An upper and a lower bound for the distance of a manifold to a nearby point

Paardekooper, M.H.C.

Publication date:
1988

TILBURG UNIVERSITY
KATHOLIEKE UNIVERSITEIT BRABANT
POSTBOX 90153
5000 LE TILBURG
THE NETHERLANDS

[redacted]

DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

AN UPPER AND A LOWER BOUND FOR THE DISTANCE OF A MANIFOLD TO A NEARBY POINT

M.H.C. Paardekooper

FEW 321

AN UPPER AND A LOWER BOUND FOR THE DISTANCE OF A MANIFOLD TO A NEARBY POINT

M.H.C. Paardekooper

Department of Econometrics
Tilburg University
Tilburg, The Netherlands

This research is part of the VF-program "Econometrie van Micro-economische gedragsmodellen", which has been approved by the Netherlands Ministry of Education and Sciences.

ABSTRACT

A generalization for underdetermined systems of the wellknown Newton-Kantorovich theorem gives bounds for the distance of a point, say 0, in Hilbert space X to a nearby manifold $S = \{x \in X \mid f(x) = 0\}$. Here $f: X \to Y$ is a differentiable mapping such that $Df(0)$ is surjective; $f(0)$, Df together with right inverse $Df(0)^+$ satisfies typical Kantorovich-like conditions. Analysis in the normal space at 0 of $\tilde{S} = \{x \in X \mid f(x) = f(0)\}$ gives an upperbound of $d(0,S)$. Furthermore the Kantorovich conditions effect S to be locally in a convex cone. The distance of 0 to that cone gives a lowerbound of $d(0,S)$.

1. INTRODUCTION

The purpose of this paper is to derive bounds, both upper bounds as lower bounds, for the distance of a manifold to a nearby point.
The starting point and main tool in the construction of the bounds is the classical Newton-Kantorovich theorem.

THEOREM 0.[2]. Let Y,Z be Banach spaces. Let B(0,r) be an open ball in Banach space Z and let be $\varphi: B(0,r) \subset Z \to Y$ Frechet differentiable on B(0,r) with

$$\|D\varphi(x) - D\varphi(y)\| \leqq L \|x - y\|,\ x,y \in B(0,r). \tag{1.1}$$

Assume that $D\varphi(0)^{-1} \in \mathcal{L}(Z,Y)$ exists,

$$\|D\varphi(0)^{-1}\| \leqq \lambda^{-1},\ \|D\varphi(0)^{-1}\varphi(0)\| = \tilde{\gamma} \leqq \gamma,\ \kappa := L\gamma\lambda^{-1} < \tfrac{1}{2}$$

and

$$M := \lambda(1-\sqrt{1-2\kappa})/L < r. \tag{1.2}$$

Then the equation $\varphi(x) = 0$ has a solution $z \in B(0,M) \subset Z$ and z is a unique zero of φ in $B(0,\rho_1) \subset Z$, where

$$\rho_1 = \lambda(1+\sqrt{1-2\kappa})/L$$ □

For the formulation of the distance problem in section two we consider the Hilbert spaces X and Y and we investigate $f: B(0,r) \subset X \to Y$, a Frechet differential mapping. We assume that

(i) $A := Df(0) \in \mathcal{L}(X,Y)$ is surjective and $\|A^+\| \leqq \lambda^{-1}$, where $A^+ \in \mathcal{L}(X,Y)$ is the right inverse of A,

(ii) $\|Df(x) - Df(y)\| \leqq L\|x-y\|,\ x,y \in B(0,r),$ (1.4)

(iii) $\|A^+ f(0)\| = \tilde{\gamma} \leq \gamma,$ (1.5)

(iv) $\kappa = L\gamma\lambda^{-1} < \tfrac{1}{2},$ (1.6) and

(v) $M := \lambda(1-\sqrt{1-2\kappa})/L < r.$ (1.7)

We use the following notation:

$$S = \{x \in B(0,r) \mid f(x) = 0\} \tag{1.8}$$

and

$$\tilde{S} = \{x \in B(0,r) \mid f(x) = f(0)\}. \tag{1.9}$$

N_1 denotes Ker(A) being the tangent space of $\tilde{S}$ in 0. Let be N_2 the orthogonal complement of N_1.

