Two notes on the joint replenishment problem under constant demand

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Publication date:
1990

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DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

TWO NOTES ON THE JOINT REPLENISHMENT PROBLEM UNDER CONSTANT DEMAND

M.J.G. van Eijs

FEW 453

TWO NOTES ON THE JOINT REPLENISHMENT PROBLEM UNDER CONSTANT DEMAND

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This note considers the joint replenishment inventory problem for N items under constant demand. We investigate the frequently used cyclic strategy $(T;k_1,..,k_N)$: a family replenishment is made every T time units and item i is included in each k_i'th replenishment. It it known that the overall optimal strategy for the joint replenishment problem is not necessarily of this type. Goyal proposed a solution to find the global optimum within the class of cyclic strategies. However, among other issues we will show that the algorithm of Goyal does not always lead to the optimal cyclic strategy. A simple correction is suggested.

Key words: inventory, multi-item

INTRODUCTION

The joint replenishment problem is an extensively studied problem in inventory theory. The problem is to construct a replenishment strategy for (a family of) N items which interact because of a special set-up cost structure: a major set-up cost is incurred for each joint replenishment, independent of which items are involved. In addition, a minor set-up cost is incurred for each item included in the replenishment. So, cost savings can be achieved by coordinating the replenishments of several items. We refer to Aksoy and Erenguc[1] and Goyal and Satir[2] for a more detailed introduction to the problem. A commonly used strategy is to place an order every T time units and to include the i'th item into every k_i'th replenishment. This leads to a replenishment interval of Tk_i time units for the i'th item. Note that (actual) joint replenishments are equally spaced under a cyclic strategy only if $k_{min}=1$, where k_{min} is the minimum value of k_i over all items i.

Example 1

Let $N=2$, $k_1=2$, $k_2=3$, $T=1$. Then the orders (above the time-axis) on subsequent time intervals are as follows:

```
{1,2}     { }      {1}      {2}      {1}      { }     {1,2}
  |--------|--------|--------|--------|--------|--------|-- time
  0        1        2        3        4        5        6
```

Note that no order is placed on $t=1$ and $t=5$. The actual joint replenishments are not equally spaced.

The cyclic strategies $(T;k_1,..,k_N)$ with $k_{min}=1$ are called strict cyclic. We assume that for a cyclic strategy with $k_{min}>1$ the major set-up cost is also incurred at multiples of T at which no actual replenishment is performed. Dagpunar[3] reformulated the objective function of the $(T;k_1,..,k_N)$ strategies under the assumption that no major set-up cost is charged if no order is placed. However, as pointed out by Goyal,[4] the minimalisation of the cost function provided by Dagpunar, is considerably more complex than that of the orginal problem.

Most solution procedures solve a subproblem in which the k_i's are not necessarily restricted to integer variables. Schweitzer and Silver[5] have shown for this continuous variable case that the problem is ill-posed if the restriction $k_i \geq 1$ is deleted. We will show that under this restriction, $k_{min}=1$ in the optimal solution.

Andres and Emmons[6] have shown by a counter example that the class of cyclic strategies does not always contain an optimal strategy. Among others, Chakravarty[7] considered a class of strategies under which the items of the family are divided into a number of groups. The items of each group have the same replenishment cycle, but the replenishment cycles of the groups are not an integer multiple of the shortest (basic) cycle. These strategies are referred to as "direct grouping strategies". Van Eijs, Heuts and Kleijnen[8] compared the performance of cyclic and direct grouping strategies with help of simulation. They concluded that the non-cyclic class of strategies proposed by Chakravarty outperforms the cyclic $(T;k_1,..,k_N)$ strategies only in situations under which the ratio of the major set-up cost and the average minor set-up cost is low (lower than 0.50, depending on the number of items in the family)[8]. However, this result also points out that the class of cyclic strategies is not always optimal. Another class of strategies which permit unequal time spacing between joint replenishments is suggested by Goyal and Soni.[9] They provide an extension of the $(T,k_1,..,k_N)$ strategies by permitting multiple basic cycles.

