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Borm, P.E.M.; Cao, R.; Garcia-Jurado, I.

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TILBURG UNIVERSITY
KATHOLIEKE UNIVERSITEIT BRABANT
POSTBOX 90153
5000 LE TILBURG
THE NETHERLANDS

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RESEARCH MEMORANDUM

**MAXIMUM LIKELIHOOD EQUILIBRIA
OF RANDOM GAMES**

Peter E.M. Borm,
Ricardo Cao and Ignacio García-Jurado

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MAXIMUM LIKELIHOOD EQUILIBRIA OF RANDOM GAMES*

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Peter E.M. Borm, Ricardo Cao and Ignacio García-Jurado

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Abstract

In this paper we introduce the concept of random game in order to incorporate the possible random structure of a game in an explicit way. Two definitions of maximum likelihood Nash equilibrium (MLNE) are given depending on the fact that the random structure is discrete or continuous. Existence theorems of MLNE are proved in both setups.

1 Introduction

In many practical situations, a group of agents have to take strategical decisions in an environment of risk. The traditional approach of game theory to this problem is to embody such a risk (and the attitude of the players towards it) in the utility functions of the players and, then, to solve the problem as a deterministic one. Although this can be a fruitful approach in a number of occasions, sometimes it will be more appropriate to address the situation in a way which pays more attention to its random structure and to explicitily incorporate such a structure in the proposed solution.

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Consider, for instance, the following game: two persons, who cannot communicate, have to say an integer smaller or equal than one million and then, without knowing the other's number, have to choose between *even* or *odd*. If they have not said the same number, they are paid 1,000 dollars each if both have chosen *even*, nothing if both have chosen *odd* and 500 dollars each otherwise. If they have resulted to say the same number, they are paid nothing if both have chosen *even*, M dollars each (a big amount of money) if both have chosen *odd* and M/2 dollars each otherwise. If we know nothing about the players (but that they are rational) we could analyze the game, in accordance to the classical approach, in the following way (see, for instance, [2] page 5): we define the payoff functions for the players as the expected payoff they obtain; hence, if M is large enough, the only equilibrium of the resulting game is (*odd,odd*). However, we think that, in the setting described above, the only rational suggestion for the players is to play *even* because, doing that, they are playing a Nash equilibrium with a very high probability or, in other words, if any of the players intends to choose *odd*, he has an incentive to deviate with a very high probability (in this example 0.999999).

In this paper we study some solutions for these *random games* based on a statistically-oriented thinking. They consist of combinations of strategies which are the most likely ones to be equilibria looking at the random structure of the situation. We do not think that this is always the only admissible approach to this kind of conflicts. However, we believe that it provides a new and enlightening point of view of the random conflict situations.

It is interesting to remark that a very simple class of random games, approached from this statistical thinking, has been used in [1] to introduce a new solution concept for noncooperative games. This shows again the interest of the statistical point of view when analyzing random games.

2 Random Games and Maximum Likelihood Equilibria

In this section we present and study a solution concept for random games. We begin introducing our model.

Definition 1 *A random game is a three-tuple*

$$< (\Omega, \mathcal{A}, P), X, H >$$

where:

1. $(\Omega, \mathcal{A}, P)$ *is a probability space,*

2. X *is the set of combination of strategies of the* n *players. It has the form*

$$X = \prod_{i=1}^{n} X_i$$

where each X_i*, the set of strategies of player* i*, is a separable topological space (i.e. each* X_i *is a topological space containing a countable subset* S_i *which verifies that* $\bar{S}_i = X_i$*, where* $\bar{S}_i$ *denotes the smallest closed set containing* S_i*), and*

3. H *is the payoff function given by:*

$$\begin{aligned} H: \quad X \times \Omega &\longrightarrow \quad \mathbf{R}^n \\ (x, \omega) &\longrightarrow \quad H(x,\omega) := (H_1(x,\omega), H_2(x,\omega), \ldots, H_n(x,\omega)) \end{aligned}$$

where, for every $i \in \{1, \ldots, n\}$, $x \in X$ *and* $\omega \in \Omega$, $H_i(x,\omega)$ *is the payoff for the i-th player if* x *is played and the state of nature is* ω*. We suppose that* H *is measurable as a function of* ω *(for all* x*) and continuous as a function of* x *(for all* ω*).*

Observe that the model described above is quite general. The condition of separability for the X_i is only a technical one. The properties of measurability and continuity of H are not very restrictive but necessary if we want the model to be reasonably handy.

