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INTERPRETATION AND GENERALIZATION OF THE LEMKE-HOWSON ALGORITHM

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Abstract

In this paper we present a game-theoretic interpretation of the Lemke-Howson algorithm for computing a Nash equilibrium in a noncooperative bi-matrix game. This also gives insight into the reason why that algorithm cannot find certain Nash equilibria. Furthermore, from this interpretation it is easy to derive an extended algorithm for computing all Nash equilibria in a nondegenerated (2×n) bi-matrix game.

1. Introduction

The Lemke-Howson method (see (2)) is the standard method for finding a Nash equilibrium in a noncooperative bi-matrix game. It finds such an equilibrium by solving a related linear complementarity problem. This is done by making complementary pivoting steps in a system of linear equations. The main drawback of the method is that it cannot find certain positively indexed equilibria (see (3)). Furthermore, it is not immediately clear how the method operates in terms of strategies and payoffs.

In this paper we provide a game-theoretic interpretation of the path generated by the Lemke-Howson procedure. We first rewrite their algorithm into an equivalent procedure which directly operates on the strategy space. Then we derive that the Lemke-Howson algorithm can be interpreted as a strategy adjustment process, i.e., as a process which reaches a Nash equilibrium through a sequence of adjustments of the strategy vectors. Also it clarifies why that method might not find all positively indexed equilibria. Finally, this equivalent algorithm leads to an extended procedure with which we can find all Nash equilibria in a (2×n) bi-matrix game which is nondegenerated as defined in (2).

The organization of the paper is as follows. In Section 2 we present the algorithm on the strategy space and discuss its features. The extended algorithm is described in Section 3.

2. Game-theoretic interpretation of the Lemke-Howson algorithm

A noncooperative bi-matrix game is a game with two players in which the payoffs are represented in matrices. Formally, it is a tuple (n_1,n_2,A,B), where n_j, $j = 1,2$, denotes the number of pure strategies of player j, whereas A and B are the $(n_1 \times n_2)$-payoff matrices of player 1 and 2 respectively. An element a_{jk} in A (b_{jk} in B) denotes the payoff to player 1 (2) if player 1 plays his j-th pure strategy while player 2 plays his k-th pure strategy. In the sequel (j,k) denotes action k of player j. The strategy space of player j, $j = 1,2$, is the (n_j-1)-dimensional unit simplex $S^{n_j-1} := \{x_j \in \mathbb{R}_+^{n_j} | \sum_{k=1}^{n_j} x_{jk}=1\}$, describing all possible mixed strategies of player j. Finally, $S := S^{n_1-1} \times S^{n_2-1}$ is the strategy space of the game. A strategy vector x in S can be denoted as $x = (x_1,x_2)$ with $x_j \in S^{n_j-1}$, $j = 1,2$. The standard equilibrium concept for a noncooperative game is that of a Nash equilibrium (N.E.). A Nash equilibrium strategy vector is a strategy vector $\bar{x}$ in S at which no player can improve upon his situation by unilateral changes. Observe that $\bar{x}_1A\bar{x}_2$ ($\bar{x}_1B\bar{x}_2$) is the payoff to player 1 (2) at $\bar{x}$. Thus, $\bar{x}$ is a N.E. if $\bar{x}_1A\bar{x}_2 = \max\{x_1A\bar{x}_2 | x_1 \in S^{n_1-1}\}$ and $\bar{x}_1B\bar{x}_2 = \max\{\bar{x}_1Bx_2 | x_2 \in S^{n_2-1}\}$.

The Lemke-Howson method (L-H) finds a N.E. by solving a related linear complementarity problem (LCP) on $\mathbb{R}_+^{n_1+n_2}$. More precisely, it searches for a $y = (y_1,y_2) \in \mathbb{R}_+^{n_1+n_2}$, $y_1 \in \mathbb{R}_+^{n_1}$, $y_2 \in \mathbb{R}_+^{n_2}$, s.t.

