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Abstract

By a cooperative game in coalitional structure or shortly coalitional game we mean the standard cooperative non-transferable utility game described by a set of payoffs for each coalition being a nonempty subset of the grand coalition of all players. It is well-known that balancedness is a sufficient condition for the nonemptiness of the core of such a cooperative non-transferable utility game. In this paper we consider non-transferable utility games in which for any coalition the set of payoffs depends on a permutation or ordering upon any partition of the coalition into subcoalitions. We call such a game a cooperative game in permutational structure or shortly permutational game. Doing so we extend the scope of the standard cooperative game theory in dealing with economic or political problems. The core of a permutational game consists of all payoff vectors attainable for the grand coalition such that no coalition has a partition with permutation on the elements of this partition through which the coalition can improve upon the payoffs of all players in the coalition. Introducing a concept of balancedness for ordered partitions of coalitions, we prove the nonemptiness of the core of a balanced non-transferable utility permutational game. Moreover we show that the core of a permutational game coincides with the core of a corresponding game in coalitional structure, but that balancedness of the permutational game does not imply balancedness of the corresponding coalitional game. This leads also to a weakening of the conditions for the existence of a nonempty core of a game in coalitional structure, induced by a game in permutational strucuture.

The proof of the nonemptiness of the core of a permututational game is based on a new intersection theorem on the unit simplex, which generalizes the well-known intersection theorme of Shapley. We also give a simplicial algorithm to compute an element of this core.

Key words: non-transferable utility game, balancedness, core, unit simplex, closed covering, intersection theorem

1 Introduction

It is well-known that balancedness is a sufficient condition for the nonemptiness of the core of the standard cooperative non-transferable utility game described by a set of payoffs for each coalition being a nonempty subset of the grand coalition of all players. In the following we call such a non-transferable utility game a game in coalitional structure or shortly coalitional game. We also speak about coalitional balancedness if we mean the well-known concept of balancedness of a family of coalitions. Scarf [9] gave a constructive proof of the nonemptiness of the core of a coalitionally balanced game in coalitional structure based on the complementary pivoting technique introduced by Lemke and Howson [6]. Shapley [11] generalized the intersection theorem of Knaster-Kuratowski-Mazurkiewicz on the unit simplex in order to prove the nonemptiness of the core, see also Shapley [10]. In Kamiya and Talman [4] a simplicial algorithm was proposed to find a core element of a coalitional game.

In this paper we generalize the concept of a cooperative non-transferable utility game to a non-transferable utility game in which for any nonempty coalition a (possibly empty) set of attainable payoffs is given for any permutation or ordering upon a partition of the coalition into subcoalitions. This dependency on an ordered partition of the coalition reflects the situation in which the payoff set of a coalition depends on the sequence in which the coalition is formed or on the hierarchy of the members of the coalition. In this way it is possible to differentiate between the players in the coalition, for instance between the player who takes the initiative to form the coalition, or is the most powerful player in the coalition, and the other players in the coalition. Another example is a situation when there is need for players to stand in a queue in order to get their payoff and waiting costs are involved. In such an environment it is necessary to differentiate the players in a coalition according to some ordering of subsets of the coalition. Also for scheduling problems the outcome depends very much on the ordering of machines (i.e., players) to be processed. In Nowak and Radzik [8] such a cooperative game is considered in case of transferable utilities and each subset consists of only one player, i.e., only permutations on the set of elements of a coalition are considered. For these TU games the value of the characteristic function depends on the ordering of the members of the coalition. Nowak and Radzik generalize the concept of the Shapley value for such games. We also refer to the work of Myerson [7], who used undirected graphs to model communication structures in cooperative games.

In this paper we consider non-transferable utility games with payoff sets for any permutation upon each possible partition of the coalitions. We call such a game a non-transferable utility game in permutational structure or shortly permutational game. The core of a permutational game consists of all payoff vectors attainable for the grand coalition such that there is no coalition having a partition and permutation on the elements of this partition through which the coalition can improve upon the payoffs of all players in the coalition. Generalizing the concept of coalitional balancedness to balancedness for ordered partitions of coalitions, we prove the nonemptiness of the core of a balanced permutational game by applying a new intersection theorem on the unit simplex. Moreover we prove that the core of a permutational game coincides with the core of a corresponding game in coalitional structure. We also give an example showing that balancedness of the permutational game does not imply balancedness of the corresponding coalitional game. This therefore leads to a weakening of the conditions for the existence of a nonempty core of a game in coalitional structure.

In Section 2 we introduce the concept of non-transferable utility permutational games. We also define for any permutational game a corresponding coalitional game and show that the core of the permutational game coincides with the core of the corresponding coalitional game. In Section 3 we define the concept of permutational balancedness and show that permutational balancedness of a permutational game does not imply coalitional balancedness of the corresponding coalitional game. In Section 4 we prove that balancedness of a permutational game is a sufficient condition for the nonemptiness of the core. This proof follows from a new intersection theorem on the unit simplex. If the corresponding coalitional game is not balanced, the nonemptiness of the core of this game follows from the nonemptiness of the core of the permutational game. In Section 5 we give a simplicial algorithm to compute a core element of permutational games. In Section 6 we make some concluding remarks.

2 Permutational games

In an n-player cooperative non-transferable utility game introduced by Aumann and Peleg [2] each nonempty subset of players, called a coalition, can obtain any vector out of a certain subset of $\mathbf{R}^n$ as payoff vector. An attainable payoff vector lies in the core of the game if no coalition can improve upon this vector, see Aumann [1]. In this paper we introduce a cooperative non-transferable utility game in which the set of payoff vectors of a coalition is allowed to depend on the permutation or ordering on a partition of subcoalitions of the players in the coalition.

