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### Optimal Monetary Policy in a Sudden Stop

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OPTIMAL MONETARY POLICY IN A 'SUDDEN STOP'

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Fabio Braggion, Lawrence J. Christiano, and Jorge Roldos  
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**ABSTRACT**

In the wake of the 1997-98 financial crises, interest rates in Asia were raised immediately, and then reduced sharply. We describe an environment in which this is the optimal monetary policy. The optimality of the immediate rise in the interest rate is an example of the theory of the second best: although high interest rates introduce an inefficiency wedge into the labor market, they are nevertheless welfare improving because they mitigate distortions due to binding collateral constraints. Over time, as various real frictions wear off and the collateral constraint is less binding, the familiar Friedman forces dominate, and interest rates are optimally set as low as possible.

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# 1. Introduction

The Asian financial crises of 1997-98 triggered a sharp debate over the appropriate response of policy to a financial crisis. The hallmark of the crises was a “sudden stop” (Calvo, 1998): capital inflows turned into outflows and output suddenly collapsed. Some argued, appealing to the traditional monetary transmission mechanism, that a cut in the interest rate was required to slow or reverse the drop in output. Others argued that because of currency mismatches in balance sheets, the exchange rate depreciation associated with a cut in the interest rate might exacerbate the crisis. They argued for an increase in interest rates. Interestingly, a look at the data indicates that *both* pieces of advice were followed in practice. Figure 1 shows what happened to short term interest rates in each of four Asian crisis countries. Initially they rose sharply. Within six months or so, the policy was reversed and interest rates were ultimately driven to below their pre-crisis levels. A casual observer might infer that policy was simply erratic, with policymakers trying out different advice at different times.

In this paper, we argue that the observed policy may have served a single coherent purpose. We describe a model in which the optimal response to a financial crisis is an initial sharp rise in the interest rate, followed by a fall to below pre-crisis levels.

In our model, because of the presence of real frictions, resources are slow to respond in the immediate aftermath of a shock. Over time, resource allocation becomes more flexible.<sup>1</sup> We characterize a financial crisis as a shock in which collateral constraints unexpectedly bind and are expected to remain in place permanently. Our model has the property that when there is a binding collateral constraint and real frictions hinder resource allocation, then the monetary transmission mechanism is the reverse of what it would otherwise be. In particular, a rise in the interest rate increases economic activity and welfare. Over time, as the real frictions wear off, the monetary transmission mechanism corresponds to the traditional one in which low interest rates stimulate output and raise welfare.

We now briefly explain the real and financial frictions in the model, and describe how they shape optimal policy in the wake of a financial crisis. We adopt a small, tradable/non-tradable goods open economy model. The real friction is that labor in the tradeable sector is chosen prior to the realization of the current period shock.<sup>2</sup> Thus, when the financial shock occurs, the allocation of labor to the tradeable sector cannot respond in the current period, although it can respond in subsequent periods.

We adopt two forms of financial friction.<sup>3</sup> First, to capture the non-neutrality of money our

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<sup>1</sup>In effect, we combine into one model, the two studied in Christiano, Gust and Roldos (2004). In one model of that paper, labor in the traded good sector was fixed in each period. In another model, labor was completely flexible.

<sup>2</sup>A similar friction is used by Fernandez de Cordoba and Kehoe (2001) to study the role of capital flows following Spain’s entry to the European Community.

<sup>3</sup>Other studies have examined the relationship between optimal interest rates and financial crises. Aghion,

model incorporates the portfolio allocation friction in the limited participation model.<sup>4</sup> In the absence of collateral constraints, our model reproduces the traditional monetary transmission mechanism: when the domestic monetary authority expands the money supply, the liquidity of the banking system increases and interest rates fall, leading to an expansion in output and a depreciation of the exchange rate. Second, our model assumes firms make use of labor and a foreign intermediate input, and that these must be financed in advance. The collateral constraint that is imposed during the crisis applies to these loans. Our collateral constraint captures the balance sheet mismatch problems often emphasized in the context of currency crises, because liabilities are denominated in foreign currency while assets are denominated in domestic currency.<sup>5</sup>

The surprising feature of optimal policy in our model is that the nominal interest rate rises sharply in the period of the collateral shock. That this is optimal is a consequence of the interaction of the financial and real frictions. A rise in the interest rate acts like a tax on the employment of labor in the nontraded good sector, and raises the marginal cost of production in that sector. Other things the same, this slows down economic activity. However, when collateral constraints are binding, there is another effect that dominates. Because the employment of labor by firms in the traded sector is predetermined in the period of the shock, the interest rate rise does not increase the marginal cost of production in that sector. With the marginal cost of non-traded goods rising relative to the marginal cost of traded goods, the relative price of nontraded goods increases. Other things the same, this increase raises the traded-good value of the physical capital stock in the non-traded sector. Because this capital is used as collateral in the import of intermediate goods, the collateral constraint is relaxed. Imports of intermediate goods increase and the production of tradeable goods expands. Because tradeable and non-tradeable goods are complements in domestic production, the demand for non-tradables increases and overall economic activity expands. Welfare is increased by the high interest rate, despite the fact that it introduces a distortionary wedge in the labor market. The reason welfare increases is that the policy has the effect of sharply reducing another wedge, the one that is associated with the

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Bacchetta and Banerjee (2000) present a model with multiple equilibria, in which a currency crisis is the bad equilibrium. The possibility of the bad equilibrium is the outcome of the interplay between credit constraints on private firms and nominal price rigidities. The authors show that the monetary authority should tighten monetary policy after any shock that results in the possibility of the currency crisis equilibrium. Our analysis differs from this analysis in three ways. First, equilibrium multiplicity plays no role in this paper. Second, our model emphasizes a different set of rigidities. Third, Aghion, Bacchetta and Banerjee focus on the prevention of crises, while we focus on their management after they occur. Similarly, Caballero and Krishnamurthy (2002) show that when the economy faces a binding international collateral constraint, a monetary expansion that would redistribute funds from consumers to distressed firms has no real effects. Given this lack of effectiveness, a monetary authority that trades-off output and an inflation target focuses on the latter and tightens monetary policy to achieve the inflation objective.

<sup>4</sup>For closed economy analyses of this model, see Lucas 1990, Fuerst 1992, Christiano 1991, Christiano and Eichenbaum, 1992, 1995.

<sup>5</sup>The relevance of balance sheet effects during sudden stops for emerging markets—but not for developed countries—is documented in Calvo, Izquierdo and Mejia (2004).

collateral constraint.

The mechanism by which the higher interest rate produces higher output is novel, and so to further highlight its workings, we construct and analyze a simple example.<sup>6</sup> The example represents a dramatic simplification of our dynamic model. There is no money, and there is only one period. In the example, a tax rate on labor plays the role of the interest rate in our dynamic, monetary model. We are able to prove that whenever the collateral constraint is binding and the equilibrium is unique, a rise in the labor tax rate must stimulate output, consumption, employment and welfare. This result may be of interest beyond the sudden stop episodes that we study here. In particular, it may be useful for shedding light on the empirical literature on the “non-Keynesian effects of fiscal policy” or “Expansionary Fiscal Consolidations”. We return to this issue in our concluding remarks.

We now briefly discuss the interaction of monetary policy and sudden stop in our model. The sudden stop is triggered by a tightening of collateral constraints. The effect of the collateral shock is to increase the shadow cost of foreign borrowing, since international debt limits - via the collateral constraint - the ability of firms to purchase foreign intermediate inputs. As a result, imports of intermediate inputs drop and, because they are crucial for domestic production, the latter falls. In addition, the sharp rise in the shadow cost of debt induces agents to pay down that debt by running a current account surplus. This process continues until the debt falls to the point where the collateral constraint is non-binding and the economy is in a new steady state. Monetary policy has no impact on how much collateral lenders require, nor does it have an important impact on real variables in the new steady state. Monetary policy affects real variables and welfare primarily by its impact on the nature of the transition from the old to the new steady state. The sharp rise in the interest rate in the immediate aftermath of the crisis has the effect of resisting (not reversing) the fall in nominal and real exchange rates, asset prices, output, employment and consumption, caused by the initial "sudden stop".

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<sup>6</sup>There exist other examples in the literature of how financial frictions may have the consequence that a high interest rate is desirable. For example, Kocherlakota (2002, 2003) shows that a high interest rate may be part of a socially efficient mechanism to help individuals smooth consumption intertemporally, in the face of binding borrowing constraints. In private communication, Kocherlakota has provided us with a very simple example that illustrates the point. Consider a two period economy, in which 1/2 the population (‘borrowers’) has a sequence of endowments,  $y^L$  in the first period and  $y^H$  in the second period, where  $y^L < y^H$ . Suppose the other half of the population (‘lenders’) has the opposite lifetime sequence of endowments,  $y^H, y^L$ . Suppose everyone has the same utility function,  $u(c_1) + u(c_2)$ , where  $u$  is strictly concave and  $c_1$  and  $c_2$  are periods 1 and 2 consumption, respectively. Suppose also that borrowing is not permitted. Then the unique equilibrium is that everyone consumes their endowment. The borrowers are forced to do so by the non-negativity constraint on private bonds, and the lenders are prevented from lending by a very low interest rate,  $R = u'(y^H)/u'(y^L)$ . An optimal policy is for the government to issue bonds in the first period, and redistribute the proceeds to everyone (suppose the government cannot see who is constrained and who is not) in lump sum form. In the second period, the government taxes everyone in order to pay back the bonds. This policy in effect allows borrowers and lenders to exchange amongst themselves. A side effect of this policy is that the interest rate is lower. Although this example has some of the flavor of our analysis (optimal policy under binding financial constraints is associated with a high interest rate), in its details it is very different.

We compare the dynamic behavior of the variables in the model with data drawn from the Korean crisis experience. Qualitatively, the model reproduces the Korean experience reasonably well. In particular, the model reproduces the observed transitory rise in the current account, and fall of real quantities such as employment, consumption and output. The model also captures the evolution of asset prices, the real and nominal exchange rate and the behavior of the interest rate. Taken together, this evidence suggests that our model may provide a useful interpretation of the apparently erratic behavior of monetary policy exhibited in Figure 1.

The model does have quantitative empirical shortcomings. Although it captures the direction of movement in the current account, it understates the magnitude. We suspect that this reflects the absence of physical investment in the model. A reduction in investment provides agents with another margin from which to draw resources that can be used to pay off the international debt. Also, though the inflation response of the model to the financial shock matches qualitatively, it misses on magnitude.

The paper is organized as follows. First, we provide empirical evidence to support the main assumptions of the model. In particular, we show that collateral constraints were increased during the Asian financial crisis, and that it is not unreasonable to assume that at least a fraction of the assets used in the nontradable sector could be used to secure foreign borrowing by tradable sector firms. We also show that imported intermediate inputs are a large fraction of imports, and that they fell sharply during the crisis. Second, we present the simplified example discussed above. The third section presents our dynamic, monetary model. Section 4 discusses model calibration and section 5 present our simulation results. Section 6 concludes.

## **2. Evidence on Key Assumptions**

This section discusses empirical evidence related to key features of our model. We begin by displaying evidence that collateral requirements play a role in emerging markets generally, as well as evidence that collateral constraints tightened at the onset of the Asian financial crises of 1997. Table 1 shows that up until 1996, approximately 20 percent of syndicated loans to emerging markets were secured by collateral. At the time of the financial crises of 1997, this fraction doubled to over 40 percent. Also, Edison, Luangaram and Miller (2000) show that in Thailand, banks loaned up to 70 to 80 percent of collateral before the Asian crisis, and only 50 to 60 percent after the crisis. According to Gelos and Werner (1999), survey evidence from the Bank of Thailand indicates that more than 80 percent of loans are collateralized in Thailand. Gelos and Werner (1999) also report that around 60 percent of loans are collateralized in Mexico. Finally, a review of financial conditions of the Asian crises countries (IMF 1999) notes that lending against collateral was a widespread practice also in these countries.

There is some indirect evidence which provides support for the notion that collateral consid-

erations matter. Baek, Kang and Park (2004) find that the stock prices of Korean firms with higher foreign ownership suffered less during the crisis. This is consistent with our model if the foreign ownership in effect provided firms with more access to collateral for borrowing purposes. Baek, Kang and Park (2004) also report evidence that firms with better disclosure rules experienced a smaller drop in asset prices. This is consistent with our model, if we suppose that greater transparency reduces the need for collateral. If collateral constraints are not binding on firms with better disclosure rules, then the logic in our model implies that they would have suffered less with the onset of the crisis.

In our model analysis, we assume that collateral in the non-traded good sector is available for borrowing by firms in the traded sector. Although our assumption is admittedly extreme, the evidence suggests that some sharing of collateral across sectors does occur. In several emerging markets a large share of the economy is dominated by groups of firms ('chaebols' in Korea) that can use internal capital markets to allocate credit among firms in the group. For example, Shin and Park (1999) report that firms in Korean chaebols guarantee bank loans taken by other firms in the same chaebol.<sup>7</sup> Groups typically encompass both traded and nontraded good sectors. For example, the Samsung group (one of the largest chaebols in Korea), which has member firms in the electricity, heavy machinery, chemical and financial sectors (see Shin and Park, 1999). Shin and Park (1999) also show that the sensitivity of investment to cash flow of a chaebol firm (a common measure of liquidity constraints) is significantly affected by the cash flow of other firms within the same chaebol. This is consistent with the notion that internal credit markets allow firms in chaebols to share collateral. Significantly, chaebol firms make up a large fraction of the Korean economy. For example, at the end of 1998, the top 30 chaebols in Korea accounted for 12 percent of total GNP, 48 percent of total corporate assets and 47 percent of corporate revenues (see Baek, Kang and Park, 2004). According to Claessens, Djankov, Fan, and Lang (1999), the average number of firms that belong to a group of firms in Southeast Asia was 75 percent in 1991-1996.<sup>8 9</sup>

In our analysis, imports are composed of intermediate goods. Because these require finance,

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<sup>7</sup>In Korea a large business group is often referred as a chaebol. The Korea Fair Trade Commission (KFTC) defines a business group as a "group of companies of which more than 30% of shares are owned by group's controlling shareholder and its affiliated companies". Chaebol firms operate in many different industries, are bound together by a nexus of explicit and implicit contracts, and maintain substantial business ties with other firms in their group. They are also characterized by an extensive arrangement of pyramidal or multi-layered share-holding arrangements and the existence of cross-debt guarantees among member firms Baek, Kang and Park (1999).

<sup>8</sup>According to Claessens, Djankov, Fan, and Lang (1999, page 2), 'A group can be described as a corporate organization where a number of firms are linked through cross-ownership or where a single individual, family or coalition of families owns a number of different firms.'

<sup>9</sup>The percentages for each country break down as follows: Hong Kong, 60; Indonesia, 69; Japan, 83; South Korea, 57; Malaysia, 57; Philippines, 74; Singapore, 67; Taiwan, 53; Thailand, 42. The average over all countries is 75.



the ‘credit crunch’ associated with a tightening of collateral constraints inhibits the ability of firms to import intermediate goods. Because intermediate goods are assumed to be important in production, this results in a fall in production and in exports. To see that intermediate goods are an important component of imports, see Table 2. According to Table 2, intermediate good imports are 50 percent of total imports for Korea and 70 percent of total imports for Indonesia and Malaysia. Figure 2 shows real GDP and intermediate good imports and shows the close correlation between the two. To see how imports fall during a sudden stop, consider Figure 3, which displays exports and imports, measured in dollars, for four Asian countries.<sup>10</sup> Note how imports fall more than exports (of course, this is what produces the positive swing in the current account). The fact that exports fall, despite the tremendous depreciation of the currency that occurs in a sudden stop, is consistent with the notion in our model that the fall in imports creates problems for domestic production.