Analysis in the normal space N_2 at 0 of $\tilde{S}$ leads to an upperbound of d(0,S), theorem 1. The Kantorovich condition (1.6) effects S to be locally in a convex cone. Theorem 2 gives the distance of 0 to that cone as a lower bound of d(0,S).

This general approach leads to a manageable method to determine sharp error bounds for an approximate solution of an undetermined system.

2. BOUNDS FOR d(0,S)

THEOREM 1. The mapping $f\colon B(0,r) \subset X \to Y$ described in the introduction has a zero z in $B(0,M) \cap N_2$; z is the unique zero of $\varphi = f|N_2$ in $B(0,\rho_1) \cap N_2$ where

$$\rho_1 = \lambda(1 + \sqrt{1-2\kappa})/L. \tag{2.1}$$

PROOF. The surjectivity of A implies that the restriction $A|N_2$ is bijective. Its inverse is also continuous and equals the right inverse $A^+ = A^*(AA^*)^{-1}$ of A [1].
Let be $\varphi = f|N_2$. This mapping $\varphi\colon N_2 \cap B(0,r) \to Y$ satisfies the conditions of the Newton-Kantorovich theorem 0 formulated above. Hence $\varphi(x) = 0$ has a solution z in $B(0,M) \cap N_2$ and z is the unique zero of φ in $B(0,p_1) \cap N_2$. □

LEMMA 1. For the zero z of $\varphi = f|N_2$ in $B(0,M) \cap N_2$ holds

$$\beta := \|z\| \geqq \rho_2 \,, \tag{2.2}$$

where

$$\rho_2 = \lambda(-1 + \sqrt{1+2\tilde{\kappa}})/L, \tag{2.3}$$

with $\tilde{\kappa} = L\tilde{\gamma}\lambda^{-1}$.

PROOF. Since Df is Lipschitz continuous on $B(0,r)$ we have $\|f(z) - f(0) - Az\| \leqq \frac{1}{2} L\, \beta^2$ and consequently [3]

$$\tilde{\gamma} = \|A^+f(0)\| = \|z + A^+(f(z) - f(0) - Az)\| \leqq \beta + \tfrac{1}{2} L\, B^2\lambda^{-1},$$

for $z = A^+Az$ as follows from $z \in R(A^*)$.
Hence the positive zero ρ_2 of the quadratic function $t \to \frac{1}{2} L\, \lambda^{-1} t^2 + t - \tilde{\gamma}$ is majorized by β. That proves (2.3). □

REMARK. As a consequence of (1.7) we get $\beta < M < 2\gamma$. Hence

$$L\beta < 2L\gamma < \lambda. \quad \square \tag{2.4}$$

In the sequel we use the following notation

$$V = \{x \in X \mid |x| < \beta\},\ P(x) := Df(x)|N_1,\ Q(x) := Df(x)|N_2,\ x \in V, \tag{2.5}$$

$$\alpha = \frac{L\,\beta}{\lambda - L\,\beta}, \tag{2.6}$$

where, as above $\beta = |z|$.

LEMMA 2. $Q(x)$ is regular for $x \in V$ and

$$|Q(x)^{-1}\,P(x)| \leqq \alpha, \quad x \in V. \tag{2.7}$$

PROOF. Let be $x \in V$ and $y = y_1 + y_2$, $y_i \in N_i$, $i = 1,2$. Then $Df(x)y = P(x)y_1 + Q(x)y_2$. Since $P(0) = 0$, $|Px| \le |Df(x) - Df(0)| \leqq L\beta$, $x \in V$. Similarly $|Q(x) - Q(0)| \leqq |Df(x) - Df(0)| \leqq L\beta$, $x \in V$. Since $Q(0) = D\varphi(0)$,

$$|(Q(x) - Q(0))\,Q(0)^{-1}| \leqq L\,\beta\lambda^{-1} < 1$$

as follows from $\beta < M$ and (2.4). This implies that

$$Q(x) = (I + (Q(x) - Q(0))\,Q(0)^{-1})Q(0)$$

is invertible for each $x \in V$ and

$$|Q(x)^{-1}| \leqq |Q(0)^{-1}|\,(1 - L\beta\,\lambda^{-1})^{-1} \leqq (\lambda - L\beta)^{-1}.$$

