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RESEARCH MEMORANDUM

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An inventory model: what is the influence of the shape of the lead time demand distribution?

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Abstract

In this paper the influence of the shape of the lead time demand distribution is studied for a specific inventory model which is described in a preceding paper by Heuts and van Lieshout [4]. This continuous review inventory model uses as lead time demand distribution a Schmeiser - Deutsch distribution (S-D distribution) [9]. In a previous paper [4] an algorithm was given to solve the decision problem.
In the literature attention is given to the following problem: what information on the demand during the lead time is necessary and sufficient to obtain "good" decisions. Using a (s,S) policy; Naddor [8] concluded that the specific form of the lead time demand distribution is negligible, and that only its first two moments are essential. For a simple (s,q) control system Fortuin [3] comes to the same conclusion. Both authors analysed the case with known lead times and with given demand distributions from the class of two parameter distributions. So in fact their results are obvious, as the lead time demand distributions resulting from their suppositions are all nearly symmetric. We shall demonstrate that the skewness of the lead time demand distribution in our inventory model is also an important measure, which should be taken into account, as the cost differences with regard to the case where this skewness measure is not used, can be considerable.

1. Introduction

In this paper we analyse an inventory model with stochastic lead time demand under the following assumptions [4]:

a. The system is of the continuous review type.
b. The order quantity is not restricted.
c. The purchase cost b(q) is a continuously differentiable function of the order quantity q.
d. The lead time demand distribution has distribution function F(z).
e. The expectation of the demand per unit of time is r.
f. The holding cost per unit inventory per unit of time is c_1.
g. Unfilled demand during the lead time is backlogged. The shortage cost per shortage unit per unit of time is c_2.

The criterion used is minimization of the average cost per unit ordered. The cost function is:

$$(1.1) \qquad K(x,q) = (c_1/(r.q)) \int_0^q \left[\int_0^{x+y} (x+y-z)f(z)dz \right] dy + (c_2/(r.q)) \int_0^q \left[\int_{x+y}^{\infty} (z-x-y)f(z)dz \right] dy + b(q)/q,$$

where:

$f(z)$: the density function of the demand during the lead time;

x : the reorder point expressed in terms of units of economic inventory;

q : the order quantity;

$b(q)$: the ordering cost. We assume that $b(q) = c_0 + q.a(q)$, with $a(q)$ a twice differentiable function.

We model the density function of the lead time demand through the so-called Schmeiser - Deutsch distribution [9], which is defined as follows:

$$(1.2) \qquad f(z) = \frac{1}{\ell_2 \ell_3} \left| \frac{\ell_1 - z}{\ell_2} \right|^{(1-\ell_3)/\ell_3}, t < z < p,$$

where,

$$t = \ell_1 - \ell_2 \ell_4^{\ell_3}; \; p = \ell_1 + \ell_2(1-\ell_4)^{\ell_3};$$

$$\ell_2, \ell_3 > 0, \; 0 < \ell_4 < 1, \; -\infty < \ell_1 < \infty.$$

This four-parameter type of distribution can take many different shapes, including U-shapeness. Table I shows the relation between the shape of the distribution and the parameters. On page 5 and 6 some figures representing S.D. density distribution, all with the <u>same</u> mean and variance value of 9 are given.

Table I: Different shapes for the S-D distribution

	$\ell_3 > 1$ (bell-shaped)	$\ell_3 = 1$	$\ell_3 < 1$ (U-shaped)
$\ell_4 < 0,5$	skewed to the right	uniform distributions	skewed to the left
$\ell_4 = 0,5$	symmetric	" "	symmetric
$\ell_4 > 0,5$	skewed to the left	" " " "	skewed to the right

Further it can be shown [9] that ℓ_1 and ℓ_2 satisfy the following relations:

(1.3)

$$\ell_2 = \sqrt{\frac{\sigma^2(2\ell_3 + 1)\,(\ell_3 + 1)^2}{(\ell_3+1)^2\,\{\ell_4^{2\ell_3+1} + (1-\ell_4)^{2\ell_3+1}\} - (2\ell_3+1)\,\{(1-\ell_4)^{\ell_3+1} - \ell_4^{\ell_3+1}\}^2}}$$

$$(1.4)\quad \ell_1 = \mu - \ell_2\,\{\frac{(1-\ell_4)^{\ell_3+1} - \ell_4^{\ell_3+1}}{\ell_3 + 1}\},$$

with μ and σ^2 as symbols for the expected value resp. variance for the lead time demand.

