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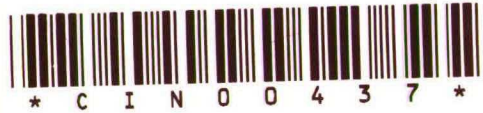
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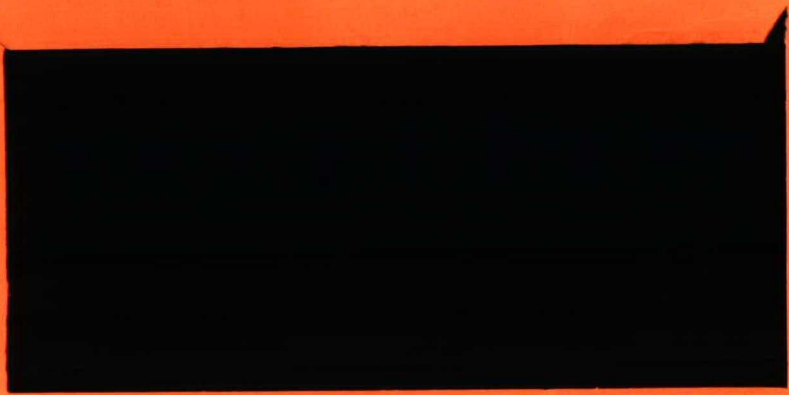
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INTERDEPENDENT PREFERENCES:

AN ECONOMETRIC ANALYSIS

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INTERDEPENDENT PREFERENCES: AN ECONOMETRIC ANALYSIS

Abstract

The theoretical model of Gaertner (1974) and Pollak (1976) to analyze
interdependence of preferences in the Linear Expenditure System is esti-
mated for a cross-section of households. The interdependence of consumption
of different households has implications for the stochastic structure of
the model and for the identifiability of its parameters. Both aspects are
dealth with.

The empirical results indicate a significant role played by the interde-
pendence of preferences. Consequently, individual income changes and aggre-
gate income changes have markedly different effects on the consumption of
different goods. Especially the consumption of conspicuous goods responds
more strongly to changes in aggregate income than to changes in individual
income.

1.Introduction

In his pioneering study, Duesenberry (1949) gave several kinds of evidence based on aggregate data to indicate the importance of preference interdependence for the explanation of consumer behavior. At about the same time Leibenstein (1950) extensively discussed various types of interdependencies in consumption behavior of individuals. Of course, these two authors were not the first ones to discuss preference interdependence. Leibenstein notes, for example, that the notion of 'conspicuous consumption' can be traced back as far as the works of Horace. Since the time the papers by Duesenberry and Leibenstein were published, some further work has been done on what has been called alternatively variable preferences, endogenous preferences, or interdependent preferences. In Kapteyn et al. (1980) we have given a brief review of most of this literature.

It seems fair to say, however, that mainstream economics takes preferences as constant. This applies in particular to empirical work within the systems apporoach to consumer demand, with the exception of some attempts to incorporate habit formation (e.g. Houthakker and Taylor, 1970, Phlips, 1972 and 1974, Manser, 1976, Darrough, Pollak and Wales, 1983, Blanciforti and Green, 1983).

In two rather closely related papers Gaertner (1974) and Pollak (1976) have studied some theoretical implications of the incorporation of preference interdependence in the Linear Expenditure System (LES). Darrough, Pollak and Wales (1983) estimate a Quadratic Expenditure System for three separate time series (one British and two Japanese) of grouped household budget data. They also consider specifications where some parameters depend on lagged consumption. Since the data are grouped according to income-demographic cells, one may interpret the dependence of parameters on lagged consumption (i.e. the consumption of other people in the same cell, one period ago) as representing interdependent preferences. The authors find the specification with lagged consumption included to be empirically superior to static versions of their model.

Thus we have two theoretical papers within the systems approach that deal with preference interdependence and one empirical paper that can be interpreted as supporting the notion of interdependent preferences (but a habit formation interpretation is possible as well). In our paper we follow the lead of Gaertner and Pollak, but focus entirely on the econometric and empirical aspects of preference interdependence in the LES. The choice of the LES as our framework of analysis is mainly motivated by a desire for simplicity in this pioneering stage. Future work should extend to other systems.

The paper is organized as follows. Section 2 presents the LES with interdependence incorporated. Section 3 and appendix A concentrate on the stochastic assumptions required to render the model amenable to estimation on the basis of a cross-section. In section 4 we consider issues of identification. Section 5 contains the results of estimating the model for a household expenditure survey in The Netherlands. Section 6 concludes with some qualifications, and points at future research.

2. The deterministic part of the model

Our starting point is the Linear Expenditure System (LES):

$$x_{gn} = b_{gn} + \gamma_g (y_n - \sum_{h=1}^{G} b_{hn}) , \tag{2.1}$$

where the index n, n=1,...,N, indicates the N consumers (or households) in society;[1] the index g, g=1,...,G, indicates goods; x_{gn} denotes the quantity of good g consumed by individual n; γ_g, with $\sum_g \gamma_g = 1$, and b_{gn} are parameters. The system (2.1) arises from the maximization of the utility function

$$U_n(x_1,...,x_G) = \sum_{g=1}^{G} \gamma_g \log(x_g - b_{gn}) \tag{2.2}$$

subject to the budget constraint

$$\sum_{g=1}^{G} x_g = y_n , \qquad (2.3)$$

with y_n income (or total expenditures) of household n. [2]

We incorporate interdependence of preferences by expressing the parameters b_{gn} as a function of consumption by others:

$$b_{gn} = b_{g0} + \beta_g \sum_{k=1}^{N} w_{nk} x_{gk} , \qquad w_{nn}=0, \ w_{nk} \geqslant 0, \ \sum_k w_{nk} = 1, \ 0 \leqslant \beta_g < 1 , \qquad (2.4)$$

with b_{g0} a good-specific intercept and β_g a good-specific coefficient, and the w_{nk} a set of <u>reference weights</u>, representing the importance attached by consumer n to consumer k's expenditures. Intuitively, β_g measures the <u>conspicuousness</u> of good g. The higher β_g is, the more one's consumption of good g is influenced by the consumption of others. The expression $\sum_{k=1}^{N} w_{nk} x_{gk}$ represents mean expenditures on good g in the <u>reference group</u> of consumer n, where the reference group of individual n is defined as the set of individuals k for whom $w_{nk} > 0$.

The model is closely related to the ones analyzed by Gaertner (1974) and Pollak (1976). Both authors mainly consider dynamic specifications in which the x_{gk} on the right hand side of (2.4) are lagged one period. Gaertner also specifies a relation for γ_g, where an individual's γ_g depends on relative changes in his permanent income. Furthermore, he considers various specifications in which the reference weights depend on consumption patterns of individuals. Both Gaertner and Pollak allow the reference weights to vary according to goods and also to be non-zero for k=n. Since for our empirical work we only have cross-section data available, a dynamic specification is ruled out. Assuming that the weight w_{nn} an individual gives to his own consumption is the same for everyone, it is impossible in a cross-section to distinguish empirically between $w_{nn}=0$ or $w_{nn} \neq 0$. So, taking $w_{nn}=0$ ('no habit formation') does not entail loss of generality (although the interpretation of parameter estimates depends on it).

Also, we do not follow Gaertner's lead to specify a model for γ_g and the w_{nk}, and in contrast to both Gaertner and Pollak the reference weights w_{nk} are assumed identical across commodities up to a constant of proportionality. These are major simplifications, inspired by our wish to have a model that can be estimated empirically.