Goyal[10,11] has proposed an algorithm to find the optimal solution within the class of cyclic strategies. We will show that this solution is optimal within the class of strict cyclic strategies, but is not necessarily optimal within the class of cyclic strategies. We propose to adjust the algorithm of Goyal slightly to make it possible to obtain cyclic strategies $(T;k_1,..,k_N)$ with $k_{min}>1$.

THE JOINT REPLENISHMENT PROBLEM

The joint replenishment problem can be described as follows: a (family of) N items are purchased from the same supplier. When one of the items of the family is ordered a major set-up cost (A) is incurred, regardless of which item is ordered. In addition, a minor set-up cost (a_i) is charged for each particular item which is included in the replenishment. Item i has constant demand D_i per time unit and holding cost h_i per dollar per time unit. Stock-outs are forbidden and the rate of replenishment is assumed to be infinite. The objective is to minimize the total cost per time unit over an infinite horizon. For an arbitrary cyclic strategy $(T;k_1,..,k_N)$ the total average cost per time unit is given by (1):

$$TRC(T;k_1,...,k_N) = \frac{1}{T}\left(A + \sum_i \frac{a_i}{k_i}\right) + \frac{T}{2}\sum_i D_i h_i k_i \qquad (1)$$

For fixed $(k_1,..,k_N)$ the optimal value $T^*(k_1,..,k_N)$ of the basic cycle equals:

$$T^*(k_1,...,k_N) = \left[\frac{2\left(A + \sum_i a_i/k_i\right)}{\sum_i D_i h_i k_i}\right]^{1/2} \qquad (2)$$

If $T^*(k_1,..,k_N)$ is substituted back in the orginal cost function (1) the following cost function is obtained:

$$TRC(k_1,...,k_N) = \left[2\left(A + \sum_i \frac{a_i}{k_i}\right) * \left(\sum_i D_i h_i k_i\right)\right]^{1/2} \qquad (3)$$

In this note we consider two cases: (i) the continuous case where both k_i and T are allowed to be continuous variables, and (ii) the mixed integer variable case where k_i is restricted to an integer value, while T is a continious variable.

THE CONTINUOUS CASE

As mentioned before, most solution procedures try to find the minimum of (3) by minimalisation over the positive orthant. So, $(k_1,..k_N)$ is treated as a continuous variable. In the appendix we show that $TRC(k_1,..,k_N)$ is neither convex, nor concave in $(k_1,..,k_N)$. In fact, (3) is convex in one single k_i.

Schweitzer and Silver[5] have shown that the constraints $k_i \geq 1$ $(i=1,..,N)$ are needed to avoid an ill-posed statement of the problem. If these constraints are deleted the resulting minimalisation problem has no optimal solution. If the objective function is minimized over the positive orthant under the restriction that $k_i \geq 1$ for every item i, at least one of the items of the family has to be replenished each T time periods. Hence, $k_{min}=1$.

Result 1 : $k_{min}=1$ in the optimal solution of the continuous case.

Proof: Let solution 1 be of the form: $k_i>1$ for all items i, and let T be the corresponding basic cycle. The cost is denoted by C_1. Construct a new solution 2 $k'_i=k_i/k_{min}$, i=1,..,N, and $T'=Tk_{min}$. Note that $k'_{min}=1$. C_2, the cost of solution 2, is (see (1)):

$$C_2 = \frac{1}{T^*k_{min}} \left(A + \sum_i \frac{a_i}{k_i/k_{min}} \right) + \frac{T^*k_{min}}{2} \sum_i D_i h_i \frac{k_i}{k_{min}}) =$$

$$\frac{A}{T^*k_{min}} + \frac{1}{T} \sum_i \frac{a_i}{k_i} + \frac{T}{2} \left(\sum_i D_i h_i k_i \right)$$

$C_2 < C_1$, since $1/T^*k_{min} < 1/T$ (note that $k_{min}>1$).