These results will be used in section 2.

Apart from the many different shapes the use of the S-D distribution is justified by the following interesting properties: The distribution function, the inverse distribution function and the conditional expectations, can be specified explicitly. Using gamma, Weibull or beta distribution, evaluation of the cost function, would require evaluation of incomplete gamma or beta functions, which is troublesome (see e.g. Burgin [1], Kottas and Lau [6], and Tadikamalla [10]. Reference [4] gives a solution procedure for the model with cost function K(x,q) given by (1.1) and lead time demand distribution represented by the S-D distribution. The optimisation method is a Newton-like algorithm.

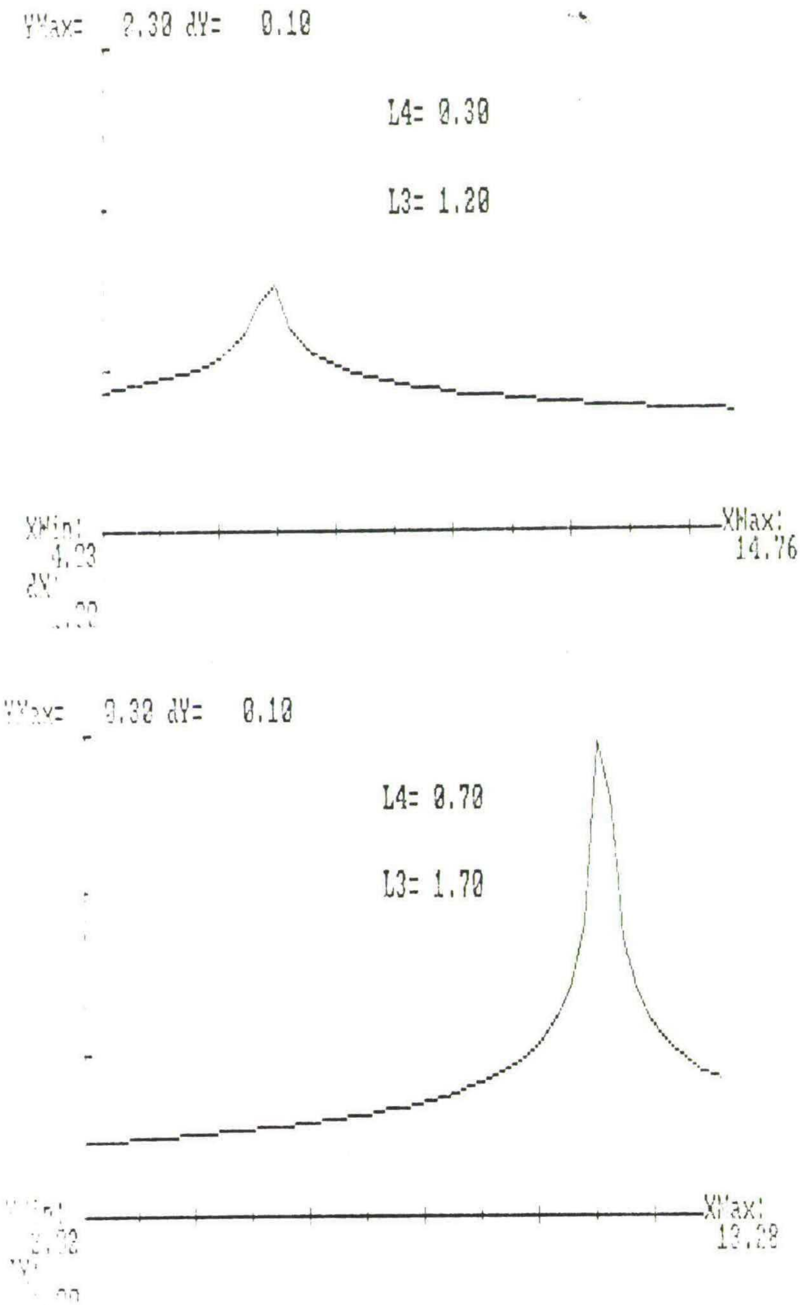
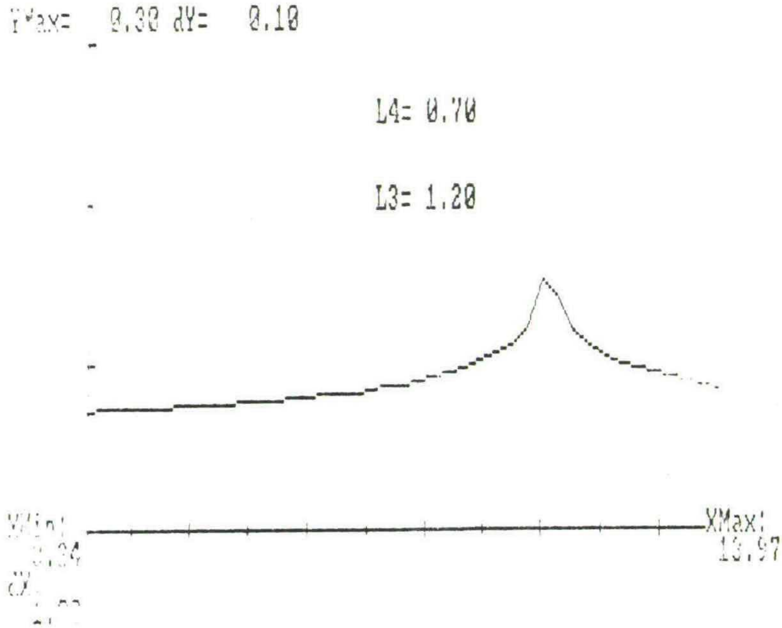