In a different respect we generalize the models of Gaertner and Pollak somewhat by incorporating the effect of household size. This is done in the following simple way. Let f_n be the size of household n, however defined. It is assumed that the household's committed expenditures on good g increase with $\mu_g f_n$, where $\mu_1, \ldots, \mu_G$ are parameters. This corresponds to 'translating' as defined by Pollak and Wales (1981). Combining preference interdependence with translating leads to the following adaptation of the basic model. Let $\tilde{x}_{gn}$ be defined as

$$\tilde{x}_{gn} \equiv x_{gn} - \mu_g f_n \, , \tag{2.5}$$

then we replace (2.4) by

$$b_{gn} = b_{g0} + \mu_g f_n + \beta_g \sum_{k=1}^{N} w_{nk} \tilde{x}_{gk} \, . \tag{2.6}$$

Notice that (2.6) reduces to $b_{gn} = b_{g0} + \mu_g f_n$ in either of two cases: $\beta_g = 0$ or all $\tilde{x}_{gk} = 0$. We may call $b_{g0} + \mu_g f_n$ the __basic needs__ of household n, because it it represents committed expenditures if the household does not refer to other households at all, or if all other households are just able to satisfy their own basic needs. It is only the excess of other households' expenditures on good g over their basic needs which raises committed expenditures.[3]

It is worth noticing that the b_{gn} are often interpreted as subsistence levels, so that (2.6) implies that subsistence levels are subject to social influences. In this connection it is of interest to mention some pieces of evidence collected by Smolensky (1965), Ornati (1966), and Mack (published in Miller, 1965, and quoted by Kilpatrick, 1973). In various budget studies, from 1903 till 1960, experts have estimated minimum subsistence levels for the U.S. It turns out that the regressions of the log of these

subsistence levels on the log of real disposable income per capita in the same year yields elasticities between 0.57 and 0.84. This suggests strongly that, indeed, subsistence levels are subject to social influences.

3. Stochastic Specification

Combining (2.1), (2.5) and (2.6) and adding an i.i.d. disturbance term, ε_{gn}, representing all effects on x_{gn} not captured by the systematic part of the model, yields

$$x_{gn} = b_{g0} + \mu_g(f_n - \beta_g \sum_{k=1}^{N} w_{nk}f_k) + \beta_g \sum_{k=1}^{N} w_{nk}x_{gk} +$$

$$+ \gamma_g[y_n - \sum_{h=1}^{G} \beta_h \sum_{k=1}^{N} w_{nk}x_{hk}] + \gamma_g[\sum_{h=1}^{G} \beta_h\mu_h \sum_{k=1}^{N} w_{nk}f_k -$$

$$- \sum_{h=1}^{G} b_{h0} - \tilde{\mu}f_n] + \varepsilon_{gn} , \qquad (3.1)$$

where $\tilde{\mu} \equiv \sum_{g=1}^{G} \mu_g$. Thus, the model relates expenditures x_{gn} to own income and family size (y_n and f_n) and expenditures and family size of others (x_{gk} and f_k) through a linear model with parameters b_{g0}, β_g, γ_g, μ_g and w_{nk}. The main problem in estimating the model is of course created by the large number of reference weights w_{nk}. A related problem is the simultaneity in the system caused by the presence of the x_{gk} on both the left and the right hand side.

In earlier work, in a different context, we have adopted the following approach to the estimation of the reference weights w_{nk}. It is intuitively plausible that consumers with a given set of personal characteristics (education, job, age, etc.) will on average attach a higher weight to expenditures of consumers sharing the same characteristics, than to those of consumers who have different characteristics. This notion can be used to parametrize the weights w_{nk} such that they become a function of the similarity in characteristics between consumers n and k. This function should of course contain a much lower number of parameters than $N(N-1)$, the number of

reference weights. Given such a parametrization, estimation of the newly introduced parameters along with the other ones becomes feasible, and does not only yield estimates of the demand system parameters but also of the reference pattern between groups in society.

Although the results of this approach are of interest (cf. Kapteyn, 1977, Kapteyn, Van Praag and Van Herwaarden, 1978), it leads to very complicated models which are costly to estimate. The estimates of the parameters describing the pattern of reference weights tend to be unreliable. In this paper we opt for a different, simpler approach: the reference weights are considered to be drawings from a multivariate probability distribution. We do not specify this distribution completely, but make a few assumptions that partly characterize the distribution.

A central concept in our approach is the notion of a <u>social group</u>, i.e. a set of people who share certain characteristics like education, age, type of job, etc. The idea is to use the social group to which an individual belongs as a proxy for his reference group. To make clear under what circumstances such a procedure is justified and what errors of approximation may be involved, we make four explicit assumptions. These four assumptions are listed and discussed in appendix A. Here we only mention the main implication of the assumptions.

The parameters in (3.1) are estimated by first deriving the reduced form. It turns out that in this reduced form $\Sigma_k w_{nk} y_k$ and $\Sigma_k w_{nk} f_k$ appear as exogenous variables. The assumptions in appendix A allow us to approximate these variables as follows (cf. (A.17) and (A.18)):

$$\sum_k w_{nk} y_k = \kappa \eta + (1-\kappa)\bar{y}_n + \hat{v}_n \ , \tag{3.2}$$

$$\sum_k w_{nk} f_k = \kappa \xi + (1-\kappa)\bar{f}_n + \hat{v}_n \ , \tag{3.3}$$

where $\bar{y}_n$ and $\bar{f}_n$ are the mean income and mean family size in the social group of individual n, η and ξ are mean income and family size in society as a whole, $\hat{v}_n$ and $\hat{v}_n$ are error terms that up to terms of $o_p(1)$ [4] are uncorrelated with $\bar{y}_n$ and $\bar{f}_n$ and have mean zero. The interpretation of the

parameter κ is that $(1-\kappa)$ is the share of the total reference weight that people assign, on average, to others <u>within</u> the same social group, whereas κ is the share given, on average, to all people in society, irrespective of whether they are within or outside an individual's social group. So, if $\kappa=0$, reference groups do not extend beyond one's own social group. If $\kappa=1$, the social group contains no information whatever on one's reference group. In other words, the smaller κ is, the better a proxy one's social group is for one's reference group. Of course, even if $\kappa=0$ the social group is not a perfect proxy as long as $\hat{u}_n$ and $\hat{v}_n$ are not identically zero.

Given the approximations (3.2) and (3.3) the reduced form of (3.1) takes a simple form, as will appear in the next section.

4. The reduced form and identification

It is shown in appendix B that under the assumptions listed in appendix A, (3.1) implies the following reduced form

$$x_{gn} = d_g + \gamma_g y_n + \alpha_g f_n + r_g \bar{y}_n - r_g \tilde{\mu} \bar{f}_n + u_{gn} .$$ (4.1)

The reduced-form parameters can be expressed in the structural parameters as follows:

$$r_g \equiv (1-\kappa)\rho_g$$ (4.2)

$$\rho_g \equiv \frac{\beta_g - p}{1-(1-\kappa)\beta_g} \gamma_g$$ (4.3)

$$p \equiv \sum_{g=1}^{G} \frac{\beta_g \gamma_g}{1-(1-\kappa)\beta_g} \Big/ \sum_{g=1}^{G} \frac{\gamma_g}{1-(1-\kappa)\beta_g}$$ (4.4)

$$d_g \equiv \frac{s_g - \phi\gamma_g}{1-\beta_g}$$ (4.5)

$$s_g \equiv b_{0g} - \gamma_g \sum_{h=1}^{G} b_{0h} + \kappa \rho_g (\eta - \tilde{\mu}\xi) \tag{4.6}$$

$$\phi \equiv \sum_{h=1}^{G} \frac{\beta_h s_h}{1-\beta_h} \bigg/ \sum_{h=1}^{G} \frac{\gamma_h}{1-\beta_h} \tag{4.7}$$

$$\alpha_g \equiv \mu_g - \tilde{\mu}\gamma_g . \tag{4.8}$$

The corresponding formulae in appendix B are (B.19) and (B.45) (for d_g, s_g and ϕ), (B.46) for α, (B.18) for ρ_g and p, and (B.47) for r_g. It is easy to see that α_g, r_g, ρ_g and d_g add up to zero, when summing over goods. The error term u_{gn} is well-behaved in the sense that up to terms of $O(N^{-1})$ it has mean zero and is uncorrelated with the other variables on the right hand side of (4.1).