For every $\omega \in \Omega$, we denote by H_ω the function which assigns $H(x,\omega)$ to every $x \in X$. Obviously, $< X, H_\omega >$ is a normal form game for every ω . Bearing this in mind, we give the following definition.

Definition 2 *For every* $x \in X$*, the Nash equilibrium indicator of* x *is the function* $N_x : \Omega \longrightarrow \{0, 1\}$ *given by:*

$$N_x(\omega) = \begin{cases} 1 & \text{if and only if } x \text{ is a Nash equilibrium of } < X, H_\omega > \\ 0 & \text{otherwise} \end{cases}$$

This Nash equilibrium indicator function describes the possibilities of a particular combination of strategies x to be a Nash equilibrium. In fact, it would be desirable that $N_x \equiv 1$. If such an x exists, it could be our proposal for the random game. Nevertheless, that will rarely be the case, so we should propose a concept which can be used as a solution in general. This is what we do next but first let us prove that, in a random game as in Definition 1, the set $\{\omega \in \Omega \ / \ N_x(\omega) = 1\}$ is in $\mathcal{A}$ (for all $x \in X$). Namely, fix $x \in X$ and denote $(x_1, ..., x_{i-1}, x_i', x_{i+1}, ..., x_n)$ by (x_{-i}, x_i'), for any $x_i' \in X_i$. Now, taking into account the continuity of every H_ω, we can write:

$$\begin{aligned}
&\{\omega \in \Omega / N_x(\omega) = 1\} = \\
&= \{\omega \in \Omega / \forall i \in \{1, ..., n\}, \forall x_i' \in X_i, H_i(x, \omega) \geq H_i((x_{-i}, x_i'), \omega)\} = \\
&= \{\omega \in \Omega / \forall i \in \{1, ..., n\}, \forall x_i' \in S_i, H_i(x, \omega) \geq H_i((x_{-i}, x_i'), \omega)\} = \\
&= \bigcap_{i=1}^{n} \bigcap_{x_i' \in S_i} \{\omega \in \Omega / H_i(x, \omega) \geq H_i((x_{-i}, x_i'), \omega)\}.
\end{aligned}$$

which clearly belongs to $\mathcal{A}$ because H is measurable as a function of ω and every S_i is a countable set.

Now we can define our equilibrium concept.

Definition 3 *A combination of strategies $x \in X$ is said to be a maximum likelihood Nash equilibrium (MLNE) if and only if*

$$N(x) \geq N(y) \ \forall y \in X,$$

where the function $N : X \longrightarrow [0, 1]$ is given by:

$$N(x) := P\{\omega \in \Omega / N_x(\omega) = 1\}.$$

Clearly, the only MLNE in the example proposed in the introduction of this paper is, as desired, $(even, even)$. However, we can ask ourselves when a certain random game has at least one MLNE. This is what we deal with next. First we prove the following lemma.

Lemma 1 *Let $< (\Omega, \mathcal{A}, P), X, H >$ be a random game as in Definition 1 and $\{x_n\}_{n \geq 1}$ a sequence in X such that $\lim_{n \to \infty} x_n = x_0 \in X$. Then, if the sequence $\{N(x_n)\}$ converges,*

$$N(x_0) \geq \lim_{n \to \infty} N(x_n)$$

Proof. For all $n \in \{0, 1, 2, ...\}$ define $A_n := \{\omega \in \Omega / N_{x_n}(\omega) = 0\}$, and take $\omega_0 \in A_0$. Clearly, x_0 is not an equilibrium of the game $G_{\omega_0} =< X, H_{\omega_0} >$. Besides, the continuity of H_{ω_0} implies that the set of Nash equilibria of G_{ω_0} is closed in X and, hence, there exists a neighbourhood E_{ω_0} of x_0 such that

$$N_x(\omega_0) = 0 \ \ \forall x \in E_{\omega_0}.$$

On the other hand, the convergence of $\{x_n\}$ to x_0 implies that

$$\exists k \in \mathbb{N} \ \forall n \geq k; x_n \in E_{\omega_0}.$$

Now, the last two conditions lead to

$$\exists k \in \mathbb{N} \ \forall n \geq k; N_{x_n}(\omega_0) = 0.$$

In summary:

$$\forall \omega_0 \in A_0 \ \exists k(\omega_0) \in \mathbb{N} \ \forall n \geq k(\omega_0); \omega_0 \in A_n.$$