Figure 1: Graphs of S.D. distributions with an expected value of 9 and a variance of 9.



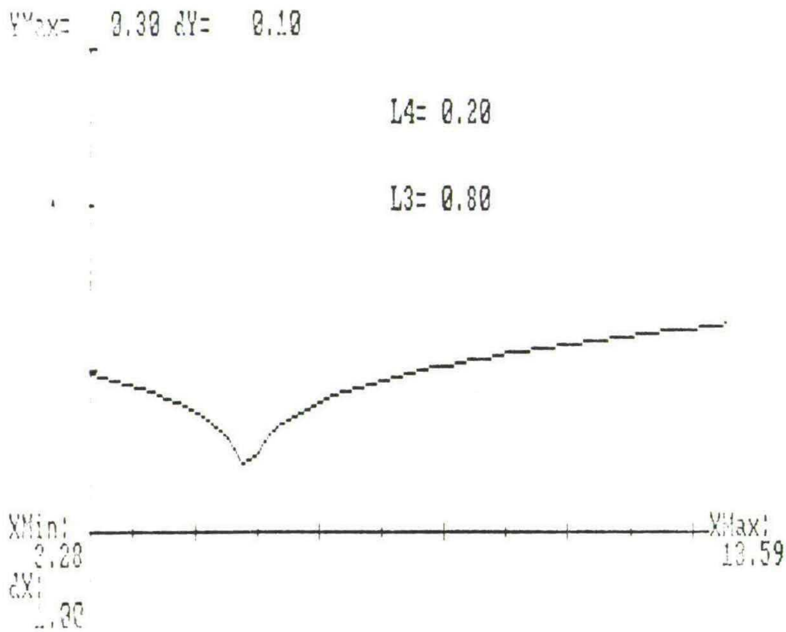


Figure 2: Graphs of S.D. distributions with an expected value of 9 and a variance of 9.

2. How important is the shape of the lead time demand distribution?

In several studies on inventory problems it is stated that the optimal decisions are strongly dependent on the mean and variance of the lead time demand and not on the specific shape of the distribution. (see e.g. Naddor [8] and Fortuin [3]). In this literature examples are constructed for which the optimal decisions are compared with those from models with the same mean and variance of the lead time demand but with different shapes. In these articles this comparison leads to neglible cost differences. However, it is remarkable that only one or two parameter distributions are compared, which are completely determined by their mean and variance. The examples in the article of Naddor [8] will be further analysed, but now with a four-parameter Schmeiser-Deutsch distribution for the lead time demand distribution. We will see that the optimal solution may significantly depend on the shape of the distribution even if the mean and variance are the same in two situations. In case of a S-D distribution this can be achieved by using the formulas (1.3) and (1.4). For a suitable comparison of the results, two transformations have to be done.

Firstly, as Naddor uses an (s, S) model and we use an (s, q) model, we have to consider the following relation: q = S-s. Secondly, in the article of Naddor the total costs per period are minimized, whereas in our model cost per unit ordered is minimized. It can be shown that the transformation of total cost per unit ordered to total cost per period is as follows:

$K_1 = r\,K_2$, where

K_1 = cost per period and K_2 = cost per unit ordered.