Hence $|Q(x)^{-1}\,P(x)| \leqq L\beta(\lambda - L\beta)^{-1} = \alpha$. □

For reasons of shortness we define

$$W = \{x = x_1 + x_2 \in X \mid \alpha\,|x_1| + |x_2| < \beta, x_i \in N_i,\ i = 1,2\} \tag{2.8}$$

and with $w = w_1 + w_2 \in X$, $w_i \in N_i$, $i = 1,2$ (2.9)

$$K(w) = \{(1-\tau)w_1 + x_2 \mid \tau \in [0,1],\ x_2 \in N_2,\ |x_2 - w_2| \leqq \alpha\,|w_1|\,\tau\}.$$

The lines along which the proof of theorem 2 will be given can be explained with a figure.

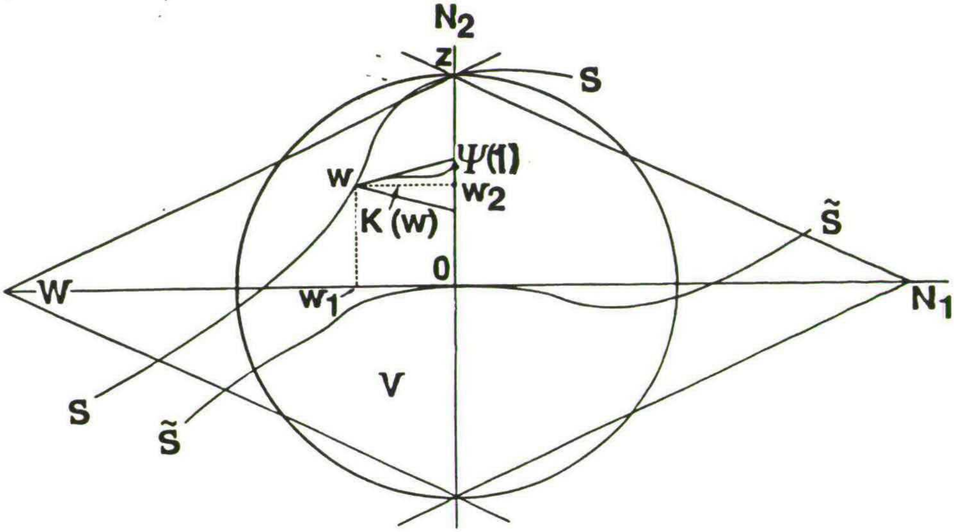

fig. 1. $w \in S \cap W \cap V$ contradicts $S \cap V \cap N_2 = \emptyset$.

In lemma 3 we prove that $w \in W \cap V$ implies $K(w) \subset W \cap V$. In lemma 4 indirectly we prove that $w \in W \cap V$ implies $f(w) \neq 0$. So $S \subset V^c \cup W^c$ and $d(0,S) \geq d(0,W^c)$. In this manner we get $m := d(0,W^c)$ as an upper bound for the distance of 0 to the manifold S.

LEMMA 3. If $w = w_1 + w_2 \in W \cap V$ then $K(w) \subset V \cap W$.

PROOF. Let be $x = x_1 + x_2 \in K(w)$. Then there exist $\varepsilon,\ \tau \in [0,1]$ and a unit vector $v \in N_2$ such that $x_1 = (1-\tau)w_1$ and $x_2 = w_2 + \varepsilon\alpha\, |w_1|\, \tau v$. Thus

$$|x|^2 = (1-\tau)^2|w_1|^2 + |w_2 + \varepsilon\alpha|w_1|\tau v|^2 \leq (1-\tau)^2|w_1|^2 + (|w_2| + \alpha\tau|w_1|)^2 = g(\tau).$$

Now $g(0) = |w|^2 < \beta^2$ and $g(1) = (|w_2| + \alpha|w_1|)^2 < \beta^2$ for $w \in V$ and $w \in W$ respectively. So $x \in V$.
Similarly we have $\alpha|x_1| + |x_2| \leq \alpha|w_1|(1-(1-\varepsilon)\tau) + |w_2| < \beta$. Thus also $x \in W$. □

LEMMA 4. The function f has no zero in $W \cap V$.