The set $\{1,\ldots,n\}$ of the n players in the game is denoted by $\mathcal{N}$, while for positive k the set of indices $\{1,\ldots,k\}$ is denoted by I_k. For a nonempty subset S of $\mathcal{N}$, called a coalition of players, let P_S^t be a partition $\{S_1,\ldots,S_t\}$ of S into t subcoalitions of S and let $\pi(P_S^t) = (\pi_1(P_S^t),\ldots,\pi_t(P_S^t))$ denote a permutation or ordering of the elements of P_S^t. In the sequel a partition into t subcoalitions is called a t-partition and a permutation $\pi(P_S^t)$ on a t-partition of S is called an ordered t-partition of S. Let $\mathcal{P}^{\mathcal{N}}$ denote the set of all ordered partitions of subsets of $\mathcal{N}$, i.e.,

$$\mathcal{P}^{\mathcal{N}} = \{\pi(P_S^t) \mid P_S^t = \{S_1,\ldots,S_t\},\ 1 \leq t \leq s,\ S \subset \mathcal{N},\ S \neq \emptyset\},$$

where $s = |S|$ denotes the number of elements of the set S. For some coalition $S \subset \mathcal{N}$ and permutation $\pi(P_S^t)$ of a t-partition of S, we define the n-vector $m^{\pi(P_S^t)}$ by

$$m_j^{\pi(P_S^t)} = 0, \text{ if } j \notin S$$

and

$$m_j^{\pi(P_S^t)} = \frac{2(t-r+1)}{t(t+1)s_r}, \text{ if } j \in \pi_r(P_S^t),$$

where $s_r = |\pi_r(P_S^t)|$. Observe that $\sum_{j=1}^n m_j^{\pi(P_S^t)} = 1$. Furthermore, let m denote the vector all of whose components are equal to n^{-1}, i.e., $m = m^{\pi(P_{\mathcal{N}}^1)}$.

Example 2.1
Take $n = 3$ and consider the ordered 2-partition $\pi(P_{\{1,2\}}^2) = (\{1\}, \{2\})$ of the subset $\{1,2\}$. Then $m^{\pi(P_S^t)} = (\frac{2}{3}, \frac{1}{3}, 0)^\top$. For the ordered 3-partition $\pi(P_{\mathcal{N}}^3) = (\{1\}, \{2\}, \{3\})$ we obtain $m^{\pi(P_{\mathcal{N}}^3)} = (\frac{1}{2}, \frac{1}{3}, \frac{1}{6})^\top$. The ordered 2-partition $\pi(P_{\mathcal{N}}^2) = (\{1,2\}, \{3\})$ of $\mathcal{N}$ gives $m^{\pi(P_{\mathcal{N}}^2)} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})^\top$ and the ordered 2-partition $\pi(P_{\mathcal{N}}^2) = (\{3\}, \{1,2\})$ of $\mathcal{N}$ gives $m^{\pi(P_{\mathcal{N}}^2)} = (\frac{1}{6}, \frac{1}{6}, \frac{2}{3})^\top$. Observe that only the components $j \in S$ of the vector $m^{\pi(P_S^t)}$ get a positive weight, the weights of two components is equal if they are in the same subset of the partition and that the total weight of the components in some subset becomes greater if the subset has a higher priority in the ordering.

Definition 2.2 Permutational Game
A non-transferable utility game in permutational structure or permutational game with n players is a function V from $\mathcal{P}^{\mathcal{N}}$ to the collection of subsets of $\mathbf{R}^n$ satisfying that for every $\pi(P_S^t) \in \mathcal{P}^{\mathcal{N}}$, the set $V(\pi(P_S^t)) \subset \mathbf{R}^n$ is a cylinder in the sense that for any two vectors u and v in $\mathbf{R}^n$ with $u_i = v_i$ for all $i \in S$, it holds that $u \in V(\pi(P_S^t))$ if and only if $v \in V(\pi(P_S^t))$.

In the sequel we denote a permutational game with n players and function V by the pair $(\mathcal{P}^{\mathcal{N}}, V)$. We call V the payoff function of the game $(\mathcal{P}^{\mathcal{N}}, V)$. If $u \in V(\pi(P_S^t))$ for some t-partition $\{S_1, \ldots, S_t\}$ of the coalition S, the members of S can guarantee themselves a payoff u_i for member $i \in S$, independent of what the players outside the coalition do, by agreeing on the permutation $\pi(P_S^t)$ of the t-partition $P_S^t = \{S_1, \ldots, S_t\}$ of S. In case S is the grand coalition $\mathcal{N}$, $V(\pi(P_{\mathcal{N}}^t))$ denotes the set of payoff vectors the whole set of players can guarantee itself when the players coordinate according to the permutation $\pi(P_{\mathcal{N}}^t)$. For ease of notation we define for any $S \subset \mathcal{N}$ the set of payoffs $V(S)$ by $V(S) = V(\pi(P_S^1))$, i.e., $V(S)$ is the set of payoff vectors the coalition S can guarantee itself without partitioning itself into subcoalitions.

For any permutational game $(\mathcal{P}^{\mathcal{N}}, V)$, let the function V' from the collection of subsets of $\mathcal{N}$ to the collection of subsets of $\mathbf{R}^n$ be defined by

$$V'(S) = \cup_{t=1}^{|S|} \cup_{\pi(P_S^t)} V(\pi(P_S^t)),\ S \subset \mathcal{N}.$$

Then the function V' induces a non-transferable utility n-player game in coalitional structure, denoted by $(\mathcal{N}, V')$. Observe that $V(S) \subset V'(S)$, but that generally $V'(S)$

is not equal to $V(S)$. Moreover in Definition 2.2 we allow for empty payoff sets. Hence, it might be possible that some of the payoff sets $V'(S)$ are also empty. In the next example we illustrate the concept of permutional game and give also the corresponding coalitional game.

Example 2.3
Take $n = 3$. Then we have to consider three subsets consisting of one player, three subsets consisting of two players, and the grand coalition of all the three players. In case of a one-player coalition $S = \{i\}$, t can only be equal to 1 and we have $P_S^1 = \{\{i\}\}$ with payoff set $V(\{i\})$ and $V'(\{i\}) = V(\{i\})$ for all $i \in \mathcal{N}$. In case of a two-player coalition $S = \{i, j\}$ we have to consider two partitions, namely the 1-partition $P_S^1 = \{\{i, j\}\}$ and the 2-partition $P_S^2 = \{\{i\}, \{j\}\}$. The first case gives the payoff set $V(S)$. In the second case we have payoff sets for both permutations $\pi(P_S^2) = (\{i\}, \{j\})$ and $\pi(P_S^2) = (\{j\}, \{i\})$. So, totally there are three payoff sets for the two-player coalition, for instance