In table II Naddor's, recalculated results can be found and in table III the results of our model for various types of S-D distributions can be found. Both table II and III use the same fixed mean and variance. The values of the parameters which are not listed in the tables are: $c_0 = 20$, $c_1 = 1$, the service level is 90% resp. 99%, the average demand per period is 3, the variance of the demand per period is 3, the lead time is 3 periods.

Table II: Naddor's recalculated results

distribution	s^*	q^*	total cost per period
$c_2 = 9$			
Poisson	10	12	13.26
Beta	10	12	13.31
Uniform	10	12	13.13
2-point	11	9	12.61
$c_2 = 99$			
Poisson	15	11	17.50
Beta	15	11	17.70
Uniform	15	10	16.81
2-point	15	9	14.50

Table III: Results for fixed mean and variance as in Naddor's examples, but with different shapes of the S-D distribution

parameters: ℓ_3	ℓ_4	x^*	q^*	total cost per period
$c_2 = 9$				
0.8	0.2	9.0	12.6	12.64
0.8	0.8	8.9	13.1	13.04
2.5	0.2	8.6	13.9	13.51
2.5	0.8	9.0	12.1	12.13
$c_2 = 99$				
0.8	0.2	12.3	11.4	14.66
0.8	0.8	13.1	11.5	15.64
2.5	0.2	14.4	11.9	17.27
2.5	0.8	11.1	11.1	13.23

The results in table III clearly demonstrate that the value of the cost function in the optimum is much more dependent on the shape of the distribution than table II suggests. That is not surprising as the probability distributions which Naddor uses are more or less alike when the mean and variance are fixed. The probability distributions which Naddor uses concern the demand per period. Table IV gives some information about it.

Table IV: Central moments, skewness and kurtosis for the demand per period

	μ_1	μ_2	μ_3	μ_4	skewness	kurtosis
Poisson	3	3	3,1177	29,7	0,6	3,3
Beta	3	3	4,1569	31,5	0,8	3,5
Uniform	3	3	0	16,2	0	1,8

As Naddor uses a lead time of 3 periods, the demand during the lead time is the convolution of the demand per period. In the appendix a set of formulas is derived for computing the first four moments of the lead time demand given the first four moments of the demand per period and of the lead time.
Table V gives some information about the lead time demand distributions.

Table V: Central moments, skewness and kurtosis for the demand during the lead time

	μ_1	μ_2	μ_3	μ_4	skewness	kurtosis
Poisson	9	9	9,3531	251,1	0,3464	3,10
Beta	9	9	12,4707	256,5	0,4619	3,17
Uniform	9	9	0	210,6	0	2,60

By using a S-D distribution as lead time demand distribution, with the same mean and variance as the distributions in table V, and by varying the shape parameters ℓ_3 and ℓ_4 as in table III, we are able to calculate the skewness and kurtosis values for these situations (see table VI).

Table VI: Parameters, central moments, skewness and kurtosis for S-D distributions representing lead time demand

ℓ_1	ℓ_2	ℓ_3	ℓ_4	μ_1	μ_2	μ_3	μ_4	skewness	kurtosis
5,838	9,267	0,8	0,2	9	9	-8,33	154,83	-0,31	1,91
12,160	9,267	0,8	0,8	9	9	-116,68	605,86	-4,32	7,48
6,62	18,31	2,5	0,2	9	9	30,81	248,12	1,14	3,06
11,38	18,31	2,5	0,8	9	9	-30,81	248,14	-1,14	3,06

In Naddor's article the optimal decision variables s^* and S^* are derived via discrete dynamic programming. In our model the decision variables are assumed continuous as the S-D distribution is continuous.
To improve the comparability we have derived the optimal integer solutions in our model. These solutions are given in table VII and its consequences for the cost are calculated for the results in table III.