PROOF. Assume $w = w_1 + w_2 \in W \cap V$, $w_i \in N_i$, $i = 1,2$ and $f(w) = 0$.
Define a function G as follows

$$(\tau,x_2) \to G(\tau,x_2) = f((1-\tau)w_1+x_2)\ ,\ x_2 \in N_2,\ (1-\tau)^2\ |w_1|^2 + |x_2|^2 < r^2.$$

Then $G(0,w_2) = 0$ and the derivative $D_2G(0,w_2)$ of G in $(0,w_2)$ with respect to x_2 equals $Q(w)$. By lemma two $Q(w)$ is regular. According to the implicit function theorem there exists a $\delta>0$ and a differentiable function $\psi:(-\delta,\delta)\to N_2 \cap V$ such that $\psi(0) = w_2$ and for $\tau \in (-\delta,\delta)$ holds

$$G(\tau,\psi(\tau)) = 0, \; D\psi(\tau) = -D_2G(\tau,\psi(\tau))^{-1} D_1G(\tau,\psi(\tau))$$

where D_1 and D_2 denote differentiation with respect to τ and x_2 respectively. Since

$$D_1G(\tau,x_2) = -P((1-\tau)w_1+x_2)w_1, \; D_2G(\tau,x_2) = Q((1-\tau)w_1+x_2)$$

we have

$$\|D\psi(\tau)\| \leqq \| Q((1-\tau)w_1 + \psi(\tau))^{-1} P((1-\tau)w_1 + \psi(\tau))\| \, \|w_1\| \leqq \alpha \|w_1\|, \; |\tau| < \delta,$$

as follows from lemma 2. Consequently

$$\|\psi(\tau) - w_2\| = \| \int_0^\tau D\psi(\sigma)d\sigma\| \leqq \alpha \|w_1\|\tau, \; 0 < \tau < \delta.$$

Hence $(1-\tau)w_1 + \psi(\tau) \in K(w)$ if $\tau \in [0,\delta) \subset [0,1]$. If $\tau_1, \tau_2 \in [0,\delta)$ then $\|\psi(\tau_1) - \psi(\tau_2)\| \leqq \alpha \|w_1\| \, |\tau_1-\tau_2|$ which implies, by the Cauchy criterion, that $\tilde{w} = \lim_{\tau\uparrow\delta}((1-\tau)w_1+\psi(\tau))$ exists in the closed $K(w)$ and thus $\tilde{w} \in V \cap W$ as follows from lemma 3. So the function ψ can be prolonged and extended until τ equals 1, i.e. $G(1,\psi(1)) = 0$. Thus $f(\psi(1) = 0$. That means $\varphi(\psi(1)) = 0$, with $\psi(1) \in V \cap N_2$ and $\varphi = f|N_2$. This contradicts theorem 1. Hence $w \in W \cap V$ implies $f(w) \neq 0$. □

THEOREM 2. Let $f: B(0,r) \subset X \to Y$ satisfy the conditions given in the introduction and let be

$$g(\tau):= \tau(\lambda/L-\tau)(\tau^2+(\lambda/L-\tau)^2)^{-\frac{1}{2}} \quad , \; 0 < \tau < \frac{\lambda}{L} . \tag{2.10}$$

Then

$$d(0,S) \geqq m:= \begin{cases} g(M) & ,\frac{1}{4}\sqrt{3} \leqq \kappa < \frac{1}{2} \text{ and } \sqrt{1-2} + \frac{1}{2}(1-2\kappa) \leqq \tilde{\kappa} \leqq \kappa \\ g(\rho_2) & ,0 < \kappa < \frac{1}{2} \text{ and } \tilde{\kappa} \leqq \min\{\kappa,\sqrt{1-2\kappa}+\frac{1}{2}(1-2\kappa)\} \end{cases} \tag{2.11}$$

where M and ρ_2 as given in (1.7) and (2.3) respectively.