THE MIXED INTEGER CASE

We have shown that $k_{min}=1$ in the continuous case, under the restriction that $k_i \geq 1$ for each item i. However, the variables k_i are not continuous but integer in the orginal problem. The question is whether $k_{min}=1$ also holds in the mixed integer case.

Andres and Emmons[6] have shown that the class of (strict) cyclic strategies is not always optimal. Andres and Emmons illustrate their statement with a two product example, for which the optimal solution is obtained by a special algorithm.[12] Moreover, the problem setting of Andres and Emmons is different from that of the joint replenishment problem in the following sense: a major set-up cost has to be paid at every replenishment in which not all items are involved. The only way to avoid the major set-up cost is to perform a joint replenishment for all items simultaneously (the problem settings are only equivalent if the number of items is two).

In literature regarding to the joint replenishment procedures for the cyclic $(T;k_1,..,k_N)$ strategies the algorithm of Goyal[10] is used to find the optimal cyclic strategy. However, the algorithm only guarantees an optimal strict cyclic strategy. We note a shortcomming on this algorithm:

Result 2 : The optimal solution within the class of cyclic strategies does not necessarily belong to the class of strict cyclic strategies.

Counter examples are given in example 2 and 3. Note that if $k_{min} > 1$ and the strategy is cyclic, then it seems profitable to order nothing on some specific replenishment dates, whereas the major set-up cost has to be paid (see also example 1).

Below, we shortly review the algorithm of Goyal.[10] Then a simple extension is proposed to the orginal algorithm. Goyal has derived that for a fixed value of T the variable costs of item i are minimised by selecting the integer $k_i(T)$ which satisfies:

$$k_i(T)(k_i(T)-1) < (2a_i/D_ih_i)/T^2) \leq k_i(T)(k_i(T)+1) \quad (4)$$

Remark 1

Formula (4) provides lower and upper bounds for T for different values of k_i. For example, if $k_i = 1$ then T has to belong to the interval $[(a_i/D_ih_i)^{½}, \infty >$.

Starting with fixed $(k_1,..,k_N)$, the optimal basic cycle $T^*(k_1,..,k_N)$ is given by (2). Goyal's algorithm obtaines a minimum (T_{min}) and a maximum value (T_{max}) for T and then it determines all intervals of T within this range for which $(k_1,..,k_N)$ is unchanged. It can be shown that only a finite number of intervals have to be considered. Hence the global optimum can be obtained by taking the minimum of all local minima after explicit enumeration of all the intervals. The algorithm is outlined below.

Algorithm of Goyal[10]

Step 1

Determination of T_{min} and T_{max}:

(i) Set $T_{max} := T^*(1,..,1)$ with (2).

(ii) Set $T_{min} := \min_i (a_i/D_ih_i)^{½}$.

(see remark 2)

Step 2

Initialisation:

(i) Set $(k_1^*,..,k_N^*) := (k_1(T_{max}),..,k_N(T_{max}))$ with (4) ($(k_1^*,..,k_N^*)$ is a vector which keeps up with the best relative replenishment frequencies, which are found so far). Set $TRC^* := TRC(k_1^*,..,k_N^*)$ with (3) (TRC^* is de cost corresponding to $(k_1^*,..,k_N^*)$).

(ii) Set $T := T_{max}$, and set $(k_1,..,k_N) := (k_1^*,..,k_N^*)$.

(iii) Set $T_{ch}(i):= (2a_i/(D_i h_i * k_i(k_i+1)))^{½}$ for all items i ($T_{ch}(i)$ is the basic cycle T at which the value of $k_i(T)$ of item i changes to $k_i(T)+1$).

Step 3

Set $T:= \max_i T_{ch}(i)$. If $T \leq T_{min}$, then go to step 5. Otherwise go to step 4.

