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Research memorandum

TILBURG INSTITUTE OF ECONOMICS
DEPARTMENT OF ECONOMETRICS

ON THE ABSOLUTE MOMENTS OF A NORMALLY DISTRIBUTED RANDOM VARIABLE.

by

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Summary.

Estimation of shortage and excess of supply on the housing market leads to the evaluation of the absolute moments of a normally distributed variable. A formula for these moments is given in section 2; since for the first two central moments the function $H(x) = \phi(x) - x\Phi(-x)$ appears to be of considerable interest, its properties are investigated in section 3. The function $K(x) = H(x)\ H(-x)$, that appears in the formula for the second central moment, is the subject of section 4. The consequences of the derived properties for the estimation of housing shortage and excess are discussed. After multiplication by a constant, $K(x)$ can be interpreted as a density which is nearly standard normal. A formula for the moments corresponding with this density is presented.

1. INTRODUCTION.

In view of the stil existing housing shortage in The Netherlands, an optimal use of the housing stock is necessary. This can be achieved by trying to allocate households to dwellings in such a way that each houshold occupies a house that suits it best. Of course, this is possible only if the number of households wanting a dwelling of a certain type (related to rent and number of rooms, for example) is known, as well as the number of available dwellings of that type. Comparing demand and supply will reveal in what categories of dwellings a shortage or an excess of supply exists, which is of course of considerable importance for building programs. Denoting supply and demand by S and D respectively, we have for the excess E of supply:

$$E = \max(S-D,0)$$

and for the shortage F:

$$F = \max(D-S,0)$$

A related concept is the potential shift P, defined as the number of households able to find suitable dwellings on the existing housing market. This important quantity indicates to what extent the housing needs could be solved if no new building activities were started.
The following relations are immediately clear:

$$P = \min(S,D) = \tfrac{1}{2}(S+D) - \tfrac{1}{2}|S-D|$$

$$E = S-P;\ F = D-P \qquad (1.1)$$

To estimate these quantities a survey is needed in which the actual as well as the favoured housing situation is asked for. Denoting the sample totals of the above quantities by the corresponding minuscules, the most obvious estimator in a simple random sample with sampling fraction f is p/f, where

$$p = \min(s,d) = \tfrac{1}{2}(s+d) - \tfrac{1}{2}|s-d| \qquad (1.2)$$

Since s/f and d/f are unbiased estimators of S and D respectively, the bias B of the estimator p/f equals

$$B = E\{p/f\} - P = \tfrac{1}{2}|E\{\frac{s-d}{f}\}| - \tfrac{1}{2}\,E|\frac{s-d}{f}| \qquad (1.3)$$

as follows from formulae (1.1) and (1.2).
Now, if the sample is large enough (and the sampling fraction small enough), (s-d)/f will be nearly normally distributed and the problem is reduced to finding the first absolute moment of a normally distributed variable u. Similarly, calculating the variance of p/f involves evaluation of the variance of $|u|$. See for details [1].
It is therefore necessary to investigate the absolute moments of a normally distributed variable, which is the subject of the present paper. The major part is denotes to the study of the function H(x) defined by

$$H(x) = \phi(x) - x\Phi(-x) \qquad (1.4)$$

which almost completely determines the first two central moments of $|u|$. In equation (1.4) $\phi(x)$ and $\Phi(x)$ denote respectively the density and the distribution function of a standard normal variable $x \sim N(0,1)$.
Of course, the results presented here may be of interest whenever a variate following a normal distribution occurs, the sign of which is irrelevant. Moreover, the function H(x) arises in other contexts as well.
It can be shown for example, that the number of times a vehicle with given speed is overtaken per kilometer, can be expressed under certain conditions by the function H(x) [2].

2. THE ABSOLUTE MOMENTS OF A NORMALLY DISTRIBUTED VARIABLE.

In this section an expression is derived for the absolute moments of a normally distributed random variable $u \sim N(\mu,\sigma)$. The density of u is given by

$$f(u) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\tfrac{1}{2}(u-\mu)^2/\sigma^2\right] \tag{2.1}$$

By means of the transformation

$$x = (u-\mu)/\sigma \tag{2.2}$$

a standard normal variable $x \sim N(0,1)$ may be obtained with the density

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}x^2) \tag{2.3}$$

Some wellknown properties of ϕ which will be used in the sequel are

$$\left.\begin{array}{l} \lim\limits_{x\to-\infty} \phi(x) = 0 \,;\, \phi(0) = \frac{1}{\sqrt{2\pi}};\ \lim\limits_{x\to\infty} \phi(x) = 0 \\ \\ \phi(x) > 0 \,; \qquad \phi(-x) = \phi(x);\ \phi'(x) = -x\phi(x) \end{array}\right\} \tag{2.4}$$