PROOF. With simple computations we find

$$d(0,W^c) = \beta(1+\alpha^2)^{-\frac{1}{2}} = g(\beta) < \beta,$$

where α and β are given in (2.6) and (2.2) respectively. So

$$d(0,S) \geqq d(0,W^c \cup V^c) = g(\beta).$$

It is easy to see that $\tau = \frac{1}{2}\lambda/L$ is the axis of symmetry of the graph of g. The function g increases on $(0, \frac{1}{2}\lambda/L]$ from 0 until its maximum $\frac{1}{4}\lambda\sqrt{2}/L$ and decreases on $[\frac{1}{2}\lambda/L, \lambda/L)$ to zero. Since $\rho_2 \leqq \beta \leqq M$ as we know from theorem 1 and lemma 1,

$$d(0,S) \geqq m := \min\{g(\rho_2), g(M)\}.$$

The symmetry of g implies that

$$m = g(M) \Leftrightarrow M \geqq \tfrac{1}{2}\lambda/L \text{ and } \rho_2 \geqq \lambda/L - M \tag{2.12}$$

and

$$m = g(\rho_2) \Leftrightarrow M < \tfrac{1}{2}\lambda/L \text{ or } (M \geqq \tfrac{1}{2}\lambda/L \text{ and } \rho_2 < \lambda/L-M). \tag{2.13}$$

With (1.7) we find $M \geqq \frac{1}{2}\lambda/L$ iff $\kappa \geqq \frac{3}{8}$ and with (2.3) we get that $\rho_2 \geqq \lambda/L - M$ iff $\sqrt{1-2\kappa} + \frac{1}{2}(1-2\kappa) \leqq \tilde{\kappa} \leqq k$. Since $\kappa \geq \sqrt{1-2\kappa} + \frac{1}{2}(1-\kappa)$ for $\kappa \geq \frac{1}{4}\sqrt{3}$, (2.11) can be concluded. □

COROLLARY. If $\gamma = \tilde{\gamma}$, i.e. $\kappa = \tilde{\kappa}$, then

$$m = d(0,S) \geqq \begin{cases} g(M) & , \frac{1}{4}\sqrt{3} \leqq \kappa < \frac{1}{2} \\ g(\rho_2) & , \kappa < \frac{1}{4}\sqrt{3} \end{cases} \tag{2.14}$$

PROOF. The two conditions (2.12) and (2.14) lead to the two cases of (2.14) with the same means as in the theorem. □

3. REFERENCES

1. Aubin, J.B.; Applied Functional Analysis, Wiley, New York, xv + 423 pp., 1979.

2. Gragg, W.B. and R.A. Tapia; Optimal Error Bounds for the Newton-Kantorovich Theorem, SIAM J. Numer.Anal., 17, 1980, pp. 883-893.

3. Tapia, R.A.; Differentiation and Integration of Nonlinear Operators, in Nonlinear Functional Analysis and Applications, L.B. Rall, ed., Academic Press, New York, 1971, pp. 45-103.

IN 1987 REEDS VERSCHENEN

242 Gerard van den Berg
Nonstationarity in job search theory

243 Annie Cuyt, Brigitte Verdonk
Block-tridiagonal linear systems and branched continued fractions

244 J.C. de Vos, W. Vervaat
Local Times of Bernoulli Walk

245 Arie Kapteyn, Peter Kooreman, Rob Willemse
Some methodological issues in the implementation of subjective poverty definitions

246 J.P.C. Kleijnen, J. Kriens, M.C.H.M. Lafleur, J.H.F. Pardoel
Sampling for Quality Inspection and Correction: AOQL Performance Criteria

