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Abstract

In this paper we propose a simplicial algorithm to find a core element for balanced games without side payments. The algorithm subdivides an appropriate simplex into smaller simplices and generates from an arbitrarily chosen point a sequence of adjacent simplices of variable dimension. Within a finite number of iterations the algorithm finds a simplex yielding an approximating core element. If the accuracy of approximation is not satisfactory the algorithm can be restarted with a smaller mesh size in order to improve the accuracy.

Keywords: Core, triangulation, simplicial algorithm

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1. Introduction

It is well-known that a cooperative game need not to have an outcome which cannot be improved upon by any subset of players. In case of a game with side payment the core, consisting of all outcomes which cannot be improved, is nonempty if and only if the game is balanced, see Bondareva [4] and Shapley [8]. A core element in a balanced game with side payments can be easily calculated by solving a sequence of linear programming problems.

Games without side payments were introduced by Aumann and Peleg [2], and Aumann [1] developed the core concept for such games. Scarf [7] proved the nonemptiness of the core for such a game if it is balanced. Scarf gave a constructive proof based on the complementary pivoting technique introduced by Lemke and Howson [6]. Shapley [9] generalized the well-known Knaster-Kuratowski-Mazurkiwicz Theorem on the unit simplex in order to give a constructive proof of the nonemptiness of the core. In an arbitrary subdivision of the (unit) simplex into simplices, a sequence of adjacent simplices is generated, which is initiated at one of the corners of the big simplex. The terminal simplex yields an approximating core element.

In this paper we propose a simplicial algorithm which can be initiated at any point of the unit simplex. From that point the algorithm generates a sequence of adjacent simplices of varying dimension. The algorithm leaves the starting point along one out of $2^n - 2$ directions in case there are n players. This number corresponds to the number of proper coalitions in the game. The algorithm is based on the simplicial algorithm developed by Doup, van der Laan and Talman [5] for computing economic equilibria. Along the path of simplices generated by the algorithm, coalitions are added and sometimes deleted until a balanced set of coalitions has been found. Once such a set is obtained, an approximating core element has been found. If the accuracy of approximation at that point is not satisfactory, the algorithm can be restarted at that point with a smaller mesh size of the triangulation in order to improve the accuracy. Within a finite number of restarts any accuracy of approximation can be reached.

2. Balanced game and core

Let N denote the set $\{1,\ldots,n\}$ and 2^N the set of all nonempty subsets of N. We call the elements of N and the elements of 2^N, players and coalitions, respectively. A game is a pair (N,v) where v is a mapping from 2^N to the set of subsets of the n-dimensional euclidean space, R^N. The set $v(S)$ represents the set of payoff or utility vectors that the players of coalition S can ensure by themselves, regardless of the actions of players outside the coalition. For S in 2^N, let R^S denote the $|S|$-dimensional subspace of R^N with coordinates indexed by the elements of S. If $x \in R^N$ and $S \in 2^N$, then $x^S \in R^S$ will denote the projection of x on R^S.

Assumption 2.1: For each $S \in 2^N$, the set $v(S)$ satisfies

i) if $x \in v(S)$ and $x_i = y_i$ for all $i \in S$, then $y \in v(S)$,

ii) if $x \in v(S)$ and $y \leqq x$, then $y \in v(S)$,

iii) $v(S)$ is closed,

iv) $\{x^S | x \in v(S)\}$ is nonempty and bounded from above.

Without loss of generality we assume that each set $v(\{i\})$, $i \in N$, has been normalized to the half space $\{x | x_i \leqq 0\}$ and that the other $v(S)$'s have been shifted accordingly. The core of a game represents the set of feasible utility vectors that cannot be improved upon by any coalition.

Definition 2.1: The core of the game (N,v) is the set
$C(N,v) = \{x \in v(N) | \nexists\ S \in 2^N \text{ and } y \in v(S) \text{ such that } y_i > x_i \text{ for all } i \in S\}$.

Under Assumption 2.1, the core is a closed and bounded set but may, however, be empty. It is a well known fact that every balanced game has a nonempty core.

Let B be a collection of nonempty subsets of 2^N, and let $B_i = \{S \in B | i \in S\}$. The set B is said to be balanced if there exist nonnegative numbers δ_S, $S \in B$, such that

$$\Sigma_{S \in B_i} \delta_S = 1 \qquad \text{for all } i \in N.$$

A game (N,v) is said to be balanced if for every balanced set B

$\cap_{S \in B} v(S) \subset v(N)$.

Theorem 2.1 (Scarf [7]): Every balanced game has a nonempty core.