In effect, the credit crunch brings on a shortage of tradeable goods according to our model. The shortage is acute, because lack of substitution in production between traded and non-traded goods causes output to slow. One expects such a shortage to manifest itself in the form of a price rise. For evidence on this, consider the data on exchange rates in Figure 4. Note that in each of the Asian crisis countries considered there is a dramatic depreciation in the aftermath of the crisis. The smallest depreciation is 143 percent (Philippines) and the largest is 169 percent (Korea). Given the relatively small movements in inflation in these countries, these movements in the nominal exchange rate correspond to movement in the real exchange rate. Assuming rough purchasing power parity in traded goods, this corresponds to a very dramatic jump in the price of traded relative to nontraded goods.

We now turn to a key assumption that causes a rise in the interest rate to be optimal in the immediate aftermath of a sudden stop. This is the assumption that labor in the tradable sector is difficult to adjust quickly. We have not found evidence that bears directly on this assumption. However, there is some indirect evidence. Botero, Djankov, La Porta, Lopez-de-Silanes and Shleifer (2003) report that there is a significant amount of labor market regulation in emerging market countries. Also, Caballero, Cowan, Engel and Miccod (2004) report that with more labor market regulation in emerging markets, employment flexibility is reduced. If the evidence found by Melitz (2003) and others for the US applies to crisis economies, then the traded sector has higher value-added, more capital per worker, higher wages, etc. All these factors are likely to be associated with greater transparency for the traded sector, which may imply that labor market regulations are applied more effectively in the traded good sector than in the non-traded good sector. If this is so, then we can suppose that labor in the traded good

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<sup>10</sup>The data were obtained from the International Monetary Fund’s, ‘International Financial Statistics’ data base. Imports are imports of goods, services and payments associated with domestic assets issued to foreigners. Exports are defined analogously. The data for Korean, Malaysia, Phillipines ad Thailand. For all countries except the Phillipines, we used annual data.

sector reacts less flexibly to shocks than does labor in the nontraded good sector.

### 3. Example

A basic result in the dynamic simulations reported in later sections is that a rise in the domestic interest rate in the period of a collateral shock places upward pressure on employment and welfare. At first glance, this result will seem puzzling since the rise in the interest rate effectively operates like a rise in the tax rate on labor. Partial equilibrium reasoning suggests such a distortion should lead to a decrease in employment and welfare, not an increase. In our model, these partial equilibrium effects are overwhelmed by a general equilibrium effect that relaxes the collateral constraint. In this section we present a drastically simplified version of our dynamic model, which allows us to show how these effects work. In the simplified example, there is no money and there is only one period.

The first subsection below displays the model. The second subsection derives the model's qualitative properties. Here, we state our proposition and provide a heuristic proof (details are provided in Appendix A). The third subsection provides a numerical example.

#### 3.1. Model

A final good sector produces a non-traded consumption good,  $c$ , for domestic households, whose utility is as follows:

$$u(c, L) = c - \frac{\psi_0}{1 + \psi} (L^N + L^T)^{1+\psi}. \quad (3.1)$$

Here,  $L^N$  and  $L^T$  denote labor in the nontraded and traded good sectors, respectively. The household's budget constraint is:

$$pc \leq w (L^N + L^T) + \pi + T, \quad (3.2)$$

where  $p$  is the price of consumption,  $w$  is the wage rate,  $\pi$  denotes lump-sum profits and  $T$  denotes a lump-sum transfer payment from the government. Here, we have imposed a property of the equilibrium of the model, namely that the wage rate in the non-traded and traded good sectors must be the same. All the quantities in (3.2) are measured in units of the traded good.

The consumption good is produced using intermediate goods, of which there are two types. One is a tradeable good and the other is non-traded. Each of these intermediate goods is essential in the production of the final good. The final good production function is Leontieff in terms of traded and nontraded intermediate goods:

$$c = \min \{(1 - \gamma)c^T, \gamma c^N\}. \quad (3.3)$$

The one period in our example model is the analog of period 0 in our dynamic model. In that model, the economy is in a steady state before period 0, and then in period 0 a collateral

constraint suddenly and unexpectedly becomes binding. Since employment in the traded good sector is chosen by intermediate good firms at the very beginning of the period, in period 0 employment is predetermined at the time of the collateral shock. Thus, for purposes of the analysis in this section, we treat intermediate good firms' choice of  $L^T$  as a fixed constant, not subject to their choice. As a result, the only variable input in traded good production, from the point of view of intermediate good firms, is the imported intermediate good,  $z$ . This good must be financed at the beginning of the period by foreign borrowing, and is subject to a collateral constraint. The imported intermediate good,  $z$ , is essential to overall economic activity by the Leontieff assumption, (3.3). We suppose that non-traded goods are produced using a Cobb-Douglas function of labor,  $L^N$ , and capital,  $K^N$ . The production functions for traded and non-traded goods is given by:

$$\begin{aligned} y^T(z) &= V^\theta z^{1-\theta}, \quad y^N(L^N) = (K^N)^\alpha (L^N)^{1-\alpha} \\ 0 &< \theta, \alpha < 1, \end{aligned} \tag{3.4}$$

respectively, where  $y^T$  and  $y^N$  denote gross output of traded and non-traded goods, respectively. Value-added in the traded good sector,  $V$ , is a Cobb-Douglas function of capital and labor in that sector:

$$V = A (K^T)^\nu (L^T)^{1-\nu}, \quad 0 < \nu < 1.$$

Production of traded and non-traded intermediate goods is carried out by a single, representative, competitive firm. This assumption allows us to sidestep potential technical complications arising from the fact that some of the economy's collateral, the capital stock in the non-traded good sector, exists in a sector different from the sector that requires collateral for borrowing. By locating all production in a single firm, we ensure that all the economy's collateral is available to the agents who need it for borrowing.<sup>11</sup> To some extent our assumption about firms resembles the situation of actual firms in some emerging economies. See, for example, our discussion of chaebols in section 2. An alternative interpretation of our assumption about firms is that it is a stand-in for the existence of financial institutions and markets that distribute collateral among domestic agents.

As indicated in the previous paragraph, the representative intermediate good firm operates the two technologies, (3.4), and seeks to maximize profits, which we denote by  $\pi$  :

$$\pi = p^N y^N + y^T - q^N (K^N - K_0^N) - q^T (K^T - K_0^T) - w(1 + \tau)L^N - wL^T - R^*z.$$

Here,  $p^N$  denotes the price of non-traded goods,  $q^i$  denotes the price of physical capital in sector  $i$ , and  $\tau$  denotes the labor tax rate. This tax is rebated in lump sum form to households via

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<sup>11</sup>For an analysis of situations in which collateral is not equally distributed in the economy, see Caballero and Krishnamurthy (2001).

$T$  in their budget constraint. In addition,  $K_0^i$  is the representative firm's initial endowment of sector  $i$  capital. It is convenient to express the firm's profits in non-traded goods units:

$$\frac{\pi}{p^N} = y^N + \frac{1}{p^N} [y^T - R^*z] - \frac{q^N}{p^N}(K^N - K_0^N) - \frac{q^T}{p^N}(K^T - K_0^T) - \frac{w}{p^N}(1 + \tau)L^N - \frac{w}{p^N}L^T. \quad (3.5)$$

Foreign borrowing is subject to the constraint that a fraction of the value of the firm's assets must be no less than the firm's end-of-period international obligations:

$$\begin{aligned} \tau^N q^N K^N + \tau^T q^T K^T &\geq R^*z \\ 0 < \tau^N \leq 1, \quad 0 \leq \tau^T &\leq 1, \end{aligned} \quad (3.6)$$

where  $\tau^N$  and  $\tau^T$  are the fractions of capital in the indicated sectors that can be used for collateral.

The timing of the intermediate good firm's decisions is as follows. First, the labor tax rate,  $\tau$ , becomes known. Then, a market opens in which intermediate good firms trade capital among themselves at prices,  $q^N$  and  $q^T$ . Then  $z$ ,  $L^N$ ,  $c$ ,  $y^N$  and  $y^T$  are determined and production occurs. Immediately after paying its wage bill, the intermediate good firm decides whether to default on its international loans. If it does, then the creditors can seize from the firm an amount of output equal to the firm's obligations. It is easy to verify that the firm's revenues, after paying the wage bill, are sufficient for this.<sup>12</sup>

The resource constraints in our economy are as follows:

$$y^N = c^N, \quad y^T = c^T + zR^*.$$

The first of these expressions states that all the output of the non-traded good sector,  $y^N$ , is used as inputs in the production of non-traded goods. The second says that the gross output of the traded good sector is divided between inputs into the production of final goods,  $c^T$ , and gross interest payments abroad for borrowing to finance the imported intermediate good,  $z$ .

### 3.2. Qualitative Analysis

We list 8 equations that characterize 8 equilibrium variables -  $w$ ,  $p$ ,  $p^N$ ,  $q^N$ ,  $q^T$ ,  $L^N$ ,  $z$  and the Lagrange multiplier on (3.6) - for our example. Consider the representative final good producer. As long as input prices are strictly positive, the final good producer always sets  $c^T = [\gamma/(1 - \gamma)]y^N$ . Combining (3.3), (3.4) and the resource constraint, this implies:

$$y^T(z) - zR^* = \frac{\gamma}{1 - \gamma} (K^N)^\alpha (L^N)^{1-\alpha}. \quad (3.7)$$

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<sup>12</sup>Implicitly, we suppose that  $z$  has no value to the intermediate good producer other than as an input to production. For example, the producer has no incentive to abscond with  $z$  without producing anything.

If the price of, say,  $c^T$ , were zero, then the final good producer would be indifferent between purchasing an amount of  $c^T$  consistent with (3.7), or purchasing more. In such a case, we suppose that the producer resolves the indifference by imposing (3.7). Competition in final goods implies that price equals marginal cost:

$$p = \frac{1}{1-\gamma} + \frac{1}{\gamma}p^N, \quad (3.8)$$

The representative intermediate good firm's optimal choice of  $K^N$  and  $K^T$  leads to the following expressions for the price of capital in each sector:

$$q^N = \frac{\alpha p^N (K^N)^{\alpha-1} (L^N)^{1-\alpha}}{1 - \lambda \tau^N} \quad (3.9)$$

$$q^T = \frac{\theta \left(\frac{z}{\bar{v}}\right)^{1-\theta} A \nu \left(\frac{L^T}{K^T}\right)^{1-\nu}}{1 - \lambda \tau^T} \quad (3.10)$$

These are the first order necessary conditions for optimization in the Lagrangian representation of the representative intermediate good firm's problem. In (3.9) and (3.10),  $\lambda \geq 0$  is the multiplier on the collateral constraint, (3.6). Note that when the collateral constraint is binding, the price of capital exceeds its marginal value product. This reflects the services the capital provides in relaxing the collateral constraint.

The labor demand choice by the intermediate good firm leads it to equate the marginal cost,  $(1 + \tau)w$ , and value marginal product of labor in the production of non-traded goods to obtain (after making use of (3.8)),

$$\frac{1 - \alpha}{\left(\frac{1}{1-\gamma} \frac{1}{p^N} + \frac{1}{\gamma}\right) (1 + \tau)} (K^N)^\alpha (L^N)^{-\alpha} = \frac{w}{p}. \quad (3.11)$$

Optimization in the choice of  $z$  leads to the following first order condition:

$$\frac{1}{p^N} [y_z^T(z) - R^*(1 + \lambda)] = 0. \quad (3.12)$$

Evidently, for  $p^N < \infty$ , (3.12) corresponds to setting the expression in square brackets to zero. However, we will also consider the possibility  $p^N = \infty$  (this corresponds to a zero price on  $c^T$ ), in which case (3.12) does not require the expression in square brackets to be zero. Finally, the complementary slackness condition on  $\lambda$  for intermediate good firm optimization is:

$$\lambda [\tau^N q^N K^N + \tau^T q^T K^T - R^* z] = 0, \quad \lambda \geq 0, \quad \tau^N q^N K^N + \tau^T q^T K^T - R^* z \geq 0. \quad (3.13)$$

Market clearing requires that prices be strictly positive:

$$q^N, q^T, w, p^N > 0. \quad (3.14)$$

The latter, in combination with (3.9), impose an upper bound on  $\lambda$ ,  $\lambda \leq \bar{\lambda}$ , where

$$\bar{\lambda} \equiv \min [1/\tau^N, 1/\tau^T].$$

Household optimization of employment leads to the following labor supply curve:

$$\psi_o (L^N + L^T)^\psi = \frac{w}{p}. \quad (3.15)$$

The 8 equations that characterize equilibrium are (3.7), (3.8), (3.9), (3.10), (3.11), (3.12), (3.13), (3.15), together with the non-negativity constraints, (3.14), and  $0 \leq \lambda \leq \bar{\lambda}$ .

In Appendix A, we establish the following proposition:

**Proposition 3.1.** *Consider a parameterization of the model in which the equilibrium is unique and the collateral constraint is binding ( $\lambda > 0$ ). Generically, a small increase in  $\tau$  leads to an increase in  $p^N$ ,  $z$ ,  $L^N$ , the value of total assets and welfare.*

This proposition establishes that an increase in the tax on labor raises the real exchange rate ( $p^N$ ), asset values ( $\tau^N q^N K^N + \tau^T q^T K^T$ ), intermediate good imports ( $z$ ), employment ( $L^N$ ) and welfare in the static version of our model. This is so, if the initial equilibrium is unique and the collateral constraint binds.

We provide a sketch of the proof to this proposition here. If we drop the complementary slackness condition, (3.13), and fix the value of the multiplier,  $\lambda$ , we are able to compute the remaining 7 equilibrium variables in the model uniquely. We denote the asset values and level of intermediate good imports computed in this way by  $q^N(\lambda; \tau)$ ,  $q^T(\lambda)$ , and  $z(\lambda)$ , respectively. The variable,  $\tau$ , is not included in the argument of  $z(\cdot)$  and  $q^T(\cdot)$  because, conditional on a fixed value of  $\lambda$ , the equilibrium value of these variables are not a function of  $\tau$ . In the case of  $z$ , this is obvious, since  $z(\lambda)$  is defined by the requirement that the object in square brackets in (3.12) is zero. With this notation, we define the following function:

$$C(\lambda; \tau) = \tau^N q^N(\lambda; \tau) K^N + \tau^T q^T(\lambda) K^T - R^* z(\lambda).$$

Let  $\lambda^*$  and  $\tau^*$  denote the multiplier and labor tax rate in the type of equilibrium considered in the proposition. In addition to uniqueness, that proposition supposes  $\lambda^* > 0$ , so that by (3.13),  $C(\lambda^*, \tau^*) = 0$ . The proof requires establishing that a small increase in  $\tau$  above  $\tau^*$  results in a fall in the equilibrium value of the multiplier. That employment, asset values and utility are all higher in the new equilibrium then follows trivially.