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} -e_1 \\ -e_2 \end{bmatrix} + \begin{bmatrix} 0 & -A \\ -B^T & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \geq 0, \; w_1^Ty_1 = 0, \; w_2^Ty_2 = 0. \qquad (2.1)$$

The vectors e_j, $j = 1,2$, are vectors of ones of appropriate length. Without loss of generality it is assumed that $A, B < 0$ in (2.1). Now to each solution of (2.1) corresponds a N.E. and visa versa. In particular, if $\hat{y} = (\hat{y}_1,\hat{y}_2)$ solves (2.1) then $\bar{x} = (\bar{x}_1,\bar{x}_2)$ with $\bar{x}_j = (e^T\hat{y}_j)^{-1}.\hat{y}_j$, $j = 1,2$, is a N.E..

Let us consider the Lemke-Howson method (see also (4)). First note that (2.1) is equivalent to the problem of finding a $y = (y_1, y_2)$ s.t.

$$F_{jk}(y) = \min \{y_{jk}, w_{jk}\} = 0,\ k = 1,\dots,n_j,\ j = 1,2. \tag{2.2}$$

L-H starts at a specific vector y satisfying $F_{11}(y) \geq 0$ while $F_{jk}(y) = 0$, $(j,k) \neq (1,1)$. Starting from y, a piecewise linear path of vectors $\tilde{y}$ is generated for which also $F_{11}(\tilde{y}) \geq 0$ and $F_{jk}(\tilde{y}) = 0$, $(j,k) \neq (1,1)$. This is continued till a $\hat{y}$ is reached at which $F_{11}(\hat{y}) = 0$. For the starting point y holds that $y_{11} = (-b_{1s})^{-1}$ (> 0), with $b_{1s} = \max_j b_{1j}$, and $y_{2s} = (-a_{rs})^{-1}$ (> 0), with $a_{rs} = \max_i a_{is}$, whereas all other components of y are zero. Observe that y solves (2.2) if r = 1. Otherwise, L-H starts by increasing y_{1r} from zero till a vector $\tilde{y}$ is reached at which $w_{2k} = 0$, for some $k \neq s$, or $\tilde{y}_{11} = 0$. In the latter case $\tilde{y}$ is a solution, otherwise L-H continues by increasing $\tilde{y}_{2k}$ from zero. In general , if $\tilde{y}_{jk}$ or w_{jk}, $(j,k) \neq (1,1)$, becomes zero then the complementary variable w_{jk} ($\tilde{y}_{jk}$) is increased from zero. The procedure stops with a solution when $\tilde{y}_{11}$ becomes 0 or w_{11} becomes 0.

The game-theoretic interpretation of L-H is not obvious because it operates on $\mathbb{R}_+^{n_1+n_2}$. To obtain an interpretation in terms of strategies we define a payoff function $z: S \to \mathbb{R}^{n_1+n_2}$ by

$$z(x) = (z_1(x), z_2(x)) \text{ with } z_1(x) = Ax_2 \text{ and } z_2(x) = B^T x_1. \tag{2.3}$$

Observe that $z_{jk}(x)$ denotes the payoff to player j if he plays his k-th pure strategy while player i, $i \neq j$, plays x_i. Stated in these terms $\bar{x}$ is a N.E. iff for all (j,k), $\bar{x}_{jk} = 0$ or $z_{jk}(\bar{x}) = \max_\ell z_{j\ell}(\bar{x})$. Furthermore, we interprete each vector $\hat{y}$ in $\mathbb{R}_+^{n_1+n_2}$ generated by L-H as corresponding to the vector x in S with $x_j = (e^T\hat{y}_j)^{-1}.\hat{y}_j$, j = 1,2. With all of this together we can view upon L-H as generating a piecewise linear path of strategy vectors x in S, starting from v with $v_{11} = v_{2s} = 1$ and $v_{jk} = 0$, $(j,k) \neq (1,1),(2,s)$, for which the following conditions hold

$$x_{11} \geq 0 \text{ and } z_{11}(x) \leq \max_{\ell} z_{1\ell}(x)$$
$$x_{jk} = 0 \text{ or } z_{jk}(x) = \max_{\ell} z_{j\ell}(x) \quad \text{if } (j,k) \neq (1,1). \tag{2.4}$$

Thus, along the path all actions except (1,1) are in equilibrium, i.e. they satisfy the conditions for a N.E.. More specifically, player 2 is in equilibrium and therefore plays optimal given the strategy of player 1. Consequently, L-H moves along the best reply set of player 2. At the start player 1 plays just his first strategy and player 2 plays his best pure reply, being pure strategy s for which $b_{1s} = \max_{\ell} b_{1\ell}$. The best pure reply of player 1 against (2,s) is the pure strategy r for which $a_{rs} = \max_{\ell} a_{\ell s}$. In case r = 1 a N.E. is found, otherwise the probability of action (1,r) is initially increased from zero.