Table VII: Integer solutions for the results in table III

parameters: ℓ_3	ℓ_4	x^*	q^*	total cost per period
$c_2 = 9$				
0.8	0.2	9	13	12.65
0.8	0.8	9	13	13.04
2.5	0.2	9	14	13.53
2.5	0.8	9	12	12.13
$c_2 = 99$				
0.8	0.2	12	12	14.71
0.8	0.8	13	12	15.66
2.5	0.2	14	12	17.33
2.5	0.8	11	11	13.25

To test the influence which the shape of the lead time demand distribution has on the cost, more detailed information is needed, concerning the difference between on the one hand the cost of the optimal strategy, and on the other hand, the cost of a strategy which is based solely on the mean and variance of the lead time demand. For that purpose we shall formulate a null strategy, which is defined as the optimal strategy (x_0, q_0) in the case of a symmetric lead time demand distribution $(\ell_4 = 0.5)$ with a specific mean and variance.
So for every combination of mean and variance we have a null strategy.
In our simulation study we compare this null strategy with the optimal strategies for the following cases:

1) For a given cost structure the form of the lead time demand distribution is varied by means of varying the distribution parameters resulting from (1.3) and (1.4) given the mean and variance.

2) The cost structure is varied for the same cases. We measure the cost parameters in units inventory cost per unit per unit of time. For the analysis of the influence of the cost parameters on the above

inventory control system see [5]. In that paper it is proved that only the quotient of the order cost and the back sales cost influences the results. In this case we vary only the back sales cost.

For the above cases the optimal decisions x^* and q^* with its cost function K^* are determined, together with the cost $\hat{K}$ of the null strategy (x_0, q_0). The values of these cost are compared with each other.

We have the following formulas:

$$\min_{x,q} K(x,q)]_{(\mu = \mu_0;\ \sigma^2 = \sigma_0^2;\ \ell_4 = 0.5;\ \ell_3 = \ell_3)} = K(x_0, q_0) = K_0$$

$$\min_{x,q} K(x,q)]_{(\mu = \mu_0;\ \sigma^2 = \sigma_0^2;\ \ell_4 = \ell_4;\ \ell_3 = \ell_3)} = K(x^*, q^*) = K^*$$

$$K(x_0, q_0)]_{(\mu = \mu_0;\ \sigma^2 = \sigma_0^2,\ \ell_4 = \ell_4,\ \ell_3 = \ell_3)} = \hat{K}.$$

The results are summarized in table VIII.

Table VIII: Null strategy versus optimal strategy

$\mu_0 = 100,\ \sigma_0^2 = 400,\ r = 100,\ c_1 = 1$

$c_0 = 10,\ c_2 = 10$

$\ell_3 = 0.4$ $\quad x_0 = 110.1$ $\quad q_0 = 51.2$ $\quad K_0 = 0.6134$

ℓ_4	x^*/x_0	q^*/q_0	reduction in %
0.2	0.96	0.98	2.0
0.4	0.99	0.99	0.3
0.6	1.01	1.02	0.2
0.8	0.99	1.18	1.1

$\ell_3 = 1.8$ $x_0 = 106.2$ $q_0 = 57.7$ $K_0 = 0.6391$

ℓ_4	x^*/x_0	q^*/q_0	reduction in %
0.2	1.03	1.01	0.5
0.4	1.02	1.02	0.2
0.6	1.00	0.93	0.3
0.8	1.01	0.87	0.6

<u>$c_0 = 10,\ c_2 = 100$</u>

$\ell_3 = 0.4$ $x_0 = 122.2$ $q_0 = 46.3$ $K_0 = 0.6853$

ℓ_4	x^*/x_0	q^*/q_0	reduction in %
0.2	0.95	1.00	6.5
0.4	0.98	1.00	1.8
0.6	1.03	1.00	3.2
0.8	1.11	1.02	29.1

$\ell_3 = 1.8$ $x_0 = 130.9$ $q_0 = 48.8$ $K_0=0.7978$

ℓ_4	x^*/x_0	q^*/q_0	reduction in %
0.2	1.03	1.00	1.7
0.4	1.03	1.00	1.8
0.6	0.95	0.99	4.3
0.8	0.90	0.95	15.4

reduction in % $:= \dfrac{\hat{K} - K^*}{K^*} \cdot 100$

From table VIII it follows that a strategy which takes into account only the mean and variance of the lead time demand distribution is in many cases not optimal at all. Moreover, if distributions with a mode are compared with U-shaped distributions the differences will increase. From the table it also follows that differences in cost very much depend on a combination of the shape of the distribution and the cost parameters.

3. The influence of the skewness of the lead time demand distribution on the cost function

We also investigated the effect of left and right skewness on the cost function and came to the following conclusions:

i) $c_2 \gg c_1$:
If the shortage cost per unit per time unit is much greater than the holding cost per unit per time unit, then the optimal cost is lower for a left skewed distribution than it is for a right skewed one.

ii) $c_1 \ll c_2$:
In the reverse case the optimal cost is much lower for a right skewed distribution than for a left skewed one.

iii) $c_1 = c_2$:
When both costs are equal, then the skewness has little influence on the value of the optimal cost.