We establish that the equilibrium value of  $\lambda$  is decreasing in  $\tau$  for  $\tau \geq \tau^*$  in two steps. First, we show that  $C(\lambda, \tau)$  is increasing in  $\lambda$  in a neighborhood of  $\lambda^*$  for given  $\tau$ . Second, we show that  $q^N(\lambda, \tau)$  (and, hence,  $C(\lambda, \tau)$ ) is increasing in  $\tau$  for fixed  $\lambda$ .

To establish that  $C$  is increasing in  $\lambda$ , the Appendix shows that for  $\lambda$  approaching its upper bound, at least one of  $q^N$  or  $q^T$  diverges to  $+\infty$ . To see the economic motivation for this result, suppose  $\tau^T < \tau^N$ . The benefit of a marginal unit of  $K^N$  is its collateral value,  $\lambda q^N \tau^N$ , plus its marginal value product. When  $\lambda \rightarrow 1/\tau^N$ , then  $\lambda q^N \tau^N = q^N$ , and the collateral value of capital equals its purchase price. In this case,  $K^N$  is a ‘money-pump’: a \$1 purchase of  $K^N$  generates \$1 in value as collateral *plus* the value marginal product of capital in production. Consequently, as  $\lambda \rightarrow 1/\tau^N$  the demand for  $K^N$  approaches infinity, as does its market clearing price,  $q^N$ . If  $\tau^T > \tau^N$ , then  $\bar{\lambda} = 1/\tau^T$ . In this case, if  $\lambda \rightarrow 1/\tau^T$ , then  $q^T \rightarrow \infty$ . Because  $z(\lambda)$  is bounded above, it follows that  $C > 0$  for  $\lambda$  sufficiently large. This implies that, generically,  $C$  must be increasing in  $\lambda$  at  $\lambda = \lambda^*$ . It may be possible to construct an example where the slope of  $C$  at  $\lambda = \lambda^*$  is zero, but to avoid contradicting our assumption of a unique equilibrium, that slope would have to be zero at only the point,  $\lambda = \lambda^*$ . Such an example is non-generic. The slope of  $f$  cannot be negative at  $\lambda = \lambda^*$  because in this case,  $C > 0$  for sufficiently high values of  $\lambda$  would require that there be a second  $\lambda$  with  $f = 0$ , and such a scenario contradicts the hypothesis of equilibrium uniqueness. Thus, we conclude that, generically,  $C$  is strictly increasing in  $\lambda$  for  $\lambda$  near  $\lambda^*$ .

That  $q^N$  is increasing in  $\tau$  for fixed  $\lambda$  is also intuitive. The requirement that the expression in square brackets in (3.12) be zero has the effect of associating a unique  $z$  with each  $\lambda > 0$ , independent of the value of  $\tau$ . By (3.7) the given value of  $\lambda > 0$  also implies a unique  $L^N$ , independent of  $\tau$ . Under perfect competition,  $p^N$  must be equal to the marginal cost of producing the nontraded good. For a given value of  $L^N$ , a higher value of  $\tau$  raises that marginal cost, and so  $p^N$  is increasing in  $\tau$  for given  $\lambda$ . In view of (3.9), we conclude that  $q^N$  increases in  $\tau$  for given  $\lambda$ .

Since  $C$  has a positive slope at  $\lambda = \lambda^*$  and shifts up with a rise in  $\tau$ , it follows immediately that equilibrium  $\lambda$  is falling in  $\tau$  (see Figure 5). From this discussion, it is clear that what is crucial in the result is that  $\tau^N > 0$ . If  $\tau^N = 0$ , so that capital in the non-traded good sector is useless in the collateral constraint, then an increase in  $\tau$  has no impact on the equilibrium. So, although our result requires that some physical capital in the nontraded sector be available as collateral for borrowing by the traded sector, it does not require that this be the only or even the largest component of that collateral.

### 3.3. Quantitative Analysis

We illustrate the proposition in the previous subsection with a numerical example. We report equilibrium outcomes for a range of values of the labor tax rate. We adopt the following

parameter values:

$$\begin{aligned} A &= 2, R^* = 1.06, \theta = 0.8, \gamma = 0.43, \alpha = 0.25, \tau^N = \tau^T = 0.1, \\ \psi_0 &= 0.06, \psi = 1, K^N = K^T = 1, \nu = 0.3 \end{aligned}$$

We computed equilibrium allocations corresponding to  $\tau$  in the range, 0.00 to 0.85. The upper bound on this range is just below the tax rate that would drive the price of  $c^T$  to zero (see  $1/p^N$  in Figure 6).<sup>13</sup> The admissible set of equilibrium values of  $\lambda$  belongs to the compact set,  $J = [0, \bar{\lambda}]$ . By considering a fine grid of  $\lambda \in J$ , we found that, for each value of  $\tau$  considered, the equilibrium is unique. The values of utility,  $1/p^N$ ,  $\tau^N q^N K^N + \tau^T q^T K^T$ ,  $\lambda$ ,  $z$ ,  $L^N$  corresponding to each  $\tau$  are displayed in Figure 6. Note that for  $\tau$  in the range of 0 to 0.7,  $\lambda > 0$ . Consistent with the proposition, utility is strictly increasing in this range. The increase in  $\tau$  also raises  $p^N$ ,  $L^N$ ,  $z$  and  $\tau^N q^N K^N + \tau^T q^T K^T$ . The latter has the effect of relaxing the collateral constraint, which is reflected in the fall in  $\lambda$ . Note that the initial value of  $\lambda$  is extremely high. According to (3.12),  $\lambda$  is equivalent to a tax on the purchase of the foreign intermediate input. When  $\tau = 0$  this tax wedge is about 250%. By increasing the labor tax rate, the shadow tax rate on foreign borrowing is completely eliminated.

For  $\tau$  beyond 0.7, utility and employment are invariant to additional increases in  $\tau$ . This is because in this range,  $z$  is in a sense the binding constraint on domestic production. The amount of  $z$ , which is now pinned down by  $V$  and  $R^*$  in (3.12), determines  $L^N$  through (3.7).

## 4. The Dynamic, Monetary Model

Our model builds on the structure analyzed in the previous section, and so we limit explanations and motivations to what is new here.

### 4.1. Households

Household preferences over consumption and leisure are the dynamic version of the preferences in the previous section:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, L_t), \quad (4.1)$$

where the subscript  $t$  denotes the time  $t$  realization of the variable and:

$$u(c, L) = \frac{\left[ c - \frac{\psi_0}{1+\psi} L^{1+\psi} \right]^{1-\sigma}}{1-\sigma}. \quad (4.2)$$

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<sup>13</sup>As a result, the scarcity assumption on  $z$  discussed in Appendix A is satisfied for each  $\tau$  considered in the example.



The household begins the period with a stock of liquid assets,  $\tilde{M}_t$ . Of this, it allocates deposits,  $D_t$ , with the financial intermediary, and the rest,  $\tilde{M}_t - D_t$ , to consumption expenditures. The household faces the following cash constraint on consumption expenditures:

$$P_t^T p_t c_t \leq P_t^T w_t L_t + \tilde{M}_t - D_t, \quad (4.2)$$

where  $w_t$  denotes the wage rate and  $p_t$  denotes the price of final goods, both denominated in units of the tradable good. In addition,  $P_t^T$  denotes the domestic currency prices of traded goods.

The law of motion of the household's assets is:

$$\tilde{M}_{t+1} = R_t(D_t + X_t) + P_t^T \pi_t + \left[ P_t^T w_t L_t + \tilde{M}_t - D_t - P_t^T p_t c_t \right]. \quad (4.3)$$

Here,  $R_t$  denotes the gross domestic nominal rate of interest,  $\pi_t$  denotes firm profits and  $X_t$  is a liquidity injection from the monetary authority. Profits,  $\pi_t$ , are measured in units of traded goods. According to (4.3), the household's liquid assets at the beginning of period  $t + 1$  include interest earnings and principal on  $D_t + X_t$ , profits, and any cash that may be left unspent in the period  $t$  goods market.

The household maximizes (4.1) subject to (4.2)-(4.3), and a particular timing constraint. The household's deposit decision is made before the realization of the collateral shock and before the realization of the current period monetary action.

## 4.2. Firms

The structure of production is the same as in the static example. One representative, competitive firm produces the final good,  $c_t$ , and another representative, competitive firm produces intermediate goods.

### 4.2.1. Final Good Firms

Final goods are produced from intermediate goods using the following constant elasticity of substitution (CES) production function:

$$c = \left\{ [(1 - \gamma) c^T]^{\frac{\eta-1}{\eta}} + [\gamma c^N]^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}}, \quad \eta \geq 0, \quad 0 < \gamma < 1. \quad (4.4)$$

Here,  $\eta \geq 0$  denotes the elasticity of substitution between tradeable,  $c^T$ , and nontradable intermediate goods,  $c^N$ , respectively. Equation (4.4) reduces to (3.3) in the previous section in the Leontieff case,  $\eta = 0$ . The final good firm maximizes profits:

$$p_t c_t - c_t^T - p_t^N c_t^N,$$

where  $p_t^N = P_t^N / P_t^T$  and  $P_t^N$  denotes the domestic currency price of non-traded goods. The final good firm takes prices as given.

### 4.2.2. Intermediate Inputs

The representative firm that produces the traded and non-traded intermediate inputs manages three types of debt, two of which are short-term. The firm borrows at the beginning of the period to finance its wage bill and to purchase a foreign input, and repays these loans at the end of the period. In addition, the firm holds the outstanding stock of external (net) indebtedness,  $B_t$ .<sup>14</sup>

The firm's optimization problem is:

$$\max \sum_{t=0}^{\infty} \beta^t \Lambda_{t+1} \pi_t, \quad (4.5)$$

where

$$\pi_t = p_t^N y_t^N + y_t^T - w_t R_t L_t^N - w_t R_t L_t^T - R^* z_t - r^* B_t + (B_{t+1} - B_t). \quad (4.6)$$

Here,  $\pi_t$  denotes dividends, denominated in units of the traded good. Also,  $B_t$  denotes the stock of external debt at the beginning of period  $t$ , denominated in units of the traded good;  $R^*$  is the gross rate of interest (fixed in units of the traded good) on loans for the purpose of purchasing  $z_t$ ; and  $r^*$  is the net rate of interest (again, fixed in terms of the traded good) on the outstanding stock of external debt. The price,  $\Lambda_{t+1}$ , is taken parametrically by firms. In equilibrium, this price is the multiplier on  $\pi_t$  in the (Lagrangian representation of the) household problem.

The intermediate good firm production functions are:

$$\begin{aligned} y_t^T &= \left\{ \theta V_t^{\frac{\xi-1}{\xi}} + (1-\theta) [\mu z_t]^{\frac{\xi-1}{\xi}} \right\}^{\frac{\xi}{\xi-1}}, \\ V_t &= A (K^T)^\nu (L_t^T)^{1-\nu}, \\ y_t^N &= (K^N)^\alpha (L_t^N)^{1-\alpha}, \end{aligned} \quad (4.7)$$

where  $\xi$  is the elasticity of substitution between value-added,  $V_t$ , in the traded good sector and the imported intermediate good,  $z_t$ . In the production functions,  $K^T$  and  $K^N$  denote capital in the traded and non-traded good sectors, respectively. They are owned by the representative intermediate input firm. The stock of capital is assumed to be fixed throughout the analysis.

Total employment of the firm,  $L_t$ , is:

$$L_t = L_t^T + L_t^N.$$

In equilibrium, borrowing must satisfy the following restriction:

$$\frac{B_{t+1}}{(1+r^*)^t} \rightarrow 0, \text{ as } t \rightarrow \infty. \quad (4.8)$$

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<sup>14</sup>One implication of our assumptions is that all financial assets and liabilities in the economy are concentrated in the hands of a single (representative) firm. For a discussion of this property of our model, recall section 3.

We suppose that international financial markets impose that this limit cannot be positive. That it cannot be negative is an implication of firm optimality.

The intermediate good firm's problem at time  $t$  is to maximize (4.5) by choice of  $B_{t+j+1}$ ,  $y_{t+j}^N$ ,  $y_{t+j}^T$ ,  $z_{t+j}$ ,  $L_{t+j}^T$ ,  $L_{t+j}^M$  and  $L_{t+j}^N$ ,  $j = 0, 1, 2, \dots$  and the indicated technology. In addition, the firm takes all prices and rates of return as given and beyond its control. The firm also takes the initial stock of debt,  $B_t$ , as given. This completes the description of the firm problem in the pre-crisis version of the model, when collateral constraints are ignored.

The crisis brings on the imposition of the following collateral constraint:

$$\tau^N q_t^N K^N + \tau^T q_t^T K^T \geq R^* z_t + (1 + r^*) B_t. \quad (4.9)$$

Here,  $q^i$ ,  $i = N, T$  denote the value (in units of the traded good) of a unit of capital in the nontraded and traded good sectors, respectively. Also,  $\tau^i$  denotes the fraction of these stocks accepted as collateral by international creditors. The left side of (4.9) is the total value of collateral, and the right side is the payout value of the firm's external debt. Before the crisis, firms ignore (4.9), and assign a zero probability that it will be implemented. With the onset of the crisis, firms believe (correctly) that (4.9) must be satisfied in every period henceforth, and do not entertain the possibility that it will be removed.

Note that we do not include the firm's working capital loans in (4.9). One interpretation is that there are no collateral requirements on domestic loans. An alternative interpretation of the absence of working capital loans in (4.9) is that (i) domestic lenders accept a broader range of assets as collateral than do foreign lenders and (ii) this broader range of assets exists in such a large quantity that the collateral constraint on domestic loans is never binding.<sup>15</sup>

We obtain  $q_t^N$  and  $q_t^T$  by differentiating the Lagrangian representation of the firm optimization problem with respect to  $K^N$  and  $K^T$ , respectively. The equilibrium value of the asset prices,  $q_t^i$ ,  $i = N, T$ , is the amount that a potential firm would be willing to pay in period  $t$ , in units of the traded good, to acquire a unit of capital and start production in period  $t$ . We let  $\lambda_t \geq 0$  denote the multiplier on the collateral constraint ( $= 0$  in the pre-crisis period) in firm problem. Then,  $q_t^i$  satisfies

$$q_t^i = \frac{VMP_{k,t}^i + \beta \frac{\Lambda_{t+2}}{\Lambda_{t+1}} q_{t+1}^i}{1 - \lambda_t \tau^i}, \quad i = N, T. \quad (4.10)$$

Here,  $VMP_{k,t}^i$  denotes the period  $t$  value (in terms of traded goods) marginal product of capital in sector  $i$ . When  $\lambda_t \equiv 0$ , so that the collateral constraint is not binding, then  $q_t^i$  is the present discounted value of the marginal physical product of capital. Asset prices are higher when  $\lambda_t > 0$  reflecting that in this case capital is also valuable for alleviating the collateral constraint.

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<sup>15</sup>The assumption that more assets can be used as collateral against domestic borrowing than foreign borrowing in emerging markets is a basic assumption of Aoki, Benigno and Kiyotaki (2007).

In our model capital is never actually traded since all firms are identical in equilibrium. Out of equilibrium, the firm might default on its external debt, and foreign creditors would then force the sale of (a fraction of) the firm's physical assets. The price,  $q_t^i$ , is how many traded goods a domestic resident would be willing to pay in exchange for a unit of the  $i^{\text{th}}$  type of capital. Foreign creditors would receive those traded goods in the event of a default. We assume that with these consequences for default, default never occurs in equilibrium.