The probability distribution of x is denoted by

$$\Phi(x) = \int_{-\infty}^{x} \phi(y)\,dy = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp(-\tfrac{1}{2}y^2)\,dy \tag{2.5}$$

with the properties

$$\left.\begin{array}{l} \lim\limits_{x\to-\infty} \Phi(x) = 0 \,;\, \Phi(0) = \tfrac{1}{2} \,;\, \lim\limits_{x\to\infty} \Phi(x) = 1 \\ \\ \Phi(x) > 0 \,; \qquad \Phi(-x)+\Phi(x) = 1 \,;\, \Phi'(x) = \phi(x) \end{array}\right\} \tag{2.6}$$

Integrals of the type $\int_0^\infty u^r f(u)du$ will be needed, which may be evaluated by use of transformation (2.2); defining

$$b = \mu/\sigma \tag{2.7}$$

we find

$$\int_0^\infty u^r f(u)du = \frac{1}{\sigma\sqrt{2\pi}}\int_0^\infty u^r \exp[-\tfrac{1}{2}(u-\mu)^2/\sigma^2]\,du$$

$$= \frac{1}{\sigma\sqrt{2\pi}}\int_0^\infty (\sigma x+\mu)^2 \exp(-\tfrac{1}{2}x^2)d(\sigma x+\mu)$$

$$= \int_{-b}^\infty (\sigma x+\mu)^2 \phi(x)dx$$

$$\int_0^\infty u^r f(u)du = \sum_{j=0}^{r} \binom{r}{j}\sigma^j\mu^{r-j}\int_{-b}^\infty x^j\phi(x)dx \tag{2.8}$$

An expression for the integrals on the right follows from lemma 2.1, which can easily be proved by taking derivatives.

<u>Lemma 2.1</u> The primitive function of $x^j\phi(x)$ is given by

$$-\phi(x)\sum_{i=0}^{m} 2^{m-i}\,\frac{m!}{i!}x^{2i} \qquad \text{for } j = 2m+1$$

$$-\phi(x)\sum_{i=1}^{m} 2^{i-m}\,\frac{(2m)!i!}{(2i)!m!}x^{2i-1} + 2^{-m}\,\frac{(2m)!}{m!}\Phi(x) \qquad \text{for } j = 2m$$

with $m = 0,1,2,\ldots$

Denoting

$$F_j(-b) = \int_{-b}^\infty x^j\phi(x)dx$$

it follows that

$$F_j(-b) = \begin{cases} \phi(b) \sum_{i=0}^{m} 2^{m-i} \frac{m!}{i!} b^{2i} & \text{for } j=2m+1 \\ -\phi(b) \sum_{i=1}^{m} 2^{i-m} \frac{(2m)!i!}{(2i)!m!} b^{2i-1} + 2^{-m} \frac{(2m)!}{m!} \Phi(b) & \text{for } j=2m \end{cases} \tag{2.9}$$

For the r^{th} moment m_r of u is follows that

$$m_r = \sum_{m=0}^{[r/2]} \frac{r!}{(r-2m)!m!} (\sigma^2/2)^m \mu^{r-2m} \tag{2.10}$$

where [a] denotes the largest integer smaller than or equal to a. Expressions for the absolute moments β_r of u are now easily derived. For the even moments we have of course

$$\beta_{2k} = m_{2k} \tag{2.11}$$

and for the odd moments (2.8) gives:

$$\beta_{2k-1} = \int_{-\infty}^{\infty} |u|^{2k-1} f(u)du = 2\int_0^{\infty} u^{2k-1} f(u)du - m_{2k-1}$$

$$= 2 \sum_{j=0}^{2k-1} \binom{2k-1}{j} \sigma^j \mu^{2k-j-1} F_j(-b) - m_{2k-1} \tag{2.12}$$

where $F_j(-b)$ and m_{2k-1} are given by (2.9) and (2.10) respectively. For the first two absolute moments we find in this way

$$\beta_1 = \mu - 2\mu\Phi(-b) + 2\sigma\phi(b) \tag{2.13}$$

$$\beta_2 = \mu^2 + \sigma^2 \tag{2.14}$$

If the function H(b) is defined as

$$H(b) = \phi(b) - b\Phi(-b) \tag{2.15}$$

we get for the expectation and variance of $|u|$:

$$E|u| = \beta_1 = \mu + 2\sigma H(b) \tag{2.16}$$

$$\mathrm{Var}|u| = \beta_2 - \beta_1^2 = \sigma^2 - (2\sigma)^2 H(b)\{H(b)+b\}$$

