory model with stochastic lead time demand will
be analysed under the following assumptions:

a. The inventory model is a continuous review system.

b. The order quantity is not restricted.

c. The purchase cost $b(q)$ is a continuous differentiable function of the
order quantity q.

d. The lead time demand distribution has distribution function $F(z)$.

e. The expected value of the demand per unit of time is r.

f. The holding cost per unit inventory per unit of time is c_1.

g. Unfilled demand during the lead time is backlogged. The shortage cost
per shortage unit per unit of time is c_2.

The criterion used is minimization of the average cost per unit ordered.
The cost function looks as follows:

$$(1.1) \quad K(x,q) = (c_1/(r.q)) . \int_0^q [\int_0^{x+y} (x+y-z) f(z) dz] dy +$$

$$(c_2/(r.q)) . \int_0^q [\int_{x+y}^{\infty} (z-x-y) f(z) dz] dy + b(q)/q .$$

where:

$f(z)$: the density function of the demand during the lead time;

x \qquad : the reorder point expressed in terms of units of economic
inventory;

q \qquad : the order quantity;

$b(q)$: the ordering cost. We assume that $b(q) = c_0 + q.a(q)$, with $a(q)$
a two times differentiable function.

Details of a derivation of (1.1) can be found in Heuts and v.Lieshout [3].
As a density function of the demand during the lead time we have chosen
the so-called Schmeiser-Deutsch distribution [5], which is defined in the
following way:

$$(1.2) \quad f(z) = \frac{1}{\ell_2 \ell_3} \left| \frac{\ell_1 - z}{\ell_2} \right|^{\frac{1-\ell_3}{\ell_3}} , \quad t \leq z \leq p ,$$

where

$$t = \ell_1 - \ell_2 \ell_4^{\ell_3} \quad ; \quad p = \ell_1 + \ell_2 (1-\ell_4)^{\ell_3} \quad ; \quad \ell_2, \ell_3 \geq 0 ; 0 \leq \ell_4 \leq 1 .$$

This four-parameter type of distribution can have many different shapes. Table I shows the relation between the shape of the distribution and the parameters.

Table I: Different shapes for the S-D distribution

	$\ell_3 > 1$ (bell-shaped)	$\ell_3 = 1$	$\ell_3 < 1$ (U-shaped)
$\ell_4 < 0,5$	skewed to the right	uniform distr.	skewed to the left
$\ell_4 = 0,5$	symmetric	" "	symmetric
$\ell_4 > 0,5$	skewed to the left	" "	skewed to the right

Further it can be shown [5] that ℓ_1 and ℓ_2 satisfy the following relations:

$$(1.3) \quad \ell_2 = \frac{\sigma^2 (2\ell_3+1)(\ell_3+1)^2}{(\ell_3+1)^2 \{\ell_4^{2\ell_3+1} + (1-\ell_4)^{2\ell_3+1}\} - (2\ell_3+1)\{(1-\ell_4)^{\ell_3+1} - \ell_4^{\ell_3+1}\}^2}$$

$$(1.4) \quad \ell_1 = \mu - \ell_2 \frac{(1-\ell_4)^{\ell_3+1} - \ell_4^{\ell_3+1}}{\ell_3+1}$$

These results will be used in section 3. The reason why we have chosen the S-D distribution is because of the following interesting properties: The distribution function, the inverse distribution function and the conditional expectations can be determined explicitly.

Using gamma, Weibull or beta distributions, evaluation of the cost function would require evaluation of incomplete gamma or beta functions, which is troublesome (see e.g. Kottas and Lau [4], Tadikamalla [6] and Burgin [2]).

As the S-D distribution has four parameters, different types of skewness can be better described than two-parameter distributions, such as the Weibull or the gamma distribution. The importance of the type of skewness

on the cost function will be demonstrated in a next paper.

In | 3| a solution procedure for the model with cost function $K(x,q)$ given by (1.1) and as lead time demand distribution the S-D distribution is given.

The optimization method used is a Newton-like algorithm.