Step 4

Evaluation of the cost in the new interval:

(i) Set $p:=\{i| \max_i T_{ch}(i)\}$ (p denotes the item for which $k_i(T)$ changes at T to $k_i(T)+1$); ties are broken arbitrary.
Set $k_p:=k_p+1$, and set $T_{ch}(p):= (2a_p/(D_p h_p * k_p(k_p+1)))^{½}$.

(ii) Calculate $TRC(k_1,..,k_N)$ with (3). If $TRC(k_1,..,k_N) < TRC^*$, then $(k_1^*,..,k_N^*):=(k_1,..,k_N)$ and $TRC^*:=TRC(k_1,..,k_N)$. Go back to step 3.

Step 5

Termination of the algorithm. The optimal strict cyclic strategy is $(T^*,k_1^*,..,k_N^*)$ with corresponding minimal cost TRC^*, where $T^*:=T^*(k_1^*,..,k_N^*)$ (with (2)). (see remark 3).

Remark 2

Since $T^*(k_1,..,k_N)$ is monotone decreasing in $(k_1,..,k_N)$ the maximum value of $T^*(k_1,..,k_N)$ occurs in $(k_1,..,k_N)=(1,..,1)$. Goyal[10] stated that the minimum of T is equal to the minimum of $(2a_i/D_i h_i)^{½}$ over all i. Andres and Emmons[6] already noted that T_{min} has to be equal to the minimum of $(a_i/D_i h_i)^{½}$ over all i (see also the remark below (4)). In a recent paper Goyal[11] gives an example where the algorithm of [10] does not give the optimal solution. He provides another algorithm which provides the optimal solution for the class of strict cyclic strategies. However, if the correct T_{min} is used the same solution is found with [10] and [11].

Remark 3

Goyal's algorithm determines $k_i(T^*)$ for each item with (4) after T^* has been obtained. This additional step is unnecessary if the correct T_{min} is used.

Our criticism on the algorithm of Goyal is that the choice of T_{min} is based on the assumption that at least one of the k_i's has an optimal value of one, and therefore it excludes an cyclic strategy $(T;k_1,..,k_N)$ with $k_{min}>1$. Such a strategy is not strict cyclic. As mentioned before, the optimum of the class of cyclic strategies does not necessarily belong to the class of strict cyclic strategies. As a consequence, the strategy obtained from Goyal's algorithm may not be the optimal cyclic strategy.

The problem is to construct a new lower bound for T. Note that $TRC^* > A/T^*$, where TRC^* and T^* are respectively the cost and the basic cycle of the optimal strategy, and A is the major set-up cost. Since $T^* > A/TRC^* \geq A/TRC$, where TRC is the cost of a feasible $(T,k_1,..,k_N)$ strategy, a lower bound for T is A/TRC. The lower this bound is the more intervals of T with an unchanged solution $(k_1,..,k_N)$ have to be distinguished. Hence, the lower T_{min}, the more computation time will be required to find the best cyclic strategy. The best choice for T_{min} is obtained when TRC is as low as possible. The minimum cost TRC^* is unknown, but a good upper bound is given by the orginal algorithm of Goyal. The following extended version of Goyal's algorithm will always find the optimal cyclic strategy.

Adjusted algorithm

Step 1

Use the orginal algorithm of Goyal[10] with $T_{min} := \min (a_i/D_i h_i)^{½}$.
The result is the optimal strict cyclic strategy $(T^*;k_1^{i*},..,k_N^*)$ with corresponding cost TRC^*, the current values of T ($\leq T_{min}$), and the vectors $(k_1,..,k_N)$ and $(T_{ch}(1),..T_{ch}(N))$.

Step 2

Set $T_{min} := A/TRC^*$, and proceed with Goyal's algorithm at step 4.

We will give two examples for which the adjusted algorithm affects the orginal solution.