Under our assumptions, the reduced form parameters d_g, γ_g, α_g, r_g and $\tilde{\mu}$ can be estimated consistently from cross-section data (some details follow in section 5). Knowing, or consistently estimating, the reduced form parameters does not suffice, however, to determine all structural parameters. This can be seen as follows. Use (4.3) to solve for β_g:

$$\beta_g = \frac{\rho_g + p\gamma_g}{\gamma_g + (1-\kappa)\rho_g} , \tag{4.9}$$

or

$$1-(1-\kappa)\beta_g = \frac{\gamma_g(1-(1-\kappa)p)}{\gamma_g + r_g} . \tag{4.10}$$

It follows from the analysis in appendix B (last paragraph) that, even with κ known, p is unidentified. Since κ is unknown as well, we are lacking two pieces of information for the identification of the β_g. Assuming that $0 \leqslant \kappa < 1$, we are able, however, to infer a <u>ranking</u> of β_g's from the reduced form estimates:

$$\beta_g < \beta_h \quad <=> \quad \frac{r_g}{\gamma_g + r_g} < \frac{r_h}{\gamma_h + r_h} . \tag{4.11}$$

The structural parameters μ_g can be identified from the α_g and $\tilde{\mu}$. Notice that without interdependence the μ_g would not be identified, since the α_g sum to zero. Consequently, we would have had only G-1 independent pieces of information to identify the G parameters μ_g. It is the presence of $\bar{f}_n$ which makes it possible to identify the sum of the μ_g, $\tilde{\mu}$, which provides the extra piece of information required.

The G parameters b_{0g} cannot be identified from (4.5), because the d_g sum to zero. Since the b_{0g} are of no particular interest we do not pay further attention to either the b_{0g} or the d_g.

5. Estimation results

Model (3.1) has been estimated using data on 1669 households from the Consumer Expenditure Survey 1974/1975 conducted by the Netherlands Central Bureau of Statistics. As mentioned in section 2, households have been assigned to social groups with identical characteristics. The characteristics considered are the following ones:

(a) Educational attainment of head of household (3 categories distinguished);
(b) Age of head of household (5 categories);
(c) Size of town of residence (3 categories).

This leads to a maximum of 45 distinct social groups, 37 of which appeared to be represented in the sample.

The variables $\bar{y}_n$ and $\bar{f}_n$ in (3.1) refer to population means in the social group to which individual n belongs. Obvious proxies for $\bar{y}_n$ and $\bar{f}_n$ are the corresponding sample means. Care has been taken, however, for each individual n to base the estimate of $\bar{y}_n$ and $\bar{f}_n$ only on the incomes and family sizes of all other sample households in the social group. Of course, replacement of $\bar{y}_n$ and $\bar{f}_n$ by sample means introduces measurement errors, but the variance-covariance matrix of measurement errors in $\bar{y}_n$ and $\bar{f}_n$ corresponding

to group t can be estimated unbiasedly by $1/(\tilde{N}_t-1)$ times the sample covariance matrix of y_n and f_n corresponding to group t, where $\tilde{N}_t$ is the number of consumers in the sample belonging to group t.

The model has been estimated by means of the LISREL program (Jöreskog and Sörbom, 1981, Aigner, Hsiao, Kapteyn and Wansbeek, 1983, Bentler, 1983). Under the conditions given in lemma 4 in appendix B the LISREL output provides consistent estimates of the reduced form parameters, and the printed standard errors can serve as asymptotic approximations of the true standard errors of the estimates.

Two sets of estimates of model (3.1) wil be presented, one ignoring the measurement error caused by the use of proxies for $\bar{y}_n$ and $\bar{f}_n$, and one taking into account this measurement error. In the latter case the estimated variance-covariance matrices of measurement errors per group have been averaged over the groups. (Correlation of measurement error across individuals in the same group has been ignored.) This average error variance-covariance matrix indicates that measurement error accounts for 2.3% of the observed variance of $\bar{y}_n$, for 22.1% of that of $\bar{f}_n$, and for 1.3% of the covariance of $\bar{y}_n$ and $\bar{f}_n$.

Seven expenditure categories are distinguished:

(1) Food;
(2) Housing;
(3) Clothing;
(4) Medical care;
(5) Education, entertainment;
(6) Transportation;
(7) Savings.

The correlation matrix of all variables involved plus their sample means and standard deviations, are given in appendix C.

Because of adding up restrictions the variance-covariance matrix of the $u_{1n}, \ldots, u_{Gn}$ is singular and the parameters satisfy restrictions across equations. As usual, these problems can be accounted for by dropping arbitrarily one of the seven equations (cf. Barten, 1969). We have chosen to drop the savings equation.

The survey records money outlays. In the case of durables these may have an investment character, so that recorded outlay is only a poor proxy of the true consumption of this durable. This is a case of measurement error in an endogenous variable, which worsens the fit of the model, but does not affect the consistency of the parameter estimates, assuming that the measurement error is distributed independent of the exogenous variables.

Income is taken to be after tax disposable household income. All money amounts are measured in thousands of guilders per annum. Family size f_n is defined as the logarithm of the number of members of household n. The variance-covariance matrix of the reduced form disturbances of the six maintained equations has been left unrestricted.

The results for different specifications of the model are given in table 1. The χ^2-statistic is an indicator of the extent to which the model is compatible with the data. The χ^2-statistic has also been used to investigate the possibility of specifying f_n as a linear rather than a loglinear function of the number of family members, but the log-linear specification appears to provide a better fit.

Let us first consider the column 'complete model', which presents the results for the model which takes into account measurement errors in $\bar{y}_n$ and $\bar{f}_n$. According to the χ^2-statistic the model describes the data well. The estimate of ν has the correct sign and differs significantly from zero. All six estimated r_g are positive, four of them significantly different from zero. As the <u>seven</u> r_g introduced in the model add up to zero this implies a negative estimate for r_7 (savings).

The column headed 'no measurement error' presents the estimates of the model for the case that the proxies for $\bar{y}_n$ and $\bar{f}_n$ are assumed accurate. This neglect of measurement error does not affect the estimates of the γ_g or the values of the χ^2-statistic up to two decimal places.

