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Constrained optimization in simulation

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Online appendix of "Constrained Optimization in Expensive Simulation: Novel Approach"

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In this appendix we present computational results for the so-called "toy problem". We generate the outputs of this problem through second-order polynomials augmented with noise. This problem is based on (Angün et al., 2002). However, to increase the integer-valued search area, we transform the original decision variables z_1 and z_2 into d_1 and d_2 as follows:

$$z_1 = \frac{d_1}{10},\ z_2 = \frac{d_2}{10} - 2,\ 0 \le d_1 \le 30,\ 0 \le d_2 \le 30. \tag{1}$$

This gives the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & E(w_0) = E[5(\tfrac{d_1}{10} - 1)^2 + (\tfrac{d_2}{10} - 7)^2 + 4\tfrac{d_1}{10}(\tfrac{d_2}{10} - 2) + e_0] \\ \text{subject to} \quad & E(w_1) = E[(\tfrac{d_1}{10} - 3)^2 + (\tfrac{d_2}{10} - 2)^2 + \tfrac{d_1}{10}(\tfrac{d_2}{10} - 2) + e_1] \le 4 \\ & E(w_2) = E[\left(\tfrac{d_1}{10}\right)^2 + 3\left(\tfrac{d_2}{10} - 0.939\right)^2 + e_2] \le 9 \\ & 0 \le d_1 \le 30,\ 0 \le d_2 \le 30;\ d_1, d_2 \in \mathbb{N}. \end{aligned} \tag{2}$$

The random errors e_0, e_1, and e_2 are normally distributed with means 0, variances $\sigma_{0,0} = 0.7500$ (so $\sigma_0 = \sqrt{0.75} = 0.87$), $\sigma_{1,1} = 0.0169$, $\sigma_{2,2} = 0.1200$, and correlations $\rho_{0,1} = 0.82$, $\rho_{0,2} = 0.30$, $\rho_{1,2} = -0.07$. Because the simulation noise is purely additive in this problem, we do not use CRN to sample the multivariate normal noise at different input combinations. It is easy to derive that the true optimum for this integer problem is (12, 24), with $E(w_0) = 23.28$, $E(w_1) = 3.88$, $E(w_2) = 7.8436$.

Table 1 shows the results of our heuristic for 10 macroreplicates. The number of replicates per point is either fixed at $m_i = m = 110$ or determined by the relative precision requirement with $\gamma = 0.15$. This table also shows the OptQuest results for the same 10 macroreplicates with $m_i = m = 110$, starting from the initial solution (15, 15).

ARENA Version 12 finds the optimum after only 9 points in each macroreplicate (we conjecture that OptQuest's scatter search happens to select a point at 2/5 and 4/5 respectively, of each input's domain; this point is actually optimal). Strangely enough, Version 11 gives very different results; it finds the optimum only in 2 of the 10 macroreplicates. Our heuristic finds the optimum after 12 to 30 points; our heuristic uses a pilot sample with 9 points, which explains why it needs more than 9 points to find the optimum.

Table 2 summarizes the performance results of the different heuristics, measured through the average, minimum, and maximum of the estimated goal and constraint values over the ten macroreplicates.

