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A NOTE ON THE τ-VALUE AND τ-RELATED SOLUTION CONCEPTS

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A Note on the τ-Value and τ-Related Solution Concepts*

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Abstract

The *τ-value* is a solution concept for a special class of *cooperative games with transferable utilities* (TU-games). Other solution concepts can be defined and axiomatized in a similar way as the τ-value. We discuss an example of such a τ-related solution concept which is defined for all TU-games.

1 Introduction

A situation in which a set of players N can obtain certain payoffs by cooperation can be described by a *cooperative game with transferable utilities* (or simply a TU-game) being a function $v: 2^N \to \mathbb{R}$ such that $v(\emptyset) = 0$. We denote the class of all TU-games on N by $\mathcal{G}^N$. An *efficient allocation* for $v \in \mathcal{G}^N$ is a vector $x \in \mathbb{R}^N$ such that $\sum_{i \in N} x_i = v(N)$. An efficient *solution concept* for a subclass of games $\mathcal{G} \subset \mathcal{G}^N$ is a function $f: \mathcal{G} \to \mathbb{R}^N$ which assigns to every $v \in \mathcal{G}$ an efficient allocation $f(v) \in \mathbb{R}^N$.

In Tijs (1981) an efficient solution concept for a special class of TU-games is introduced, namely the *τ-value*. First the following two functions on $\mathcal{G}^N$ are defined. The *marginal contribution* is the function $M: \mathcal{G}^N \to \mathbb{R}^N$ given by

$$M_i(v) = v(N) - v(N \setminus \{i\}) \text{ for all } i \in N \text{ and } v \in \mathcal{G}^N.$$

The *minimal right* is the function $m: \mathcal{G}^N \to \mathbb{R}^N$ given by

$$m_i(v) = \max_{E \ni i} \left(v(E) - \sum_{j \in E \setminus \{i\}} M_j(v) \right) \text{ for all } i \in N \text{ and } v \in \mathcal{G}^N.$$

A game $v \in \mathcal{G}^N$ is called *quasi-balanced* if the following two conditions are satisfied

$$m(v) \leq M(v) \text{ and } \sum_{i \in N} m_i(v) \leq v(N) \leq \sum_{i \in N} M_i(v). \tag{1}$$

The class of all quasi-balanced games on N is denoted by Q^N. If $v \in Q^N$ then the allocations $m(v)$ and $M(v)$, respectively, can be seen as lower and upper bounds for the distribution of the payoffs over the players in N. The τ-value is the function $\tau: Q^N \to \mathbb{R}^N$ which assigns to every quasi-balanced game the unique efficient allocation on the line segment between $m(v)$ and $M(v)$, i.e., for every $v \in Q^N$ it holds that

$\tau(v) = m(v) + \alpha_v(M(v) - m(v))$,

where $\alpha_v = \begin{cases} \frac{v(N)-\sum_{i\in N} m_i(v)}{\sum_{i\in N} M_i(v)-\sum_{i\in N} m_i(v)} & \text{if } m(v) \neq M(v) \\ 0 & \text{else.} \end{cases}$

For every $v \in \mathcal{G}^N$, $k \in \mathbb{R}_+$ and $c \in \mathbb{R}^N$ the game $(kv + c)$ is given by $(kv + c)(E) = kv(E) + \sum_{i\in E} c_i$ for all $E \subset N$. In Tijs (1987) the following axiomatization of the τ-value is presented.

Theorem 1.1 (Tijs (1987)) *The efficient solution concept $f: Q^N \to \mathbb{R}^N$ is equal to the τ-value if and only if the following two conditions are satisfied:*

(i) *for every $v \in Q^N$ it holds that $f(v) = m(v) + f(v - m(v))$ (minimal right property);*

(ii) *for every $v \in Q_0^N := \{v \in Q^N \mid m(v) = 0\}$ the vector $f(v) \in \mathbb{R}^N$ is proportional to the vector $M(v) \in \mathbb{R}^N$. (restricted proportionality property).*