To understand the impact of a binding collateral constraint on firm decisions, it is useful to consider the Euler equations of the firm. Differentiating Lagrangian representation of the firm problem with respect to  $B_{t+1}$ :

$$1 = \beta \frac{\Lambda_{t+2}}{\Lambda_{t+1}} (1 + r^*) (1 + \lambda_{t+1}), \quad t = 0, 1, 2, \dots \quad (4.11)$$

Following standard practice in the small open economy literature, we assume  $\beta(1 + r^*) = 1$ . A high value for  $\lambda_{t+1}$ , which occurs when the collateral constraint is binding, raises the effective rate of interest on external debt. As a result, the price of  $\pi_t$  relative to  $\pi_{t+s}$  is increased, and we can expect  $\pi_t$  to be reduced. The firm can accomplish this by paying off the external debt, *i.e.*, running a positive current account. The other effect of  $\lambda_t > 0$  is to raise the effective interest rate cost of  $z_t$ , and so we can expect imports to drop with  $\lambda_t > 0$ . As emphasized in section 2, a drop in imports and a rise in the current account are two important features of a sudden stop.

### 4.3. Monetary Authority and Equilibrium

The financial intermediary takes domestic currency deposits,  $D_t$ , from the household at the beginning of period  $t$ . In addition, it receives the liquidity transfer,  $X_t = x_t M_t$ , from the monetary authority.<sup>16</sup> The financial intermediary then lends all its domestic funds to firms which use them to finance their employment working capital requirements,  $P^T w L$ . Clearing in the money market requires  $D_t + X_t = P_t^T w_t L_t$ , or, after scaling by the beginning-of-period  $t$  aggregate money stock,

$$d_t + x_t = p_t^T [w_t L_t^N + w_t L_t^T], \quad (4.12)$$

where  $d_t = D_t/M_t$ .

Equilibrium is a sequence of prices and quantities having the properties: (i) for each date, the quantities solve the household and firm problems, given the prices, and (ii) the labor, goods and domestic money markets clear.

Clearing in the money market requires that (4.12) hold and that actual money balances,  $M_t$ , equal desired money balances,  $\tilde{M}_t$ . Combining this with the household's cash constraint, (4.2),

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<sup>16</sup>In practice, injections of liquidity do not occur in the form of lump sum transfers, as they do here. It is easy to show that our formulation is equivalent to an alternative, in which the injection occurs as a result of an open market purchase of government bonds which are owned by the household, but held by the financial intermediary. To conserve on notation, we do not adopt this interpretation in our formal model.

we obtain the equilibrium cash constraint:

$$p_t^T p_t c_t = 1 + x_t. \quad (4.13)$$

According to this the total end of period stock of money must equal the value of final output,  $c_t$ . Market clearing in the traded good sector requires:

$$y_t^T - R^* z_t - r^* B_t - c_t^T = -(B_{t+1} - B_t). \quad (4.14)$$

The left side of this expression is the current account of the balance of payments, i.e., total production of traded goods, net of foreign interest payments, net of domestic consumption. The right side of (4.14) is the change in net foreign assets. Equation (4.14) reflects our assumption that external borrowing to finance the intermediate good,  $z_t$ , is fully paid back at the end of the period. That is, this borrowing resembles short-term trade credit. Note, however, that this is not a binding constraint on the firm, since our setup permits the firm to finance these repayments using long term debt. Market clearing in the nontraded good sector requires:

$$y_t^N = c_t^N. \quad (4.15)$$

Our procedure for computing the equilibrium of the model is described in details in Appendix B and corresponds to a variation on the procedure applied in Christiano, Gust and Roldos (2004)

## 5. Quantitative Analysis

In this section we begin with a discussion of the parameterization of the model. We then report the model's implications for optimal monetary policy.

### 5.1. Parameter Values and Steady State

The time period of the model is one-half year and the values of the model parameters are displayed in Table 3. These values were selected so that the model's steady state in the absence of collateral constraints (i.e., the 'pre-crisis steady state' in Table 4) roughly matches features of Korean data (and, to a lesser extent, Argentina) during the first semester of 1997. Tradables were about one-third of total production for Korea before the crisis, assuming that tradables correspond to the non-service sectors. Combining this share estimate with estimates of labor's share from Young (1995), we estimate capital income shares for the tradable and nontradable sector in Korea to be 0.48 and 0.21, respectively. These shares are similar to what Uribe (1997) and Rebelo and Vegh (1995) report for Argentina. They estimate that capital's share is 0.52 and 0.37 in the tradable and nontradable sectors of Argentina, respectively. We take an intermediate point between all these estimates by specifying  $\nu = 0.50$  and  $\alpha = 0.36$ . Reinhart and Vegh (1995)

estimate the elasticity of intertemporal substitution in consumption for Argentina to be equal to 0.2. We adopt the somewhat higher elasticity of 0.25 by setting  $\sigma = 4$ . We take the foreign interest rate to be equal to 6 percent and we assume a rate of money growth that implies an annual nominal domestic interest rate of 12 percent, roughly in line with the experience of Korea in the years before the crises. We set  $\psi = 1$ , implying a labor supply elasticity of 1.

To determine a value for  $\mu$  we considered the 1995 Korean input-output tables. According to those tables, the ratio of imported intermediate inputs to value added in manufacturing, construction, services and agriculture were 0.40, 0.04, 0.02, 0.01, respectively. Assuming that most tradables are in manufacturing, these findings are roughly consistent with our model specification that all imported intermediate goods are used in the tradable sector. We selected a value of  $\mu$  that implies  $z/V = 0.3$ , a number that corresponds closely with the manufacturing number in Korea (see Table 4 for the properties of the pre-crisis steady state of our model).

As noted above, we estimate that the share of tradable goods in Korean production is roughly one third. This is reasonably close to our model, where the analogous figure is 0.275. Finally, our initial stock of debt is 13.6, or 32 percent of annual GDP. This percent lies very close to Korea's stock of external debt on the eve of the crisis, which was 33 percent of annual GDP. The Korean debt to annual GDP ratio was around 26.8 percent of annual GDP at the end of the year 2000. This corresponds closely to the model, which implies a debt to annual GDP ratio of 27 percent in the post crisis steady state, the steady state associated with the collateral constraint (see Table 5).

## 5.2. Optimal Monetary Policy

We now consider the optimal monetary policy response to the unexpected imposition of the collateral constraint in period 0. In the periods before  $t = 0$  the economy is in the pre-crisis steady state in which there is no collateral constraint. At the start of period 0, the household makes its deposit decision and the intermediate good firm makes its employment decision in the traded good sector. Agents make these decisions in the belief that the economy will remain in the pre-crisis steady state. Immediately afterward, the collateral constraint on borrowing is imposed and agents correctly expect the constraint to remain in place forever. The monetary authority announces a sequence of (optimal) monetary actions from period 0 and on. The deterministic equilibrium is characterized by convergence to a new steady state.

The quantitative properties of the equilibrium are displayed in Figure 7. The thick line indicates the optimal equilibrium, while the thin line indicates a feasible equilibrium in which the interest rate increases by a smaller amount. The feasible equilibrium will be discussed later. Note first how the nominal rate of interest rises sharply in the period of the shock, jumping from a 12 percent annual rate in the initial steady state, to 78 percent in period 0. We denote

the real exchange rate by  $\epsilon$  :

$$\epsilon \equiv \frac{P^T}{P} \equiv \frac{1}{p} = \frac{1}{\left[ \left( \frac{1}{1-\gamma} \right)^{1-\eta} + \left( \frac{p^N}{\gamma} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}}, \quad (5.1)$$

where the last equality makes use of zero profits and productive efficiency in the final goods sector.<sup>17</sup> In (5.1) we have the familiar result that the real exchange rate is a monotone decreasing function of the relative price of nontraded versus traded goods,  $p^N$ . According to Figure 7, there is a substantial, 34 percent, real exchange rate depreciation in the period of the crisis, so that we can infer that  $p^N$  falls. The real exchange rate is virtually back at its pre-crisis level in period 1, the period after the collateral shock. The nominal exchange rate depreciates roughly as much (30 percent) as the real exchange rate in the period of the shock, though the impact on the nominal exchange rate is much more persistent.

Turning to asset prices, we consider the value of assets in the nontraded sector,  $q^N$ , and an index of all asset prices ('Stock market index'):

$$q_t^N \frac{q_s^N K^N}{q_s^N K^N + q_s^T K^T} + q_t^T \frac{q_s^T K^T}{q_s^N K^N + q_s^T K^T}.$$

Here, the subscript,  $s$ , denotes base year which we take to correspond to the initial steady state. Both the stock market index as well as assets in the non-traded sector increase in value by nearly 3 percent in the period of the shock and then settle at a 2 percent increase in value thereafter.

Note how the current account rises sharply in the optimal equilibrium, to over 5 percent of the initial steady state level of output. This reflects in part the 36 percent decline in imports of intermediate goods,  $z$ . Gross output drops by a very large 15 percent relative to its initial steady state level. Consumption and employment fall even more than output. The greater fall in employment reflects diminishing returns in production. In the new steady state, imports, employment, output and consumption are higher than they are in the initial steady state. This reflects that optimal policy drives the interest rate lower in the new steady state, and this reduces the inefficiency of the labor market. In addition, the lower external debt produced by the positive current account has a positive wealth effect on consumption. Inflation jumps from a 3 percent annual rate in the initial steady state to about 30 percent in the period after the shock, before stabilizing at -2.5 percent. The 4 percent higher level of employment in the new

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<sup>17</sup>We assume purchasing power parity in traded goods, so that  $P^T = SP^*$ , where  $S$  denotes the nominal exchange rate and  $P^*$  is the foreign price index. We assume that  $P^*$  is exogenous with respect to the events in the small open economy we study. Also

$$p^N = \frac{\gamma}{1-\gamma} \left( \frac{(1-\gamma)c^T}{\gamma c^N} \right)^{\frac{1}{\eta}}.$$

steady state raises the marginal productivity of capital and helps account for the permanently higher level of asset prices.

To understand the role of monetary policy, as opposed to the collateral shock itself, in these results we compare the optimal and benchmark equilibria. The results indicate that the sharp rise in the interest rate in the optimal policy has effects much like those in the simple static example in the previous section. The rise in the interest rate drives up  $p^N$  (note how the real exchange rate appreciates going from the benchmark to the optimal policy). The rise in  $p^N$  produces a rise in  $q^N$ , the value of assets in the nontraded sector. In the benchmark equilibrium,  $q^N$  falls 1.7 percent in the period of the shock, while - as noted above - it rises by 2 percent in the optimal equilibrium. The sharp rise in the interest rate has a similar positive impact on the overall value of assets. The rise in asset values alleviate the collateral constraint, so that the multiplier in the optimal equilibrium is substantially smaller than it is in the benchmark equilibrium. The improvement in the collateral constraint permits an expansion in imports and this in turn produces an expansion in employment, output and consumption. In the process, the exchange rate depreciation - both real and nominal - are less severe. In effect, the sharp rise in the interest rate slows - but does not reverse - the exchange rate depreciation.

Our model is too simple to justify formal econometric testing against the data. It is nevertheless important to see whether the model conforms qualitatively with actual currency crisis data. Credibility of the analysis also requires that the quantitative magnitude of the mechanisms analyzed here lie at least within an order of magnitude of the actual data. To investigate these issues, we compare the model's implications with the Korean data. Figure 8 shows the dynamic simulation of the model when policy in the model roughly replicates the interest rate in Korea. We see that, with one exception, the model's qualitative predictions correspond well with the actual data. The model captures the basic direction of movement of each of our 10 variables in the Korean currency crisis. The exception is that labor productivity fell during the Korean crisis whereas labor productivity rises in the wake the crisis in our model. Reductions in labor productivity and total factor productivity are often associated with severe economic recessions, and exploring the reasons for this is an important topic for research. Aoki, Benigno and Kiyotaki (2007)'s theoretical analysis explores the possibility that the international credit disruptions that are the focus of our analysis may be accompanied by domestic credit disruptions. Aoki, et al show how in principle the misallocation of resources induced by disruptions in domestic credit can produce a decline in labor productivity. We do not know whether integrating these considerations would substantially alter the conclusions of this paper.

Consider now the quantitative implications of the model. Figure 8 indicates that the quantitative effects of the mechanisms we explore are large. For example, consumption, employment, output and inflation substantially overshoot their empirical counterparts. At the same time, our model understates the movements in the current account, real and nominal exchange rate,



and asset prices. Still, in view of our model’s simplicity, we interpret the evidence in Figure 8 as broadly favorable to the notion that the model captures key aspects of the Korean currency crisis episode. This is a necessary condition for taking its policy implications seriously.

## 6. Conclusion

In this paper we studied the optimal monetary policy response to a financial crisis of the kind experienced by the Asian economies in 1997-98. These crises, as many other emerging market crises, were characterized by a sudden reversal in capital inflows. Using a particular open economy model with collateral constraints, we found that the optimal monetary response to such a crisis involves an initial increase in interest rates, followed by a relatively sharp and rapid reduction in rates in the aftermath of the crisis. Interestingly, this is the policy that was actually followed.

In our model, increasing the interest rate is very much like raising a tax. As a result, our analysis may also yield insight into the episodes of “expansionary fiscal consolidations” emphasized by a large literature initiated by Giavazzi and Pagano (1990). For example, Perotti (1999) presents some evidence that large tax increases are more likely to stimulate the economy when levels of debt are high. Based on this, he argues that a model is required in which the response of the economy to tax changes depends on the initial conditions, such as the level of debt. Our model is very much in this spirit.

To keep the analysis simple, our model abstracts from investment. In principle, including investment could improve the model’s empirical implications. However, whether it does so remains an important, open question. Because capital appears in the collateral constraint, investment in physical capital represents an alternative strategy - relative to that of paying off international debt - by which agents can reduce the burden of the collateral constraint. In effect, a binding collateral constraint creates incentives to pay off the external debt, as well as to invest in domestic capital.<sup>18</sup> Thus, in principle one cannot rule out the possibility that in an environment in which investment is a choice variable, a binding collateral constraint could lead to an increase in investment, and to a *fall* in the current account.<sup>19</sup> Clearly, this would deal a blow to the hypothesis that tightening collateral constraints were the proximate cause of the Asian financial crises. We suspect, however, that with reasonable investment adjustment costs and other frictions, paying off the international debt would dominate investment in physical capital as a strategy for reducing the burden of the collateral constraint. If so, then the introduction of variable investment would improve our model’s empirical implications, by magnifying the rise

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<sup>18</sup>For a recent statement of this conjecture, see Chari, Kehoe and McGrattan (2005).

<sup>19</sup>Mendoza (2005) provides an example of a sudden stop similar to ours, except that he also includes investment. He finds that when collateral constraints tighten, investment drops. (Mendoza does not study the implications of sudden stop for monetary policy, which is our central focus.)

in the current account in the wake of a financial crisis.

At a methodological level, this paper adds to the literature that studies the impact of financial frictions on the monetary transmission mechanism. In traditional models, financial frictions have the effect of magnifying - through an 'accelerator effect' - the effects of monetary actions, without changing their sign. In this model we have shown that financial frictions could actually have a 'reverse accelerator effect', in that they reverse the sign of the effect of a monetary action.

