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**EVALUATION OF MOMENTS OF QUADRATIC FORMS IN NORMAL VARIABLES**



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ABSTRACT

We present and describe a subroutine, called CUM, which calculates the cumulants and moments of a quadratic form in normal variables. That is, if we let $Q = x'Ax$, then CUM can calculate $E(Q^s)$, where x is an n x 1 vector of normally distributed variables with some mean and a positive definite (hence non-singular) covariance matrix , and A is an n x n symmetric matrix. A diskette containing the Fortran code of CUM and some test programmes is available on request.

### Language

Fortran 77

### Description and Purpose

The subroutine CUM calculates the exact cumulants and moments of the quadratic form x'Ax, where x is an nx1 vector of normally distributed variables with some mean $\mu$ and a positive definite (hence non-singular) variance-covariance matrix $\Omega$ and A is an nxn symmetric matrix. The routine is based on theory developed by Magnus(1978, 1979) for the central case where $\mu$=0, and Magnus(1986, Lemmas 2 and 3) for the general case.

### Parameter statements

The following parameters have been set in subroutines CUM and PARINT :

| | |
|---|---|
| ISPAR=77<br>&<br>ISDIM=12 | The dimensions of the array ISPRTN (ISPARxISDIM) where all possible partitions for a particular s are stored. This two dimensional array is set up by subroutine PARINT. |
| MAXMOM=24 | The maximum of s allowed. |

If 12<s≤24 is to be calculated, then both ISPAR and ISDIM should be increased.

Working out cumulants and moments for s>24 is possible. In that case not only ISPAR and ISDIM should be changed in the parameter statements, but also MAXMOM. Furthermore, the DATA statement in subroutine PARINT should be extended to include MAXMOM numbers. In the data statement the vector NUM has been set up to contain the number of all possible partitions of numbers up to 24. If a MAXMOM bigger than 24 is specified, the data statement for NUM should be extended accordingly. For a table containing the partitions for integers up to 100 see Hall(1986, p.38).

**Structure**

SUBROUTINE CUM(IMOM, N, LS, A, IEMU, EMU, IOMEGA, OMEGA, RKUM, RMOM, WORK1, WORK2, WORK3, VEC, IFAIL)

Formal Parameters

| | | | |
|---|---|---|---|
| IMOM | integer | input: | =0 if only cumulants are required<br>=1 if both cumulants and moments are required. |
| N | integer | input: | No of observations n |
| LS | integer | input: | order of the highest cumulant or moment required. |
| | | output: | unchanged unless LS>M where M=MIN(MAXMOM,ISDIM) in which case LS is set equal to M. |
| A | real array of dimension at least Nx(N+1)/2 | input: | symmetric matrix in the quadratic form x'Ax. Only the lower part of A is stored as:<br>$a_{11}, a_{21}, a_{22}, a_{31}, a_{32}, a_{33}$ etc. |
| IEMU | integer | input: | =0 if $\mu$=0<br>$\neq$0 if $\mu \neq 0$ |
| EMU | real array of dimension at least N | input: | vector $\mu$. Values required only if IEMU$\neq$0 though storage should be allocated to it. |
| IOMEGA | integer | input: | = -1 if $L^{-1}$ is supplied where $\Omega = LL'$, L lower triangular<br>= 1 if L is supplied where $\Omega = LL'$, L lower triangular<br>= 2 if $\Omega$ is supplied |
| OMEGA | real array of dimension at least Nx(N+1)/2 | input: | either L or $L^{-1}$ or $\Omega$ where only lower part of these are stored |

| | | | |
|---|---|---|---|
| RKUM | real array of dimension at least LS | output: | the required cumulants with i-th cumulant stored in RKUM(i). |
| RMOM | real array of dimension at least LS | output: | the required moments with i-th moment stored as RMOM(i). |
| WORK1 | real array of dimension at least Nx(N+1)/2 | | work space |
| WORK2 | real array of dimension at least Nx(N+1)/2 | | work space |
| WORK3 | real array of dimension at least Nx(N+1)/2 | | work space |
| VEC | real array of dimension at least N | | work space |
| IFAIL | integer | output: | a fault indicator where: |