$$V(ij) = \{x \in \mathbf{R}^3 \mid 3x_i + 2x_j \leq 6\},$$

$$V(i, j) = \{x \in \mathbf{R}^3 \mid x_i + 2x_j \leq 3\},$$

and

$$V(j, i) = \{x \in \mathbf{R}^3 \mid 4x_i + x_j \leq 4\},$$

where $V(ij)$ denotes $V(\pi(P^1_{\{i,j\}}))$ and $V(i, j)$ denotes $V(\pi(P^2_{\{i,j\}}))$ with $\pi(P^2_{\{i,j\}}) = (\{i\}, \{j\})$. The set $V'(\{i, j\})$ is the union over these three sets. The projection of the three sets in the (x_i, x_j)-plane is given in Figure 1. The shaded area in this figure is the projection of the set $V'(\{i, j\})$ in the (x_i, x_j)-plane. For the grand coalition we have one payoff set for the 1-partition $P_{\mathcal{N}}^1 = \{\{i, j, k\}\}$, two payoff sets for any of the three 2-partitions $P_{\mathcal{N}}^2 = \{\{i, j\}, \{k\}\}$ and six payoff sets for the unique 3-partition $P_{\mathcal{N}}^3 = \{\{i\}, \{j\}, \{k\}\}$. So, totally there are 13 payoff sets for the grand coalition in a permutational game with three players. The payoff set of the grand coalition in the corresponding coalitional game $(\mathcal{N}, V')$ is the union over all these 13 payoff sets.

The core of the corresponding coalitional game $(\mathcal{N}, V')$, denoted by $C(\mathcal{N}, V')$, is as usual defined by the set of vectors $u \in V'(\mathcal{N})$ such that there do not exist a coalition $S \subset \mathcal{N}$ and a vector $v \in V'(S)$ such that $v_i > u_i$ for all $i \in S$. Analogously we say that a payoff vector u is in the core of the permutational game if $u \in V'(\mathcal{N})$ and there is no coalition S and permutation $\pi(P_S^t)$ of a t-partition of S in which the coalition S can improve upon u.

Definition 2.4 Core of a Permutational Game
The core of a non-transferable utility permutational game $(\mathcal{P}^{\mathcal{N}}, V)$ is the set of vectors $u \in \mathbf{R}^n$ satisfying that $u \in V'(\mathcal{N})$ and there do not exist a coalition S with ordered partition $\pi(P_S^t) \in \mathcal{P}^{\mathcal{N}}$ and a vector $v \in V(\pi(P_S^t))$ such that $v_i > u_i$ for all $i \in S$.

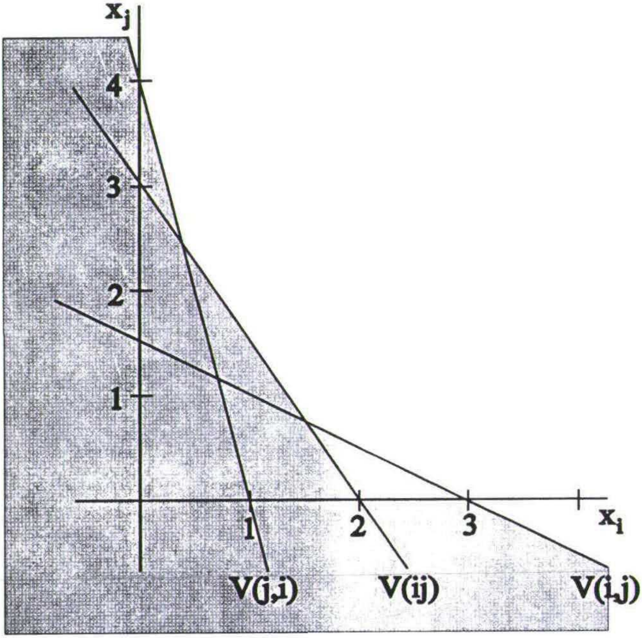

Figure 1: Example 2.3, the projection of the payoff sets of coalition $\{i, j\}$

Observe that a core element is an element of $V'(\mathcal{N})$ because any vector u lying in a set $V(\pi(P^t_{\mathcal{N}}))$ of some permutation of some t-partition $P^t_{\mathcal{N}}$ of the grand coalition is attainable and hence the payoff set of the grand coalition is not restricted to the set $V(\mathcal{N})$. In the sequel we denote the core of a permutational game $(\mathcal{P}^{\mathcal{N}}, V)$ by $C(\mathcal{P}^{\mathcal{N}}, V)$. Now we have the following lemma.

Lemma 2.5 Equivalence of the Cores
For any permutational game $(\mathcal{P}^{\mathcal{N}}, V)$ and the corresponding coalitional game $(\mathcal{N}, V')$ it holds that $C(\mathcal{P}^{\mathcal{N}}, V) = C(\mathcal{N}, V')$.

Proof.
For some $u \in \mathbf{R}^n$, first suppose $u \notin C(\mathcal{N}, V')$. Then there exists a coalition $S \subset \mathcal{N}$ and a vector $v \in \mathbf{R}^n$ such that $v \in V'(S)$ and $v_i > u_i$ for all $i \in S$. By the definition of $V'(S)$ this implies that there is some ordered partition $\pi(P^t_S)$ such that $v \in V(\pi(P^t_S))$. Hence $u \notin C(\mathcal{P}^{\mathcal{N}}, V)$. Secondly, suppose that $u \notin C(\mathcal{P}^{\mathcal{N}}, V)$. Then there exist an ordered partition $\pi(P^t_S)$ of some coalition S and some vector $v \in V(\pi(P^t_S))$ such that $v_i > u_i$ for all $i \in S$. By definition we have that $v \in V'(S)$. Hence $u \notin C(\mathcal{N}, V')$. Q.E.D.

3 Balanced permutational games

The core of a non-transferable utility permutational game might be empty. However, it will be shown that the core is nonempty if the permutational game satisfies some balancedness condition and every set $V(\pi(P^t_S))$, $\pi(P^t_S) \in \mathcal{P}^{\mathcal{N}}$, is comprehensive, closed and bounded from above in its projection space $\mathbf{R}^S$ defined by $\mathbf{R}^S = \{(x_i)_{i \in S} | x \in \mathbf{R}^n\}$. The balancedness condition differs from the well-known concept of balancedness of coalitions, in the sequel to be called coalitional balancedness. In this section we introduce the concept of permutational balancedness for ordered partitions of coalitions and define the related concept of a permutationally balanced game. Moreover we show by an example that permutational balancedness of the permutational game does not imply coalitional balancedness of the corresponding coalitional game. Since it will be proved in Section 4 that balancedness of the permutational game is sufficient for the nonemptiness of the core, it also follows that it is sufficient for the nonemptiness of the core of a coalitional game induced by a permutational game that the underlying permutational game is balanced.