Let U be the (n-1)-dimensional subset of R^N defined by $U = \text{conv}\{-Mne(j) | j = 1, \ldots, n\}$ where e(j) is the j-th unit vector in R^N and the number $M > 0$ is such that $x \in v(S)$ implies $x_i < M$ for every $i \in S$. Let e be the n-vector of ones. The function $\tau: U \to R_+$ is defined by

$$\tau(u) = \max\{r \in R | u + re \in \cup_{S \subset N} v(S)\}.$$

Clearly, τ is a continuous function on U, for example see Berge [3]. For $S \in 2^N$ we now define the set C_S by

$$C_S = \{u \in U | u + \tau(u)e \in v(S)\}.$$

Since v(S) is closed, the set C_S is also closed. The algorithm will compute a point u* in U such that for some balanced collection B*

$$u^* \in \cap_{S \in B^*} C_S.$$

Then $x^* = u^* + \tau(u^*)e \in \cap_{S \in B^*} v(S) \subset v(N)$ and x* lies in the core since x* lies on the (upper) boundary of $\cup_{S \subset N} v(S)$.

Lemma 2.2 (Shapley [9]): For all $u \in U$, if $u \in C_S$ then $S \subset \{i \in N | u_i \neq 0\}$.

Proof: Let $u \in C_S$ and $T = \{i \in N | u_i \neq 0\}$. The lemma is trivial if T = N. So assume that $|T| < n$. Because $u_i = 0$ for all $i \notin T$, we have $\Sigma_{i \in T} u_i = -Mn$, so there exists a $k \in T$ for which $u_k < -M$. Since $u + \tau(u)e \in R^N_+$, we have $u_k + \tau(u) \geq 0$, and hence $\tau(u) > M$. On the other hand, $u + \tau(u)e \in v(S)$, so for every $j \in S$, $u_j + \tau(u) < M$. Therefore $u_j < 0$ for every $j \in S$, from which it follows that $S \subset T$. Q.E.D.

The lemma will guarantee the algorithm never hits the boundary of the set U.

3. The algorithm

To describe the algorithm, let p be an arbitrarily chosen starting point in the relative interior of U. Next, let s be a sign vector in R^N, i.e., $s_j \in \{0,-1,+1\}$ for all $j \in N$. We call a sign vector s feasible if s contains at least one -1 and one +1. For a feasible sign vector s let the subset A(s) of U be defined by

$$A(s) = \{u \in U | u_j/p_j = \max_h u_h/p_h \quad \text{if } s_j = -1$$

$$u_j/p_j = \min_h u_h/p_h \quad \text{if } s_j = +1\}.$$

Clearly, the dimension of A(s) is equal to $t = |I^0(s)| + 1$ where

$$I^0(s) = \{i \in N | s_i = 0\}.$$

In particular, if the sign vector s does not contain zeros then A(s) is a 1-dimensional set, being the line segment connecting p and the point p(s) in the boundary of U given by $p_j(s) = 0$ for all j with $s_j = +1$ and $p_j(s) = -Mnp_j/\Sigma_{s_h=-1}\, p_h$ for all j with $s_j = -1$. For n = 2 the subdivision of U into sets A(s) for an arbitrary p is illustrated in Figure 3.1. Next U is subdivided into n-dimensional simplices such that each A(s) is triangulated into t-dimensional simplices, for example see Doup, van der Laan and Talman [5]. A t-dimensional simplex or t-simplex σ can be represented by its t+1 vertices $w^1,\dots,w^{t+1}$. To each vertex w of the simplicial subdivision we assign a vector label a(S) corresponding to some fixed coalition S for which w lies in C_S, where $a_j(S) = 1 - |S|/n$ for $j \in S$ and $a_j(S) = -|S|/n$ for $j \notin S$. For g = t or t-1, let $\sigma(w^1,\dots,w^{g+1})$ be a g-simplex with vertices $w^1,\dots,w^{g+1}$ in A(s) for some feasible sign vector s. Let $a(S^j)$ be the vector label of vertex w^j, then we call σ s-complete if the system of linear equations

$$\Sigma_{j=1}^{g+1} \lambda_j \begin{bmatrix} a(S^j) \\ 1 \end{bmatrix} - \Sigma_{s_h\neq 0}\, \mu_h s_h \begin{bmatrix} e(h) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad (3.1)$$