0: no error
1: N ≤ 1
2: IOMEGA out of range
3: LS < 1
4: if IOMEGA=2 Ω not pos. definite

if IOMEGA=1 diagonal elements of L not all positive

if IOMEGA=-1 diagonal elements of $L^{-1}$ not all positive

5: if IOMEGA=2 or 1, L can't be inverted

if IOMEGA=-1, $L^{-1}$ can't be inverted

6: ISPAR in the parameter statement is too small

**Auxiliary Algorithms**

CUM calls various functions and subroutines listed below:

| | | |
|---|---|---|
| (F1) | FUNCTION INX(I,J): | picks out the appropriate element of a symmetric matrix stored in lower triangular form |
| (F2) | REAL*8 FUNCTION FACT(N): | calculates N! A real*8 function is used because large values of N are passed as the argument. |
| (S1) | SUBROUTINE PARINT: | constructs the matrix containing all partitions of an integer |
| (S2) | SUBROUTINE SEP: | decomposes A (pos. def.) into LL' (L lower triangular) and replaces A by L |
| (S3) | SUBROUTINE LOWINV: | inversion (in place) of a lower triangular matrix |
| (S4) | SUBROUTINE MULT1: | computes C=B'AB (A symmetric, B lower triangular) |

**Constants**

The DATA statement in CUM sets EPS = 1.0-11 as a small number. Any number with an absolute value below EPS will be treated as zero.

**Precision**

The version of the routines listed below is in double precision (Real*8). In order to change the program to single precision the following changes should be made:

(1) Change all IMPLICIT REAL*8 to IMPLICIT REAL*4

(2) Change the constants in the DATA statements to single precision versions.

(3) Change DSQRT, DABS to SQRT, ABS, in the statement functions appearing in the beginning of routines SEP and LOWINV.

**Time**

Using the VAX 6330 at the London School of Economics, typical CPU times for the double precision version are reported in Table 1 for different combinations of LS(4, 8 and 12) and n(10, 20, 30, 40 and 50). Other parameters were kept fixed at IMOM=1 and IEMU=1 as the variations in these did not affect the CPU time significantly. It should be pointed out that the effect of increase in LS is cumulative, i.e. when LS=4 all of the first 4 cumulants and moments are calculated while for LS=8 all first 8 cumulants and moments are evaluated by CUM.

**Table 1.** **Typical CPU times**

| n | LS=4 | LS=8 | LS=12 |
|---|---|---|---|
| 10 | 0.30 | 0.32 | 0.47 |
| 20 | 1.07 | 1.47 | 1.98 |
| 30 | 3.74 | 5.17 | 6.39 |
| 40 | 10.43 | 13.24 | 16.35 |
| 50 | 23.81 | 29.52 | 35.26 |

**Accuracy**

In order to check the accuracy of calculations we used CUM to evaluate the first 4 cumulants and moments of the $\chi^2$ distribution with 4 degrees of freedom. This was achieved by using an arbitrary (20x4) matrix R and by setting $A=R(R'R)^{-1}R'$ and $\Omega=I$. Setting IEMU=0 the first 4 cumulants and moments of $x'Ax$ were calculated.

The exact rth cumulant of a $\chi^2$ distribution with $\nu$ degrees of freedom can be calculated using the formula

$$\kappa_r = \nu \, 2^{r-1} \, (r-1)! \tag{1}$$

while the moments of a $\chi^2$ distribution with $\nu$ degrees of freedom can be worked out using its moment generating function which is

$$\mathrm{MGM}(t) = (1 - 2t)^{-\nu/2} \tag{2}$$

Table 2 allows the comparison between the exact results using (1) and (2) and the calculated values using CUM.