247 D.B.J. Schouten
Algemene theorie van de internationale conjuncturele en strukturele afhankelijkheden

248 F.C. Bussemaker, W.H. Haemers, J.J. Seidel, E. Spence
On (v,k,λ) graphs and designs with trivial automorphism group

249 Peter M. Kort
The Influence of a Stochastic Environment on the Firm's Optimal Dynamic Investment Policy

250 R.H.J.M. Gradus
Preliminary version
The reaction of the firm on governmental policy: a game-theoretical approach

251 J.G. de Gooijer, R.M.J. Heuts
Higher order moments of bilinear time series processes with symmetrically distributed errors

252 P.H. Stevers, P.A.M. Versteijne
Evaluatie van marketing-activiteiten

253 H.P.A. Mulders, A.J. van Reeken
DATAAL - een hulpmiddel voor onderhoud van gegevensverzamelingen

254 P. Kooreman, A. Kapteyn
On the identifiability of household production functions with joint products: A comment

255 B. van Riel
Was er een profit-squeeze in de Nederlandse industrie?

256 R.P. Gilles
Economies with coalitional structures and core-like equilibrium concepts

257 P.H.M. Ruys, G. van der Laan
Computation of an industrial equilibrium

258 W.H. Haemers, A.E. Brouwer
Association schemes

259 G.J.M. van den Boom
Some modifications and applications of Rubinstein's perfect equilibrium model of bargaining

260 A.W.A. Boot, A.V. Thakor, G.F. Udell
Competition, Risk Neutrality and Loan Commitments

261 A.W.A. Boot, A.V. Thakor, G.F. Udell
Collateral and Borrower Risk

262 A. Kapteyn, I. Woittiez
Preference Interdependence and Habit Formation in Family Labor Supply

263 B. Bettonvil
A formal description of discrete event dynamic systems including perturbation analysis

264 Sylvester C.W. Eijffinger
A monthly model for the monetary policy in the Netherlands

265 F. van der Ploeg, A.J. de Zeeuw
Conflict over arms accumulation in market and command economies

266 F. van der Ploeg, A.J. de Zeeuw
Perfect equilibrium in a model of competitive arms accumulation

267 Aart de Zeeuw
Inflation and reputation: comment

268 A.J. de Zeeuw, F. van der Ploeg
Difference games and policy evaluation: a conceptual framework

269 Frederick van der Ploeg
Rationing in open economy and dynamic macroeconomics: a survey

270 G. van der Laan and A.J.J. Talman
Computing economic equilibria by variable dimension algorithms: state of the art

271 C.A.J.M. Dirven and A.J.J. Talman
A simplicial algorithm for finding equilibria in economies with linear production technologies

272 Th.E. Nijman and F.C. Palm
Consistent estimation of regression models with incompletely observed exogenous variables

273 Th.E. Nijman and F.C. Palm
Predictive accuracy gain from disaggregate sampling in arima - models

274 Raymond H.J.M. Gradus
The net present value of governmental policy: a possible way to find the Stackelberg solutions

275 Jack P.C. Kleijnen
A DSS for production planning: a case study including simulation and optimization

276 A.M.H. Gerards
A short proof of Tutte's characterization of totally unimodular matrices

277 Th. van de Klundert and F. van der Ploeg
Wage rigidity and capital mobility in an optimizing model of a small open economy

278 Peter M. Kort
The net present value in dynamic models of the firm

279 Th. van de Klundert
A Macroeconomic Two-Country Model with Price-Discriminating Monopolists

280 Arnoud Boot and Anjan V. Thakor
Dynamic equilibrium in a competitive credit market: intertemporal contracting as insurance against rationing

281 Arnoud Boot and Anjan V. Thakor
Appendix: "Dynamic equilibrium in a competitive credit market: intertemporal contracting as insurance against rationing

282 Arnoud Boot, Anjan V. Thakor and Gregory F. Udell
Credible commitments, contract enforcement problems and banks: intermediation as credibility assurance

283 Eduard Ponds
Wage bargaining and business cycles a Goodwin-Nash model

284 Prof.Dr. hab. Stefan Mynarski
The mechanism of restoring equilibrium and stability in polish market

285 P. Meulendijks
An exercise in welfare economics (II)

286 S. Jørgensen, P.M. Kort, G.J.C.Th. van Schijndel
Optimal investment, financing and dividends: a Stackelberg differential game