2. Approximation formulas for the reorder point and order quantity

A simple approximation for the order quantity is the well-known lot size formula:

$$(2.1) \quad q_0 = \left(\frac{2c_0 r}{c_1}\right)^{\frac{1}{2}}$$

whereas for the reorder point x two approximations are used. The first one is based on the fact that the service level α is defined as the probability that the demand during the lead time is smaller than the order level:

$$(2.2) \quad F(x_1) = \alpha = \frac{c_2}{c_1 + c_2} \quad ,$$

Remarkable upon these approximation formulas is, that they are completely independent of each other.

Baken [1] has shown that this independent calculation of x and q will lead to inferior quality of these variables in certain cases.

A second set of approximation formulas for q and x, which is dependent of each other, is:

$$(2.3) \quad q_0 = \left(\frac{2c_0 r}{c_1}\right)^{\frac{1}{2}}$$

$$(2.4) \quad \frac{1}{q_0} \int_{x_2}^{\infty} (z - x_2) f(z) dz = \frac{c_1}{c_1 + c_2} \quad ,$$

where (2.4) means that the fraction expected shortages with regard to the total sales per order cycle is equal to $1 - \alpha = \frac{c_1}{c_1 + c_2}$, where α is the service level.

It will be shown that if $f(z)$ is a S-D distribution, then the following dependence between x and q exists:

If the order quantity q_0 increases, then the order point x_2 decreases, because of the fact that the value of the integral in (2.4) is non-increasing as function of x_2.

Equation (2.2) is equivalent with (see e.g. Schmeiser-Deutsch [5]):

$$(2.5) \quad x_1 = \begin{cases} \ell_1 - \ell_2 \left(\ell_4 - \dfrac{c_2}{c_1 + c_2}\right)^{\ell_3} & \text{if } \dfrac{c_2}{c_1 + c_2} \leq \ell_4 \\[4mm] \ell_1 + \ell_2 \left(\dfrac{c_2}{c_1 + c_2} - \ell_4\right)^{\ell_3} & \text{if } \dfrac{c_2}{c_1 + c_2} > \ell_4 \end{cases}$$

The integral in equation (2.4) can be written as

$$\int_{x_2}^{\infty} (z-x_2) f(z) dz := G(x_1) =$$

i) $\quad \ell_1 + \dfrac{\ell_2}{\ell_3+1} \{(1-\ell_4)^{\ell_3+1} - \ell_4^{\ell_3+1}\} - x_2 = \mu - x_2$ for $x_2 \leq t$

ii) $\quad \ell_1(1-\ell_4) + \dfrac{\ell_2}{\ell_3+1} (1-\ell_4)^{\ell_3+1} + x_2(\ell_4-1) + \dfrac{\ell_2\ell_3}{\ell_3+1} \left(\dfrac{\ell_1-x_1}{\ell_2}\right)^{(\ell_3+1)/\ell_3}$

$\quad$ for $t \leq x_2 \leq \ell_1$

iii) $\quad \ell_1(1-\ell_4) + \dfrac{\ell_2}{\ell_3+1} (1-\ell_4)^{\ell_3+1} + x_2(\ell_4-1) + \dfrac{\ell_2\ell_3}{\ell_3+1} \left(\dfrac{x_2-\ell_1}{\ell_2}\right)^{(\ell_3+1)/\ell_3}$

$\quad$ for $\ell_1 \leq x_2 \leq p$

iv) $\qquad\qquad 0 \qquad\qquad$ for $x_2 \geq p$

Properties of $G(x)$:

a) $G(0) = \mu$ $\quad$ b) $G(x_2) = 0$ for $x_2 \geq p$

c) $G(x_2)$ is a decreasing function of x_2 on the interval $[0,p)$.

$\quad$ On the intervals $[0,t]$ and $[t, \ell_1]$ this is trivial as $(\ell_4-1) \leq 0$.

On the interval $[\ell_1,p)$ it follows from the inequality:

$$\frac{\partial G(x_2)}{\partial x_2} = (\ell_4-1) + \left(\frac{x_2-\ell_1}{\ell_2}\right)^{1/\ell_3} < (\ell_4-1) + \left(\frac{p-\ell_1}{\ell_2}\right)^{1/\ell_3} = 0 \quad ,$$

because $p = \ell_1 + \ell_2 (1-\ell_4)^{\ell_3}$.