**Table 2. Accuracy of cumulants and moments of a $\chi^2$ distribution calculated by CUM**

| | Cumulants | | Moments | |
|---|---|---|---|---|
| r | Exact | Calculated | Exact | Calculated |
| 1 | 4 | 4.00000000 | 4 | 4.0000000 |
| 2 | 8 | 8.00000000 | 24 | 24.00000000 |
| 3 | 32 | 32.00000000 | 192 | 192.00000000 |
| 4 | 192 | 192.00000000 | 1920 | 1920.00000000 |

Examining Table 2 shows that in this example an accuracy of at least eight decimal points is achieved.

**Related Algorithms**

The algorithm for the evaluation of moments of ratios of quadratic forms in normal variables and related statistics by Magnus and Pesaran (1990).

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```
      SUBROUTINE CUM(IMOM,N,LS,A,IEMU,EMU,IOMEGA,
     +OMEGA,RKUM,RMOM,WORK1,WORK2,WORK3,VEC,IFAIL)
C
C        ALGORITHM AS??? APPL. STATIST. (1990).
C        CALCULATES CUMULANTS AND MOMENTS OF A
C        QUADRATIC FORM IN NORMAL VARIABLES
C
      IMPLICIT REAL*8(A-H,O-Z)
      PARAMETER(ISDIM=12,ISPAR=77,MAXMOM=24)
      DIMENSION A(*),OMEGA(*),WORK1(*),WORK2(*),WORK3(*)
      DIMENSION EMU(*),RKUM(*),RMOM(*),VEC(*)
      DIMENSION ISPRTN(ISPAR,ISDIM),TR(ISDIM)
      DATA ZERO,ONE,TWO,EPS/0.0D0,1.0D0,2.0D0,1.D-11/
      IFAIL=1
      IF(N.LE.1) RETURN
      IFAIL=2
      IF(IOMEGA.NE.-1.AND.IOMEGA.NE.1.AND.IOMEGA.NE.2) RETURN
      IFAIL=3
      IF(LS.LT.1) RETURN
      NN=N*(N+1)/2
      LS=MIN(LS,MAXMOM,ISDIM)
C
C        SET WORK2 = L WHERE OMEGA = LL'
C
      DO 10 I=1,NN
10    WORK2(I)=OMEGA(I)
      IF(IOMEGA.EQ.1.OR.IOMEGA.EQ.-1) THEN
        IFAIL=4
        DO 20 I=1,N
        R=WORK2(INX(I,I))
        IF(R.LT.EPS) RETURN
20      CONTINUE
      END IF
```

```
      IFAIL=0
      IF(IOMEGA.EQ.2) CALL SEP(WORK2,N,ONE,EPS,IFAIL)
      IF(IOMEGA.EQ.-1) CALL LOWINV(WORK2,N,ZERO,ONE,EPS,IFAIL)
      IF(IFAIL.NE.0) THEN
        IF(IOMEGA.EQ.2) IFAIL=4
        IF(IOMEGA.EQ.-1) IFAIL=5
        RETURN
      END IF
C
C       SET UP L'AL IN WORK1
C
      CALL MULT1(A,WORK2,WORK1,N,ZERO)
C
C       CALCULATE LINV*EMU IN VEC IF EMU IS NOT ZERO
C
      IF(IEMU.NE.0) THEN
        IF(IOMEGA.GT.0) THEN
          CALL LOWINV(WORK2,N,ZERO,ONE,EPS,IFAIL)
          IFAIL=IFAIL+4
          IF(IFAIL.NE.4) RETURN
        END IF
        DO 40 I=1,N
        SUM=ZERO
        DO 30 J=1,I
        IF(IOMEGA.GT.0) THEN
          R=WORK2(INX(I,J))
        ELSE
          R=OMEGA(INX(I,J))
        END IF
30      SUM=SUM+R*EMU(J)
40      VEC(I)=SUM
      END IF
```