2 A τ-related solution concept for all TU-games

A disadvantage of the τ-value is that it does not exist for games that are not quasi-balanced. This is because there are games for which the marginal contribution and minimal right are inadequate as upper and lower bounds for the distribution of payoffs. But for such games other bounds can be appropriate. If we take two functions $L: \mathcal{G}^N \to \mathbb{R}^N$ and $U: \mathcal{G}^N \to \mathbb{R}^N$ such that for game v the conditions stated under (1) are satisfied with m and M replaced by L and U, respectively, then the values $L(v)$ and $U(v)$ can be seen as lower and upper bounds for the distribution of payoffs in game v. Let $\mathcal{G}^N(L, U) \subset \mathcal{G}^N$ denote the class of games for which the conditions under (1) are satisfied in terms of L and U. Then we define $t^{\{(L,U\}}: \mathcal{G}^N(L, U) \to \mathbb{R}^N$ as the function which assigns to every $v \in \mathcal{G}^N(L, U)$ the unique efficient allocation on the line segment between $L(v)$ and $U(v)$. (Thus $\tau = t^{\{m,M\}}$.) Moreover, if L and U satisfy the S-equivalence property* then $t^{\{L,U\}}$ can be axiomatized similarly as the τ-value by

*The function $f: \mathcal{G} \to \mathbb{R}^N$, satisfies the *S-equivalence* property on $\mathcal{G} \subset \mathcal{G}^N$ if for every $v, w \in \mathcal{G}$, $k \in \mathbb{R}_+$ and $c \in \mathbb{R}^N$ such that $w = kv + c$, it holds that $f(w) = kf(v) + c$.

replacing m and M in Theorem 1.1 by L and U, respectively. (The proof of this result is similar to the proof of Theorem 1.1 as given in Tijs (1987)[†].) We refer to solution concepts that can be obtained in this way as *τ-related solution concepts.*

Most τ-related solution concepts are only defined for specific subclasses of games that satisfy the conditions stated under (1). However, next we present a τ-related solution concept that is defined for all TU-games. As lower and upper bounds for the final payoff of a player we take the minimal and maximal contribution of that player to any coalition. Thus we define $\widehat{L}: \mathcal{G}^N \to \mathbb{R}^N$ and $\widehat{U}: \mathcal{G}^N \to \mathbb{R}^N$ by

$$\widehat{L}_i(v) = \min_{E \ni i}(v(E) - v(E \setminus \{i\})) \text{ for all } i \in N \text{ and } v \in \mathcal{G}^N,$$

and

$$\widehat{U}_i(v) = \max_{E \ni i} (v(E) - v(E \setminus \{i\})) \text{ for all } i \in N \text{ and } v \in \mathcal{G}^N.$$

The lower bound $\widehat{L}$ has been considered in Kikuta (1980), and the upper bound $\widehat{U}$ in Milnor (1952). This pair of functions forms reasonable upper and lower bounds for any game on N.

Theorem 2.1 *For every $v \in \mathcal{G}^N$ it holds that*

$$\widehat{L}(v) \leq \widehat{U}(v) \text{ and } \sum_{i \in N} \widehat{L}_i(v) \leq v(N) \leq \sum_{i \in N} \widehat{U}_i(v).$$

PROOF

Let $v \in \mathcal{G}^N$ and let $\pi: N \to N$ be a permutation on N. Further, let $P(i,\pi) := \{j \in N \mid \pi(j) < \pi(i)\}$, being the collection of players that precede player i in permutation π. Then we define $m^\pi(v) \in \mathbb{R}^N$ by

$$m_i^\pi(v) = v(P(i,\pi) \cup \{i\}) - v(P(i,\pi)) \text{ for all } i \in N.$$

It is easy to verify that (i) $\widehat{L}(v) \leq m^\pi(v)$, (ii) $\widehat{U}(v) \geq m_i^\pi(v)$, and (iii) $\sum_{i \in N} m_i^\pi(v) = v(N)$.

[†]In that proof only twice use is made of the specific formula's of $m(v)$ and $M(v)$. This is to show that for every $v \in \mathcal{G}^N$, $m(v - m(v))$ is the null vector, and to show that M satisfies the S-equivalence property.

From this it follows that $\widehat{L}(v) \leq \widehat{U}(v)$ and $\sum_{i \in N} \widehat{L}_i(v) \leq v(N) \leq \sum_{i \in N} \widehat{U}_i(v)$.

$\square$

Next we define the function $\widehat{f}: \mathcal{G}^N \rightarrow \mathbb{R}^N$ which assigns to every game $v \in \mathcal{G}^N$ the unique efficient vector on the line segment between $\widehat{L}$ and $\widehat{U}$. Since $\widehat{U}$ and $\widehat{L}$ satisfy the S-equivalence property the function $\widehat{f}$ is a τ-related solution concept, and thus can be axiomatized as in Theorem 1.1 by replacing m and M by $\widehat{L}$ and $\widehat{U}$, respectively.