So on the interval $x_2 \in [0,p)$ the function $G(x)$ is uniquely reversable, which means that the inverse function of $G(x_2)$ exists. However, the inverse function is not easily calculated. For the determination of x_2 we therefore used procedure CO5ADF from the NAG-library, which calculates the zero point of a function via a search procedure.

3. Some results on the quality of the two sets of approximation formulas

In this section we will investigate the quality of the approximation formulas, which we have defined in section 2. The first set of approximation formulas in equations (2.1) and (2.2) will be denoted by q_0 and x_1, the second one (see equations (2.3) and (2.4)) by q_0 and x_2, and the optimal solution will be denoted by q^* and x^*. For the cost function the following notations are used:

$$K_1 := K(x_1, q_0) \; ; \; \text{red } 1 := \frac{K(x_1, q_0) - K(x_2, q_0)}{K(x_1, q_0)} * 100$$

$$\text{red } * := \frac{K(x_2, q_0) - K(x^*, q^*)}{K(x_2, q_0)} * 100 \; .$$

In the next tables the following variables are held constant:
- the mean and variance of the demand during the lead time
  ($\mu = 100$ and $\sigma^2 = 600$)
- the demand p. unit of time and the holding cost p. unit of inventory
  p. unit of time ($r = 100$ and $c_1 = 1$).

The shape parameters ℓ_3 and ℓ_4 are varied together with the cost parameters c_0 and c_2. The variation in the shape parameters is denoted as follows:

(1) $\ell_3 = 0,4 \wedge \ell_4 = 0,2$ (2) $\ell_3 = 0,4 \wedge \ell_4 = 0,8$

(3) $\ell_3 = 1,8 \wedge \ell_4 = 0,2$ (4) $\ell_3 = 1,8 \wedge \ell_4 = 0.8$

Table II contains information about the reorder points, Table III about the order quantities and Table IV about the cost differences.

Table II: Decisions on the reorder points

	(1)	(2)	(3)	(4)
$c_0=10$ $c_2=1$				
x_1	109.3	90.7	91.4	108.6
x_2	83.3	77.9	78.2	82.4
x^*	63.8	60.2	60.1	62.3
$c_0=10$ $c_2=10$				
x_1	123.1	146.8	141.5	124.3
x_2	110.0	124.5	120.4	112.2
x^*	108.9	116.9	114.8	111.0
$c_0=10$ $c_2=100$				
x_1	125.2	152.4	155.2	128.1
x_2	120.6	145.4	144.6	122.3
x^*	120.5	145.1	144.0	122.1
$c_0=100$ $c_2=10$				
x_1	123.1	146.8	141.5	124.3
x_2	95.7	92.5	94.6	96.2
x^*	94.3	88.0	90.4	94.3
$c_0=100$ $c_2=100$				
x_1	125.2	152.4	155.2	128.1
x_2	116.7	138.5	135.1	118.9
x^*	116.6	138.2	134.6	118.8

Table III: Decisions on the order quantities

	(1)	(2)	(3)	(4)
$c_0=10$ $c_2=1$				
q_0	44.7	44.7	44.7	44.7
q^*	76.0	76.0	77.5	77.5
$c_0=10$ $c_2=10$				
q_0	44.7	44.7	44.7	44.7
q^*	50.9	61.6	60.5	50.9
$c_0=10$ $c_2=100$				
q_0	44.7	44.7	44.7	44.7
q^*	46.4	47.5	49.2	46.7
$c_0=100$ $c_2=10$				
q_0	141.4	141.4	141.4	141.4
q^*	153.3	167.0	164.0	154.7
$c_0=100$ $c_2=100$				
q_0	141.4	141.4	141.4	141.4
q^*	144.5	146.8	149.0	144.2

Table IV: Cost differences for the resp. decisions

	(1)	(2)	(3)	(4)
$c_0=10$ $c_2=1$				
K_1	0.5511	0.4912	0.4904	0.5508
red1	20.2	9.3	7.9	18.8
red*	9.5	10.7	11.8	10.9
$c_0=10$ $c_2=10$				
K_1	0.6784	0.9172	0.8711	0.6911
red1	11.4	12.5	11.5	9.9
red*	0.6	2.2	2.3	0.6
$c_0=10$ $c_2=100$				
K_1	0.6992	0.9716	0.9994	0.7281
red1	4.2	4.5	6.6	5.3
red*	0.2	0.1	0.2	0.1
$c_0=100$ $c_2=10$				
K_1	1.6453	1.8832	1.8322	1.6576
red1	10.1	16.7	15.0	9.8
red*	0.3	1.2	0.9	0.4
$c_0=100$ $c_2=100$				
K_1	1.6662	1.9386	1.9663	1.6951
red1	3.3	4.5	6.6	3.8
red*	0.0	0.1	0.1	0.0