Definition 3.1 Permutational Balancedness
A family $\mathcal{P} = \{\pi^1(P^{t^1}_{S^1}), \ldots, \pi^k(P^{t^k}_{S^k})\}$ of k ordered partitions in $\mathcal{P}^{\mathcal{N}}$ is permutationally balanced if there exist positive numbers λ^*_j, $j = 1, \ldots, k$, such that

$$\sum_{j=1}^{k} \lambda^*_j m^{\pi^j(P^{t^j}_{S^j})} = m.$$

Permutational balancedness of a family $\mathcal{P} = \{\pi^1(P^{t^1}_{S^1}), \ldots, \pi^k(P^{t^k}_{S^k})\}$ of k ordered partitions in $\mathcal{P}^{\mathcal{N}}$ can be interpreted as follows. In the ordered partition $\pi^j(P^{t^j}_{S^j}) = (\pi^j_1(P^{t^j}_{S^j}), \ldots, \pi^j_{t_j}(P^{t^j}_{S^j}))$, $j \in \{1, \ldots, k\}$, the power or weight of subcoalition $\pi^j_h(P^{t^j}_{S^j})$ is a fraction $\frac{t_j - h + 1}{t_j}$ of the power of subcoalition $\pi^j_1(P^{t^j}_{S^j})$, while the total weight of a subcoalition is equally divided over the members of the subcoalition. This is reflected in the vector $m^{\pi^j(P^{t^j}_{S^j})}$. Now the family $\mathcal{P}$ is permutationally balanced if the ordered partitions $\pi^j(P^{t^j}_{S^j})$, $j = 1, \ldots, k$, can be assigned weights λ^*_j in such a way that the total power of every player $i \in \mathcal{N}$ is the same and therefore equal to $m_i = \frac{1}{n}$. Geometrically it means that $\mathcal{P}$ is permutationally balanced if and only if the vector m lies in the convex hull of the vectors $m^{\pi^j(P^{t^j}_{S^j})}$, $j = 1, \ldots, k$. Notice that in Definition 3.1 it must hold that $\sum_{j=1}^k \lambda^*_j = 1$.

Example 3.2
Take $n = 3$. Then the family $\{\pi^1, \pi^2, \pi^3\}$ of two ordered 2-partitions and one 1-partition given by $\pi^1 = (\{1\}, \{2\})$, $\pi^2 = (\{2\}, \{3\})$ and $\pi^3 = (\{3\})$ is permutationally balanced. Since $m^{\pi^1} = (\frac{2}{3}, \frac{1}{3}, 0)^\top$, $m^{\pi^2} = (0, \frac{2}{3}, \frac{1}{3})^\top$ and $m^{\pi^3} = (0, 0, 1)^\top$, this family is permutational balanced with weights $\lambda^*_1 = \frac{1}{2}$, $\lambda^*_2 = \frac{1}{4}$, and $\lambda^*_3 = \frac{1}{4}$. Observe that the ordered 2-partition $(\{1, 2\}, \{3\})$ of $\mathcal{N}$ is permutationally balanced, but that the family of the ordered 2-partition $(\{1\}, \{2\})$ and the ordered 1-partition $(\{3\})$ is not permutationally balanced.

In case $\mathcal{P}$ is a family of 1-partitions we have that $\pi^j(P^{t^j}_{S^j}) = (S^j)$ and hence the system of equations in the balancedness condition reduces to

$$\sum_{j=1}^k \lambda^*_j m^{S^j} = m,$$

with $m^{S^j}_h = \frac{1}{|S^j|}$ if $h \in S^j$ and $m^{S^j}_h = 0$ if $h \notin S^j$, which is equal to the well-known concept of coalitional balancedness of the family of subsets $\{S^1, \ldots, S^k\}$ of $\mathcal{N}$. Therefore, the concept of permutational balancedness contains the concept of coalitional balancedness for a family of 1-partitions as a special case.

Definition 3.3 Balanced Permutational Game
A non-transferable utility permutational game $(\mathcal{P}^{\mathcal{N}}, V)$ is permutationally balanced if for every permutationally balanced family $\mathcal{P} = \{\pi^1(P^{t^1}_{S^1}), \ldots, \pi^k(P^{t^k}_{S^k})\}$ of ordered partitions in $\mathcal{P}^{\mathcal{N}}$ it holds that

$$\cap_{i=1}^k V(\pi^i(P^{t^i}_{S^i})) \subset V'(\mathcal{N}).$$

In the sequel we speak shortly about a balanced permutational game and a balanced coalitional game if we mean a permutationally (respectively coalitionally) balanced non-transferable utility permutational (respectively coalitional) game. For a given permutational game $(\mathcal{P}^{\mathcal{N}}, V)$ any vector in the set $V'(S)$ is attainable for coalition S. Since $V(S) \subset V'(S)$, and generally $V'(S) \neq V(S)$, the induced coalitional game $(\mathcal{N}, V')$ need not to be coalitionally balanced if $(\mathcal{P}^{\mathcal{N}}, V)$ itself is permutationally balanced. This fact is shown in the next example.