Similarly as shown in Tijs (1981) for the τ-value, it can be shown that $\widehat{f}$ is continuous, and satisfies the anonimity and zero player properties‡. Besides these properties it turns out that the value that $\widehat{f}$ assigns to a TU-game is equal to the value that it assigns to its *dual* game.

Theorem 2.2 *Let $v \in \mathcal{G}^N$ and let $v^* \in \mathcal{G}^N$ be the dual game of v, i.e.,*

$$v^*(E) = v(N) - v(N \setminus E) \text{ for all } E \subset N.$$

Then $\widehat{f}(v) = \widehat{f}(v^*)$.

PROOF

Let $v \in \mathcal{G}^N$ and let v^* be the dual game of v. For every $i \in N$ it then holds that

$$\widehat{L}_i(v^*) = \min_{E \ni i} (v^*(E) - v^*(E \setminus \{i\})) = \min_{E \ni i} (v(N) - v(N \setminus E) - v(N) + v(N \setminus (E \setminus \{i\})))$$

$$= \min_{E \ni i} (v((N \setminus E) \cup \{i\}) - v(N \setminus E)) = \min_{F \ni i} (v(F) - v(F \setminus \{i\})) = \widehat{L}_i(v).$$

Similarly we can prove that $\widehat{U}(v^*) = \widehat{U}(v)$, and thus $\widehat{f}(v^*) = \widehat{f}(v)$.

$\square$

To conclude this section we show that for *convex* games the function $\widehat{f}$ coincides with the τ-value.

‡The function $f: \mathcal{G} \rightarrow \mathbb{R}^N$ satisfies the *anonimity* property on $\mathcal{G} \subset \mathcal{G}^N$ if for every permutation $\pi: N \rightarrow N$ it holds that $f_i(v) = f_{\pi(i)}(v)(\pi v)$ for all $i \in N$ and $v \in \mathcal{G}$, where $\pi v: 2^N \rightarrow \mathbb{R}$ is given by: $\pi v(E) = v(\bigcup_{i \in E} \pi(i))$ for all $E \subset N$.

The f function $f: \mathcal{G} \rightarrow \mathbb{R}^N$ satisfies the *zero player* property on $\mathcal{G} \subset \mathcal{G}^N$ if for every $v \in \mathcal{G}$ and every player $i \in N$ with $v(E) = v(E \setminus \{i\})$ for all $E \subset N$, it holds that $f_i(v) = 0$.

Proposition 2.3 *Let $v \in \mathcal{G}^N$ be convex, i.e., $v(E \cup F) + v(E \cap F) \geq v(E) + v(F)$ for all $E, F \subset N$. Then $\widehat{f}(v) = \tau(v)$.*

PROOF
Let $v \in \mathcal{G}^N$ be convex, and $i \in N$ and $F \supset E \ni i$.
By convexity of v it holds that

$$v(F) + v(E \setminus \{i\}) = v(E \cup (F \setminus \{i\})) + v(E \cap (F \setminus \{i\})) \geq v(E) + v(F \setminus \{i\}).$$

Thus $v(F) - v(F \setminus \{i\}) \geq v(E) - v(E \setminus \{i\})$ for all $F \supset E \ni i$.
From this it follows that

(i) $\widehat{U}_i(v) = \max_{E \ni i} (v(E) - v(E \setminus \{i\})) = v(N) - v(N \setminus \{i\}) = M_i(v)$, and

(ii) $\widehat{L}_i(v) = \min_{E \ni i} (v(E) - v(E \setminus \{i\})) = v(\{i\})$.

In Driessen and Tijs (1985) it is shown that for a convex game v it holds that $m_i(v) = v(\{i\})$ for all $i \in N$.
Thus $\widehat{f}(v) = \tau(v)$.

□

3 Other τ-related solution concepts

We conclude this note by mentioning some examples of other τ-related solution concepts. In van den Brink (1989) a reasoning is given why for some games the marginal contribution and minimal right can be seen as lower and upper bounds, respectively (so their roles are reversed compared to their roles in determining the τ-value). Consider, for example, the simple majority game v on $N = \{1, 2, 3\}$ which assigns the value 1 to all coalitions that contain two or three players, and the value zero to all other coalitions. In this game the 'grand coalition' N is a 'winning' coalition. If an individual player leaves the grand coalition then the remaining players still form a winning coalition. Thus the marginal contribution of every player equals zero. The minimal right of every player then equals one. Thus the marginal contribution and minimal right, respectively, can be seen as lower and upper bounds for the distribution of payoffs. The function $t^{\{M,m\}}: \mathcal{G}^N(M,m) \to \mathbb{R}^N$ that assigns to every $v \in \mathcal{G}^N(M,m)$ the unique efficient vector on the line segment between M and m, is a τ-related solution concept.