Example 2 (Andres and Emmons [6])

Let N=2, A=1, D_1=400, D_2=900, a_1=50, a_2=50, h_1=1, h_2=1
Goyal's algorithm yields $k_1^*=2$, $k_2^*=1$, and $T^*=0.30$. The cost of the this strict cyclic strategy is 508.33. The adjusted algorithm yields $k_1^*=3$, $k_2^*=2$ and $T^*=0.17$. The corresponding cost is 505.96. This solution is the optimal cyclic $(T^*;k_1^*,..,k_N^*)$ strategy. The overall optimal strategy for the joint replenishment problem is obtained by the algorithm of Andres and Emmons.[12] The strategy is to order item 1 every 1/2 time unit and item 2 every 1/3 time unit.[6] Note that this solution corresponds with the cyclic strategy with T=1/6, $k_1=3$ and $k_2=2$. The cost of this strategy, 504, is lower than the cost obtained by the adjusted algorithm, because we assume that every T time units the major set-up cost is charged regardless whether an actual order is placed. In the model of Andres and Emmons the major set-up cost is only charged when any of the items of the family is actually ordered.

Example 3 (an extension of the example of Andres and Emmons)

We add an extra item to the family with D_3=850, h_3=1 and a_3=50. The solution for the orginal algorithm of Goyal is $k_1^*=2$, $k_2^*=1$, $k_3^*=1$, $T^*=0.31$. The cost is 801.62. The adjusted algorithm yields $k_1^*=3$, $k_2^*=2$, $k_3^*=2$. The time between two replenishments equals 0.17 time units and the corresponding cost is 797.54. We already mentioned that the computer time needed increases strongly. To establish the optimal strict cyclic strategy three intervals have to be evaluated However, 941 intervals have to be evaluated to find the optimal cyclic strategy. Note that the adjusted value of T_{min} in our algorithm depends heavily on the data.

The optimal cyclic strategy does not belong to the class of strict cyclic strategies if the major set-up cost is low relative to the average minor set-up cost. Otherwise it will be too costly to order nothing on a particular multiple of T. As mentioned before we prefer to use a class of non-cyclic strategies (with unequal spaced family replenishments) if the ratio $A/a<0.5$ (depending on N)[8], where a is the average minor set-up cost. In table 1 the solution of the cyclic $(T,k_1,..,k_N)$ strategy is compared with the solution of the direct grouping strategy proposed by Chakravarty[7] and the multi-cycle strategy proposed by Goyal and Soni[9] for example 2 and 3. Note that the cost obtained by [9] are lower than that obtained by [6]. This difference is caused by rounding off errors of [6].

Table 1. *Comparison of the numeric average cost for different classes of strategies.*

	cyclic strategies		non-cyclic strategies	
example	Goyal	our algorithm	Chakravarty	Goyal and Soni
2	508.33	505.96	504.98	503.98
3	801.62	797.54	796.55	795.57

CONCLUSIONS AND DISCUSSION

In this note we considered the class of cyclic $(T;k_1,..,k_N)$ strategies for the joint replenishment problem. The smallest value k_i-value is denoted by k_{min}. The class of cyclic strategies with $k_{min}=1$ are called strict cyclic. Under a strict cyclic strategy the actual family replenishments and the replenishments of the individual items in the family are equally spaced (by respectively T and Tk_i time units). Under a cyclic strategy with $k_{min}>1$ fake replenishments are made at some multiples of T. The major set-up cost is incurred at all multiples of T, even if there is is no actual replenishment. In literature it is assumed that the optimal cyclic strategy belongs to the class of strict cyclic strategies (so, $k_{min}=1$). We have shown that this is correct if the variables k_i are treated as continuous variables. However, if the variables k_i are restricted to integer values, we have illustrated that the optimal cyclic strategy is not necessarily strict cyclic. Goyal's algorithm,[10] which is commonly used to find the optimal cyclic strategy, does not allow solutions with $k_{min}>1$. As a consequence, this algorithm yields the optimal strict cyclic strategy, but it does not always yield the best cyclic strategy. We proposed a simple adjustment of Goyal's algorithm, which always finds the optimal cyclic $(T;k_1,...,k_N)$ strategy. A drawback of this algorithm is that the computer time needed increases strongly. The reader should note that the optimal cyclic strategy does not belong to the class of strict cyclic strategies only in situations when the ratio of the major set-up cost and the average minor set-up cost is very low. One can imagine that in such situations joint replenishments of items do not make much sense. In this cases it may be better to use classes of strategies with unequally spaced family replenishments.