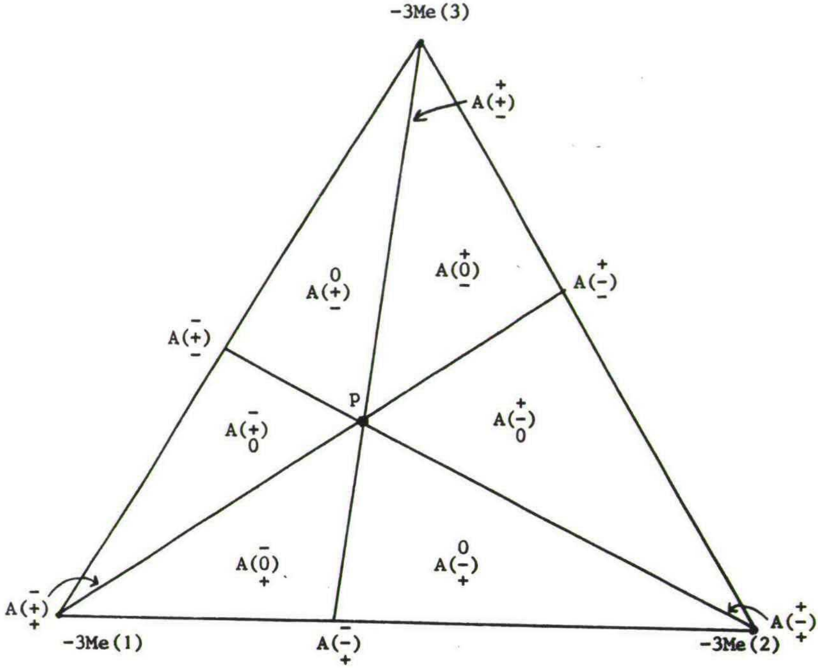

Figure 3.1

has a nonnegative solution λ^*_j, $j = 1,\ldots,g+1$, μ^*_h for $h \notin I^0(s)$. In particular, for $t = 1$ and $g = 0$, the zero-dimensional simplex consisting of the point p is s^0-complete with $s^0_i = +1$ if $i \in S^0$ and $s^0_i = -1$ if $i \notin S^0$, where S^0 is such that $a(S^0)$ is the vector label of p. If S^0 equals N, the point $p + \tau(p)e$ lies in the core. Suppose now that S^0 unequals N. Clearly, s^0 is feasible and does not contain zeros. Notice that there are 2^n-2 feasible sign vectors not containing zeros and that each such sign vector corresponds in this way to one of the 2^n-2 proper coalitions.

The starting point p of the algorithm is an end point of a uniquely determined 1-dimensional simplex $\sigma(p,p^1)$ in $A(s^0)$ and therefore $\sigma(p,p^1)$ is also s^0-complete. Let $a(S^1)$ be the vector label of p^1 then the algorithm is initiated by making a linear programming pivot step with $(a(S^1)^\top,1)^\top$ in the system

$$\lambda\begin{bmatrix}a(S^0)\\1\end{bmatrix} - \Sigma_{h=1}^{n}\ \mu_h s_h\begin{bmatrix}e(h)\\0\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix}. \tag{3.2}$$

If by this pivot step λ becomes first 0, the algorithm moves to the 1-simplex $\sigma(p^1,p^2)$ in $A(s^0)$ adjacent to $\sigma(p,p^1)$ and continues with making a pivot step with $(a(S^2)^\top,1)^\top$, where $a(S^2)$ is the vector label of p^2. Otherwise, one of the μ_h's must become first 0.

In general the algorithm generates, for varying feasible sign vectors s, a sequence of adjacent t-dimensional simplices in $A(s)$, having s-complete common facets. In each simplex $\sigma(w^1,\dots,w^{t+1})$ a pivot step is made in (3.1) in order to determine which variable becomes first 0. To prevent degeneracy we perturb the right hand side of (3.1). If for some $j \in \{1,\dots,t+1\}$, λ_j becomes 0, then the facet τ opposite vertex w^j of σ is also s-complete. If this facet does not lie in the boundary of $A(s)$, there is exactly one t-simplex $\bar{\sigma}$ in $A(s)$ having τ also as a facet. Let $\bar{w}$ be the vertex of $\bar{\sigma}$ opposite to τ, then the algorithm continues by making a pivot step in (3.1) with $(a(\bar{S})^\top, 1)^\top$, where $a(\bar{S})$ is the vector label of $\bar{w}$. If τ lies in the boundary of $A(s)$ then either τ is a (t-1)-simplex in $A(\bar{s})$ with $|I^0(\bar{s})| = |I^0(s)| - 1$ or τ lies in the boundary of U.

<u>Lemma 3.1</u>: An s-complete facet in $A(s)$ does not lie in the boundary of U.