It is rather straightforward to derive from (2.4) an algorithm which operates the same as L-H but on S instead of $\mathbb{R}_+^{n_1+n_2}$. Let us substitute (2.3) in (2.4) obtaining for the vectors x in S on the path

$$\begin{array}{llll} x_{1k} > 0 & \text{and} & A_k x_2 = \max_{\ell} A_{\ell} x_2, & (1,k) \in T_1 \\ x_{1k} = 0 & \text{and} & A_k x_2 < \max_{\ell} A_{\ell} x_2, & (1,k) \notin T_1 \\ & & & \\ x_{2k} > 0 & \text{and} & B_k^{\top} x_1 = \max_{\ell} B_{\ell}^{\top} x_1, & (2,k) \in T_2 \\ x_{2k} = 0 & \text{and} & B_k^{\top} x_1 < \max_{\ell} B_{\ell}^{\top} x_1, & (2,k) \notin T_2, \end{array} \tag{2.5}$$

where T_j denotes the set of strategies (j,k) for which $z_{jk}(x) = \max_{\ell} z_{j\ell}(x)$ while A_k ($B_k^{\top}$) is the k-th row of A ($B^{\top}$). Next we introduce slack variables for the inequalities at the right hand side of (2.5). Then we obtain that the vectors x in S on the path have to satisfy the system of linear equations given by

$$\Sigma_{(2,k)\in T_2} x_{2k} A^k + \Sigma_{(1,k)\notin T_1} \mu_{1k} e_1(k) - \beta_1 e_1 = 0$$
$$\Sigma_{(1,k)\in T_1} x_{1k} B^{k\top} + x_{11} B^{1\top} + \Sigma_{(2,k)\notin T_2} \mu_{2k} e_2(k) - \beta_2 e_2 = 0, \tag{2.6}$$

where $x_{jk} \geq 0$, $\Sigma_{(1,k)\in T_1} x_{1k} + x_{11} = 1$, $\Sigma_{(2,k)\in T_2} x_{2k} = 1$, $\mu_{jk} \geq 0$, $\beta_j = \max_k z_{jk}(x)$, $e_j(k)$ is the k-th unit vector in $\mathbb{R}^{n_j}$, j =1,2, and A^k ($B^{k\top}$)

the k-th column of A (B^T). The final system is then obtained by substituting $x_{11} = 1 - \sum_{k\neq 1} x_{1k}$ and $x_{2s} = 1 - \sum_{k\neq s} x_{2k}$. At the start $T_1 = \emptyset$, $T_2 = \{(2,s)\}$, $\beta_1 = z_{1r}(v)$, $\beta_2 = z_{2s}(v)$, and $\mu_{jk} = \beta_j - z_{jk}(x), (j,k) \notin T$, where $T = T_1 \cup T_2$. If the starting vector v is not a N.E. then since $\mu_{1r} = 0$, T_1 becomes $\{(1,r)\}$ and x_{1r} enters (2.6). If along the path x_{jk} (μ_{jk}), $(j,k) \neq (1,1)$, becomes zero, then the complementary variable μ_{jk} (x_{jk}) enters the system and T becomes $T/\{(j,k)\}$ ($T \cup \{(j,k)\}$). The algorithm stops if x_{11} becomes 0 or μ_{11} becomes 0. Thus, along the path action (1,1) is played with positive probability while it is not an optimal action for player 1, i.e., both x_{11} and μ_{11} are greater than zero. However, all other actions are only played with positive probability if they are optimal for the relevant player, i.e. $x_{jk} > 0$ iff $\mu_{jk} = 0$. If an action $(j,k) \neq (1,1)$ becomes optimal, i.e. μ_{jk} becomes 0, then the related probability x_{jk} is increased from 0. On the other hand, if a probability with which an optimal action $(j,k) \neq (1,1)$ is played becomes zero, i.e. x_{jk} becomes 0, then the corresponding action is made non-optimal (μ_{jk} is made positive). A N.E. is reached whenever also action (1,1) gets into equilibrium.