Acknowledgment - The author would like to thank Professor Frank van der Duyn Schouten for several fruitful discussions and for his helpful comments.

APPENDIX

Lemma: Objective function (3) is neither convex nor concave in $(k_1,..,k_N)$

Proof: It is sufficient to prove that the function

$$f(k_1,..k_N) = \left(A + \sum_i \frac{a_i}{k_i} \right) \left(\sum_i D_i h_i k_i \right) \qquad (a1)$$

is not convex or concave. We will prove the lemma by showing that the Hessian matrix of $f(k_1,..,k_N)$ with respect to $(k_1,..,k_N)$ is neither positive definite (in this case $f(k_1,..,k_N)$ is convex) nor negative definite (then $f(k_1,..,k_N)$ is concave). In the case $N=2$ the Hessian matrix is given by:

$$H_2 = \begin{bmatrix} \dfrac{2a_1D_2h_2k_2}{k_1^3} & -\dfrac{a_1D_2h_2}{k_1^2} - \dfrac{a_2D_1h_1}{k_2^2} \\ -\dfrac{a_1D_2h_2}{k_1^2} - \dfrac{a_2D_1h_1}{k_2^2} & \dfrac{2a_2D_1h_1k_1}{k_2^3} \end{bmatrix} \qquad (a2)$$

(i): $f(k_1,..,k_N)$ is not convex.

Since the Hessian matrix of $f(k_1,..,k_N)$ is symmetric, $f(k_1,..,k_N)$ will be positive definite if all submatrices H_k $(k=1,..,N)$ have an positive determinant. We will show that the determinant of H_2, denoted by D_2, is negative $(b_i := D_ih_i,\ i=1,2)$:

$$D_2 = 4 \cdot \frac{a_1a_2b_1b_2}{k_1^2\,k_2^2} - \left[2 \cdot \frac{a_1a_2b_1b_2}{k_1^2k_2^2} + \left[\frac{a_2b_1}{k_2^2}\right]^2 + \left[\frac{a_1b_2}{k_1^2}\right]^2 \right] =$$

$$- \left[-2 \cdot \frac{a_1a_2b_1b_2}{k_1^2k_2^2} + \left[\frac{a_2b_1}{k_2^2}\right]^2 + \left[\frac{a_1b_2}{k_1^2}\right]^2 \right] = - \left[\frac{a_2b_1}{k_2^2} - \frac{a_1b_2}{k_1^2} \right]^2 < 0$$

(ii): $f(k_1,..,k_N)$ is not concave.

The Hessian matrix H_2 is negative definite if $x^TH_2x<0$ for all nonzero vectors $x^T=(x_1,x_2)^T$.

$$x^T H_2 x = 2\cdot\frac{a_1 D_2 h_2 k_2}{k_1^3}\cdot x_1^2 + 2\cdot\left[-\frac{a_1 D_2 h_2}{k_1^2} - \frac{a_2 D_1 h_1}{k_2^2}\right]\cdot x_1 x_2 + 2\cdot\frac{a_2 D_1 h_1 k_1}{k_2^3}\cdot x_2^2$$

Consider the vector $(x_1,x_2)^T=(1,0)^T$, then we have $x^T H_2 x>0$ and hence H_2 is not concave.

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