Example 3.4
Take $n = 3$ and define the permutational game $(\mathcal{P}^{\mathcal{N}}, V)$ by

$$V(i) = \{x \in \mathbf{R}^3 \mid x_i \leq 0\},\ i = 1, 2, 3,$$

$$V(1,2) = \{x \in \mathbf{R}^3 \mid 2x_1 + x_2 \leq 3\},$$

and

$$V(2,1) = \{x \in \mathbf{R}^3 \mid x_1 + 2x_2 \leq 3\},$$

where $V(i)$ denotes $V(\{i\})$ and $V(i,j)$ denotes $V((\{i\},\{j\}))$. Furthermore,

$$V(\mathcal{N}) = V(3) \cap V(1,2) \cap V(2,1),$$

and

$$V(\pi(P_S^t)) = \emptyset, \text{ otherwise.}$$

Observe again that we allow for empty payoff sets. The corresponding coalitional game is given by

$$V'(\{i\}) = V(i),\ i = 1, 2, 3,$$

$$V'(\{1,2\}) = V(1,2) \cup V(2,1) \cup V(\{1,2\}) = V(1,2) \cup V(2,1),$$

since $V(\{1,2\}) = \emptyset$,

$$V'(\{1,3\}) = V'(\{2,3\}) = \emptyset,$$

and

$$V'(\mathcal{N}) = V(\mathcal{N}).$$

The projection of the sets $V(1,2)$ and $V(2,1)$ on the (x_1, x_2)-space is given in Figure 2. The shaded area in this figure is the projection of the set $V(\{1,2,3\}) = V'(\{1,2,3\})$ on the (x_1, x_2)-space. Both the permutational game $(\mathcal{P}^{\mathcal{N}}, V)$ and the coalitional game $(\mathcal{N}, V')$ have the point $(1,1,0)^{\top}$ as the unique core element. For the permutational game this point lies in $V(\mathcal{N})$ and there is no coalition having an ordered partition through which the coalition can improve upon this outcome. The coalition $\{1,2\}$ can improve on each other point in $V(\mathcal{N})$ through the ordered 2-partition $(\{1\},\{2\})$ or the ordered 2-partition $(\{2\},\{1\})$. Also for the coalitional game the outcome $(1,1,0)^\top$ is the unique element of $V'(\mathcal{N})$ on which the coalition $\{1,2\}$ cannot improve upon. Clearly the coalitional game is not balanced, since the family of coalitions $\{1,2\}$ and $\{3\}$ is coalitionally balanced, whereas the point $x = (\frac{1}{2}, 2, 0)^\top$ lies in $V'(\{1,2\}) \cap V'(\{3\})$ but not in $V'(\mathcal{N})$. On the other hand the permutational game is permutationally balanced. In fact there are only four relevant families of ordered partitions to consider, namely the family of the three ordered 1-partitions $(\{1\})$, $(\{2\})$, $(\{3\})$, the family of two ordered 2-partitions and one ordered 1-partition $(\{1\},\{2\})$, $(\{2\},\{1\})$, $(\{3\})$, the family of one ordered 2-partition and two ordered 1-partitions $(\{1\},\{2\})$, $(\{2\})$, $(\{3\})$, and the family of one ordered 2-partition and two ordered 1-partitions $(\{2\},\{1\})$, $(\{1\})$, $(\{3\})$. For each of these families we have that the intersection of the sets of payoffs of the members of the family is a subset of $V'(\mathcal{N})$, for instance $V(1,2) \cap V(2) \cap V(3) \subset V'(\mathcal{N})$. For all other permutationally balanced families we have that the intersection of the payoff sets of the members of the family is empty and hence is a subset of $V'(\mathcal{N})$.

4 Nonemptiness of the core of a balanced permutational game

In order to prove the nonemptiness of the core of a balanced permutational game we first introduce an intersection theorem on the $(n-1)$-dimensional unit simplex S^n defined by

$$S^n = \{x \in \mathbf{R}^n_+ \mid \sum_{j=1}^{n} x_j = 1\}.$$

In this theorem the simplex S^n is covered by closed subsets $C^{\pi(P^t_S)}$, $\pi(P^t_S) \in \mathcal{P}^{\mathcal{N}}$, satisfying some boundary condition. Under this condition there exists a balanced collection of permutations for which the corresponding subsets of S^n have a nonempty intersection.

Theorem 4.1
Let $\{C^{\pi(P^t_S)} \mid \pi(P^t_S) \in \mathcal{P}^{\mathcal{N}}\}$ be a collection of closed sets covering S^n satisfying that if x lies in the boundary of S^n and $x \in C^{\pi(P^t_S)}$, then $S \subset \{i \in I_n \mid x_i > 0\}$. Then there is a permutationally balanced family $\mathcal{P} = \{\pi^1(P^{t^1}_{S^1}), \ldots, \pi^k(P^{t^k}_{S^k})\}$ of k ordered partitions in $\mathcal{P}^{\mathcal{N}}$ for which it holds that

$$\cap_{j=1}^{k} C^{\pi^j(P^{t^j}_{S^j})} \neq \emptyset.$$

Proof.
For any ordered partition $\pi(P^t_S) \in \mathcal{P}^{\mathcal{N}}$, define the vector $c^{\pi(P^t_S)} = m - m^{\pi(P^t_S)}$. For $x \in S^n$, define the set $F(x)$ by

$$F(x) = \mathrm{Conv}(c^{\pi(P^t_S)} \mid x \in C^{\pi(P^t_S)}\}),$$

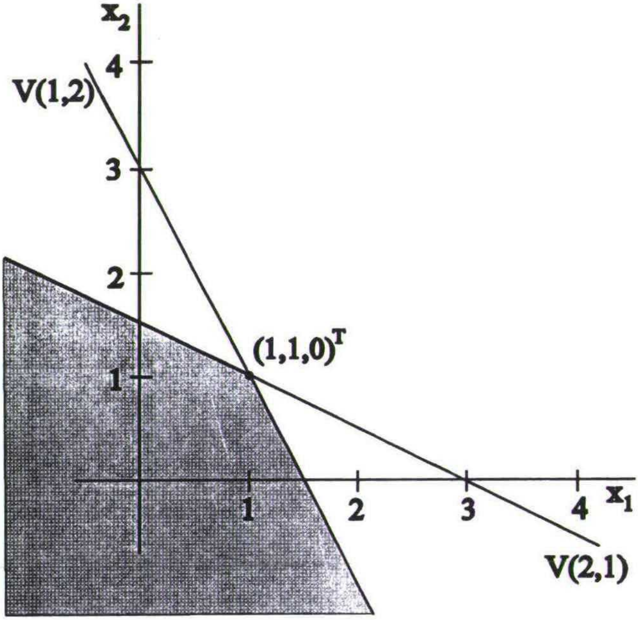

Figure 2: Example 3.4, balanced permutational game

where Conv(X) denotes the convex hull of a set $X \subset \mathbf{R}^n$. Clearly, for every $x \in S^n$, the set $F(x)$ is nonempty, convex, and compact. Moreover, $\cup_{x \in S^n} F(x)$ is bounded and F is an upper hemi-continuous mapping from the set S^n to the collection of subsets of the set Y^n defined by

$$Y^n = \{y \in \mathbf{R}^n \mid m^\top y = 0 \text{ and } y_i \geq -1 \text{ for } i = 1, \ldots, n\}.$$