Table 1
Results for the toy problem obtained by the heuristic

Macrorep.	Heuristic	$\mathbf{d_{opt}}$	w_0	w_1	w_2	i_{max}	Rank
1	OptQuest (Arena 12) $m_i = 110$	(12, 24)	23.31	3.89	7.82	105	9
	OptQuest (Arena 11) $m_i = 110$	(13, 24)	23.504	NA	NA	105	55
	DOE-Kri-MP $m_i = 110$	(12, 24)	23.31	3.89	7.82	105	12
	DOE-Kri-MP $\gamma = 0.15$	(11, 23)	22.64	3.93	6.72	56	14
2	OptQuest (Arena 12) $m_i = 110$	(12, 24)	23.41	3.89	7.85	103	9
	OptQuest (Arena 11) $m_i = 110$	(12, 24)	23.41	NA	NA	103	90
	DOE-Kri-MP $m_i = 110$	(12, 24)	23.41	3.89	7.85	103	13
	DOE-Kri-MP $\gamma = 0.15$	(13, 24)	22.83	3.44	7.97	62	30
3	OptQuest (Arena 12) $m_i = 110$	(12, 24)	23.34	3.89	7.85	93	9
	OptQuest (Arena 11) $m_i = 110$	(12, 24)	23.34	NA	NA	93	90
	DOE-Kri-MP $m_i = 110$	(12, 24)	23.34	3.89	7.85	93	15
	DOE-Kri-MP $\gamma = 0.15$	(12, 24)	22.49	3.79	7.74	61	13
4	OptQuest (Arena 12) $m_i = 110$	(12, 24)	23.31	3.88	7.81	104	9
	OptQuest (Arena 11) $m_i = 110$	(13, 24)	23.84	NA	NA	104	55
	DOE-Kri-MP $m_i = 110$	(12, 24)	23.31	3.88	7.81	104	12
	DOE-Kri-MP $\gamma = 0.15$	(12, 24)	23.05	3.82	7.86	60	12
5	OptQuest (Arena 12) $m_i = 110$	(12, 24)	23.28	3.89	7.79	105	9
	OptQuest (Arena 11) $m_i = 110$	(13, 24)	23.67	NA	NA	105	55
	DOE-Kri-MP $m_i = 110$	(12, 24)	23.28	3.89	7.79	105	13
	DOE-Kri-MP $\gamma = 0.15$	(11, 23)	22.67	3.93	6.95	47	18
6	OptQuest (Arena 12) $m_i = 110$	(12, 24)	23.22	3.87	7.81	71	9
	OptQuest (Arena 11) $m_i = 110$	(13, 24)	23.65	NA	NA	71	55
	DOE-Kri-MP $m_i = 110$	(12, 24)	23.22	3.87	7.81	71	12
	DOE-Kri-MP $\gamma = 0.15$	(12, 24)	23.08	3.85	7.86	49	12
7	OptQuest (Arena 12) $m_i = 110$	(12, 24)	23.28	3.88	7.83	91	9
	OptQuest (Arena 11) $m_i = 110$	(13, 24)	23.64	NA	NA	91	55
	DOE-Kri-MP $m_i = 110$	(12, 24)	23.28	3.88	7.83	91	16
	DOE-Kri-MP $\gamma = 0.15$	(12, 24)	23.21	3.86	7.66	63	15
8	OptQuest (Arena 12) $m_i = 110$	(12, 24)	23.29	3.88	7.84	98	9
	OptQuest (Arena 11) $m_i = 110$	(13, 25)	23.15	NA	NA	98	78
	DOE-Kri-MP $m_i = 110$	(13, 25)	23.15	3.77	8.97	98	16
	DOE-Kri-MP $\gamma = 0.15$	(12, 23)	23.22	3.65	6.96	56	17
9	OptQuest (Arena 12) $m_i = 110$	(12, 24)	23.27	3.87	7.89	75	9
	OptQuest (Arena 11) $m_i = 110$	(13, 24)	23.55	NA	NA	75	55
	DOE-Kri-MP $m_i = 110$	(12, 24)	23.27	3.87	7.89	75	11
	DOE-Kri-MP $\gamma = 0.15$	(11, 23)	23.08	3.94	6.81	55	12
10	OptQuest (Arena 12) $m_i = 110$	(12, 24)	23.20	3.86	7.82	98	9
	OptQuest (Arena 11) $m_i = 110$	(13, 25)	23.33	NA	NA	98	78
	DOE-Kri-MP $m_i = 110$	(12, 24)	23.20	3.86	7.82	98	11
	DOE-Kri-MP $\gamma = 0.15$	(12, 24)	23.07	3.79	8.03	54	14

Table 2
Performance summary based on 10 macroreplicates for the toy problem

Heuristic	w_0			w_1			w_2		
	average	max	min	average	max	min	average	max	min
OptQuest (Arena 12) $m_i = 10$	23.29	23.41	23.20	3.88	3.89	3.86	7.83	7.89	7.79
OptQuest (Arena 11) $m_i = 10$	23.39	23.84	22.26	NA	NA	NA	NA	NA	NA
DOE-Kri-MP $m_i = 10$	23.28	23.41	23.15	3.87	3.89	3.77	7.94	8.97	7.79
DOE-Kri-MP $\gamma = 0.15$	22.93	23.22	22.49	3.80	3.94	3.44	7.46	8.03	6.72