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Table 1: Syndicated Loans to Emerging Markets  
(in billions of U.S. dollars)

Year	Total	Secured	Secured as % of Total
1993	47.5	7.9	16.5
1994	64.9	11.5	17.7
1995	93.0	16.1	17.3
1996	104.3	22.0	21.1
1997	143.7	61.4	42.7
1998	77.3	25.9	33.5
1999	73.1	26.3	35.9

Source: Capital Data, Loanware

Table 2: Intermediate Imports and Total Imports

Panel A: **Thailand**

Year	Total	Intermediate	% of Total
<b>1993</b>	45,995	17,184	37%
<b>1994</b>	54,338	19,294	36%
<b>1995</b>	70,718	25,061	35%
<b>1996</b>	72,248	24,874	34%
<b>1997</b>	63,286	21,860	35%
<b>1998</b>	42,403	14,744	35%
<b>1999</b>	49,919	18,205	36%
<b>2000</b>	62,181	23,663	38%
<b>2001</b>	61,847	22,978	37%
<b>2002</b>	64,317	24,461	38%

Panel B: **Korea**

Total	Intermediate	% of Total
83,800	43,987	52%
102,348	50,158	49%
135,119	64,611	48%
150,339	68,556	46%
144,616	69,361	48%
93,282	45,593	49%
119,752	57,253	48%
160,481	78,975	49%
141,098	71,929	51%
152,126	73,891	49%

**Panel C: Malaysia**

Year	Total	Intermediate	% of Total
<b>1993</b>			
<b>1994</b>			
<b>1995</b>	77,601	50,447	65%
<b>1996</b>	78,426	52,201	67%
<b>1997</b>	79,036	51,922	66%
<b>1998</b>	58,293	40,901	70%
<b>1999</b>	65,389	48,321	74%
<b>2000</b>	81,963	61,233	75%
<b>2001</b>	73,856	53,271	72%
<b>2002</b>	79,881	56,939	71%

**Panel D: Indonesia**

Total	Intermediate	% of Total
28,376	20,035	71%
32,222	23,146	72%
40,921	29,610	72%
44,240	30,470	69%
46,223	30,230	65%
31,942	19,612	61%
30,600	18,475	60%
40,367	26,073	65%
34,669	23,879	69%
	24,118	

**Panel E: Philippines**

Year	Total	Intermediate	% of Total
<b>1993</b>	17,597	7,855	45%
<b>1994</b>	21,333	9,559	45%
<b>1995</b>	26,538	12,174	46%
<b>1996</b>	32,427	14,015	43%
<b>1997</b>	35,933	14,663	41%
<b>1998</b>	29,660	11,586	39%
<b>1999</b>	30,726	12,596	41%
<b>2000</b>	34,491	16,747	49%
<b>2001</b>	33,058	15,121	46%
<b>2002</b>	35,427	14,791	42%

Source: CEIC Data Company Ltd

Table 3: Parameters Values of the Model

$\beta$	0.943	$\gamma$	0.26
$\psi$	1.00	$R$	1.12
$R^*$	1.06	$r^*$	0.06
$\alpha$	0.36	$K^N$	10
$\nu$	0.5	$K^T$	5
$\zeta$	0.0	$\mu$	3.5
$\tau$	0.05	$\theta$	0.7
$\psi_0$	0.0036	$\sigma$	4
$A$	1.5	$\xi$	0.9
$\eta$	0.015		

Note : Here,  $\beta$ ,  $R$  and  $R^*$  are expressed in annualized terms.

Table 4: Pre-crisis steady state

$L$	30	$z$	2.7
$L^T$	7.3	$L^N$	22.7
$c^T$	6	$c^N$	16.9
$w$	0.4	$V$	9.1
$\frac{c^T}{p^N c^N + y^T - R^* z}$	0.275	$y^T$	9.2
$p^N$	0.9	$p^T$	0.05
$q^T$	22.4	$q^N$	19.4
$B$	13.6	$\frac{B}{\frac{p^N c^N}{c^T} + y^T - R^* z}$	0.32
		$\frac{B}{c^T}$	2.57



Table 5: Post-crisis Steady State Under Optimal Monetary Policy

$L$	31.2	$z$	2.7
$L^T$	7.6	$L^N$	23.6
$c^T$	6.2	$c^N$	17.3
$w$	0.43	$V$	9.3
$\frac{c^T}{p^N c^N + y^T - R^* z}$	0.276	$y^T$	9.3
$p^N$	0.9	$p^T$	0.04
$q^T$	22.8	$q^N$	19.8
$B$	12.33	$\frac{B}{2(p^N c^N + y^T - R^* z)}$	0.27
		$\frac{p^N c^N}{c^T}$	2.56

## 7. Appendix A: Proof of Proposition 1

Following is a proof of the proposition in section 3.2. We begin by describing the details of the mapping discussed in the text, taking the multiplier,  $\lambda \geq 0$ , on the collateral constraint, into candidate equilibrium prices and quantities. An equilibrium for  $\lambda$  is a value for this parameter such that the complementary slackness condition on the collateral constraint is satisfied (see (3.13)). We then discuss a condition on model parameters implied by the assumption in our proposition that equilibrium is unique. The condition ensures that the traded intermediate good,  $c^T$ , is a ‘scarce’ factor in the production of final goods. In particular, we note that, in the absence of the collateral constraint, there is a maximum amount of  $c^T$ , we call this amount  $c_0^T$ , that can be produced, after paying for the required imported intermediate good,  $z$ . Given that employment,  $L^T$ , in the traded good sector is fixed in our static model, producing  $c_0^T$  does not require the reallocation of domestic resources from other useful activities. Under these circumstances, the domestic market price of  $c_0^T$  will be positive only if  $c_0^T$  is ‘scarce’. That is,  $c^T$  is scarce if with a zero price on  $c^T$  and in the absence of collateral constraints, domestic demand for  $c^T$  would exceed  $c_0^T$ . When  $c^T$  is not scarce, then there are at least two equilibria, if there are any. Evidently, that  $c^T$  is scarce is an implication of our assumption that equilibrium is unique.

We begin by defining a set of candidate equilibrium functions,  $z(\lambda, \tau)$ ,  $L^N(\lambda, \tau)$ ,  $p^N(\lambda, \tau)$  which satisfy, for a given  $\tau, \lambda \geq 0$ ,

$$\frac{1}{p^N} [y_z^T(z) - R^*(1 + \lambda)] = 0 \quad (7.1)$$

$$V^\theta z^{1-\theta} - R^* z = \frac{\gamma}{1-\gamma} (K^N)^\alpha (L^N)^{1-\alpha} \quad (7.2)$$

$$\frac{1}{p^N} = \frac{1-\gamma}{\gamma} \left[ \frac{\kappa}{1+\tau} - 1 \right], \quad (7.3)$$

where  $p^N \geq 0$  and

$$\kappa = \frac{\gamma(1-\alpha)(K^N)^\alpha}{\psi_0(L^N + L^T)^\psi (L^N)^\alpha}. \quad (7.4)$$

Equations (7.1) and (7.2) are (3.12) and (3.7), respectively, reproduced here for convenience. Equation (7.3) is obtained by using (3.11) and (3.15) to substitute out for  $w/p$ .

Let  $z_\lambda$  be the value of  $z$  that sets the object in square brackets in (7.1) to zero:

$$z_\lambda = \left( \frac{1-\theta}{R^*(1+\lambda)} \right)^{\frac{1}{\theta}} V, \quad \lambda \geq 0. \quad (7.5)$$

This will be our candidate equilibrium value of  $z$  in case it turns out that  $1/p^N > 0$ . The function,  $z_\lambda$ , is strictly positive and strictly decreasing for each  $\lambda \geq 0$ , and  $z_\lambda \rightarrow 0$  as  $\lambda \rightarrow \infty$ . Define the function,  $c_\lambda^T$ , by:

$$c_\lambda^T \equiv V^\theta z_\lambda^{1-\theta} - R^* z_\lambda, \quad \lambda \geq 0$$

It is readily verified that the function,  $c_\lambda^T$ , is strictly decreasing and positive for each  $\lambda \geq 0$ , and that  $c_\lambda^T \rightarrow 0$  as  $\lambda \rightarrow \infty$ . Let  $L_\lambda^N$  the value of  $L^N$  implied by (7.2) for the given value of  $c_\lambda^T$  :

$$L_\lambda^N = \left[ \frac{(1-\gamma) c_\lambda^T}{\gamma (K^N)^\alpha} \right]^{\frac{1}{1-\alpha}}.$$

Evidently,  $L_\lambda^N$  is strictly positive and strictly decreasing for each  $\lambda \geq 0$ , with  $L_\lambda^N \rightarrow 0$  as  $\lambda \rightarrow \infty$ .

Define the function,  $\kappa_{\lambda,\tau}$  :

$$\kappa_{\lambda,\tau} = \max \left[ \frac{\gamma(1-\alpha)(K^N)^\alpha}{\psi_o(L_\lambda^N + L^T)^\psi (L_\lambda^N)^\alpha}, 1 + \tau \right].$$

The first object in square brackets is strictly positive and increasing in  $\lambda \geq 0$ , converging to  $\infty$  as  $\lambda \rightarrow \infty$  and converging to a positive constant as  $\lambda \rightarrow 0$ . If that constant is less than  $1 + \tau$ , there is a value of  $\lambda$ , call it  $\tilde{\lambda}(\tau)$ , such that the first and second terms are equal. That is,  $\tilde{\lambda}(\tau)$  is defined by

$$\frac{\gamma(1-\alpha)(K^N)^\alpha}{\psi_o(L_{\tilde{\lambda}(\tau)}^N + L^T)^\psi (L_{\tilde{\lambda}(\tau)}^N)^\alpha} = 1 + \tau,$$

if such a  $\tilde{\lambda}(\tau) \geq 0$  exists. The function,  $\kappa_{\lambda,\tau}$ , is strictly positive for  $\lambda \geq 0$ . If  $\tilde{\lambda}(\tau)$  does not exist, then  $\kappa_{\lambda,\tau}$  is strictly increasing in  $\lambda$  for all  $\lambda \geq 0$ , and otherwise  $\kappa_{\lambda,\tau}$  is strictly increasing for all  $\lambda \geq \tilde{\lambda}(\tau)$ .

Let

$$\frac{1}{p^N(\lambda, \tau)} = \frac{1-\gamma}{\gamma} \left[ \frac{\kappa_{\lambda,\tau}}{1+\tau} - 1 \right]. \quad (7.6)$$

Note that if  $\kappa_{\lambda,\tau} > 1 + \tau$ , then  $1/p^N(\lambda) > 0$ . In this case, condition (7.1) requires the expression in square brackets to be zero, and so in this case we set  $z(\lambda, \tau) = z_\lambda$  and  $L^N(\lambda, \tau) = L_\lambda^N$ .

Suppose  $\kappa_{\lambda,\tau} = 1 + \tau$ . Then condition (7.1) does not require the expression in square brackets to be zero. Let  $L^N(\tau)$  be the unique solution to the following expression:

$$\frac{\gamma(1-\alpha)(K^N)^\alpha}{\psi_o(L^N(\tau) + L^T)^\psi L^N(\tau)^\alpha} = 1 + \tau. \quad (7.7)$$

Note that  $L^N(\tau)$  is strictly decreasing in  $\tau$ . If  $\kappa_{\lambda,\tau} = 1 + \tau$ , we set  $L^N(\lambda, \tau) = L^N(\tau)$ . That is:

$$L^N(\lambda, \tau) = \begin{cases} L_\lambda^N & \frac{1}{p^N(\lambda, \tau)} > 0 \\ L^N(\tau) & \frac{1}{p^N(\lambda, \tau)} = 0 \end{cases}.$$

Note,

$$L_\lambda^N \geq L(\lambda, \tau). \quad (7.8)$$

When  $\kappa_{\lambda,\tau} = 1 + \tau$ , we use (7.2) to define  $z(\lambda, \tau)$  :

$$V^\theta z(\lambda, \tau)^{1-\theta} - R^* z(\lambda, \tau) = \frac{\gamma}{1-\gamma} (K^N)^\alpha (L^N(\lambda, \tau))^{1-\alpha}.$$

Condition (7.8) and the fact that  $z_\lambda$  and  $L_\lambda^N$  both satisfy (7.2) imply that the previous equation generically has two solutions. The object,  $z(\lambda, \tau)$ , is taken to be the smaller of the two solutions. It is easy to verify that

$$z_\lambda \geq z(\lambda, \tau).$$

Thus, when  $1/p^N = 0$ , then the object in square brackets in (7.1) evaluated at  $z(\lambda, \tau)$  is zero or, possibly, positive. Either way, (7.1) is satisfied.

This completes our discussion of the candidate equilibrium functions,  $z(\lambda, \tau)$ ,  $L^N(\lambda, \tau)$ ,  $p^N(\lambda, \tau)$ . Note that these functions satisfy (7.1)-(7.3), as well as the condition,  $p^N \geq 0$ . Expressions (3.8) and (3.11) can then be used to compute candidate equilibrium functions for  $w$  and  $p$ .

Next, we define the asset price functions, based on (3.9) and (3.10):

$$q^N(\lambda, \tau) = \frac{\alpha p^N(\lambda, \tau) (K^N)^{\alpha-1} L^N(\lambda, \tau)^{1-\alpha}}{1 - \lambda \tau^N}$$

$$q^T(\lambda, \tau) = \frac{\theta \left( \frac{z(\lambda, \tau)}{V} \right)^{1-\theta} A \nu \left( \frac{L^T}{K^T} \right)^{1-\nu}}{1 - \lambda \tau^T}$$

Define:

$$C(\lambda, \tau) = \tau^N q^N(\lambda, \tau) K^N + \tau^T q^T(\lambda, \tau) K^T - R^* z(\lambda, \tau).$$

An equilibrium is a value of  $\lambda \geq 0$  such that  $C(\lambda, \tau) \geq 0$  and  $\lambda C(\lambda, \tau) = 0$ .

It is easy to see that if  $1/p^N = 0$  when  $\lambda = 0$ , then it is possible to construct two equilibria. In this case,  $\tilde{\lambda}(\tau)$  exists and as  $\lambda \rightarrow \tilde{\lambda}(\tau)$  from above,  $1/p^N \rightarrow 0$ . As a result, as  $\lambda \rightarrow \tilde{\lambda}(\tau)$  then  $q^N \rightarrow \infty$ . In particular,  $C(\lambda, \tau) > 0$  for  $\lambda$  close enough to  $\tilde{\lambda}(\tau)$ . Since, for the reasons outlined in the text,  $C(\lambda, \tau) > 0$  for  $\lambda$  close enough to  $\bar{\lambda}$ , it follows that if there is an equilibrium, there are at least two. We rule out this scenario by assuming that the traded good input,  $c^T$  is *scarce*. That is, we assume

$$\frac{\gamma}{1-\gamma} (K^N)^\alpha (L^N(\bar{\tau}))^{1-\alpha} > c_0^T,$$

where  $\bar{\tau}$  is the largest value of the labor tax rate,  $\tau$ , that we consider. The term on the left of the equality is the equilibrium demand for  $c^T$  when the collateral constraint is absent and  $1/p^N = 0$ , and the term on the right is the maximal supply. With the above assumption,  $1/p^N > 0$  for  $\lambda \geq 0$ , and the argument for multiple equilibria just described does not apply.