287 E. Nijssen, W. Reijnders
Privatisering en commercialisering; een oriëntatie ten aanzien van verzelfstandiging

288 C.B. Mulder
Inefficiency of automatically linking unemployment benefits to private sector wage rates

289 M.H.C. Paardekooper
A Quadratically convergent parallel Jacobi process for almost diagonal matrices with distinct eigenvalues

290 Pieter H.M. Ruys
Industries with private and public enterprises

291 J.J.A. Moors & J.C. van Houwelingen
Estimation of linear models with inequality restrictions

292 Arthur van Soest, Peter Kooreman
Vakantiebestemming en -bestedingen

293 Rob Alessie, Raymond Gradus, Bertrand Melenberg
The problem of not observing small expenditures in a consumer expenditure survey

294 F. Boekema, L. Oerlemans, A.J. Hendriks
Kansrijkheid en economische potentie: Top-down en bottom-up analyses

295 Rob Alessie, Bertrand Melenberg, Guglielmo Weber
Consumption, Leisure and Earnings-Related Liquidity Constraints: A Note

296 Arthur van Soest, Peter Kooreman
Estimation of the indirect translog demand system with binding non-negativity constraints

IN 1988 REEDS VERSCHENEN

297 Bert Bettonvil
Factor screening by sequential bifurcation

298 Robert P. Gilles
On perfect competition in an economy with a coalitional structure

299 Willem Selen, Ruud M. Heuts
Capacitated Lot-Size Production Planning in Process Industry

300 J. Kriens, J.Th. van Lieshout
Notes on the Markowitz portfolio selection method

301 Bert Bettonvil, Jack P.C. Kleijnen
Measurement scales and resolution IV designs: a note

302 Theo Nijman, Marno Verbeek
Estimation of time dependent parameters in lineair models using cross sections, panels or both

303 Raymond H.J.M. Gradus
A differential game between government and firms: a non-cooperative approach

304 Leo W.G. Strijbosch, Ronald J.M.M. Does
Comparison of bias-reducing methods for estimating the parameter in dilution series

305 Drs. W.J. Reijnders, Drs. W.F. Verstappen
Strategische bespiegelingen betreffende het Nederlandse kwaliteits-concept

306 J.P.C. Kleijnen, J. Kriens, H. Timmermans and H. Van den Wildenberg
Regression sampling in statistical auditing

307 Isolde Woittiez, Arie Kapteyn
A Model of Job Choice, Labour Supply and Wages

308 Jack P.C. Kleijnen
Simulation and optimization in production planning: A case study

309 Robert P. Gilles and Pieter H.M. Ruys
Relational constraints in coalition formation

310 Drs. H. Leo Theuns
Determinanten van de vraag naar vakantiereizen: een verkenning van materiële en immateriële factoren

311 Peter M. Kort
Dynamic Firm Behaviour within an Uncertain Environment

312 J.P.C. Blanc
A numerical approach to cyclic-service queueing models

313 Drs. N.J. de Beer, Drs. A.M. van Nunen, Drs. M.O. Nijkamp
Does Morkmon Matter?

314 Th. van de Klundert
Wage differentials and employment in a two-sector model with a dual labour market

315 Aart de Zeeuw, Fons Groot, Cees Withagen
On Credible Optimal Tax Rate Policies

316 Christian B. Mulder
Wage moderating effects of corporatism
Decentralized versus centralized wage setting in a union, firm, government context

317 Jörg Glombowski, Michael Krüger
A short-period Goodwin growth cycle

318 Theo Nijman, Marno Verbeek, Arthur van Soest
The optimal design of rotating panels in a simple analysis of variance model

319 Drs. S.V. Hannema, Drs. P.A.M. Versteijne
De toepassing en toekomst van public private partnership's bij de grote en middelgrote Nederlandse gemeenten

320 Th. van de Klundert
Wage Rigidity, Capital Accumulation and Unemployment in a Small Open Economy