Table 1. Estimation results

Parameter	Complete model	No measurement error	No interdependence
γ_1	0.07 (0.01)	0.07 (0.01)	0.08 (0.01)
γ_2	0.24 (0.02)	0.24 (0.02)	0.29 (0.01)
γ_3	0.04 (0.00)	0.04 (0.00)	0.04 (0.00)
γ_4	0.06 (0.00)	0.06 (0.00)	0.07 (0.00)
γ_5	0.10 (0.01)	0.10 (0.01)	0.11 (0.00)
γ_6	0.09 (0.01)	0.09 (0.01)	0.12 (0.01)
r_1	0.02 (0.01)	0.02 (0.01)	
r_2	0.13 (0.03)	0.12 (0.03)	
r_3	0.00 (0.01)	0.00 (0.01)	
r_4	0.04 (0.01)	0.04 (0.01)	
r_5	0.04 (0.01)	0.04 (0.01)	
r_6	0.08 (0.02)	0.07 (0.02)	
α_1	2.75 (0.37)	2.74 (0.37)	2.71 (0.37)
α_2	-0.04 (1.01)	-0.13 (1.01)	-0.32 (1.01)
α_3	0.99 (0.21)	0.99 (0.21)	0.99 (0.21)
α_4	0.97 (0.21)	0.94 (0.21)	0.88 (0.21)
α_5	1.34 (0.41)	1.31 (0.40)	1.25 (0.40)
α_6	-0.33 (0.72)	-0.39 (0.71)	-0.50 (0.72)
$\tilde{\mu}$	49.60 (17.69)	36.73 (14.01)	
R_1^2 a)	0.205	0.205	0.203
R_2^2	0.260	0.259	0.249
R_3^2	0.187	0.187	0.186
R_4^2	0.368	0.366	0.345
R_5^2	0.268	0.267	0.261
R_6^2	0.115	0.114	0.106
χ^2	6.37	6.37	109.40
df	5	5	12

a) R_g^2 is defined as $1-\sigma_{ug}^2/var(x_g)$

The column headed 'no interdependence' presents parameter estimates under the restriction $r_1=r_2=\ldots=r_7=0$. Although the fit of the equations, as gauged by the R^2's, hardly changes and the γ_g and α_g change only marginally, the χ^2-statistic rejects the restrictions decisively. As a final comment on the statistical quality of the results, a χ^2-test of the over-identifying restrictions on the coefficients of f_n and $\bar{f}_n$ does not lead to a rejection.

To start off a discussion of the economic significance of the result, we present information on the structural parameters in table 2.

Table 2. Values of structural parameters derived from the reduced form estimates for the complete model

expenditure category	$\dfrac{r_g}{\gamma_g + r_g}$	μ_g
1. Food	0.21	6.18
2. Housing	0.36	11.70
3. Clothing	0.04	2.19
4. Medical Care	0.43	2.80
5. Education	0.30	4.48
6. Transportation	0.46	4.22
7. Savings	-3.44	15.39

Although the β_g are not identified, we can derive their relative ranking from table 2 in conjunction with relation (4.11), assuming that $0<\kappa<1$ and all $0<\beta_g<1$. We find $\beta_6>\beta_4>\beta_2>\beta_5>\beta_1>\beta_3>\beta_7$. Interpreting β_g as a measure of the conspicuousness of good g, we have that the order of conspicuousness is: transportation, medical care, housing, education/entertaintment, food, clothing, savings. Except, maybe, for the relative ranking of medical care and clothing, the ranking seems quite plausible. As to the position of medical care: this category comprises 'domestic services', a high ranking of which seems intuitively plausible. Moreover, there is an artifact at work here, as most households in the sample were insured via the sick fund, the contributions to which depend on income and hence, statistically, also on reference group income.

It is of interest to confront predictions of the model <u>with</u> interdependence with the predictions of the model <u>without</u> interdependence. For the model without interdependence we have that $\Sigma_{g=1}^{6}\hat{\gamma}_g = 0.71$, i.e. an increase of the household's income with one dollar raises total consumption by 71 cents. For the complete model we find that $\Sigma_{g=1}^{6}\hat{\gamma}_g = 0.60$, so the complete model predicts a smaller response of consumption to an income change than the model without interdependence.[5]

On the aggregate level, this conclusion changes drastically. The model without interdependence still implies a marginal propensity to consume out of income equal to 0.71. So, if everyone's income increases with one dollar, mean consumption goes up with 71 cents, according to this model. When interdependence is taken into account, we have three upward influences on consumption, because y_n, $\bar{y}_n$ and η will increase at the same time. It follows from (4.1)-(4.8) that an increase of everyone's income with one dollar raises x_{gn} with[6]

$$\frac{\gamma_g}{1-\beta_g} \bigg/ \sum_h \frac{\gamma_h}{1-\beta_h} . \tag{5.1}$$

So the extent to which the aggregate consumption of a good responds to income changes does not only depend on the good's marginal budget share, but also on its conspicuousness. One sees that the magnitude of the response for good g is positively related to both its marginal budget share and its conspicuousness. To give one numerical example, if $\kappa=0$, it is easy to show that (5.1) is equal to $\gamma_g + r_g$. Since $\Sigma_{g=1}^{6}\hat{r}_g = 0.31$, we then have that an across-the-board income increase with one dollar raises mean aggregate consumption with 91 cents; 31 cents of this is due to a reaction to the increase in income, and hence in consumption, by others. Most of this interdependence effect is in the realm of conspicuous goods like housing (13 cents) and transportation (8 cents). In this example, neglecting interdependence in a cross-section and employing the results to make predictions of aggregate income changes on aggregate consumption changes would lead to a gross underestimation of the changes in consumption. This is, of course, a typical experience.

It is a little harder to interpret the μ's. It may help to look at an example. If a family's size increases from two persons to three, the utility function (2.2) implies that the extra expenditures required on each category to maintain the family's previous utility level are: food, Dfl. 2500 per annum; housing, Dfl. 4500; clothing, Dfl. 1200; medical care, Dfl. 1400; education/entertainment, Dfl. 2300; transportation, Dfl. 1700; savings, Dfl. 6200. (A Dutch guilder is approximately 40 dollar cents.) These are implausibly high numbers and probably should be interpreted as evidence that the LES is too restrictive a specification. Recall from section 4, moreover, that without interdependence the family size coefficients are not identified. Achieving identification of family size effects by incorporating interdependence may be asking too much from the data. Also note that, in contrast to the μ_g, the reduced form coefficients α_g have very plausible magnitudes.

6. Concluding remarks

The main purpose of this paper has been to show that preference interdependence can be incorporated in a demand system and to investigate its empirical importance. The results confirm the suspicion that preference interdependence is an important determinant of consumer behavior; not so much for the extra variance in consumption which can be thus explained nor for the parameter estimates, most of which do not change very much, but certain conclusions from the model (e.g. what is the effect of an across-the-board income change on the aggregate consumption of various goods?) do change drastically if preference interdependence is accounted for. So, to the extent that we want to use a model to predict aggregate responses to changes in exogenous variables, interdependence should not be neglected.

Although spelling out the stochastic assumptions that are required to arrive at a well-behaved reduced form asks for a fair amount of space (appendix A), and although the derivation of this reduced form is rather tedious (appendix B), the result is surprisingly simple. Estimation of the model by means of the widely available LISREL computer program is, moreover, straightforward. This suggests that there is really no practical reason to ignore preference interdependence in demand analysis or in other

empirical applications of micro-economic theory. Obvious extensions of the analysis in this paper include preference interdependence in labor-supply models and oligopolistic models of firm behavior. Lemma 1 of appendix B provides a rather general framework for the study of interdependencies in linear models of interdependent behavior. Of course, most of the simplicity may be due to the linearity of the specification. Future work should be directed towards an extension of the analysis to more flexible specifications.

A second extension is to supplement preference interdependence with habit formation. Not only will that probably increase the explanatory power of the model, it will also aid in identifying the structural parameters. This extension requires the availability of panel data.

A third extension has to be in the modelling of reference groups. In this paper we have basically described the distribution of reference weights by means of one parameter κ. It should be possible to refine this specification. Ideally, of course, one would like to have a formal theory of how reference groups are formed. To our knowledge no such theory exists at this moment.