If the lead time demand distribution has an anti-mode (U-shaped distribution) we found that the above relations are strengthened in many cases.

Remarks:

It was not possible to determine the consequences for the minimal cost when, for example, a left skewed distribution with skewness $\alpha_3 = -2$ was changed into $\alpha_3 = -1$ or $\alpha_3 = -3$. This was also not possible for a change in the kurtosis values. The reason is probably that a change in the kurtosis must be reached by a change in ℓ_3 and ℓ_4 of the S-D distribution, which simultaneously changes the skewness and the quantiles of the S-D distribution.

Appendix*)

In this appendix the first four central moments of the lead time demand distribution will be derived in terms of the moments of the lead time $\underline{N}$ and of the demand per unit of time $\underline{x}_i(i=1,\dots,N)$. The stochastic lead time demand is defined by: $\underline{S}_{\underline{N}} = \underline{x}_1 + \underline{x}_2 + \dots + \underline{x}_{\underline{N}}$.
Assuming that $\underline{N}$ is a discrete stochastic variable and that $\underline{N}$, $\underline{x}_1$, $\underline{x}_2$,... are independent and the $\underline{x}_i$ are identically distributed and all moments exist, then we will show that the following relations hold:

$$(1)\begin{cases}
\mu_1(\underline{S}_{\underline{N}}) = \mu_1(\underline{N})\ \mu_1(\underline{x}) \\[2ex]
\mu_2(\underline{S}_{\underline{N}}) = \mu_1(\underline{N})\ \mu_2(\underline{x}) + \mu_2(\underline{N})\ \mu_1^2(\underline{x}) \\[2ex]
\mu_3(\underline{S}_{\underline{N}}) = \mu_3(\underline{N})\ \mu_1^3(\underline{x}) + \mu_1(\underline{N})\ \mu_3(\underline{x}) + 3\mu_2(\underline{N})\ \mu_2(\underline{x})\ \mu_1(\underline{x}) \\[2ex]
\mu_4(\underline{S}_{\underline{N}}) = \mu_1^4(\underline{x})\ \mu_4(\underline{N}) + \mu_4(\underline{x})\ \mu(\underline{N}) + 6\mu_2(\underline{N})\ \mu_1(\underline{N})\ \mu_1^2(\underline{x})\ \mu_2(\underline{x}) \\[2ex]
\quad + 4\ \mu_2(\underline{N})\ \mu_1(\underline{x})\ \mu_3(\underline{x}) + 3\ \mu_2^2(\underline{x})\ \{\mu_1^2(\underline{N}) - \mu_1(\underline{N}) + \mu_2(\underline{N})\} + \\[2ex]
\quad 6\ \mu_3(\underline{N})\ \mu_1^2(\underline{x})\ \mu_2(\underline{x}),
\end{cases}$$

where $\mu_n(\underline{x}) := E\{\underline{x}-E\{\underline{x}\}\}^n$, for $n > 1$, and the same for $\mu_n(\underline{N})$.
It can be shown that the results of Kottas and Lau [6,7] are not correct, as the third and fourth central moment which they derived for $\underline{S}_{\underline{N}}$ are false.

*) stochastic variables will be underlined.

When the lead time N is deterministic, then the above results reduce to:

$$(2)\begin{cases} \mu_1(\underline{S}_N) = N\ \mu_1(\underline{x}) \\ \mu_2(\underline{S}_N) = N\ \mu_2(\underline{x}) \\ \mu_3(\underline{S}_N) = N\ \mu_3(\underline{x}) \\ \mu_4(\underline{S}_N) = N\ \mu_4(\underline{x}) + 3\ \mu_2^2(\underline{x})\ \{N^2-N\} \end{cases}$$

In the next part of the appendix we will derive the formulas in (1) using generating functions.

<u>Derivation of the first four central moments of the lead time demand distribution using generating functions</u>

Let $\underline{x}$ be concentrated in $\{0,1,\ldots\}$ with point probability $p_k := P(\underline{x} = k)$, $k = 0,1,\ldots$. Then the generating function of $\underline{x}$ is defined as $P_{\underline{x}}(z) := P(z) := E\{z^{\underline{x}}\} = \sum_{k=0}^{\infty} p_k\ z^k$ for those values of z for which the expectation exists (the series converges absolutely).