```
C
C       CALCULATE CUMULANTS OF QUADRATIC FORM IN RKUM
C
      DO 130 IS=1,LS
      IF(IS.EQ.1) THEN
        DO 50 I=1,NN
50      WORK2(I)=WORK1(I)
      ELSE
        DO 70 I=1,N
        DO 70 J=1,I
        IJ=INX(I,J)
        SUM=ZERO
        DO 60 K=1,N
        IK=INX(I,K)
        KJ=INX(K,J)
60      SUM=SUM+WORK1(IK)*WORK2(KJ)
70      WORK3(IJ)=SUM
        DO 80 I=1,NN
80      WORK2(I)=WORK3(I)
      END IF
      SUM=ZERO
      DO 90 I=1,N
      RI=WORK2(INX(I,I))
90    SUM=SUM+RI
      TR(IS)=SUM
      IF(IEMU.NE.0) THEN
        SUM=ZERO
        DO 100 I=1,N
        TT=TWO
        DO 100 J=1,I
        IJ=INX(I,J)
        IF(I.EQ.J) TT=ONE
100     SUM=SUM+TT*VEC(I)*WORK2(INX(I,J))*VEC(J)
        RS=IS
```

```
         TR(IS)=TR(IS)+RS*SUM
       END IF
       TT=2**(IS-1)
       RKUM(IS)=TT*FACT(IS-1)*TR(IS)
C
C        CALCULATE MOMENTS OF QUADRATIC FORM (IF REQUIRED) IN RMOM
C
       IF(IMOM.EQ.1) THEN
         CALL PARINT(IS,ISPRTN,ISPAR,ISDIM,ISROW,IFAIL)
         IF(IFAIL.NE.0) THEN
           IFAIL=6
           RETURN
         END IF
         SUM=ZERO
         DO 120 I=1,ISROW
         RB=ONE
         RA=ONE
         DO 110 L=1,IS
         R2L=2*L
         K=ISPRTN(I,L)
         IF(K.NE.0) THEN
           RB=RB*FACT(K)*(R2L**K)
           RA=RA*(TR(L)**K)
         END IF
110      CONTINUE
         RB=(FACT(IS)*(TWO**IS))/RB
120      SUM=SUM+RA*RB
         RMOM(IS)=SUM
       END IF
130    CONTINUE
       IFAIL=0
       RETURN
       END
```

```
      SUBROUTINE PARINT(M,MPRTN,MRDIM,MDIM,MR,IFAIL)
C
C       CONSTRUCTS THE MR X M MATRIX "MPRTN" CONTAINING
C       ALL MR PARTITIONS OF THE INTEGER M
C
      PARAMETER(MAXMOM=24)
      DIMENSION MPRTN(MRDIM,MDIM)
      DIMENSION NUM(MAXMOM),IWORK(MAXMOM)
      DATA NUM/1,2,3,5,7,11,15,22,30,42,56,77,101,135,176,231,
     +297,385,490,627,792,1002,1255,1575/
      IFAIL=1
      IF(M.LT.1.OR.M.GT.MAXMOM.OR.M.GT.MDIM) RETURN
      IFAIL=2
      IF(NUM(M).GT.MRDIM) RETURN
      IFAIL=0
      N1=0
      N2=1
      N3=0
      MR=1
      M1=1
      L=0
      MPRTN(1,1)=1
      IF(M.EQ.1) RETURN
      DO 10 J=2,M
10    MPRTN(1,J)=0
      DO 99 K=2,M
      IF(N2.NE.0) THEN
        DO 30 I=1,N2
        MPRTN(MR+I,1)=0
        DO 20 J=2,M
20      MPRTN(MR+I,J)=MPRTN(I+N1,J)
30      MPRTN(MR+I,2)=MPRTN(MR+I,2)+1
      END IF
      IF(N3.NE.0) THEN
```