Preliminary conclusions drawn from tables II, III and IV:

1) The second set of approximations is better than the first one:
   $K(x_2,q_0) < K(x_1,q_0)$ or red1>0.

2) The approximation x_2 for x^* is better than x_1, or
   $|x_2-x^*| < |x_1-x^*|$.

3) The approximation for q^* is an underestimation:
   $q_0<q^*$, which improves when c_2 increases.

4) The approximations for x^* are an overestimation:

$x_1 > x^*$ and $x_2 > x^*$, and they improve when c_2 increases.

5) The approximation (x_2, q_0) improves:
   - when the order cost c_0 increases
   - when the shortage cost c_2 increases.

6) The influence of the cost parameters on the quality of the approximation formulas is much more important than the influence of the shape of the probability distribution.

Some comments

- When the shortage cost is relatively high, then the tail behaviour for <u>large</u> values of the probability distribution is an important factor. In the second approximation formula for x^*, this factor is better incorporated than in the first one, explaining the results in the tables.

- When the shortage cost is relatively low, then the tail behaviour of the low values of the probability distribution is an important factor. This aspect is not incorporated in any of the approximation formulas. The mentioned approximation formulas for x^* are not suitable when the inventory cost p.unit p.unit of time is relatively high.

As the above results are only based on a limited number of examples, we further investigated the quality of the approximations for a more diversified number of cost structures. It then follows that the preliminary conclusions under points 3) 4) and 5), which describe the relation between the cost parameters and the quality of the approximation formulas, are not valid in general.
In tables V and VI examples are given to illustrate this.
However the question arises if all supposed cost structures there are realistic.

In tables V and/or VI the symbols in the respective columns are defined
as follows:

$$dx_1 := (x_1 - x^*) / x^* \quad . \quad 100\%$$

$$dx_2 := (x_2 - x^*) / x^* \quad . \quad 100\%$$

$$dq_0 := (q^* - q_0) / q^* \quad . \quad 100\%$$

$$dK_1 := (K(x_1, q_0) - K(x^*, q^*)) / K(x^*, q^*) \quad . \quad 100\%$$

$$dK_2 := (K(x_2, q_0) - K(x^*, q^*)) / K(x^*, q^*) \quad . \quad 100\%$$

x^*, q^* optimal decision variables

$K^* := K(x^*, q^*)$ minimal cost function

$$dx_2 q^* := (x_2(q^*) - x^*) / x^* \quad . \quad 100\%$$

where $x_2(q^*)$ is the approximation for x^* via (2.4),
however with the optimal q, q^*, substituted in it.

$$dKx_2 q^* := (K(x_2(q^*), q^*) - K(x^*, q^*)) / K(x^*, q^*) \quad . \quad 100\%$$

$$dKx^* q_0 := (K(x^*, q_0) - K(x^*, q_0^*)) / K(x^*, q^*) \quad . \quad 100\%$$

$$vx1 := x_1 - x^*$$

$$vx2 := x_2 - x^*$$

When $x^* = 0.0$, dx_1 and dx_2 do not exist.

In those cases absolute differences are given:

$$dx_1 := |x_1 - x^*| \quad , \quad dx_2 := |x_2 - x^*| \quad .$$

Conclusively we can state that the quality of the proposed approxi-
mations strongly depends on relative costs p. unit per unit of time as
can be seen from the tables.