Appendix A. Stochastic assumptions

Here we introduce and discuss four assumptions that justify the approxima-
tions (3.2) and (3.3) and the reduced form given in section 4.

If individual n is a member of social group t, t=1,...,T, we denote this
as $n \in G_t$, and we denote the size of social group t (i.e. the number of indi-
viduals in it) as N_t.

Assumption 1. Within each social group the y_n and f_n are random drawings
from a bivariate distribution with mean vector $(\bar{y}_n, \bar{f}_n)$, i.e.

$$y_n = \bar{y}_n + \zeta_n , \tag{A.1}$$

$$f_n = \bar{f}_n + \omega_n , \tag{A.2}$$

where $\mathbf{E}\zeta_n = \mathbf{E}\omega_n = 0$; ζ_n and ω_n are distributed independently from $\bar{y}_n$, $\bar{f}_n$ and
w_{nk} for any n and k.

As a matter of notation, notice that $\bar{y}_n$ is constant within a social
group. Sometimes we shall write $\bar{y}_t$ for the value of $\bar{y}_n$ witn $n \in G_t$.

Let y_n^*, f_n^* and p_n be defined as

$$y_n^* \equiv \frac{1}{N-N_t} \sum_{s \neq t}^{T} N_s \bar{y}_s , \qquad\qquad n \in G_t \tag{A.3}$$

$$f_n^* \equiv \frac{1}{N-N_t} \sum_{s \neq t}^{T} N_s \bar{f}_s , \qquad\qquad n \in G_t \tag{A.4}$$

$$p_n \equiv \sum_{k \in G_t} w_{nk} . \qquad\qquad n \in G_t \tag{A.5}$$

Assumption 2.

$$\sum_k w_{nk} \bar{y}_k = p_n \bar{y}_n + (1-p_n) y_n^* + o_p(1) \,, \tag{A.6}$$

$$\sum_k w_{nk} \bar{f}_k = p_n \bar{f}_n + (1-p_n) f_n^* + o_p(1) \,. \tag{A.7}$$

(The symbol $o_p(1)$ has been defined in footnote 4.) We will refer to y_n^* and f_n^* as the average income and family size outside individual n's social group. Strictly speaking this is not correct, since, according to (A.1) and (A.2) these average incomes and family sizes are random variables. Due to the law of large numbers, the observed averages will be very close to y_n^* and f_n^*, given that the social groups contain many people.

As mentioned in section 3, in the reduced form of the system (3.1), variables like $\Sigma_k w_{nk} y_k$ and $\Sigma_k w_{nk} f_k$ appear. The two assumptions made so far allow us to circumvent the problem of having to specify the reference weights w_{nk}. Instead, we only have to concern ourselves with the <u>total</u> weight p_n given by individual n to others in the same social group and the total weight $(1-p_n)$ given to all people outside this social group. This can be seen as follows. Using both assumptions we have

$$\sum_k w_{nk} y_k = \sum_k w_{nk} \bar{y}_k + \sum_k w_{nk} \zeta_k \equiv \sum_k w_{nk} \bar{y}_k + v_n =$$

$$= p_n \bar{y}_n + (1-p_n) y_n^* + v_n + o_p(1) \,; \tag{A.8}$$

$$\sum_k w_{nk} f_k = \sum_k w_{nk} \bar{f}_k + \sum_k w_{nk} \omega_k \equiv \sum_k w_{nk} \bar{f}_k + \nu_n =$$

$$= p_n \bar{f}_n + (1-p_n) f_n^* + \nu_n + o_p(1) \,, \tag{A.9}$$

where v_n and ν_n have been defined implicitly; v_n and ν_n are random variables with zero mean and distributed independently of p_n, $\bar{y}_n$, y_n^*, $\bar{f}_n$, f_n^*. Next, define η and ξ by

$$\eta \equiv \frac{1}{N} \sum_t N_t \bar{y}_t \ , \qquad \xi \equiv \frac{1}{N} \sum_t N_t \bar{f}_t \ . \qquad (A.10)$$

In the same informal terminology as above, we call η and ξ the mean income and family size in society. From (A.3) and (A.10) we have

$$N\eta = N_t \bar{y}_n + (N-N_t) y_n^* \ , \qquad (A.11)$$

so that

$$y_n^* = \frac{N}{N-N_t} \eta - \frac{N_t}{N-N_t} \bar{y}_n \ . \qquad (A.12)$$

This can be used to further simplify (A.8):

$$\sum_k w_{nk} y_k = p_n \bar{y}_n + \frac{N(1-p_n)}{N-N_t} \eta - \frac{(1-p_n)N_t}{N-N_t} \bar{y}_n + v_n + o_p(1) =$$

$$= \bar{y}_n \left[1 - \frac{N(1-p_n)}{N-N_t} \right] + \frac{N(1-p_n)}{N-N_t} \eta + v_n + o_p(1) =$$

$$= (1-\kappa_n)\bar{y}_n + \kappa_n \eta + v_n + o_p(1) \ , \qquad n \epsilon G_t \ , \qquad (A.13)$$

where κ_n has been defined implicitly. A similar expression for $\sum_k w_{nk} f_k$ is

$$\sum_k w_{nk} f_k = (1-\kappa_n)\bar{f}_n + \kappa_n \xi + v_n + o_p(1) \ , \qquad n \epsilon G_t \ . \qquad (A.14)$$

We can rewrite κ_n as

$$\kappa_n = \frac{N(1-p_n)}{N-N_t} = (1-p_n)/[(N-N_t)/N] \qquad n \epsilon G_t \ . \qquad (A.15)$$

Note that $(N-N_t)/N$ is simply the proportion of the population not in social group t, whereas $(1-p_n)$ is the total weight given to these people by individual n. So, if $\kappa_n = 0$, individual n gives no weight to people outside his or her social group. If $\kappa_n = 1$, these people get a weight proportional to

their share in the population, which means that knowledge of n's social group does not give information concerning his or her reference group. If $\kappa_n > 1$, people within social group t get even less total weight than their share in the population.

Basically, (A.13) and (A.14) reduce the number of unknown parameters from about $N(N-1)$ to about N. A further reduction of the number of unknown parameters is obtained by assumption 3.

Assumption 3.

$$\kappa_n = \kappa + \delta_n , \tag{A.16}$$

where δ_n is a random variable with mean zero; δ_n and δ_k are independently distributed for $n \neq k$, δ_n is independent of $w_{k\ell}$ for $k \neq n$, $\ell = 1, \ldots, N$.

This assumption mainly serves to further reduce the number of unknown parameters. In particular, it implies a further simplification of (A.13) and (A.14):

$$\sum_k w_{nk} y_k = (1-\kappa)\bar{y}_n + \kappa\eta + v_n - \delta_n(\bar{y}_n - \eta) + o_p(1) \tag{A.17}$$

$$\sum_k w_{nk} f_k = (1-\kappa)\bar{f}_n + \kappa\xi + v_n - \delta_n(\bar{f}_n - \xi) + o_p(1) . \tag{A.18}$$

Under the above assumptions, $v_n - \delta_n(\bar{y}_n - \eta)$ and $v_n - \delta_n(\bar{f}_n - \xi)$ are independent of $\bar{y}_n$ and $\bar{f}_n$. So, rather than having $N(N-1)$ reference weights to deal with we are left with one unknown parameter κ.

To arrive at a reduced form with a well-behaved error term we need one more assumption. Define $w_{nm}^{(2)} \equiv \Sigma_k w_{nk} w_{km}$ and $w_{nm}^{(\ell)} \equiv \Sigma_k w_{nk} w_{km}^{(\ell-1)}$ for $\ell > 2$.