This means that

$$A_0 \subset \bigcup_{k=1}^{\infty} \bigcap_{n=k}^{\infty} A_n = \liminf_{n \to \infty} A_n,$$

which implies that

$$1 - N(x_0) = P(A_0) \leq P(\liminf_{n \to \infty} A_n) = \liminf_{n \to \infty} P(A_n) = 1 - \lim_{n \to \infty} N(x_n).$$

From this fact the result can be immediately derived.□

Now we are able to prove the following theorem.

Theorem 1 *Every random game* $< (\Omega, \mathcal{A}, P), X, H >$ *satisfying that* X *is compact has at least one MLNE.*

Proof. Since the image of N is bounded, it has a supremum M. Hence, we can construct a sequence $\{x_n\}$ in X such that $\lim_{n \to \infty} N(x_n) = M$. The compactness of X ensures the existence of a subsequence $\{x_{n_k}\}$ and a point $x_0 \in X$ such that $\lim_{k \to \infty} x_{n_k} = x_0$. Then, applying Lemma 1 we conclude:

$$N(x_0) \geq \lim_{k \to \infty} N(x_{n_k}) = M \geq N(x) \text{ for every } x \in X.$$

In other terms, x_0 is an MLNE.□

Summarizing, we have introduced the MLNE and proved its existence for a rather general class of random games. In this process, we have defined a function N which measures the possibilities of every combination of strategies x to be a Nash equilibrium. Such a function allows, for instance, to give not only an MLNE but also its *level of likelihood.*

However there are situations where the function N is identically equal to zero. In that case, every $x \in X$ is trivially an MLNE. In the following theorem we present two conditions sufficient to assure that this is not the case.

Theorem 2 *If* $< (\Omega, \mathcal{A}, P), X, H >$ *is a random game and one of the following two conditions is verified*

1. X *is countable and the event* $A := \{\omega \in \Omega / < X, H_\omega >$ *has at least one Nash equilibrium*$\}$ *satisfies that* $P(A) > 0$,
2. Ω *is finite and, for an* $\omega \in \Omega$ *with* $P(\{\omega\}) > 0$, $< X, H_\omega >$ *has at least one Nash equilibrium,*

then the function N, *defined as in Definition 3, is not identically equal to zero.*

Proof. If Condition 1 is fulfilled then

$$0 < P(A) = \int_A dP \leq \int_A \sum_{x \in X} N_x(\omega)\, dP(\omega) = \sum_{x \in X} \int_A N_x(\omega)\, dP(\omega) =$$

$$= \sum_{x \in X} P\{\omega \in A / N_x(\omega) = 1\} \leq \sum_{x \in X} N(x)$$

and hence we can conclude that there exists $x \in X$ such that $N(x) > 0$. If Condition 2 is fulfilled then, if x is an equilibrium of $< X, H_\omega >$ and $P(\{\omega\}) > 0$, we can obviously assure that $N(x) > 0$.□

So, in Theorem 2 above, we have proved that, in many practical situations, the concept of MLNE is not a trivial one (in fact, we will often deal with random games of the type "*one game is going to be played out of a finite list of games (with Nash equilibria) each of them having a positive and known probability of being played*" which clearly falls in Condition 2. However, it is convenient to modify the Definition 3 if, in the corresponding random game, N is identically equal to zero. A nontrivial case when this can happen is when some of the random variables H_x (we denote by H_x the function which assigns $H(x,\omega)$ to every $\omega \in \Omega$) are absolutely continuous. Observe that, although in these situations $N(x) = 0$ for all $x \in X$, some x can be such that their corresponding events $\{\omega \in \Omega / N_x(\omega) = 1\}$ bear more density of probability than others' corresponding events do and, hence, still makes sense to select a maximum likelihood Nash equilibrium. In the next section we present the redefinition of the MLNE when $N \equiv 0$.