Both sets S^n and Y^n are nonempty, convex, and compact. Next, let G be the mapping from Y^n to the collection of subsets of S^n defined by

$$G(y) = \{x \in S^n \mid x'^\top y \leq x^\top y \text{ for every } x' \in S^n\}.$$

Clearly, for every $y \in Y^n$ the set $G(y)$ is nonempty, convex, and compact, and G is upper hemi-continuous. Hence, the mapping H from the nonempty, convex, compact set $S^n \times Y^n$ into the collection of subsets of $S^n \times Y^n$ defined by $H(x,y) = G(y) \times F(x)$ is upper hemi-continuous and for every $(x,y) \in S^n \times Y^n$, the set $H(x,y)$ is nonempty, convex, and compact. According to Kakutani's fixed point theorem the mapping H has a fixed point on $S^n \times Y^n$, i.e., there exist $x^* \in S^n$ and $y^* \in Y^n$ satisfying $y^* \in F(x^*)$ and $x^* \in G(y^*)$. Let $\alpha^* = x^{*\top} y^*$, then $x^* \in G(y^*)$ implies $y_i^* = \alpha^*$ if $x_i^* > 0$ and $y_i^* \leq \alpha^*$ if $x_i^* = 0$. On the other hand, $y^* \in F(x^*)$ implies there exist nonnegative numbers $\lambda_1^*, \ldots, \lambda_k^*$ satisfying $\sum_{j=1}^k \lambda_j^* = 1$ and $y^* = \sum_{j=1}^k \lambda_j^* c^{\pi^j(P_{Sj}^{tj})}$, where $\pi^j(P_{Sj}^{tj})$, $j = 1, \ldots, k$, are such that $x^* \in C^{\pi^j(P_{Sj}^{tj})}$. Without loss of generality we may assume that $\lambda_j^* > 0$ for every $j = 1, \ldots, k$. We now show that $y^* = \underline{0}$ and hence that the collection $\{\pi^1(P_{S1}^{t1}), \ldots, \pi^k(P_{Sk}^{tk})\}$ is permutationally balanced. Since by definition of the set Y^n, $m^\top y^* = 0$, we obtain that $\alpha^* \geq 0$. Moreover, by the boundary condition we have that $x_i^* = 0$ implies that $i \notin S^j$ for every $j = 1, \ldots, k$, and so $y_i^* = \sum_{j=1}^k \lambda_j^* n^{-1} > 0$. Therefore, $0 < y_i^* \leq \alpha^*$ if $x_i^* = 0$ and $y_i^* = \alpha^* \geq 0$ if $x_i^* > 0$. Since $\sum_{j=1}^n y_i^* = 0$, this implies that $x_i^* > 0$ for every $i \in I_n$ and $\alpha^* = 0$. So, $y^* = \underline{0}$. Consequently, $\{\pi^1(P_{S1}^{t1}), \ldots, \pi^k(P_{Sk}^{tk})\}$ is permutationally balanced. Since $x^* \in \cap_{j=1}^k C^{\pi^j(P_{Sj}^{tj})}$, this completes the proof. Q.E.D.

By applying this intersection theorem we can prove the nonemptiness of the core of a balanced permutational game.

Theorem 4.2

A non-transferable utility permutational game $(\mathcal{P}^\mathcal{N}, V)$ has a nonempty core if

i) the set $V(\{i\})$ is nonempty and the set $\{x_i \mid x \in V(\{i\})\}$ is bounded from above for every $i \in \mathcal{N}$,

ii) the game is permutationally balanced,

iii) for every $\pi(P_S^t) \in \mathcal{P}^\mathcal{N}$, the set $V(\pi(P_S^t))$ is comprehensive and closed, and the set $\{(x_i)_{i \in S} \in \mathbf{R}^S \mid x \in V(\pi(P_S^t)) \text{ and } x_i \geq \max\{y_i \mid y \in V(\{i\})\} \text{ for all } i \in S\}$ is bounded.

Proof.
Without loss of generality we may assume that $\underline{0} \in V(\{i\})$ for any $i \in \mathcal{N}$. To prove the theorem we define a closed covering $\{C^{\pi(P_S^t)} \mid \pi(P_S^t) \in \mathcal{P}^{\mathcal{N}}\}$ of S^n satisfying the conditions of Theorem 4.1 and show that an intersection point of a permutationally balanced collection of these sets induces an element in the core of the game. For given $M > 0$ and for any $x \in S^n$, let the number λ_x be determined by

$$\lambda_x = \max\{\lambda \in \mathbf{R} \mid -Mx + \lambda m \in \cup_{\pi(P_S^t) \in \mathcal{P}^{\mathcal{N}}} V(\pi(P_S^t))\}.$$

Since $\underline{0} \in V(\{i\})$ and because of iii), for every $M > 0$, λ_x exists for any $x \in S^n$. Moreover, by the boundedness condition, $M > 0$ can be chosen so large that for every $i \in I_n$ and $x \in S^n$, $x_i = 0$ implies that $i \notin S$ for any S satisfying $-Mx + \lambda_x m \in V(\pi(P_S^t))$. Now, for $\pi(P_S^t) \in \mathcal{P}^{\mathcal{N}}$ we define