<u>Lemma</u>

Let $\underline{x}$ be restricted to $\{0,1,2,\ldots\}$.

a) The probability distribution of $\underline{x}$ is completely determined by $P_{\underline{x}}$ of $\underline{x}$.

b) When P(z) of $\underline{x}$ exists for a z with $|z| > 1$ then we have for all n = 0,1,2,... :

$$P^{(n)}(z) = z^{-n}\ E\{\underline{x}(\underline{x}-1)\ldots(\underline{x}-n+1)\ z^{\underline{x}}\},\ |z| < 1,\ \text{where } P^{(n)}(z) := \frac{d^n}{dz^n}(P(z)).$$

In particular we have:

$$P^{(n)}(1) = E\ \{\underline{x}(\underline{x}-1)\ldots(\underline{x}-n+1)\}$$

$P^{(1)}(1) = E\{\underline{x}\}$, $P^{(2)}(1) = E\{\underline{x}(\underline{x}-1)\}$.

Now assume $\underline{S}_0 = 0$ and $\underline{S}_k = \underline{x}_1 + \ldots + \underline{x}_k$, $k=1,2,\ldots$, then $\underline{S}_N$ (formally defined by $\underline{S}_N = \underline{S}_n$ when $\underline{N}=n$) is the total demand during the lead time. The variables $\underline{N}$, $\underline{x}_1$, $\underline{x}_2,\ldots$ are independent and the $\underline{x}_i$ are identically distributed. Assume that the moment generating function M(s) of the $\underline{x}_i$ exists in an open internal around s=0, and let P be the generating function of $\underline{N}$.

Define $M^{(n)}(s)$ as $\frac{d^n}{ds^n}(M(s))$, then we have

$$E\{e^{s\,\underline{S}_N}\ \underline{N}=n\} = E\{e^{s\,\underline{S}_n}\} = \prod_{k=1}^{n} E\{e^{s\,\underline{x}_k}\} = (M(s))^n$$

$$E\{e^{s\,\underline{S}_N}\} = E\{E\{e^{s\,\underline{S}_N}\ \underline{N}\}\} = E\{(M(s))^{\underline{N}}\} = P(M(s)).$$

So, using the notation $\mu_n'(\underline{x}) = E\{\underline{x}^n\}$, we get

$$E(\underline{S}_N) = \frac{d}{ds}\{P(M(s))]_{s=0}\} = P^{(1)}(1)\,M^{(1)}(0) = \underline{\mu}(N)\,\underline{\mu}(x)$$

$$E(\underline{S}_N^2) = \frac{d^2}{ds^2}\{P(M(s))]_{s=0}\} = P^{(2)}(1)\,(M^{(1)}(0))^2 + P^{(1)}(1)\,M^{(2)}(0) =$$

$$\{\mu_2'(\underline{N}) - \mu(\underline{N})\}\,\mu^2(\underline{x}) + \mu(\underline{N})\,\{\mu_2'(\underline{x})\} =$$

$$\{\mu_2(\underline{N}) + \mu^2(\underline{N}) - \mu(\underline{N})\}\,\mu^2(\underline{x}) + \mu(\underline{N})\,\{\mu_2(\underline{x}) + \mu^2(\underline{x})\}, \text{ giving}$$

$$\mu_2(\underline{S}_N) = E(\underline{S}_N^2) - (E(\underline{S}_N))^2 = \mu(\underline{N})\,\mu_2(\underline{x}) + \mu_2(\underline{N})\,\mu^2(\underline{x}).$$

In an analogous way, we find after lengthly computations

$$\mu_3(\underline{S}_N) = \mu_3(\underline{N})\,\mu^3(\underline{x}) + \mu(\underline{N})\,\mu_3(\underline{x}) + 3\,\mu_2(\underline{N})\,\mu_2(\underline{x})\,\mu(\underline{x})$$

$$\mu_4(\underline{S}_N) = \mu^4(\underline{x})\,\mu_4(\underline{N}) + \mu_4(\underline{x})\,\mu(\underline{N}) + 6\,\mu_2(\underline{N})\,\mu(\underline{N})\,\mu^2(\underline{x})\,\mu_2(\underline{x})$$