It is easily seen from the definition that H(b) has the property $H(-b) = H(b)+b$, so that $\mathrm{Var}|u|$ can be written as

$$\mathrm{Var}|u| = \sigma^2 - (2\sigma)^2 H(b)H(-b) \tag{2.17}$$

The expectations of $|u|$ and u differ by a term in which the function H(b) plays a dominant part and the same holds for the variance of $|u|$ and u with respect to the function H(b)H(-b). The properties of H(b) and H(b)H(-b) are investigated in the next sections.

3. PROPERTIES OF THE FUNCTION $H(x) = \phi(x) - x\Phi(-x)$.

Some important properties of H(x) are:

$$\left.\begin{array}{l} \lim\limits_{x\to-\infty} H(x) = 0 \,;\ H(0) = \frac{1}{\sqrt{2\pi}} \,;\ \lim\limits_{x\to\infty} H(x) = 0 \\ \\ H(x) > 0 \,; \qquad H(-x) - H(x) = x \,;\ H'(x) = -\Phi(-x) \end{array}\right\} \tag{3.1}$$

Most of these relations follow immediately from the properties of $\phi(x)$ and $\Phi(x)$ mentioned in formulae (2.4) and (2.6). The limiting value of H(x) when x nears infinity is somewhat more difficult to find; it is however a special case of lemma 3.1.

Lemma 3.1 For all real numbers c relation (3.2) holds:

$$\lim_{x\to\infty} x^c \, H(x) = 0 \tag{3.2}$$

Proof.

Of course, the relation

$$\lim_{x\to\infty} x^c \, \phi(x) = 0$$

holds for all real numbers c. Further we have according to de l'Hospital's rule

$$\lim_{x\to\infty} \frac{\Phi(-x)}{\phi(x)/x} = \lim_{x\to\infty} \frac{\{\Phi(-x)\}'}{\{\phi(x)/x\}'} = \lim_{x\to\infty} \frac{-\phi(x)}{-\phi(x)-\phi(x)/x^2} = 1$$

Combination of these resultes gives:

$$\lim_{x\to\infty} x^c \, \Phi(-x) = \lim_{x\to\infty} x^{c-1}\phi(x) . \lim_{x\to\infty} \frac{\Phi(-x)}{\phi(x)/x} = 0$$

from which the lemma follows. ▲

A consequence of the lemma is that the ratio $\Phi(-x)/\phi(x)$-called Mills' ratio-is approximately $1/x$ for large x. It is always smaller, however, since $H(x)$ is positive everywhere. In the following section also a lower bound for Mills' ratio is needed, which is provided by lemma 3.2.

Lemma 3.2 For all positive x inequality (3.3) holds:

$$\phi(x) < (x+\sqrt{2/\pi})\Phi(-x) \tag{3.3}$$

Proof.

Define

$$J(x) = \phi(x)-(x+\sqrt{2/\pi})\Phi(-x)$$

The first two derivatives are then

$$J'(x) = \sqrt{2}/\pi \; \phi(x)-\Phi(-x)$$

$$J''(x) = (1 - x\sqrt{2/\pi})\phi(x)$$

Consider first the values of x with $0 < x \leqq \sqrt{\pi/2}$. For these values the second derivative of J(x) is positive and hence the first derivative is an increasing function. Since J'(0) is negative and $J'(\sqrt{\pi/2})$ positive, J(x) has local maxima in the endpoints 0 and $\sqrt{\pi}/2$.
On the other hand, for $x > \sqrt{\pi}/2$ the second derivative is negative and the first decreasing; since J'(x) tends to zero for large x, the first derivative is positive. So, J(x) is increasing for all $x > \sqrt{\pi}/2$ and has a local maximum for $x \to \infty$. Since $J(0) = J(\infty) = 0$, J(x) is negative for all positive x. ▲