Assumption 4. $Ew_{nm}^{(\ell)} = 0(N^{-1})$, for $\ell \geq 2$.

Notice that $w_{nk}^{(2)}$ is the weight assigned by n to k 'via all others'. Assumption 4 therefore states that on average the indirect influence of any individual on any other individual will tend to zero if the number of individuals in society tends to infinity.

For the derivation of the reduced form we shall employ the following implication of the assumptions:

$$\sum_k w_{nk}^{(\ell)}(\bar{y}_k - \eta)\delta_k = o_p(1) \ , \qquad \text{for } \ell \geqslant 2 \ . \tag{A.19}$$

$$\sum_k w_{nk}^{(\ell)}(\bar{f}_k - \xi)\delta_k = o_p(1) \ . \qquad \text{for } \ell \geqslant 2 \tag{A.20}$$

The proof of (A.19) and (A.20) is an application of Chebychev's Lemma.

Appendix B. Derivation of the reduced form

This appendix presents the derivation of the reduced form (4.1) under the assumption given in appendix A. For the sake of simplicity, we first derive a version of (4.1) that does not take family size into account, and then adapt the results by including family size.

Let

$$x \equiv (x_{11}, \ldots, x_{1N}, \ldots, x_{G1}, \ldots, x_{GN})' \qquad GN \times 1 \qquad (B.1)$$

$$b \equiv (b_{11}, \ldots, b_{1N}, \ldots, b_{G1}, \ldots, b_{GN})' \qquad GN \times 1 \qquad (B.2)$$

$$b_0 \equiv (b_{01}, \ldots, b_{0G})' \qquad G \times 1 \qquad (B.3)$$

$$\gamma \equiv (\gamma_1, \ldots, \gamma_G)' \qquad G \times 1 \qquad (B.4)$$

$$y \equiv (y_1, \ldots, y_N)' \qquad N \times 1 \qquad (B.5)$$

$$B \equiv \begin{bmatrix} \beta_1 & & & \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot & \beta_G \end{bmatrix} \qquad G \times G \qquad (B.6)$$

$$\varepsilon \equiv (\varepsilon_{11}, \ldots, \varepsilon_{1N}, \ldots, \varepsilon_{G1}, \ldots, \varepsilon_{GN})' \qquad GN \times 1 \qquad (B.7)$$

$$u \equiv (u_{11}, \ldots, u_{1N}, \ldots, u_{G1}, \ldots, u_{GN})' \qquad GN \times 1 \qquad (B.8)$$

$$W \equiv \begin{bmatrix} w_{11} & \cdots & w_{1N} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ w_{N1} & \cdots & w_{NN} \end{bmatrix} \qquad N \times N \qquad (B.9)$$

$$\bar{y} \equiv (\bar{y}_1, \ldots, \bar{y}_N)' \qquad N \times 1 \qquad (B.10)$$

$$\rho \equiv (\rho_1, \ldots, \rho_G)' \qquad G \times 1 \qquad (B.11)$$

$$\beta \equiv (\beta_1, \ldots, \beta_G)' \qquad G \times 1 \qquad (B.12)$$

Let ι denote a vector of ones, with a subscript that indicates its length.
(So, e.g. $B\iota_G=\beta$.) Equation (2.1), after adding a disturbance term ε_{gn}, can
now be rewritten

$$x = b + \gamma\otimes\{y - (\iota_G'\otimes I_N)b\} + \varepsilon \ , \tag{B.13}$$

and equation (2.4):

$$b = b_0\otimes\iota_N + (B\otimes W)x \ . \tag{B.14}$$

Substitute (B.14) into (B.13):

$$x = b_0\otimes\iota_N + (B\otimes W)x + \gamma\otimes\{y - (\iota_G'\otimes I_N)(b_0\otimes\iota_N + (B\otimes W)x)\} + \varepsilon =$$
$$= c\otimes\iota_N + \{(B-\gamma\beta')\otimes W\}x + \gamma\otimes y + \varepsilon \ , \tag{B.15}$$

where c is defined as

$$c \equiv b_0 - \iota_G'b_0\gamma \ . \tag{B.16}$$

Obviously, $\iota_G'c=0$.

To state our first lemma we need a number of definitions:

$$A \equiv B - \gamma\beta' \tag{B.17}$$

$$\rho \equiv [I_G - (1-\kappa)A]^{-1}A\gamma \tag{B.18}$$

$$\psi \equiv (I_G - A)^{-1}(c+\kappa\eta A\rho) \tag{B.19}$$

$$z \equiv (1-\kappa)\bar{y} + \kappa\eta\iota_N \ . \tag{B.20}$$

Furthermore, let $\delta\equiv(\delta_1,\ldots,\delta_N)'$, $\bar{Y}\equiv\text{diag}(\bar{y}_1,\ldots,\bar{y}_N)$, $v\equiv(v_1,\ldots,v_N)'$.

Lemma 1. Under assumptions 1, 2 and 3, (B.15) implies

$$x = \psi\otimes\iota_N + \gamma\otimes y + \rho\otimes z + u \ , \tag{B.21}$$

where u satisfies:

$$(I_{GN} - A \otimes W)u = \varepsilon - \rho \otimes (\bar{Y} - \eta I_N)\delta + A\gamma \otimes v + o_p(1) \ . \tag{B.22}$$

Proof. We show that substitution of (B.21) in (B.15) leads to an identity with u satisfying (B.22). Equations (B.21) and (B.15) imply

$$\psi \otimes \iota_N + \gamma \otimes y + \rho \otimes z + u = c \otimes \iota_N + (A \otimes W)\{\psi \otimes \iota_N + \gamma \otimes y + \rho \otimes z + u\} + \gamma \otimes y + \varepsilon \ , \tag{B.23}$$

or, using $W\iota_N = \iota_N$,

$$\psi \otimes \iota_N - c \otimes \iota_N - A\psi \otimes \iota_N + \rho \otimes z - A\gamma \otimes Wy - A\rho \otimes Wz + (I_{GN} - A \otimes W)u = \varepsilon \ . \tag{B.24}$$

Since, according to (B.19), $\psi - A\psi = c + \kappa \eta A\rho$, the first three terms of (B.24) are equal to $\kappa \eta A\rho \otimes \iota_N$. So we have

$$(I_{GN} - A \otimes W)u = \varepsilon + A\gamma \otimes Wy + A\rho \otimes Wz - \rho \otimes z - \kappa \eta A\rho \otimes \iota_N \ . \tag{B.25}$$

From (A.17) we have

$$Wy = (1-\kappa)\bar{y} + \kappa \eta \iota_N + v - (\bar{Y} - \eta I_N)\delta + o_p(1) =$$
$$= z + v - (\bar{Y} - \eta I_N)\delta + o_p(1) \ . \tag{B.26}$$

Since $W\bar{y} = Wy - v$, we have for Wz:

$$Wz = (1-\kappa)W\bar{y} + \kappa \eta \iota_N = (1-\kappa)\{z - (\bar{Y} - \eta I_N)\delta + o_p(1)\} + \kappa \eta \iota_N \ . \tag{B.27}$$

Collecting terms, we find for (B.25):

$$(I_{GN} - A \otimes W)u = A\gamma \otimes z + (1-\kappa)A\rho \otimes z - \rho \otimes z + \varepsilon + A\gamma \otimes v -$$
$$- [A\gamma + (1-\kappa)A\rho] \otimes [\bar{Y} - \eta I_N]\delta + o_p(1) \ . \tag{B.28}$$

It follows immediately from (B.18) that $A\gamma+(1-\kappa)A\rho=\rho$. Hence (B.28) simplifies to (B.22). ///

Lemma 2.

$$[I_{GN} - A\otimes W]^{-1} = I_{GN} + A\otimes W + A^2\otimes W^2 + A^3\otimes W^3 + \ldots \qquad (B.29)$$

Proof. For any integer $\ell>1$:

$$[I_{GN} + A\otimes W + A^2\otimes W^2 + \ldots + A^{(\ell-1)}\otimes W^{(\ell-1)}][I_{GN} - A\otimes W] =$$

$$= I - A^\ell\otimes W^\ell . \qquad (B.30)$$

So to prove (B.29) it is sufficient to prove that $A^\ell\otimes W^\ell$ converges to zero if ℓ tends to infinity. Since W is a Markov matrix, W^ℓ is a Markov matrix as well. Hence the elements of W^ℓ are bounded (they have values between zero and one). It is therefore sufficient to prove that $A^\ell\to0$ for ℓ to infinity. We show this by proving that the eigenvalues of A are all within the unit circle (Oldenburger, 1940).