The proof of the proposition in the text is now easy to summarize. The function,  $C(\lambda, \tau)$ , is continuous and bounded for each  $0 \leq \lambda < \bar{\lambda}$ . As  $\lambda$  approaches  $\bar{\lambda}$ , either  $q^N$  or  $q^T$  diverges to  $\infty$ . Hence, there is some  $\lambda$  close enough to  $\bar{\lambda}$  such that  $C(\lambda, \tau) > 0$ . Generically,  $C(\lambda, \tau)$  cuts the zero line (see Figure 5) from below.

In an equilibrium with  $\lambda > 0$ , it must be that  $1/p^N > 0$ . Suppose otherwise, that  $1/p^N = 0$ . In this case,  $q^N = \infty$  and  $C(\lambda, \tau) > 0$ , contradicting  $C(\lambda, \tau) = 0$ . From  $1/p^N > 0$ , it follows that  $L^N(\lambda, \tau)$  and  $q^T(\lambda, \tau)$  are not functions of  $\tau$ . The only way  $\tau$  enters  $C(\lambda, \tau)$  is via  $p^N(\lambda, \tau)$  in  $q^N(\lambda, \tau)$ . It is then easy to see that since  $p^N(\lambda, \tau)$  is increasing in  $\tau$ ,  $q^N(\lambda, \tau)$  is increasing in  $\tau$  too. Since  $C(\lambda, \tau)$  is increasing in  $\tau$  and  $C(\lambda, \tau)$  is increasing in  $\lambda$  at the equilibrium value of  $\lambda$ , for given  $\tau$ , it follows that equilibrium  $\lambda$  is decreasing in  $\tau$ .

To see what happens to equilibrium  $p^N$  with the increase in  $\tau$ , consider (7.6). According to that expression, the increase in  $\tau$  affects  $p^N$  in two ways. The direct channel via the denominator term drives  $p^N$  up. A second channel operates via  $\kappa_{\lambda, \tau}$ . When  $1/p^N > 0$ ,  $\kappa_{\lambda, \tau}$  is not a function of  $\tau$ , and it is an increasing function of  $\lambda$ . So, the fall in  $\lambda$  drives  $p^N$  up. With both channels driving  $p^N$  up after a rise in  $\tau$ , we conclude that equilibrium  $p^N$  rises with an increase in  $\tau$ .

To see what happens to  $z$ , note that when  $1/p^N > 0$ , then  $z$  is determined by (7.5). The fall in  $\lambda$  induced by the rise in  $\tau$  makes  $z$  increase. Because the collateral constraint is satisfied as a strict equality, we conclude that the value of assets increases. However, it is not clear whether this is because of a rise in  $q^T$  or  $q^N$ , or both.

Finally, consider utility. From (3.1):

$$c - \frac{\psi_0}{1 + \psi} (L^N + L^T)^{1+\psi} = \gamma (K^N)^\alpha (L^N)^{1-\alpha} - \frac{\psi_0}{1 + \psi} (L^N + L^T)^{1+\psi},$$

using (3.3) and (3.4). Differentiating this function, it is easy to verify that it is strictly increasing in  $L^N$  up to the point where ,

$$\frac{\gamma (1 - \alpha) K^\alpha}{\psi_0 (L^N + L^T)^\psi (L^N)^\alpha} = 1.$$

Our assumption that  $c^T$  is scarce guarantees  $\kappa > 1 + \tau$  in (7.4). We conclude that utility is increasing in  $\tau$ . Q.E.D.

It is straightforward to see what happens when the collateral function,  $C(\lambda, \tau)$ , crosses the zero line twice in Figure 5, in which case there are two equilibria. When  $\tau$  is increased there exists an equilibrium in the neighborhood of the high  $\lambda$  equilibrium, which satisfies our proposition. However, there exists an equilibrium in the neighborhood of the low  $\lambda$  equilibrium, in which the results of the proposition are reversed. These observations about comparative statistics when there are multiple equilibria but no credible equilibrium selection mechanism is available are of little practical interest.

## 8. Appendix B: Algorithm for Finding the Optimal Equilibrium

Monetary policy is characterized by a sequence of money growth rates,  $x_0, x_1, \dots$ . The optimal policy is the sequence that has an equilibrium with the highest utility associated with it. For

a given sequence of money growth rates, we compute an equilibrium for the model as follows. We impose that the steady state is achieved at a particular date,  $T + 1$ , and that the collateral constraint is non-binding thereafter. The computational strategy is a dynamic version of the strategy used to solve the static example in section 3. In particular, we find  $\lambda_0, \dots, \lambda_T$  which solve the  $T + 1$  complementary slackness conditions for  $t = 0, \dots, T$  associated with the collateral constraint in the Lagrangian representation of the firm problem. To evaluate these complementary slackness conditions for a given set,  $\lambda_0, \dots, \lambda_T$ , we proceed as follows. First, we fix a value of the new steady state debt, which we denote by  $B_s$ . Second, conditional on this value of  $B_s$ , we compute all the variables in the new steady state. Third, we use all the equilibrium conditions of the model, except (4.14) and the complementary slackness conditions, to compute a set of candidate values for variables in the dynamic equilibrium. Fourth, the current account equation, (4.14), and the initial debt are used to recursively compute  $B_t$ ,  $t = 1, \dots$ . Steps 2 to 4 define a mapping from  $B_s$  into itself. We adjust  $B_s$  until a fixed point is found. The complementary slackness equations are evaluated using the candidate equilibrium variables in step 3 together with the fixed point value of  $B_s$ . The values of  $\lambda_0, \dots, \lambda_T$  are adjusted until the complementary slackness conditions are satisfied. We set  $T = 19$ , although Figures 7 and 8 suggested that a smaller value of  $T$  would have worked just as well.

For many money growth sequences, including the optimal one, we found two equilibria. In one, the collateral constraint is satisfied as a strict equality in the new steady state, and in the other the collateral constraint is satisfied as a strict inequality in the new steady state. We always select the equilibrium that produces the higher level of utility, and this is typically the equilibrium in which the collateral constraint is satisfied as a strict inequality in the new steady state.

### 8.1. Equilibrium Conditions

We differentiate between the variables dated  $t = 0$  and  $t \geq 1$ , because the set of equations to be solved and the variables whose values are to be determined are different. The 17 variables to be solved for in period  $t \geq 1$  are:

$$VMP_{k,t}^N, VMP_{k,t}^T, q_t^N, q_t^T, c_t, c_t^N, c_t^T, L_t^N, L_t^T, p_t, p_t^N, p_{t-1}^T, R_t, z_t, d_t, \Lambda_t, y_t^T.$$

These variables must satisfy 17 equilibrium conditions. The household and firm intertemporal Euler equations imply:

$$\beta R_t = (1 + x_{t-1})(1 + \lambda_t)(p_t^T/p_{t-1}^T), \quad (8.1)$$

for  $t = 1, 2, \dots$ . We obtain equation (8.1) by combining the following three equations. The intertemporal Euler equation associated with the household deposit decision is:

$$u_{c,t} = \frac{p_t}{p_{t+1}} \frac{p_t^T}{p_{t+1}^T} \frac{\beta R_t u_{c,t+1}}{1 + x_t}, \quad t = 1, 2, \dots \quad (8.2)$$

The intertemporal Euler equation of the firm is:

$$\Lambda_t = (1 + \lambda_t) \Lambda_{t+1}, \quad t = 1, 2, \dots \quad (8.3)$$

Here,  $\Lambda_t$  is the multiplier on the household's period  $t-1$  flow budget constraint in the Lagrangian representation of the household problem. This multiplier satisfies:

$$\Lambda_t = \beta \left( \frac{u_{c,t} p_{t-1}^T}{p_t} \frac{1}{p_t^T (1 + x_{t-1})} \right), \quad (8.4)$$

for  $t = 1, 2, \dots$ . Equations (8.2)-(8.4) can be combined to produce 8.1.

The nontraded good production function is:

$$c_t^N = (K^N)^\alpha (L_t^N)^{1-\alpha}, \quad t = 0, 1, \dots \quad (8.5)$$

Optimization by final good producers implies:

$$p_t^N = \frac{\gamma}{1 - \gamma} \left( \frac{(1 - \gamma) c_t^T}{\gamma c_t^N} \right)^{\frac{1}{\eta}}, \quad t = 0, 1, \dots \quad (8.6)$$

The intermediate traded good production function is:

$$y_t^T = \left\{ \theta V_t^{\frac{\xi-1}{\xi}} + (1 - \theta) [\mu z_t]^{\frac{\xi-1}{\xi}} \right\}^{\frac{\xi}{\xi-1}}, \quad t = 0, 1, \dots \quad (8.7)$$

Optimization implies:

$$\left( \frac{y_t^T}{\mu z_t} \right)^{\frac{1}{\xi}} \mu (1 - \theta) = (1 + \lambda_t) R^*, \quad t = 0, 1, \dots \quad (8.8)$$

Labor in the traded and nontraded sectors receives the same wage, and so the value marginal product of labor in the two sectors must be the same:

$$(1 - \alpha) p_t^N \frac{c_t^N}{L_t^N} = \left( \frac{y_t^T}{V_t} \right)^{\frac{1}{\xi}} \theta (1 - v) \frac{V_t}{L_t^T}, \quad t = 1, \dots \quad (8.9)$$

Equation (8.9) does not hold for  $t = 0$  because employment in the traded good sector is predetermined then.

The value marginal product of capital in the traded good sector,  $VMPP_{k,t}^T$ , is:

$$VMPP_{k,t}^T = \left( \frac{y_t^T}{V_t} \right)^{\frac{1}{\xi}} \theta v \frac{V_t}{K^T}, \quad t = 0, 1, \dots \quad (8.10)$$

The value marginal product of capital in the non-traded good sector is:

$$VMP_{k,t}^N = \alpha p_t^N \frac{c_t^N}{K^N}, \quad t = 0, 1, \dots \quad (8.11)$$

Equation (4.2) with money market clearing condition (i.e., the wage bill equals deposits plus new money injections) implies, after scaling by the money stock:

$$p_t p_t^T c_t = (1 + x_t), \quad t = 0, 1, \dots \quad (8.12)$$

The condition that total money spend on consumption goods is equal to the wage bill plus money allocated by households to consumption goods implies:

$$p_t c_t = (1 - \alpha) \frac{p_t^N}{R_t (1 + \lambda_t)} \frac{c_t^N}{L_t^N} (L_t^T + L_t^N) + \frac{1 - d_t}{p_t^T}, \quad t = 0, 1, \dots \quad (8.13)$$

The expressions for the two asset prices are:

$$q_t^N = VMP_{k,t}^N + \lambda_t \tau^N q_t^N + \beta \frac{\Lambda_{t+2}}{\Lambda_{t+1}} q_{t+1}^N, \quad t = 0, 1, \dots \quad (8.14)$$

$$q_t^T = VMP_{k,t}^T + \lambda_t \tau^T q_t^T + \beta \frac{\Lambda_{t+2}}{\Lambda_{t+1}} q_{t+1}^T, \quad t = 0, 1, \dots \quad (8.15)$$

Equality of labor supply and labor demand in the non-traded good sector implies:

$$\psi_0 (L_t^N + L_t^T)^\psi p_t = (1 - \alpha) p_t^N \frac{c_t^N}{R_t (1 + \lambda_t) L_t^N}, \quad t = 0, 1, \dots \quad (8.16)$$

Zero profits and optimization by final good producers implies:

$$p_t = \left[ \left( \frac{1}{1 - \gamma} \right)^{1-\eta} + \left( \frac{p_t^N}{\gamma} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}, \quad t = 0, 1, \dots \quad (8.17)$$

The 17<sup>th</sup> equation is the final good production function, (4.4).

## 8.2. Steady State

The algorithm requires computing the new steady state conditional on a specified value of the steady state debt,  $B_s$ . Equation (8.1) in steady state implies:

$$R = \frac{1 + x}{\beta}, \quad (8.18)$$

where  $x$  is the money growth rate in the new steady state. Equation (8.8) implies:

$$\left( \frac{y^T}{\mu z} \right)^{\frac{1}{\xi}} \mu (1 - \theta) = R^*. \quad (8.19)$$



Equation (8.9) implies:

$$(1 - \alpha)p^N \frac{c^N}{L^N} = \left(\frac{y^T}{V}\right)^{\frac{1}{\xi}} \theta (1 - v) \frac{V}{L^T}, \quad (8.20)$$

Equations (8.10), (8.11), (8.14) and (8.15) imply

$$q^N = \frac{\alpha p^N \frac{c^N}{K^N}}{1 - \beta}, \quad (8.21)$$

$$q^T = \frac{\left(\frac{y^T}{V}\right)^{\frac{1}{\xi}} \theta v \frac{V}{K^T}}{1 - \beta}. \quad (8.22)$$

Equation (8.16) implies:

$$\psi_0 (L^N + L^T)^\psi p = (1 - \alpha) p^N \frac{c^N}{RL^N} \quad (8.23)$$

The traded goods resource constraint is:

$$y^T - R^* z - c^T = r^* B_s, \quad (8.24)$$

which says that net exports must equal the interest on the international debt.

The endogenous variables here are the following seven:  $L^N$ ,  $L^T$ ,  $p^N$ ,  $z$ ,  $q^T$ ,  $q^N$ , and  $R$ . The seven equations, (8.18)-(8.24), can be used to solve for these variables (the variables,  $c^N$ ,  $y^T$  and  $V$  are solved using the relevant production functions). The steady state value of  $p_t^T$  can be computed using (4.12) in steady state.

### 8.3. Backward Recursion

We now discuss how prices and quantities are computed based on a given set of sequences,  $\lambda_0$ ,  $\lambda_1, \dots, \lambda_T$  and  $x_0, x_1, \dots, x_T$  and  $B_s$ . We solve the equilibrium conditions recursively, beginning with the new steady state and working backwards. We start the backward iteration in period  $T$ , when  $p_t^T$  for  $t = T$  and all other variables dated  $T + 1$  and later are assumed to be in the new steady state. The calculations are done in two steps. First, we proceed for  $t = T, T - 1, \dots, 1$ . After that, we consider the variables in  $t = 0$ .

It is convenient to substitute out for  $p_t$  from (8.17) into (8.12), (8.13), (8.4), and (8.16). With this change, we have the following 16 unknowns:

$$VM P_{k,t}^N, VM P_{k,t}^T, q_t^N, q_t^T, c_t, c_t^N, c_t^T, L_t^N, L_t^T, p_t^N, p_{t-1}^T, R_t, z_t, \Lambda_t, d_t, y_t^T.$$

in 16 equations. We reduce these equations to two equations in two unknowns,  $L_t^N$  and  $c_t^T$ . Thus, fix  $L_t^N$  and  $c_t^T$ . Then,  $c_t^N$  is computed from (8.5) and  $c_t$  is computed from (4.4). The variable,  $p_t^N$  is computed using (8.6) with  $p_t$  replaced with (8.17). The variables,  $z_t$ ,  $L_t^T$ , and  $y_t^T$  are computed using (8.7), (8.8) and (8.9). We computed  $\Lambda_t$  using (8.3). We then computed

$p_{t-1}^T$  using (8.4) and the interest rate,  $R_t$ , using (8.1). The variables,  $VMP_{k,t}^T$  and  $VMP_{k,t}^N$  are computed using (8.10) and (8.11). The variable,  $d_t$ , is computed using (8.13). Then, (8.14) and (8.15) are solved for the asset prices,  $q_t^T$  and  $q_t^N$ . We adjust  $L_t^N$  and  $c_t^T$  until equation (8.12) and (8.16) are satisfied. We proceed sequentially, for  $t = T, T - 1, \dots, 1$ .