Other possible lower and upper bounds are the functions $\overline{L}: \mathcal{G}^N \to \mathbb{R}^N$ and $\overline{U}: \mathcal{G}^N \to \mathbb{R}^N$ given by

$$\overline{L}_i(v) = v(\{i\}) \text{ for all } i \in N \text{ and } v \in \mathcal{G}^N,$$

and

$$\overline{U}_i(v) = v(N) - \sum_{j \in N \setminus \{i\}} \overline{L}_j(v) = v(N) - \sum_{j \in N \setminus \{i\}} v(\{j\}) \text{ for all } i \in N \text{ and } v \in \mathcal{G}^N.$$

The function $t^{\{\overline{L},\overline{U}\}}: \mathcal{G}^N(\overline{L},\overline{U}) \to \mathbb{R}^N$ which assigns to every $v \in \mathcal{G}^N(\overline{L},\overline{U})$ the unique efficient vector on the line segment between $\overline{L}(v)$ and $\overline{U}(v)$, is equal to the *CIS-value* as considered in Driessen and Funaki (1991), i.e., $t_i^{\{\overline{L},\overline{U}\}}(v) = v(\{i\}) + \frac{v(N) - \sum_{j \in N} v(\{j\})}{\#N}$ for all $i \in N$ and $v \in \mathcal{G}^N(\overline{L},\overline{U})$.

The following theorem characterizes the class $\mathcal{G}^N(\overline{L},\overline{U})$. It also shows that for many games, in particular for superadditive games, the functions $t^{\{\overline{L},\overline{U}\}}$ and $t^{\{M,m\}}$ are related through duality.

Theorem 3.1 *Let $v \in \mathcal{G}^N$. Then $v \in \mathcal{G}^N(\overline{L},\overline{U})$ if and only if $v(N) \geq \sum_{i \in N} v(\{i\})$. Moreover, if $v(E) \geq \sum_{i \in E} v(\{i\})$ for all $E \subset N$, then $t^{\{\overline{L},\overline{U}\}}(v) = t^{\{M,m\}}(v^*)$, where v^* is the dual game of v.*

PROOF

Let $v \in \mathcal{G}^N$. It is easy to verify that $v \in \mathcal{G}^N(\overline{L},\overline{U})$ if and only if $v(N) \geq \sum_{i \in N} v(\{i\})$. Suppose that $v(E) \geq \sum_{i \in N} v(\{i\})$ for all $E \subset N$. We show that the lower and upper bounds of the CIS-value $t^{\{\overline{L},\overline{U}\}}(v)$ coincide with the corresponding bounds of $t^{\{M,m\}}(v^*)$. For every $i \in N$ we can derive that

$$M_i(v^*) = v^*(N) - v^*(N \setminus \{i\}) = v(N) - v(\emptyset) - (v(N) - v(\{i\})) = v(\{i\}) = \overline{L}_i(v)$$

and

$$m_i(v^*) = \max_{E \ni i} \left(v^*(E) - \sum_{j \in E \setminus \{i\}} M_j(v^*) \right) = \max_{E \ni i} \left(v(N) - v(N \setminus E) - \sum_{j \in E \setminus \{i\}} v(\{j\}) \right)$$

$$= v(N) - \min_{E \ni i} \left(v(N \setminus E) + \sum_{j \in E \setminus \{i\}} v(\{j\}) \right) = v(N) - \min_{F \not\ni i} \left(v(F) + \sum_{j \in N \setminus (F \cup \{i\})} v(\{j\}) \right)$$

Since $v(E) \geq \sum_{i \in E} v(\{i\})$ for all $E \subset N$ it holds that the minimum in the last expression is obtained for $F = \emptyset$. Thus

$$m_i(v^*) = v(N) - \sum_{j \in N \setminus \{i\}} v(\{j\}) = \overline{U}_i(v).$$

□

More τ-related solution concepts can be found in the survey paper by Tijs and Otten (1993).

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