Recall that L-H starts with player 1 playing his first pure strategy whereas player 2 plays his best pure reply. Along the path action (1,1) is the only action not in equilibrium. However, the role of action (1,1) can be taken over by any action of any player. This results in at most n_1+n_2 different starting vectors, which might yield different Nash equilibria. These equilibria always have a positive index due to the features of a complementary pivoting algorithm. Because the number of possible starting vectors is very limited, L-H may fail to reach all positively indexed equilibria. In Shapley (3) an example can be found. One way to solve this problem is the use of an algorithm that can start from any vector. Such an algorithm has been given by van den Elzen and Talman (1).

We conclude this section by illustrating L-H when projected on the strategy space, i.e., we consider the algorithm operating on (2.6). Let us consider the game (n_1, n_2, A, B) with $n_1 = 2$, $n_2 = 3$, and

$$A = \begin{bmatrix} 1 & -2 & 5 \\ 3 & 2 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 4 & 3 \\ 5 & -2 & 1 \end{bmatrix}.$$

Note that these matrices contain positive elements. To apply L-H we first have to subtract a positive number from each element to make A,B < 0. Our method can be applied directly to the original matrices. The strategy space $S = S^1 \times S^2$ has been drawn in Figure 2.1. The set ABCD in the figure denotes the strategy vectors x at which $z_{11}(x) = z_{12}(x)$. It is easily verified that for such an x, $x_{22} = -1 + \frac{3}{2}x_{23}$. Above that plane $z_{11} > z_{12}$ holds and below it $z_{11} < z_{12}$. The bold piecewise linear curve connecting ((1,0),(0,1,0)) and ((0,1),(1,0,0)) denotes the best reply set of player 2, i.e. the set $\{x \in S | z_2(x_1,x_2) = \max_y z_2(x_1,y)\}$. The game has three Nash equilibria, namely, $x^* = ((0,1),(1,0,0))$, $\bar{x} = ((\frac{3}{4},\frac{1}{4}),(0,\frac{1}{5},\frac{4}{5}))$ and $\hat{x} = ((\frac{4}{9},\frac{5}{9}),(\frac{1}{3},0,\frac{2}{3}))$. The latter equilibrium is the only one with a negative index. Now the algorithm related to (2.6) starts at v = ((1,0),(0,1,0)) at which player 1 plays (1,1) while player 2 plays his best reply (2,2). The best reply of 1 against (2,2) is (1,2), and hence x_{12} is increased till $x_{12} = \frac{1}{4}$. At that point also