$$C^{\pi(P_S^t)} = \{x \in S^n \mid -Mx + \lambda_x m \in V(\pi(P_S^t))\}.$$

Since every $V(\pi(P_S^t))$ is closed and comprehensive, the collection of sets $\{C^{\pi(P_S^t)} | \pi(P_S^t) \in \mathcal{P}^{\mathcal{N}}\}$ is a collection of closed sets covering the simplex S^n, and satisfying the boundary condition of Theorem 4.1. Hence there is a balanced family $\mathcal{P} = \{\pi^1(P_{S^1}^{t^1}), \ldots, \pi^k(P_{S^k}^{t^k})\}$ of elements of $\mathcal{P}^{\mathcal{N}}$ such that $\cap_{j=1}^k C^{\pi^j(P_{S^j}^{t^j})} \neq \emptyset$. Let x^* be a point in this intersection, so $x^* \in C^{\pi^j(P_{S^j}^{t^j})}$ for $j = 1, \ldots, k$. Since the game is balanced we have that $\cap_{j=1}^k V(\pi^j(P_{S^j}^{t^j})) \subset V'(\mathcal{N})$ and hence $u^* = -Mx^* + \lambda_{x^*} m \in V'(\mathcal{N})$. Now, suppose there exist a vector $v \in \mathbf{R}^n$, a coalition S, and an ordered partition $\pi(P_S^t) \in \mathcal{P}^{\mathcal{N}}$ such that $v \in V(\pi(P_S^t))$ and $v_i > u_i^*$ for all $i \in S$. Since $V(\pi(P_S^t))$ is comprehensive and cylindric, there is a $\mu > 0$ such that $u^* + \mu m \in V(\pi(P_S^t))$. However, then $-Mx^* + (\lambda_{x^*} + \mu)m \in V(\pi(P_S^t))$, which contradicts that $-Mx^* + \lambda m \notin V(\pi(P_S^t))$ for any $\lambda > \lambda_{x^*}$. Hence $u^* \in C(\mathcal{P}^{\mathcal{N}}, V)$. Q.E.D.

5 Computation of a core element of a balanced permutational game

In this section we discuss how an element of the core of a balanced permutational game can be approximated. For a game $(\mathcal{P}^{\mathcal{N}}, V)$, let $C^{\pi(P_S^t)}$ be defined as in the proof of Theorem 4.2. Let y be an arbitrary point in the relative interior of S^n and for $T \subset I_n$, $T \neq I_n$, let the subset $A(T)$ of S^n be defined by

$$A(T) = \{x \in S^n \mid x = y + \sum_{i \in T} \lambda_i (y - e(i)),\ \lambda_i \geq 0 \text{ for all } i \in T\},$$

where for $i \in I_n$, $e(i)$ is the ith unit vector in $\mathbf{R}^n$. Since $y_j > 0$ for all $j \in I_n$, the dimension of $A(T)$ equals $t = |T|$. Notice that $A(\emptyset) = \{y\}$. The vector y will be the starting point of the algorithm.

Now, let S^n be subdivided into $(n-1)$-dimensional simplices such that every subset $A(T)$, $T \subset I_n$ with $t \neq n$, is triangulated into t-dimensional simplices. A simplicial subdivision of S^n that can easily be implemented on the computer is the V-triangulation with arbitrary mesh size proposed in Doup and Talman [3]. Let x be an arbitrary vertex of a simplex in the underlying simplicial subdivision. Then we label x with the n-vector $l(x)$ equal to $l(x) = m - m^{\pi(P_S^t)}$, where $\pi(P_S^t) \in \mathcal{P}^{\mathcal{N}}$ is such that $x \in C^{\pi(P_S^t)}$. If x lies in more than one set we choose one of them. Let the t-simplex $\sigma(w^1, \ldots, w^{t+1})$ be a t-dimensional simplex of the simplicial subdivision with vertices $w^1, \ldots, w^{t+1}$. Then σ is called T-complete for some $T \subset I_n$, $t \neq n$, if the system of linear equations

$$\sum_{j=1}^{t+1} \lambda_j \begin{pmatrix} l(w^j) \\ 1 \end{pmatrix} + \sum_{h \notin T} \mu_h \begin{pmatrix} e(h) \\ 0 \end{pmatrix} - \beta \begin{pmatrix} m \\ 0 \end{pmatrix} = \begin{pmatrix} \underline{0} \\ 1 \end{pmatrix} \tag{1}$$

has a solution $(\lambda^*, \mu^*, \beta^*) = (\lambda_1^*, \ldots, \lambda_{t+1}^*, \mu_h^*, h \notin T, \beta^*)$ satisfying $\lambda_j^* \geq 0$ for all $j \in I_{t+1}$ and $\mu_h^* \geq 0$ for $h \notin T$. We assume that for any some t-dimensional simplex σ in $A(T)$ the system (1) is nondegenerate (if necessary by perturbing the system), i.e., if (1) has at least one feasible solution for some simplex $\sigma(w^1, \ldots, w^{t+1})$ in $A(T)$, then the system has a whole line segment of feasible solutions (λ, μ, β) with at each of the end points exactly one of the variables in (λ, μ) being zero. Notice that the system (1) has $n+2$ columns and $n+1$ rows. In case at an end point of a set of feasible solutions to (1) it holds that $\lambda_h = 0$ for some $h \in I_{t+1}$, then also the facet $\tau(w^1, \ldots,$ $w^{h-1}, w^{h+1}, \ldots, w^{t+1})$ opposite the vertex w^h is called T-complete. A t-simplex in $A(T)$ can have at most two T-complete facets and a T-complete facet in $A(T)$ is a facet of either two t-simplices in $A(T)$ or lies in the boundary of $A(T)$ and is facet of just one t-simplex in $A(T)$. Now, let us consider the T-complete t-simplices in $A(T)$ for some given $T \subset I_n$. They form sequences of adjacent t-simplices with T-complete common facets in $A(T)$. Such a sequence is either a loop or has two end simplices. An end simplex σ of a sequence in $A(T)$ is either a t-simplex having a solution to (1) with $\mu_k^* = 0$ for some unique $k \notin T$ or is a t-simplex with a T-complete facet τ lying in $A(T \setminus \{h\})$ for some unique $h \in T$. In the latter case we have that $\tau = \{y\}$ if $T = \{h\}$ and otherwise τ is an end simplex of a sequence of adjacent $(T \setminus \{h\})$-complete $(t-1)$-simplices in $A(T \setminus \{h\})$. Notice that τ cannot lie in the boundary of S^n because $x \in C^{\pi(P_S^t)}$ implies that $S \subset \{i \in I_n \mid x_i > 0\}$. In the first case σ is an end simplex of a sequence of adjacent $(T \cup \{k\})$-complete $(t+1)$- simplices in $A(T \cup \{k\})$, unless $T = I_n \setminus \{k\}$. If $T \cup \{k\} = I_n$, let $K = \{j \in I_n \mid \lambda_j^* > 0\}$ at the corresponding solution of the system (1), and for $j \in K$ let $\pi^j(P_{S^j}^{t^j})$ be the element of $\mathcal{P}^{\mathcal{N}}$ such that $l(w^j) = m - m^{\pi^j(P_{S^j}^{t^j})}$, so $w^j \in C^{\pi^j(P_{S^j}^{t^j})}$. Clearly, the family of permutations $\{\pi^j(P_{S^j}^{t^j}), j \in K\}$ is permutationally balanced. Therefore the vector $u^* = -Mx^* + \lambda_{x^*} m$ with$x^* = \sum_{j \in K} \lambda_j^* w^j$ can be considered to be an approximating core element of the permutational game.