First assume that all β_g are different and strictly positive. Then the eigenvalues of A follow from the determinantal equation

$$|A - \lambda I_G| = |B - \lambda I_G - \beta\gamma'| = |\beta - \lambda I_G| \{1 - \beta'(B - \lambda I_G)^{-1}\gamma\} = 0 . \qquad (B.31)$$

The expression between braces equals

$$\iota_G'(B - \lambda I_G)(B - \lambda I_G)^{-1}\gamma - \beta'(B - \lambda I_G)^{-1}\gamma = -\lambda\iota_G'(B - \lambda I_G)^{-1}\gamma , \qquad (B.32)$$

so $\lambda=0$ or $\iota_G'(B-\lambda I_G)^{-1}\gamma=0$. In scalar notation, the latter expression reads

$$\sum_{g=1}^{G} \frac{\gamma_g}{\beta_g-\lambda} = 0. \qquad (B.33)$$

Each of the terms under the sum sign is an orthogonal hyperbola in λ with $\lambda = \beta_g$ as its vertical asymptote. So (B.33) has a solution between each two successive β_g, giving the remaining $G-1$ roots of (B.31). So all roots are non-negative and smaller than the largest β_g, which by assumption is less than 1.

This still holds when not all β_g are different or strictly positive. This follows directly from the continuity of eigenvalues of a matrix as a function of its elements.   ///

Lemma 3. Under assumptions 1, 2, 3 and 4 and ignoring terms of order $o_p(1)$, the vector u satisfying (B.22) has mean zero and

$$\mathbf{E}uy' = A\gamma \otimes \mathbf{E}W + O(N^{-1}) .$$ (B.34)

Proof. Use lemma 2 to rewrite (B.22) as

$$u = (I_{GN} - A\otimes W)^{-1}\{\varepsilon + A\gamma \otimes v\} - (I_G + A)\rho \otimes W(\bar{Y} - \eta I_N)\delta -$$

$$- \sum_{j=2}^{\infty} A^j \rho \otimes W^j(\bar{Y} - \eta I_N)\delta + o_p(1) .$$ (B.35)

(A.19) implies

$$\sum_{j=2}^{\infty} A^j \rho \otimes W^j(\bar{Y} - \eta I_N)\delta = \sum_{j=2}^{\infty} A^j \rho \otimes o_p(1) = A^2(I_G - A)^{-1}\rho \otimes o_p(1) = o_p(1) .$$
(B.36)

So we have for u,

$$u = (I_{GN} - A\otimes W)^{-1}\{\varepsilon + A\gamma \otimes v\} - (I_G + A)\rho \otimes W(\bar{Y} - \eta I_N)\delta + o_p(1) .$$ (B.37)

The first term on the right hand side involves ε and $v \equiv W\zeta$ where $\zeta \equiv (\zeta_1,\ldots,\zeta_N)'$. Since both ε and ζ are independent of W, and have expectation equal to zero, this first term has expectation zero as well. In the second term the random variables are W and δ. A typical element of $W(\bar{Y}-\eta I_N)\delta$ is

$\Sigma_{k\neq n}(\bar{y}_k-\eta)w_{nk}\delta_k$. Since δ_k has mean zero and is independent of w_{nk} for $k\neq w$, this element has mean zero. Consequently the second term has mean zero. Neglecting the $o_p(1)$ term, we conclude that u has mean zero.

To prove the second part of the lemma, we first observe that δ and ε are independent of y. So we only have to consider

$$\mathbf{E}(I_{GN} - A\otimes W)^{-1}(A\gamma\otimes v)\zeta' = \mathbf{E}(I_{GN} - A\otimes W)^{-1}(A\gamma\otimes W\zeta\zeta') =$$

$$= \mathbf{E}(I_{GN} - A\otimes W)^{-1}(A\gamma\otimes W\mathbf{E}(\zeta\zeta')) = \sigma_\zeta^2\mathbf{E}(I_G - A\otimes W)^{-1}(A\gamma\otimes W) , \quad (B.38)$$

where the second equality sign is based on the independence of W and ζ. Next we write

$$\mathbf{E}(I_G - A\otimes W)^{-1}(A\gamma\otimes W) = \mathbf{E}\{A\gamma\otimes W + A^2\gamma\otimes W^2 + A^3\gamma\otimes W^3 + \ldots\} =$$

$$= A\gamma\otimes\mathbf{E}W + A^2\gamma\otimes\mathbf{E}W^2 + A^3\gamma\otimes\mathbf{E}W^3 + \ldots =$$

$$= A\gamma\otimes\mathbf{E}W + A^2\gamma\otimes0(N^{-1}) + A^3\gamma\otimes0(N^{-1}) + \ldots =$$

$$= A\gamma\otimes\mathbf{E}W + A^2(I-A)^{-1}\gamma\otimes0(N^{-1}) = A\gamma\otimes\mathbf{E}W + 0(N^{-1}) , \quad (B.39)$$

where the third equality follows from assumption 4. ///

Note that the diagonal elements of W are identically equal to zero. As a result, an element of u corresponding to a certain observation is uncorrelated with the element of y corresponding to that same observation. Of course, any element of u does correlate with elements of y corresponding to different observations, but that does not affect the asymptotic distribution of the ML-estimator. This statement is made more precise in lemma 4.

Let us define the 'conventional' ML-estimator of the reduced form parameters ψ, γ, and ρ in (B. 21) as the estimator that maximizes the likelihood of the observations under the assumption that u follows a normal distribution (with mean zero) with a variance-covariance matrix of the form $\Sigma\otimes I_N$,

where Σ is unrestricted., This estimator provides us with consistent esti-
mates of ψ, γ, and ρ under assumptions 1-4, but in order to use the corres-
ponding conventional standard errors an extra assumption is needed. This is
summarized, somewhat informally, in lemma 4.

Lemma 4 Under assumptions 1, 2, 3, and 4, the conventional ML-estimator
of the reduced form parameters is consistent. If we strengthen assumption 4
to

$$E \, w_{nm} = O(N^{-1}) \, ,$$

then the conventional standard errors are consistent estimates of the true
standard errors.

Proof. To prove the first part, we notice that (B.21) is simply a system
of seemingly unrelated regressions where the same explanatory variables
appear in each equation. Consequently, the conventional ML-estimator is
identical to the OLS-estimator applied equation by equation. Since the
diagonal elements of W are identically equal to zero, it follows from lemma
3 that the elements of u are uncorrelated with the explanatory variables
corresponding to the same observation. It follows immediately that the OLS-
estimator is consistent.