Relation (3.3) may be written alternatively as

$$H(x) < \sqrt{2}/\pi \; \Phi(-x) \text{ for } x > 0 \tag{3.4}$$

For Mills'ratio we find for positive x the double inequality:

$$\frac{1}{x+\sqrt{2/\pi}} < \frac{\Phi(-x)}{\phi(x)} < \frac{1}{x} \tag{3.5}$$

Other inequalities for this ratio are

$$\frac{1}{x} - \frac{1}{x^3} < \frac{\Phi(-x)}{\phi(x)} < \frac{1}{x}$$

(see for example Feller [3]) and

$$\frac{1}{x+1/x} < \frac{\Phi(-x)}{\phi(x)} < \frac{1}{x}$$

due to Gordon [4]. The first part of this last relation is stronger than Feller's, but for $x < \sqrt{\pi}/2$ weaker than (3.5).
If equation (2.16) is written as

$$E|u| - Eu = 2\sigma H(b)$$

relations (3.1) imply that $|u|$ is always larger than u in expectation. For constant variance σ^2 the difference between $E|u|$ and $Eu = \mu$ decreases as μ increases in absolute value. This result is quite plausible, since the difference between the two expectations is caused by the occurrence of positive and negative values of u. If $b = \mu/\sigma$ moves farther away from the origin in either direction, positive or negative values dominate. So, the bias of the estimator p/f is largest when demand and supply are about equal, but tends to zero when $|S-D|$ is large.

4. PROPERTIES OF THE FUNCTION $k(x) = H(x)H(-x)$.

The function k(x) is defined by

$$k(x) = H(x)H(-x) = \phi^2(x)+b\phi(x)\{\Phi(x)-\Phi(-x)\}-b^2\Phi(x)\Phi(-x) \tag{4.1}$$

Some important properties are

$$\left.\begin{array}{l} \lim_{x\to-\infty} k(x) = 0;\ k(0) = \frac{1}{2\pi};\ \lim_{x\to\infty} k(x) = 0 \\ \\ k(x) > 0; \qquad k(x) = k(-x);\ xk'(x) < 0 \end{array}\right\} \tag{4.2}$$

All these relations but one follow immediately from the similar properties of H(x) given in (3.1). The last assertion concerning the sign of the first derivative, is the subject of lemma 4.1.

Lemma 4.1 k(x) is increasing for x < 0 and decreasing for x > 0.

Proof.

In view of the symmetry of k(x) the two assertions of the lemma are equivalent. So it is sufficient to prove that k'(x) is negative for x > 0. Now,

$$k'(x) = H'(x)H(-x)-H(x)H'(-x)$$

$$= -\Phi(-x)\{H(x)+x\}+H(x)\Phi(x)$$

$$= H(x)\Phi(-x)\{\Phi^{-1}(-x)-xH^{-1}(x)-2\}$$

and the expression between { } tends to 0 for $x \to 0$. So, if for $x > 0$ this expression can be shown to be decreasing, or its derivative to be negative, the proof is given. Indeed,

$$\{\Phi^{-1}(-x)-x\ H^{-1}(x)-2\}' = \frac{\phi(x)}{\Phi^2(-x)} - \frac{\phi(x)}{H^2(x)}$$

$$= \frac{\phi(x)}{\Phi(-x)H^2(x)}\{H^2(x)-\Phi^2(-x)\} < 0$$

because of inequality (3.4). ▲

If equation (2.17) is written as

$$\mathrm{Var}|u| - \mathrm{Var}\ u = -(2\sigma)^2\ H(b)H(-b)$$

relations (4.2) imply that $|u|$ has always a smaller variance than u. For constant σ^2 the difference between them decreases if μ increases in absolute value, which is again very plausible. Figure 1 shows the functions $\phi(x)$, $\Phi(x)$, $H(x)$ and $k(x)$. It appears that $k(x)$ differs from $\phi(x)$ by a factor that is reasonably constant. The following values can be obtained, for example, from tables of the normal density and distribution:

x	0	± 1	± 2	± 3
$k(x)/\phi(x)$	0.3989	0.3823	0.3158	0.2573

So, the function $g(x)$ defined as

$$g(x) = k(x)/\int_{-\infty}^{\infty} k(y)dy \tag{4.3}$$

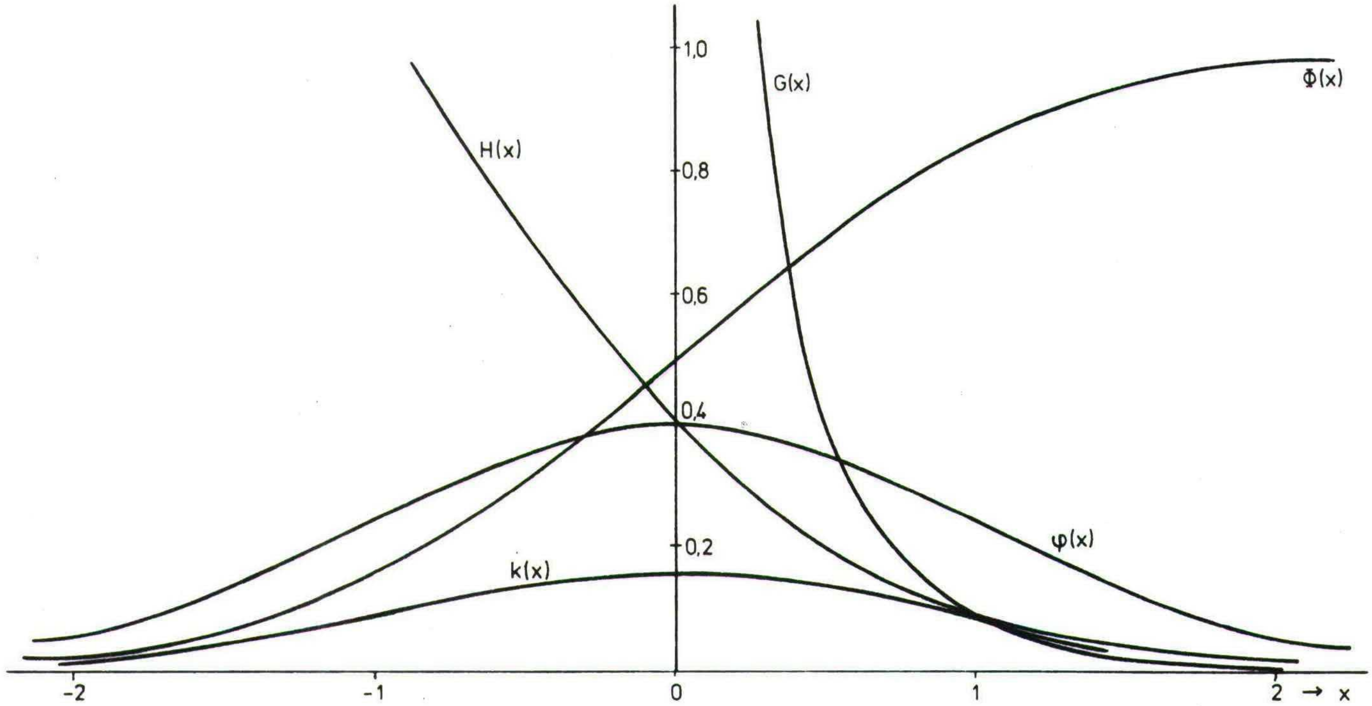

Figure 1 The functions $\varphi(x)$, $\Phi(x)$, $H(x) = \varphi(x) - x\Phi(-x)$ and $k(x) = H(x)H(-x)$

can be interpreted as a density which resembles the density of a standard normal variate. This density could be used, for example to detect numerically the influence of small deviations from normality. The moments of a variable x with density f(x) can be found with lemma 4.2.

Lemma 4.2

The primitive function of $x^{2n}\,k(x)$ is

$$\frac{1}{2n+3}\Big[-x^{2n+3}\Phi(x)\Phi(-x)+\phi(x)\{2\Phi(x)-1\}(x^{2n+2}-\sum_{i=0}^{n} 2^{n-i}\,\frac{n!}{i!}x^{2i})+$$

$$+\ \phi^2(x)x(x^{2n}+\sum_{i=0}^{n} c_i^{[2n]}x^{2i})+d^{[2n]}\,\Phi(x\surd 2)/\surd\pi\Big] \qquad (4.4)$$

for all non-negative integers. The $c_i^{[2n]}$ follow the recursion formula

$$c_{n-1}^{[2n]} = -2;\quad c_{i-1}^{[2n]} = (i+\tfrac{1}{2})c_i - 2^{n-i}\,\frac{n!}{i!} \qquad (4.5)$$

and $d^{[2n]}$ is given by

$$d^{[2n]} = \begin{cases} 2 & \text{for } n = 0 \\ 2^n n! - \tfrac{1}{2}c_o^{[2n]} & \text{for } n > 0 \end{cases} \qquad (4.6)$$

Proof.