```
         L=0
         DO 80 I=N1+N2+1,MR
         DO 80 J=2,K-1
         IF(MPRTN(I,J).EQ.0) GO TO 80
         DO 40 JJ=1,M
40       IWORK(JJ)=MPRTN(I,JJ)
         IWORK(J)=IWORK(J)-1
         IWORK(J+1)=IWORK(J+1)+1
         IF(N2.NE.0.OR.L.NE.0) THEN
           DO 60 II=MR+1,MR+N2+L
           DO 50 JJ=1,M-1
50         IF(MPRTN(II,JJ).NE.IWORK(JJ)) GO TO 60
           IF(MPRTN(II,M).EQ.IWORK(M)) GO TO 80
60         CONTINUE
         END IF
         L=L+1
         DO 70 JJ=1,M
70       MPRTN(MR+N2+L,JJ)=IWORK(JJ)
80       CONTINUE
       END IF
       DO 90 II=1,MR
90     MPRTN(II,1)=MPRTN(II,1)+1
       N1=M1
       N3=N2+L
       N2=MR-N1
       M1=MR
       MR=MR+N3
99     CONTINUE
       RETURN
       END
```

```
      INTEGER FUNCTION INX(I,J)
C
C        PICKS OUT THE APPROPRIATE ELEMENT OF A SYMMETRIC
C        MATRIX STORED IN LOWER TRIANGULAR FORM
C
      IF(I.GE.J) THEN
        INX=I*(I-1)/2+J
      ELSE
        INX=J*(J-1)/2+I
      END IF
      RETURN
      END

      REAL*8 FUNCTION FACT(N)
C
C        CALCULATES N FACTORIAL
C
      FACT=1
      IF(N.LE.1) RETURN
      DO 10 I=2,N
      RI=I
10    FACT=FACT*RI
      RETURN
      END

      SUBROUTINE SEP(A,N,ONE,EPS,IFAIL)
C
C        DECOMPOSES A (POSITIVE DEFINITE) INTO LL'
C        (L LOWER TRIANGULAR) AND REPLACES A BY L
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(*)
      ZSQRT(H)=DSQRT(H)
      IFAIL=1
```

```
      DO 20 I=1,N
      DO 20 J=I,N
      IJ=INX(J,I)
      R=A(IJ)
      IF(I.NE.1) THEN
        DO 10 K=1,I-1
10      R=R-A(INX(I,K))*A(INX(J,K))
      END IF
      IF(I.EQ.J) THEN
        IF(R.LT.EPS) RETURN
        D=ONE/ZSQRT(R)
      END IF
20    A(IJ)=R*D
      IFAIL=0
      RETURN
      END

      SUBROUTINE LOWINV(A,N,ZERO,ONE,EPS,IFAIL)
C
C       INVERSION (IN PLACE) OF N X N LOWER TRIANGULAR MATRIX
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(*)
      ZABS(H)=DABS(H)
      IFAIL=1
      DO 20 I=1,N
      R=A(INX(I,I))
      IF(ZABS(R).LT.EPS) RETURN
      D=ONE/R
      DO 20 J=1,I
      IJ=INX(I,J)
      IF(I.NE.J) THEN
        SUM=ZERO
        DO 10 K=J,I-1
```

```
10        SUM=SUM+A(INX(I,K))*A(INX(K,J))
          A(IJ)=-SUM*D
        ELSE
          A(IJ)=D
        END IF
20      CONTINUE
        IFAIL=0
        RETURN
        END

        SUBROUTINE MULT1(A,B,C,N,ZERO)
C
C         FINDS C = B'AB (A=A', B LOWER TRIANGULAR)
C         ALL MATRICES N X N
C
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION A(*),B(*),C(*)
        DO 20 I=1,N
        DO 20 J=1,I
        IJ=INX(I,J)
        SUM=ZERO
        DO 10 K=I,N
        KI=INX(K,I)
        DO 10 L=J,N
        LJ=INX(L,J)
        KL=INX(K,L)
10      SUM=SUM+B(KI)*A(KL)*B(LJ)
20      C(IJ)=SUM
        RETURN
        END
```

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