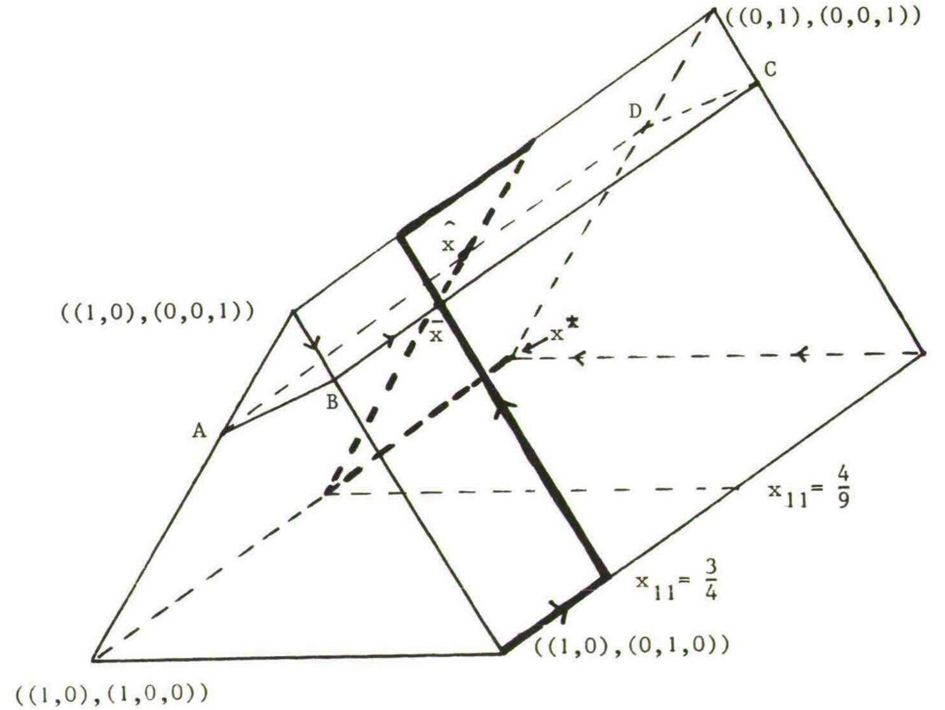

Figure 2.1. Illustration of the Lemke-Howson algorithm projected on the strategy space.

(2,3) is optimal for player 2, and x_{23} is increased from 0 till μ_{11} becomes zero. At that point the N.E. $((\frac{3}{4},\frac{1}{4}),(0,\frac{1}{5},\frac{4}{5}))$ is reached. The other possible starting vectors are ((0,1),(1,0,0)) which is a N.E., ((1,0),(0,0,1)) from which $((\frac{3}{4},\frac{1}{4}),(0,\frac{1}{5},\frac{4}{5}))$ is reached and ((0,1),(0,1,0)) from which the N.E. ((0,1),(1,0,0)) is reached.

3. Computing all Nash equilibria in a (2xn) bi-matrix game

In this section we argue that we can find all N.E. by a straightforward adaptation of the algorithm operating on system (2.6) in case one player has 2 strategies. The algorithm in Section 2 starts with player 1 playing (1,1) while player 2 plays his best reply. Via a piecewise linear path of vectors in G_2, the best reply set of player 2, a N.E. is reached. However, in case player 1 has two strategies, G_2 itself is piecewise linear. It can be parametrized by x_{11} running from 1 to 0. So, let for $0 \le x_{11} \le 1$, $G_2(x_{11}) := \{x \in S | x_{12} = 1 - x_{11},\ x_2 = \arg\max_y z_2(x_1,y)\}$. Then the algorithm in Section 2 generates $G_2(x_{11})$ starting from $x_{11} = 1$ till a N.E. is found. Since the N.E. have to lie on G_2, we can find all of them by tracing $G_2(x_{11})$ from $x_{11} = 1$ to $x_{11} = 0$. Hence we must extend the algorithm of Section 2 in such a way that it continues after having found a N.E. $\hat{x}$ at which $\hat{x}_{11} > 0$. If the starting vector v is a N.E. we continue by increasing x_{12} from zero. By doing so, action (1,2) fulfils the role of (1,1) and a N.E. is reached when μ_{12} becomes zero. In general, if μ_{11} (μ_{12}) becomes zero while $x_{11} > 0$ then a N.E. is reached and the algorithm continues by increasing μ_{12} (μ_{11}) from zero. The extended algorithm stops if x_{11} becomes 0. Of course, that point might be also a N.E..

For illustration again consider Figure 2.1. The algorithm starts at ((1,0),(0,1,0)) and follows G_2 till μ_{11} becomes zero at the N.E. $\bar{x} = ((\frac{3}{4},\frac{1}{4}),(0,\frac{1}{5},\frac{4}{5}))$. Then the algorithm continues by increasing μ_{12} and it generates vectors x at which $z_{11}(x) > z_{12}(x)$, i.e. it continues from $\bar{x}$ upwards from ABCD. In this way G_2 is followed till the N.E. $\hat{x} = ((\frac{4}{9},\frac{5}{9}),(\frac{1}{3},0,\frac{2}{3}))$ is reached at which $\mu_{12} = 0$. Then the algorithm continues by increasing μ_{11}. It now moves from $\hat{x}$ downwards from ABCD and finally terminates at the third N.E. $x^* = ((0,1),(1,0,0))$.

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