<u>Proof</u>: Suppose that τ is an s-complete (t-1)-simplex in $A(s)$, lying in bd(U). Clearly, $x_i = 0$ for all $x \in \tau$ and all i for which $s_i = +1$. Let $y^1,\dots,y^t$ be the vertices of τ. Therefore $y_i^j = 0$ for all i for which $s_i = +1$. Let $a(S^j)$ be the vector label of vertex y^j, $j = 1,\dots,t$. According to Lemma 2.2, we must have $i \notin S^j$, $j = 1,\dots,t$, for all i for which $s_i = +1$. On the other hand, τ is s-complete. Therefore

$$\Sigma_{j=1}^{t}\ \lambda_j\begin{bmatrix}a(S^j)\\1\end{bmatrix} - \Sigma_{s_h\neq 0}\ \mu_h s_h\begin{bmatrix}e(h)\\0\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix} \tag{3.3}$$

has a nonnegative solution λ_j^*, $j = 1,\ldots,t$, μ_h^* for $h \notin I^0(s)$. For all i with $s_i = +1$, since $i \notin S^j$, we have that $a_i(S^j) = -|S^j|/n$, $j = 1,\ldots,t$. Consequently, for i with $s_i = +1$, the i-th equation at the solution of (3.3) is equal to

$$-\Sigma_{j=1}^{t} \lambda_j^* |S^j|/n - \mu_i^* = 0$$

and hence $\mu_i^* < 0$, which contradicts $\mu_i^* \geqq 0$. Q.E.D.

An s-complete facet of a t-simplex in A(s) in the boundary of A(s) therefore must be a (t-1)-simplex in $A(\bar{s})$ with $\bar{s}_j \neq 0$ for some $j \in I^0(s)$ and $\bar{s}_h = s_h$ for all $h \neq j$. Then the algorithm continues with making a pivot step with $\bar{s}_j(e(j)^\top, 0)^\top$.

Finally, by making a pivot step in (3.1) for a t-simplex σ in A(s), one of the μ_h's may become first 0. Because of the perturbation of the right hand side we may assume that only one of the μ_h's, say μ_k, becomes 0. If s_k is not the only positive or negative component of s, then τ is a facet of just one (t+1)-simplex $\bar{\sigma}$ in $A(\bar{s})$ where $\bar{s}_k = 0$ and $\bar{s}_h = s_h$ for $h \neq k$. Let $\bar{w}$ be the vertex of $\bar{\sigma}$ opposite to σ and let $a(\bar{S})$ be the vector label of $\bar{w}$, then the algorithm continues by making a pivot step with $(a(\bar{S})^\top, 1)^\top$. Suppose now that s_k is the only positive or negative component of s, then system (3.1) implies that when we disregard the perturbation also the other μ_h's must be zero and hence that the system

$$\Sigma_{j=1}^{t+1} \lambda_j \begin{bmatrix} a(S^j) \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

has a nonnegative solution λ_j^*, $j = 1,\ldots,t+1$. For $j = 1,\ldots,t+1$, let δ_j^* be defined by

$$\delta_j^* = \lambda_j^*/(\Sigma_{i=1}^{t+1} \lambda_i^* |S^i|/n),$$

then we get

$$\Sigma_{i \in S^j} \delta_j^* = 1 \text{ for } i = 1,\ldots,n.$$

Hence, the subset $B^* = \{S^1,\ldots,S^{t+1}\}$ is balanced. We remark that some of the λ_j^*'s and therefore some of the δ_j^*'s might be equal to zero. In that case we restrict ourselves to the balanced subset of coalitions S^j for which $\lambda_j^* > 0$. The point $u^* = \Sigma_{j=1}^{t+1} \lambda_j^* w^j$ can be considered to approximately lie in $\cap_{S \in B^*} C_S$ in the sense that u^* lies close to a point in C_S for any $S \in B^*$. Hence, the point $u^* + \tau(u^*)e$ can be taken as an approximating core element.

For $u \in U$, let $\tau^N(u)$ be defined by

$$\tau^N(u) = \max\{r \in R \mid u + re \in v(N)\}.$$

As a measure of accuracy of approximation at u^* one could consider the non-negative number $\tau(u^*) - \tau^N(u^*)$. If the latter number is too large one may restart the algorithm with a simplicial subdivision of U having a smaller mesh size and with p equal to u^*. Now, let $(G^1, G^2,\ldots)$ be a sequence of triangulations of U with mesh size tending to zero and let $u^{k^*} + \tau(u^{k^*})e$ be the approximating core element found with the algorithm applied for the triangulation G^k, $k = 1,2,\ldots$. Let B^{k^*} be the set of balanced coalitions corresponding to the vertices of the final simplex σ^k containing u^{k^*}, for all k. Then there exists a subsequence $k_1, k_2, \ldots$, such that $B^{k_j^*} = B^*$ for some balanced set B^* and $u^{k_j^*}$ converges to some u^* in U. Since the vertices of σ^k on this subsequence also converge to u^* and each C_S is closed, we obtain that $u^* \in \cap_{S \in B^*} C_S$ and hence that $u^* + \tau(u^*)e$ lies in the core, due to the balancedness of B^*. Notice that $\tau(u^*) - \tau^N(u^*)$ must be zero.

Because the number of simplices of any triangulation G^k in the sequence is finite and due to the perturbation to avoid degeneracy, the algorithm finds for each k within a finite number of iterations an approximating core element. Moreover, within a finite number of restarts, any accuracy of approximation will be reached.

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