The proof can be given by differentiating formula (4.4). (This formula was formed with the help of (4.1).) ▲

The primitive function of $x^{2n+1}\,k(x)$ may be obtained in a similar way; it reads

$$\frac{1}{2n+4}\Big[-x^{2n+4}\Phi(x)\Phi(-x)+\phi(x)\{2\Phi(x)-1\}xp_{n+1}(x^2)\ +$$

$$+\ \phi^2(x)q_{n+1}(x^2)\Big] \qquad (4.7)$$

where $p_{n+1}(x^2)$ and $q_{n+1}(x^2)$ are polynomials in x^2 of degree $n+1$, the coefficients of which can be found by differentiating. The most important difference with formula (4.4) is the absence of the term in which $\Phi(x\sqrt{2})$ occurs; this corresponds with lemma 2.1.

Especially interesting are the integrals of $x^i k(x)$ over the whole real axis, to which only the term containing $\Phi(x\sqrt{2})$ contributes. So, it follows from formulae (4.4) and (4.7) for all non-negative integers n:

$$\int_{-\infty}^{\infty} x^i k(x)dx = \begin{cases} 0 & \text{for } i=2n+1 \\ \dfrac{d^{[2n]}}{(2n+3)\sqrt{\pi}} & \text{for } i=2n \end{cases} \tag{4.8}$$

The result for odd i follows of course immediately from the symmetry of k(x).

In particular, it follows that

$$\int_{-\infty}^{\infty} k(x)dx = \frac{2}{3\sqrt{\pi}}$$

and substituting this in equation (4.3) we find that

$$g(x) = \tfrac{3}{2}\sqrt{\pi}\ k(x) \tag{4.9}$$

is a density.

To calculate the even moments corresponding with g(x) it is necessary to evaluate $c_o^{[2n]}$. Recursion formula (4.5) gives for all $j < n$:

$$c_o^{[2n]} = c_j^{[2n]} \prod_{i=j}^{j} (i+\tfrac{1}{2}) - \sum_{k=2}^{j} \frac{2^{n-k}n!}{k!} \prod_{i=1}^{k-1} (i+\tfrac{1}{2}) - 2^{n-1}n!$$

With the wellknown relation

$$\prod_{i=1}^{k-1} (i+\tfrac{1}{2}) = \frac{(2k)!}{2^{2k-1}k!}$$

we find in particular for j = n:

$$c_o^{[2n]} = c_{n-1}^{[2n]} \frac{(2n)!}{2^{2n-1}n!} - 2^{n+1}n! \sum_{k=2}^{n-1} \binom{2k}{k} 2^{-3k} - 2^{n-1}n!$$

or, bringing terms under the sommation sign:

$$c_o^{[2n]} = -2^{n+1}n! \left[\binom{2n}{n} 2^{-3n} + \sum_{k=1}^{n} \binom{2k}{k} 2^{-3k}\right] \qquad (4.10)$$

Expression (4.6) can now be written as

$$d^{[2n]} = 2^n n! \left[\binom{2n}{n} 2^{-3n} + \sum_{k=0}^{n} \binom{2k}{k} 2^{-3k}\right] \qquad (4.11)$$

which holds under the usual conventions even for n=0.
The even moments of a variate x with density g(x) are given by

$$\int_{-\infty}^{\infty} x^{2n} g(x)\,dx = \frac{3n!\,2^{n-1}}{2n+3} \left[\binom{2n}{n} 2^{-3n} + \sum_{k=0}^{n} \binom{2k}{k} 2^{-3k}\right] \qquad (4.12)$$

for non-negative n. This gives for the first three even moments:

$$\mu_o = 1;\ \mu_2 = 9/10;\ \mu_4 = 69/28$$

Note that for large n the factor between [ ] tends to $\sqrt{2}$ since

$$\sum_{k=0}^{\infty} \binom{2k}{k} 2^{-3k} = \sum_{k=0}^{\infty} (-\tfrac{1}{2})^k \binom{-\frac{1}{2}}{k} = (1-\tfrac{1}{2})^{-\frac{1}{2}} = \sqrt{2}.$$

Finally, an easier method for calcuting $c_o^{[2n]}$ and $d^{[2n]}$ is by use of the recursion formulae

$$c_o^{[2(n+1)]} = 2(n+1)c_o^{[2n]} + \frac{(2n)!}{2^{2n-1}n!} \qquad (4.13)$$

$$d_o^{[2(n+1)]} = 2(n+1)d^{[2n]} - \frac{(2n)!}{2^{2n}n!} \qquad (4.14)$$

which can be derived immediately from (4.10) and (4.11).

M.v.d.B.

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