Concerning the second part of the lemma, we observe that the strengthened
version of assumption 4 in conjunction with lemma 3 implies that we can
neglect the correlation between u and y. Furthermore, considering (B.35) it
is clear that the only source of correlation of elements of u across obser-
vations arises from terms involving W. By assumption these terms can be
neglected. As a result u has the variance covariance matrix assumed by the
conventional ML-estimator and its standard errors are consistent estimates
of the true standard errors of the parameter estimates.   ///

The final step in the derivation of the reduced form is to account for
family size. In matrix format (2.6) reads

$$b = b_0 \otimes \iota_N + \mu \otimes f + (B \otimes W) \tilde{x} \, , \tag{B.40}$$

with $\mu \equiv (\mu_1, \ldots, \mu_G)$, $f \equiv (f_1, \ldots, f_N)'$, and

$$\tilde{x} \equiv x - \mu \otimes f , \tag{B.41}$$

i.e. (2.5) in matrix format. Define

$$\tilde{y} \equiv y - \tilde{\mu} f , \tag{B.42}$$

with $\tilde{\mu} = \iota_G' \mu$. Substitution of (B.40) into (B.13) yields

$$x = b_0 \otimes \iota_N + \mu \otimes f + (B \otimes W)\tilde{x} + \gamma \otimes \{y - (\iota_G' \otimes I_N)[b_0 \otimes \iota_N + \mu \otimes f + (B \otimes W)\tilde{x}]\} + \varepsilon =$$

$$= c \otimes \iota_N + \mu \otimes f + (B \otimes W)\tilde{x} + \gamma \otimes y - \gamma \otimes \tilde{\mu} f - (\gamma \beta' \otimes W)\tilde{x} + \varepsilon . \tag{B.43}$$

So,

$$\tilde{x} = c \otimes \iota_N + [(B - \gamma \beta') \otimes W]\tilde{x} + \gamma \otimes \tilde{y} + \varepsilon . \tag{B.44}$$

Apart from the tildes, this is exactly (B.15). This allows us to use the preceding results in deriving the reduced form. Define

$$d \equiv \psi + (\eta - \tilde{\mu}\xi)\kappa\rho \tag{B.45}$$

$$\alpha \equiv \mu - \tilde{\mu}\gamma \tag{B.46}$$

$$r \equiv (1-\kappa)\rho . \tag{B.47}$$

Lemma 5. Under assumptions 1-4, (B.44) implies

$$x = d \otimes \iota_N + \alpha \otimes f + \gamma \otimes y + r \otimes \bar{y} - r \upsilon \otimes \bar{f} + u . \tag{B.48}$$

Up to terms of $O(N^{-1})$ the error term u has mean zero and u_{gn} is uncorrelated with y_n and f_n. Lemma 4 applies.

Proof. Using the analogy between (B.44) and (B.15) and using lemma 1 gives:

$$\tilde{x} = \psi \otimes \iota_N + \gamma \otimes \tilde{y} + \rho \otimes \tilde{z} + u , \tag{B.49}$$

where

$$\tilde{z} \equiv (1-\kappa)(\bar{y}-\tilde{\mu}\bar{f}) + \kappa(\eta-\tilde{\mu}\xi)\iota_N \; . \tag{B.50}$$

Working this out gives (B.48).

The error term u satisfies an expression similar to (B.22) and the properties of u follow from arguments similar to lemma 3. It is also a matter of analogy to prove that lemma 4 applies, except for one slight complication. In (B.48) these are overidentifying restrictions on the reduced form parameters so that ML is no longer identical to OLS equation by equation. Since imposition of correct restrictions does not impair consistency, the consistency of the ML-estimator still follows from the consistency of the OLS-estimator. ///

Finally we substantiate the remark following (4.10) in section 4, by using (B.18) to express β as a function of γ, ρ and κ. First rewrite (B.18) as

$$A\gamma + (1-\kappa)A\rho = \rho \tag{B.51}$$

or

$$(I_G - \gamma\iota_G')B(\gamma + (1-\kappa)\rho) = \rho \; . \tag{B.52}$$

Let Δ be the diagonal matrix with typical diagonal element $\gamma_g+(1-\kappa)\rho_g$. Then (B.52) is equivalent to

$$(I_G - \gamma\iota_G')\Delta\beta = \rho \; . \tag{B.53}$$

As $I_G-\gamma\iota_G'$ has rank $G-1$, $\Delta\beta$ can not uniquely be inferred from (B.53). Using the algebra of singular linear systems (e.g. Searle, 1971), the general solution of (B.53) is, for arbitrary p,

$$\Delta\beta = (I_G - \gamma\iota_G')\rho + p\gamma = \rho + p\gamma \tag{B.54}$$

or

$$\beta = \Delta^{-1}(\rho + p\gamma) \; . \tag{B.55}$$

This is equivalent to (4.9).

Appendix C. The data

Below we give the correlation matrix of the variables, their sample average and their sample standard deviation. The following symbols are used:

x_1 = food
x_2 = housing
x_3 = clothing
x_4 = medical care
x_5 = education
x_6 = transportation
$\bar{y}$ = average income in social group
$\bar{f}$ = average of log family size in social group
y = income
f = logarithm of family size

	x_1	x_2	x_3	x_4	x_5	x_6	$\bar{y}$	$\bar{f}$	y	f
x_1	1.00000									
x_2	.20831	1.00000								
x_3	.34518	.27841	1.00000							
x_4	.37012	.31918	.36747	1.00000						
x_5	.37226	.24914	.34532	.36052	1.00000					
x_6	.16115	.17744	.21133	.21316	.27743	1.00000				
$\bar{y}$	.30853	.40110	.28168	.48243	.38783	.28048	1.00000			
$\bar{f}$	.18201	.17319	.16679	.18160	.16480	.11374	.43538	1.00000		
y	.42060	.49882	.41880	.58143	.50622	.32466	.64612	.28489	1.00000	
f	.23651	.08596	.18136	.18827	.15764	.04431	.12414	.21451	.18549	1.00000
average[a]	6.186	7.413	2.310	3.061	3.574	3.084	26.389	1.405	26.389	1.405
s.d.[a]	2.104	5.967	1.174	1.347	2.403	3.861	6.925	.038	10.432	.127

a) Money amounts in Dfl. 1000 per annum

Notes

1) For the purpose of this paper we use the the terms 'household', 'family', 'individual', 'consumer' as synonyms, whereas 'income' denotes after tax disposable family income.

2) Notice that in (2.3) the prices of all goods are equal to one. Since we will be dealing with a cross-section where all consumers face the same prices, this does not involve any loss of generality. As a result we will use 'consumption' and 'expenditures' as synonyms. Savings are viewed as an expenditure category so that total expenditure equals income.

3) It would be tempting to call $\tilde{x}_{gn}$ 'discretionary spending' on good g, but we prefer to adhere to the more common definition of discretionary spending as $x_{gn} - b_{gn}$.

4) The symbol $o_p(1)$ is defined as follows: the random variable x_m is $o_p(1)$ if for any $\varepsilon > 0$,

$$\lim_{m \to \infty} Pr(|x_m| > \varepsilon) = 0 .$$

We shall use the symbol $o_p(1)$ for both scalars, vectors and matrices.

5) Here and in what follows we ignore the supply side of the market for consumption goods, i.e. we assume that changing demands can be met without affecting prices. This allows us to equate demand with consumption.

6) It is somewhat tedious to show this; (5.1) can be derived more directly by using (B.18), (B.19), (B.48) and (B.49).

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     observations are missing.