By linking the sequences of t-simplices in $A(T)$ with common T-complete facets together as described above, there exists a sequence of adjacent simplices of variable dimension having T-complete facets in common in $A(T)$ for varying $T \subset I_n$, connecting the starting point y with a point x^* correspponding to an approximating core element. This sequence can be followed by alternating (lexicographic) pivot steps in system (1) and replacement steps in the underlying simplicial subdivision of S^n. If the accuracy of approximation at x^* is not satisfactory, the algorithm can be restarted at x^* with a finer simplicial subdivision. Within a finite number of steps any a priori chosen accuracy can be reached.

6 Concluding remarks

In this paper we introduced permutational games and proved that the core of such a game is nonempty if the game is permutationally balanced. This concept of balancedness is a generalization of the well-konown concept of balancedness of coalitions. Analogously the existence result concerning the nonemptiness of the core is more general than for games in coalitional structure. A game in coalitional structure is a special case in the family of games in permutational structure. Indeed, when $V(\pi(P_S^t)) = \emptyset$ for every $t \geq 2$ then the permutational game is a game in coalitional structure and permutational balancedness coincides with coalitional balancedness. Moreover, a balanced permutational game is also coalitionally balanced with respect to the sets $V(S)$, $S \subset \mathcal{N}$. However, the corresponding coalitional game $(\mathcal{N}, V')$, obtained by defining the payoff set $V'(S)$ of coalition S as the union of all payoff sets of the ordered partitions of S in the permutational game, need not to be coalitionally balanced. Since a permutational game and its corresponding coalitional game have the same core, it follows that permutational balancedness of the underlying permutational game is a sufficient condition for the nonemptiness of the core of the corresponding coalitional game.

Intersection Theorem 4.1 is a generalization of the well-known intersection theorem of Shapley [10], [11], in which only sets C^S are defined for coalitions $S \subset \mathcal{N}$. We notice that the boundary condition of this intersection result can be relaxed by applying a general intersection result given in van der Laan, Talman and Yang [5]. Instead of assuming that for every boundary point $x \in S^n$ we have that $S \subset \{i \in I_n \mid x_i > 0\}$ if $x \in C^{\pi(P_S^t)}$, it is sufficient to assume that for every boundary point x there is at least one permutation $\pi(P_S^t) \in \mathcal{P}^{\mathcal{N}}$ such that $S \subset \{i \in I_n \mid x_i > 0\}$ and $x \in C^{\pi(P_S^t)}$. This allows that there are also permutations $\pi(P_S^t) \in \mathcal{P}^{\mathcal{N}}$ such that $x \in C^{\pi(P_S^t)}$ but not $S \subset \{i \in I_n \mid x_i > 0\}$. This is stated in the following theorem.

Theorem 6.1
Let $\{C^{\pi(P_S^t)} \mid \pi(P_S^t) \in \mathcal{P}^{\mathcal{N}}\}$ be a collection of closed sets covering the simplex S^n satisfying that for every $T \subset I_n$, it holds that $\{x \in S^n \mid x_i = 0 \text{ for all } i \in T\} \subset \cup_{S \subset I_n \setminus T} \cup_{\pi(P_S^t)} C^{\pi(P_S^t)}$. Then there is a balanced family $\mathcal{P} = \{\pi^1(P_{S^1}^{t^1}), \ldots, \pi^k(P_{S^k}^{t^k})\}$ of elements of $\mathcal{P}^{\mathcal{N}}$ for which $\cap_{j=1}^k C^{\pi^j(P_{S^j}^{t^j})} \neq \emptyset$.

The proof of this theorem follows by applying the Main Theorem in [5].

Finally we point out that there are many ways to define the weights of the players for an ordered partition of a given coalition. For example, for a given permutation $\pi(P_S^t) \in \mathcal{P}^{\mathcal{N}}$, we could define the n-dimensional power vector $m^{\pi(P_S^t)}$ by

$$m_j^{\pi(P_S^t)} = 0, \text{ if } j \notin S$$

and

$$m_j^{\pi(P_S^t)} = \frac{t^{r-1}(t+1)^{1-r}}{\sum_{h=1}^{t} s_h t^{h-1}(t+1)^{1-h}}, \text{ if } j \in \pi_r(P_S^t),$$

where $s_h = |\pi_h(P_S^t)|$. It is easily seen that $\sum_{j=1}^{n} m_j^{\pi(P_S^t)} = 1$. In this case we always have that $m_k^{\pi(P_S^t)} > m_l^{\pi(P_S^t)}$ for any $k \in \pi_i(P_S^t)$ and $l \in \pi_j(P_S^t)$ if $1 \leq i < j \leq t$. This implies that every member in a higher ranked subcoalition has more power than any member in a lower ranked subcoalition. Notice however that this has consequences in forming permutationally balanced families and hence on the fact whether or not a game in permutational structure is permutationally balanced. Since the core of a game does not depend on the definition of the power vectors, this implies that for the nonemptiness of the core of a permutational game it is sufficient to have permutational balancedness with respect to some arbitrary set of power vectors. Notice that if we take $m^{\pi(P_S^t)} = m^S$ for every $\pi(P_S^t) \in \mathcal{P}^{\mathcal{N}}$, then the permutational game is balanced with respect to these constant vectors (with respect to S) if and only if the induced coalitional game is balanced. Hence, an induced coalitional game being balanced implies that the original permutational game is permutationally balanced with respect to some collection of power vectors. Clearly, the other way around is not true, i.e., a coalitional game induced by a balanced permutational game may not be coalitional balanced.

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