3 An Alternative Maximum Likelihood Equilibrium Concept

Let us consider a random game $< (\Omega, \mathcal{A}, P), X, H >$ such that its corresponding N defined as in Definition 3 is identically equal to zero. Now, let us make the following suppositions:

S1 Each X_i is a metric space (with distance d_i) which verifies that, if we denote the open and closed balls with center x_i and radius δ by $B(x_i, \delta)$ and $B[x_i, \delta]$ respectively, $\overline{B(x_i, \delta)} = B[x_i, \delta]$, for all $x_i \in X_i$ and $\delta > 0$. We denote by $C(x, \delta)$ and $C[x, \delta]$ the sets $\prod_{i=1}^{n} B(x_i, \delta)$ and $\prod_{i=1}^{n} B[x_i, \delta]$ respectively.

S2 There is a measure $\mu : X \longrightarrow [0, \infty]$ satisfying:

1. $\mu(C[x, \delta]) > 0$ for every $x \in X$ and every $\delta > 0$, and
2. For every $\varepsilon > 0$, there exist $\rho > 0$ and $r > 0$ such that, for every $\delta \in (0, \rho]$ and every $x, y \in X$ verifying that
$$\max_{1 \leq i \leq n} d_i(x_i, y_i) \leq r,$$
it results that
$$\left| \frac{\mu(C[x, \delta])}{\mu(C[y, \delta])} - 1 \right| \leq \varepsilon.$$

The existence of a measure defined on X is a necessary supposition to define a kind of probability density function containing the information about the Nash equilibria. Apart from that, S1 and S2 are only technical conditions and not very restrictive: for instance, if the sets X_i are euclidean spaces and μ is the Lebesgue measure, S1 and S2 are fulfilled.

Now we can redefine the MLNE for this particular case.

Definition 4 *Let us consider a random game* $< (\Omega, \mathcal{A}, P), X, H >$ *satisfying S1 and S2 and such that its corresponding* N *defined as in Definition 3 is identically equal to zero. Then,* $x \in X$ *is an MLNE of* R *if*

$$f(x) \geq f(y) \quad \forall y \in X,$$

where

$$f(x) := \limsup_{\delta \to 0^+} \frac{P(C[x,\delta]^*)}{\mu(C[x,\delta])}$$

(being $Y^* := \{\omega \in \Omega / \exists y \in Y, N_y(\omega) = 1\}$ *for any* $Y \subset X$*).*

Now we are able to prove an existence theorem for this new version of the MLNE. For not to enlarge the paper unnecessarily we only provide a sketch of the proof.

Theorem 3 *Every random game* R *in the conditions of Definition 4 and verifying that* X *is compact has at least one MLNE as introduced in that definition.*

Proof (sketched). First, using S1, prove that, for every $x \in X$ and every $\delta > 0$, $C(x,\delta)^*$ and $C[x,\delta]^*$ are in $\mathcal{A}$. With this and S2 prove that, if $\{x_n\}_{n\geq 1}$ a sequence in X such that $\lim_{n\to\infty} x_n = x_0 \in X$ and verifying that the sequence $\{f(x_n)\}$ converges, then

$$f(x_0) \geq \lim_{n\to\infty} f(x_n).$$

Now, using the compactness of X, we can immediately conclude the proof.□

Observe that the revised definition is only suitable for random games with $N \equiv 0$. Namely, if there exists $x \in X$ such that $N(x) > 0$, then

$$f(x) = \limsup_{\delta \to 0^+} \frac{P(C[x,\delta]^*)}{\mu(C[x,\delta])} \geq \limsup_{\delta \to 0^+} \frac{P(\{x\}^*)}{\mu(C[x,\delta])} =$$

$$= \frac{N(x)}{\mu(\{x\})} = \infty \text{ if } \mu(\{x\}) = 0.$$

This means that, if μ is a non-atomic measure, for every $x \in X$ with $N(x) > 0$, $f(x) = \infty$. Hence, in this case, f is not a good criterium to select the MLNE.

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Ricardo Cao, Departamento de Matemáticas, Universidad de La Coruña, Campus da Zapateira s/n,
15071 La Coruña, SPAIN

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