Table V: The influence of c_2 on the quality of the approximation formulas by different values of c_0 and c_1

$\mu = 100$ $\sigma^2 = 600$ $\ell_3 = 1.8$ $\ell_4 = 0.2$

	(1) dx_1	(2) dx_2	(3) dq_0	(4) dK_1	(5) dK_2	(6) x^*	(7) q^*	(8) K^*	(9) dx_2q^*	(10) dKx_2q^*	(11) dKx^*q_0
$c_0= 1$ $c_1=1$											
$c_2 = 1$	20.1	43.9	59.8	15.0	38.9	76.12	35.22	0.2433	12.22	4.75	17.2
$c_2 = 10$	10.0	5.7	45.6	6.4	3.6	128.72	26.01	0.5488	0.06	0.0004	5.5
$c_2 = 100$	4.0	0.4	16.4	4.8	0.3	149.30	16.91	0.6621	0.00	0.0000	0.4
$c_0=1$ $c_1=10$											
$c_2 = 1$	17.8	87.4	74.3	40.9	706.	64.26	17.38	0.3574	37.96	203.	56.3
$c_2 = 10$	9.6	55.4	73.5	6.8	79.5	83.37	16.88	2.1051	26.47	24.3	8.7
$c_2 = 100$	4.4	7.0	63.0	2.4	4.3	135.53	12.08	5.0361	1.5	0.28	3.1
$c_0=10000$ $c_1=1$											
$c_2 = 1$	91.40	0.00	0.5	6.4	0.0015	0.0	1422.	13.22	0.00	0.00	0.0015
$c_2 = 10$	141.53	0.00	2.8	7.4	0.0431	0.0	1455.	13.55	0.00	0.00	0.0431
$c_2 = 100$	63.46	0.42	1.5	3.0	0.0105	91.75	1435.	14.27	0.00	0.00	0.0107
$c_0=10000$ $c_1=10$											
$c_2 = 1$	75.68	0.00	2.8	17.9	0.05	0.0	460.	36.01	0.00	0.00	0.05
$c_2 = 10$	91.40	0.00	4.9	18.8	0.16	0.0	470.	37.03	0.00	0.00	0.16
$c_2 = 100$	84.86	2.68	6.2	12.8	0.20	56.66	477.	43.34	0.00	0.00	0.22

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Table VI: The influence of c_0 on the quality of the approximation formulas by different values of c_1 and c_2

$$\mu = 100 \qquad \sigma^2 = 600 \qquad \ell_3 = 1.8 \qquad \ell_4 = 0.2$$

	(1) vx_1	(2) vx_2	(3) dq_0	(4) dK_1	(5) dK_2	(6) x^*	(7) q^*	(8) K^*	(9) dx_2q^*	(10) dKx_2q^*	(11) dKx^*q_0
$c_1=1$ $c_2=1$											
$c_0 = 1$	15.28	33.38	59.8	15.0	38.9	76.12	35.22	0.2433	12.22	4.75	17.2
$c_0 = 100$	91.40	29.29	30.3	33.1	7.1	0.0	203.	1.03	*)	1.03	12.96
$c_0 = 10000$	91.40	0.0	0.5	6.4	0.0015	0.0	1422.	13.22	0.00	0.00	0.0015
$c_1=1$ $c_2=10$											
$c_0 = 1$	12.81	7.31	45.6	6.4	3.6	128.72	26.01	0.5488	0.06	0.0004	5.5
$c_0 = 100$	51.16	4.20	13.7	18.7	0.95	90.37	164.	1.5432	*)	0.00	1.2
$c_0 = 10000$	141.53	0.0	2.8	7.4	0.043	0.0	1455.	13.55	0.00	0.00	0.043
$c_1=10$ $c_2=1$											
$c_0 = 1$	11.41	56.15	74.3	40.9	706.	64.26	17.38	0.3574	37.96	203.	56.3
$c_0 = 100$	75.68	59.34	58.4	111.3	75.	0.0	107.54	1.6025	*)	1.62	88.0
$c_0 = 10000$	75.68	0.0	2.8	17.9	0.05	0.0	460.	36.01	0.00	0.00	0.05
$c_1=10$ $c_2=10$											
$c_0 = 1$	8.03	46.18	73.5	6.8	79.5	83.37	16.88	2.1051	26.47	24.3	8.7
$c_0 = 100$	31.26	18.03	42.3	23.0	13.3	60.14	77.54	3.9857	1.8	0.034	18.8
$c_0 = 10000$	91.40	0.0	4.9	18.8	0.16	0.0	470.	37.03	0.00	0.00	0.16

*) does not exist

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