We now consider  $t = 0$ . Relative to the previous list of unknowns, we drop 4 variables:  $p_{-1}^T$ ,  $\Lambda_0$ ,  $L_0^T$ ,  $d_0$ . We drop  $\Lambda_0$  because (8.3) is only satisfied for  $t = 1, 2, \dots$ . We drop  $L_0^T$  because this variable is set to its value in the initial steady state. The list of 12 unknowns for this period is:

$$VMP_{k,0}^N, VMP_{k,0}^T, q_0^N, q_0^T, c_0, c_0^N, c_0^T, L_0^N, p_0^N, R_0, z_0, y_0^T.$$

We reduce these equations to two equations in two unknowns,  $L_0^N$  and  $c_0^T$ . Fix the value of  $L_0^N$  and  $c_0^T$ .

We obtain the values of  $c_0^N$  and  $c_0$  from (8.5) and (4.4) as before. The variable,  $p_0^N$  is computed using (8.6) with  $p_0$  replaced with (8.17). The variables,  $z_0$ , and  $y_0^T$  are computed using (8.7) and (8.8). We use (8.13) to compute  $R_0$ . We then obtain  $VMP_{k,0}^T$  and  $VMP_{k,0}^N$  from equations (8.10) and (8.11). Asset prices,  $q_0^N$  and  $q_0^T$ , are found using (8.14) and (8.15). Finally,  $L_0^N$  and  $c_0^T$  are adjusted until (8.12) and (8.16) are satisfied for  $t = 0$ .

The external debt,  $B_{t+1}$ , can be obtained by simulating the traded good market clearing conditions forward:

$$y_t^T - R^* z_t - r^* B_t - c_t^T = -(B_{t+1} - B_t), \quad t = 0, 1, \dots \quad (8.25)$$

for the given value of  $B_0$ . Adjust  $B_s$  until  $B_{T+1} = B_s$ .

We adjust the  $T + 1$  numbers,  $\lambda_t \geq 0$ ,  $t = 0, \dots, T$ , until the complementarity slackness conditions are satisfied:

$$\begin{aligned} \lambda_t (\tau^N q_t^N K^N + \tau^T q_t^T K^T - [R^* z_t + (1 + r^*) B_t]) &= 0, \\ \tau^N q_t^N K^N + \tau^T q_t^T K^T - [R^* z_t + (1 + r^*) B_t] &\geq 0, \end{aligned}$$

for  $t = 0, 1, \dots, T$ . Evidently, the strategy we use to solve the model involves solving  $T + 1$  complementary slackness conditions in  $T + 1$  non-negative multipliers. We used the algorithm and code in Miranda and Fackler (2002) to do this.

#### 8.4. Optimal Monetary Policy

To solve for the optimal monetary policy, we search over sequences of  $x_t$ 's,  $t \geq 0$ . In principle this is a impractically high-dimensional space. We reduced the dimension of this space by making  $x_0$ ,  $x_1$  and  $x_2$  free parameters. We impose that the optimal monetary policy involves setting  $x_t$  for  $t \geq 3$  to a value slightly above the one implied by the Friedman rule,  $x = \beta - 1 + \varepsilon$ , where

$\varepsilon = 0.0037$ . Figure 7 indicates that the system has roughly converged into the new steady state by period 2, suggesting that our assumption that the optimal  $x_t$  has converged to its steady state by period 3 is not a problem.

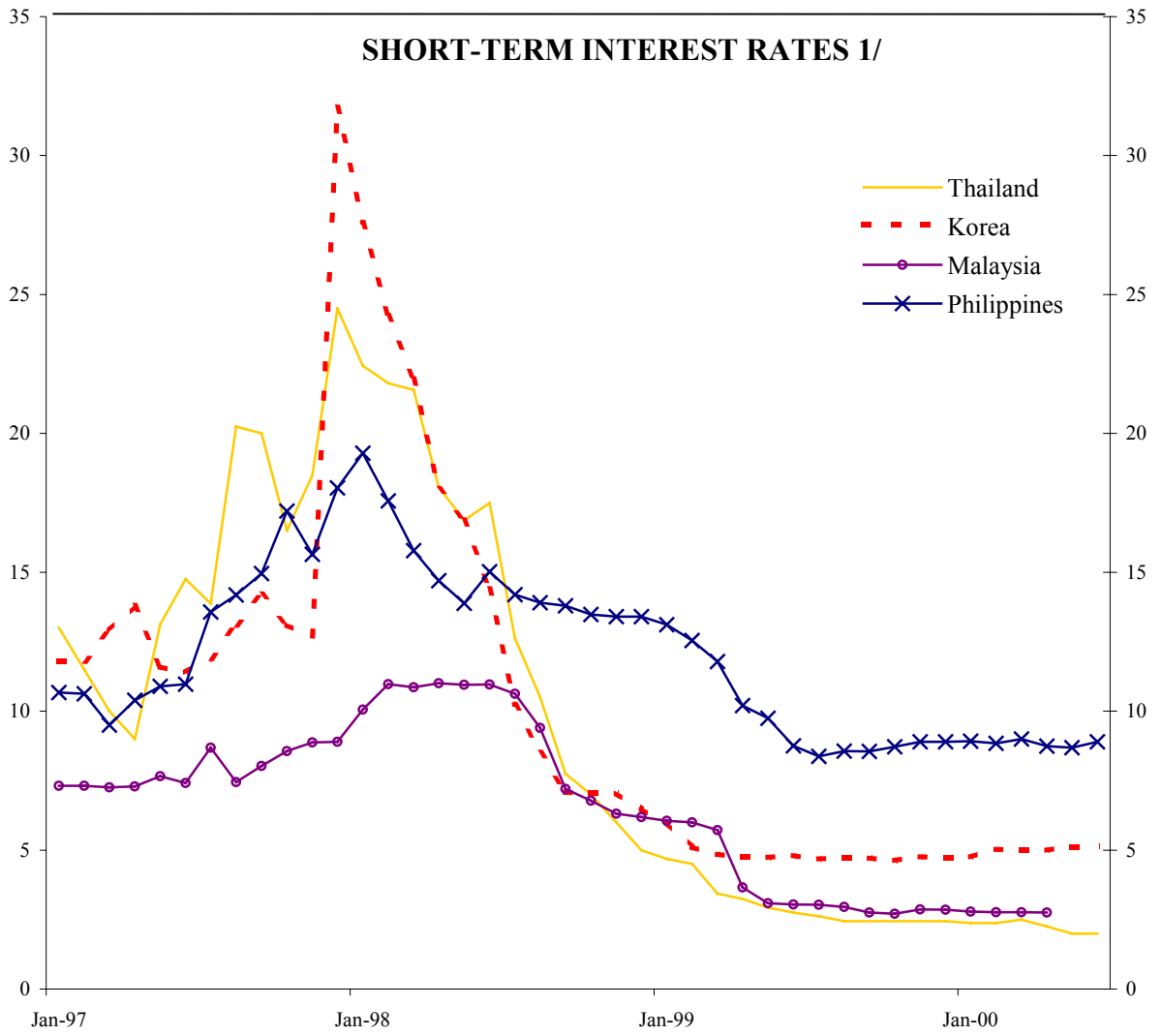
To find the optimal policy, we searched for  $x_0$ ,  $x_1$ , and  $x_2$  on a sequence of grids. The first grid is a coarse one:

$$\chi_t^0 = (-1, -0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8, 1), \text{ for } t = 0, 1, 2.$$

We computed an equilibrium for each of the 1331 points belonging to  $\chi_0^0 \times \chi_1^0 \times \chi_2^0$ . Denote the point on the first grid associated with the highest level of utility by  $(x_0^1, x_1^1, x_2^1)$ . We then computed a second grid of 1331 points around  $(x_0^1, x_1^1, x_2^1)$ . In this second grid, the grid of points for  $x_t$  is

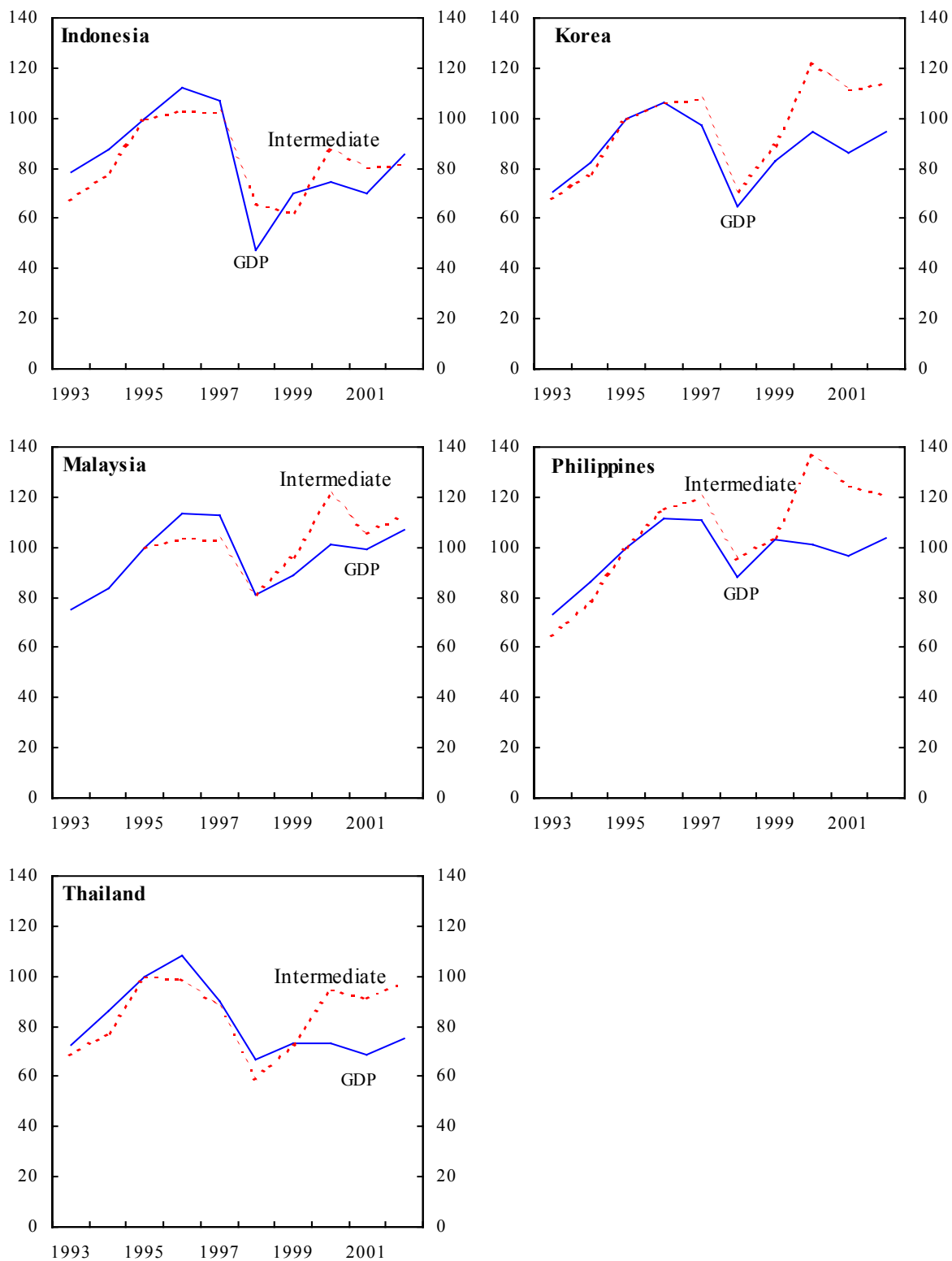
$$\begin{aligned} \chi_t^1 = & (x_t^1 - 0.1, x_t^1 - 0.2, x_t^1 - 0.3, x_t^1 - 0.4, x_t^1 - 0.5, \\ & x_t^1, x_t^1 + 0.1, x_t^1 + 0.2, x_t^1 + 0.3, x_t^1 + 0.4, x_t^1 + 0.5), \end{aligned}$$

for  $t = 0, 1, 2$ . We then computed an equilibrium for each of the 1331 points belonging to  $\chi_0^1 \times \chi_1^1 \times \chi_2^1$ . Denote the point in this grid associated with the highest level of utility by  $(x_0^2, x_1^2, x_2^2)$ . A new grid,  $\chi_t^2$ , of points was constructed as for  $\chi_t^1$ , except we did so around the point,  $x_t^2$ . We then computed equilibrium for each of the 1331 points belonging to  $\chi_0^2 \times \chi_1^2 \times \chi_2^2$ . The best point on this grid is our estimate of the globally optimal monetary policy. The money growth rates associated with the optimal policy computed in this way are  $x_0 = -0.27$ ,  $x_1 = 0.7$ ,  $x_2 = -0.03$ . Also,  $x_t = -0.02$ , for  $t > 2$ .