$$+ 4\,\mu_2(\underline{N})\,\mu(\underline{x})\,\mu_3(\underline{x}) + 3\,\mu_2^2(\underline{x})\,\{\mu^2(\underline{N}) - \mu(\underline{N}) + \mu_2(\underline{N})\}$$

$$+ 6\,\mu_3(\underline{N})\,\mu^2(\underline{x})\,\mu_2(\underline{x}).$$

These results are confirmed by those of Carlson [2], who uses cumulant generating functions. Let the cumulants of $\underline{N}$, $\underline{x}$ and $\underline{S}_{\underline{N}}$ be denoted by $\kappa_i(\underline{N})$, $\kappa_i(\underline{x})$ and $\kappa_i(\underline{S}_{\underline{N}})$. Then Carlson has found the following relationships:

$$(3)\begin{cases} \kappa_1(\underline{S}_{\underline{N}}) = \mu_1(\underline{S}_{\underline{N}}) = \mu(\underline{N})\ \mu(\underline{x}) \\ \kappa_2(\underline{S}_{\underline{N}}) = \kappa_1(\underline{N})\ \kappa_2(\underline{x}) + \kappa_2(\underline{N})\ \kappa_1^2(\underline{x}) \\ \kappa_3(\underline{S}_{\underline{N}}) = \kappa_1(\underline{N})\ \kappa_3(\underline{x}) + 3\ \kappa_2(\underline{N})\ \kappa_2(\underline{x})\ \kappa_1(\underline{x}) + \kappa_3(\underline{N})\ \kappa_1^3(\underline{x}) \\ \kappa_4(\underline{S}_{\underline{N}}) = \kappa_1(\underline{N})\ \kappa_4(\underline{x}) + \kappa_2(\underline{N})\ (4\ \kappa_3(\underline{x})\ \kappa_1(\underline{x}) + 3\ \kappa_2^2(\underline{x})) \\ + \kappa_3(\underline{N})\ (6\ \kappa_2(\underline{x})\ \kappa_1^2(\underline{x})) + \kappa_4(\underline{N})\ \kappa_1^4(\underline{x}) \end{cases}$$

Using a well-known relationship between cumulants and central moments:

$$(4)\begin{cases} \kappa_1(\cdot) = \mu_1(\cdot) \\ \kappa_2(\cdot) = \mu_2(\cdot) \\ \kappa_3(\cdot) = \mu_3(\cdot) \\ \kappa_4(\cdot) = \mu_4(\cdot) - 3\ \mu_2^2(\cdot), \end{cases}$$

and combining the above results we find:

$$
(5)\begin{cases}
\mu_1(\underline{S}_{\underline{N}}) = \mu_1(\underline{N})\ \mu_1(\underline{x}) \\
\mu_2(\underline{S}_{\underline{N}}) = \mu_1(\underline{N})\ \mu_2(\underline{x}) + \mu_2(\underline{N})\ \mu_1^2(\underline{x}) \\
\mu_3(\underline{S}_{\underline{N}}) = \mu_1(\underline{N})\ \mu_3(\underline{x}) + 3\ \mu_2(\underline{N})\ \mu_2(\underline{x})\ \mu_1(x) + \mu_3(\underline{N})\ \mu_1^3(\underline{x}) \\
\mu_4(\underline{S}_{\underline{N}}) = 3\ \mu_1^2(\underline{N})\ \mu_2^2(\underline{x}) + 6\ \mu_1(\underline{N})\ \mu_2(\underline{N})\ \mu_2(\underline{x})\ \mu_1^2(\underline{x}) \\
+ \mu_1(\underline{N})\ \mu_4(\underline{x}) - 3\ \mu_1(\underline{N})\ \mu_2^2(\underline{x}) + 4\ \mu_2(\underline{N})\ \mu_3(\underline{x})\ \mu_1(\underline{x}) \\
+ 3\ \mu_2(\underline{N})\ \mu_2^2(\underline{x}) + 6\ \mu_3(\underline{N})\ \mu_2(\underline{x})\ \mu_1^2(\underline{x}) + \mu_4(\underline{N})\ \mu_1^4(\underline{x}).
\end{cases}
$$

It can be seen that the results of Carlson correspond with our results, but not with those of Kottas and Lau [6,7]. We have checked the solution procedure of Kottas and Lau and found errors in their derivation for $\mu_3(\underline{S}_{\underline{N}})$ and $\mu_4(\underline{S}_{\underline{N}})$. Details of a correct derivation using the solution procedure of Kottas and Lau can be obtained from the authors on request.

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