**Figure 1**

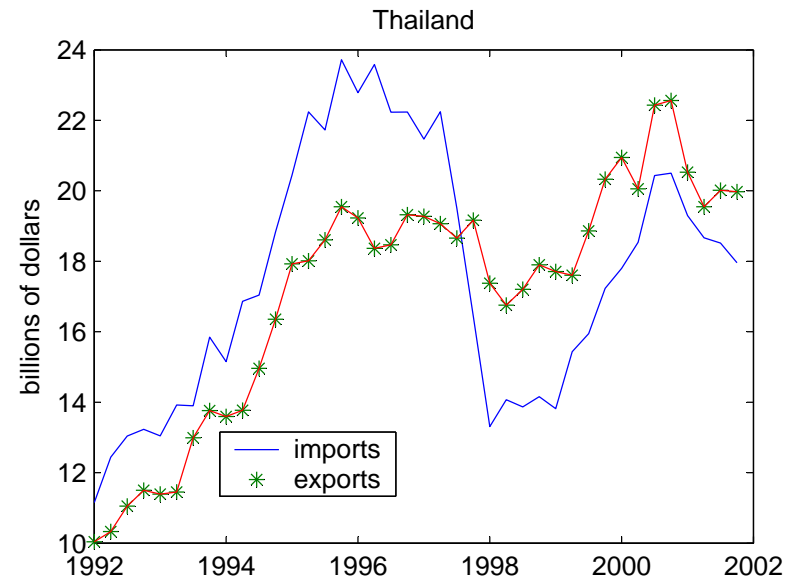
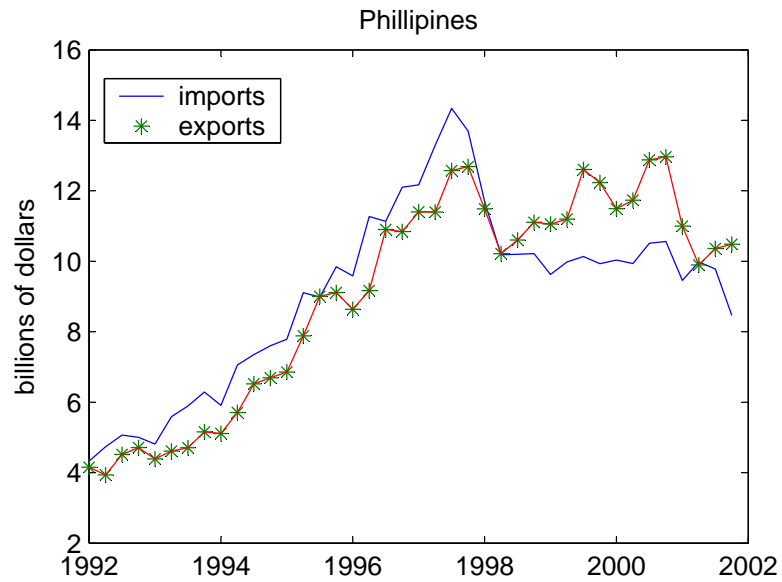
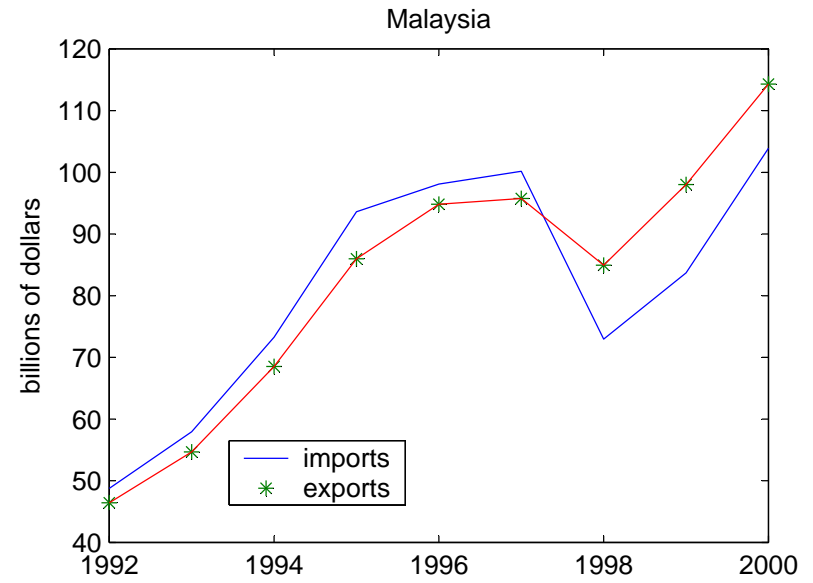
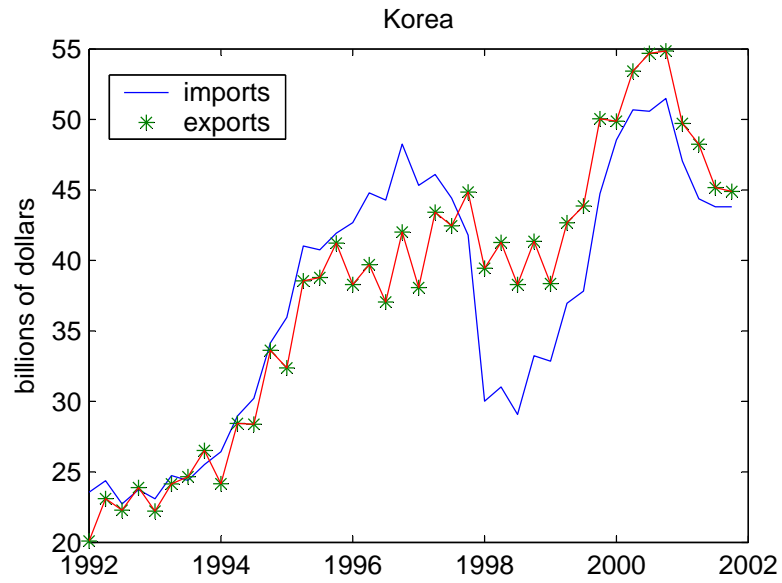
## Intermediate Goods Import vs. GDP (Index 1995 = 100)



Sources: CEIC; and WEO.

**Figure 2**

Figure 3: Exports and Imports



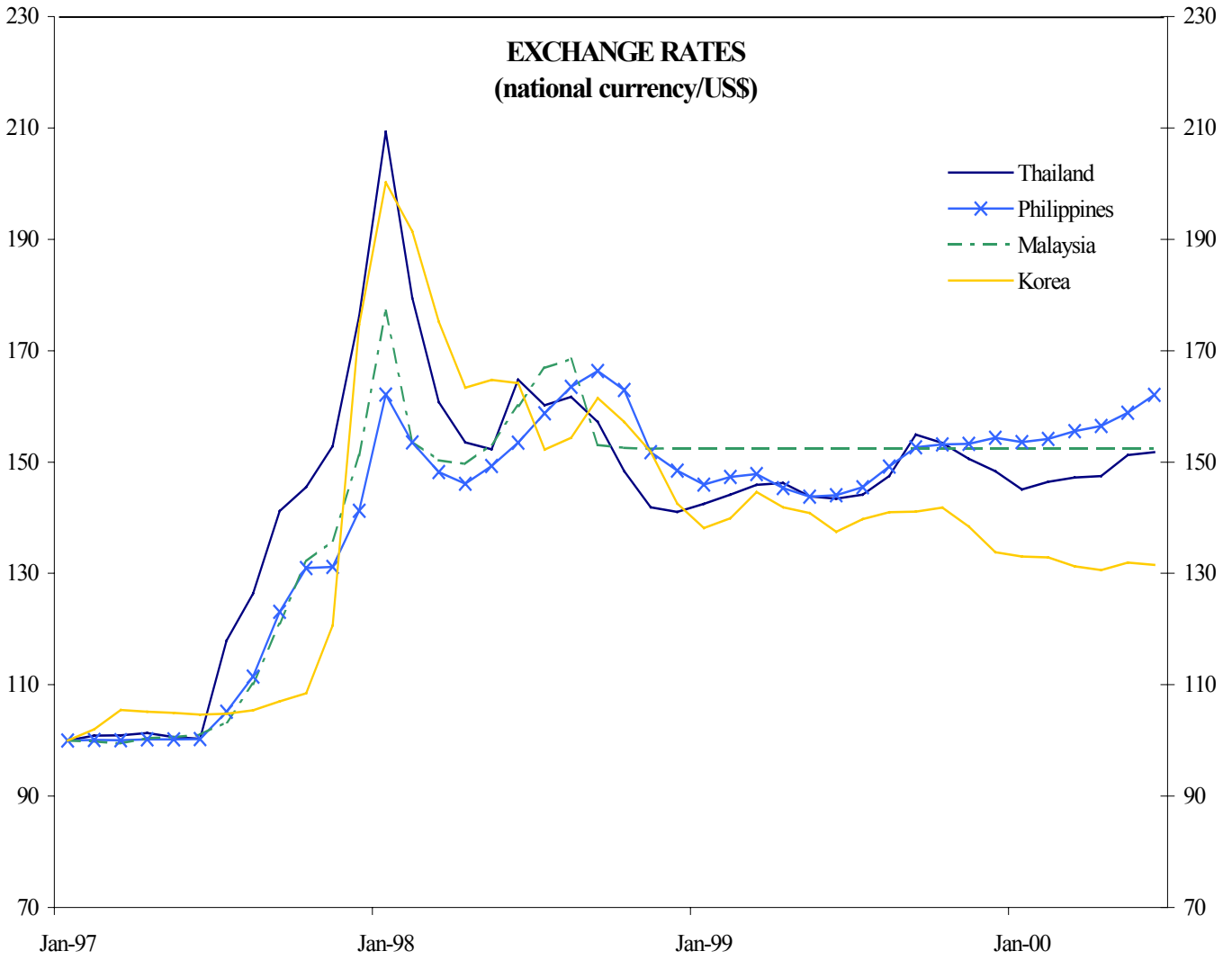


Figure 4

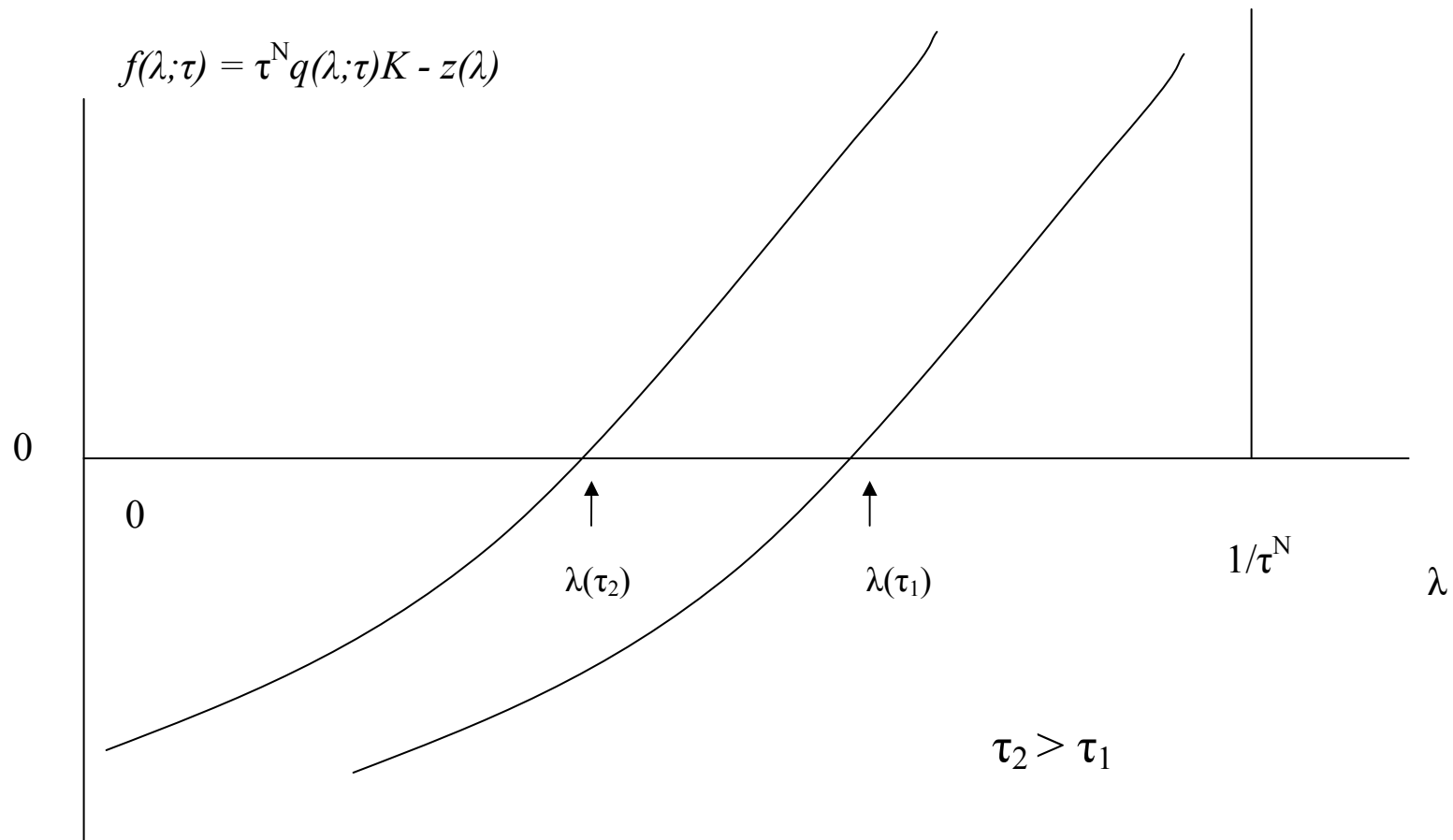


Figure 5: The Effect of An Increase in the Labor Tax Rate



Figure 6: Equilibrium Associated with Various Tax Rates

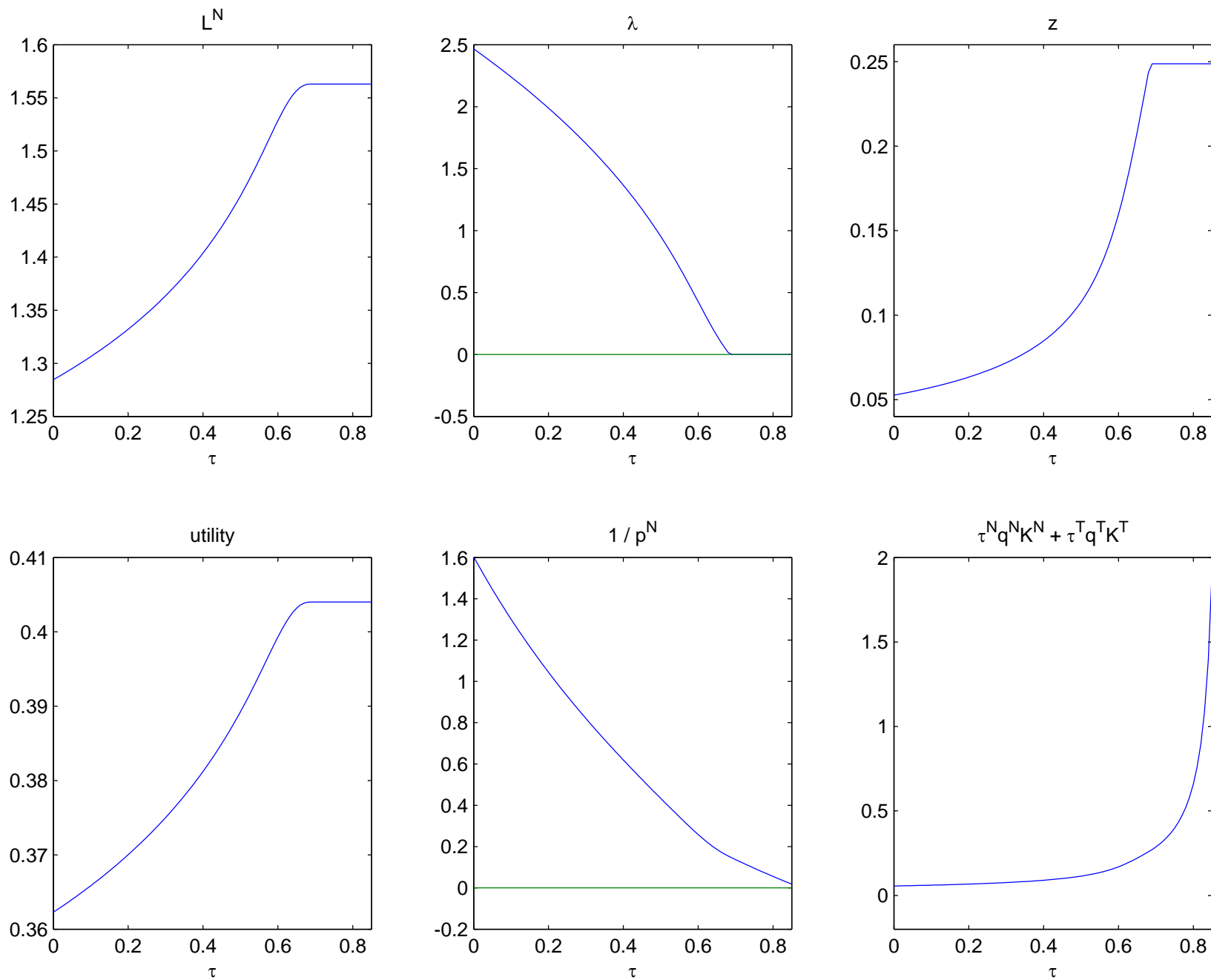


Figure 7: Optimal and Feasible Equilibrium

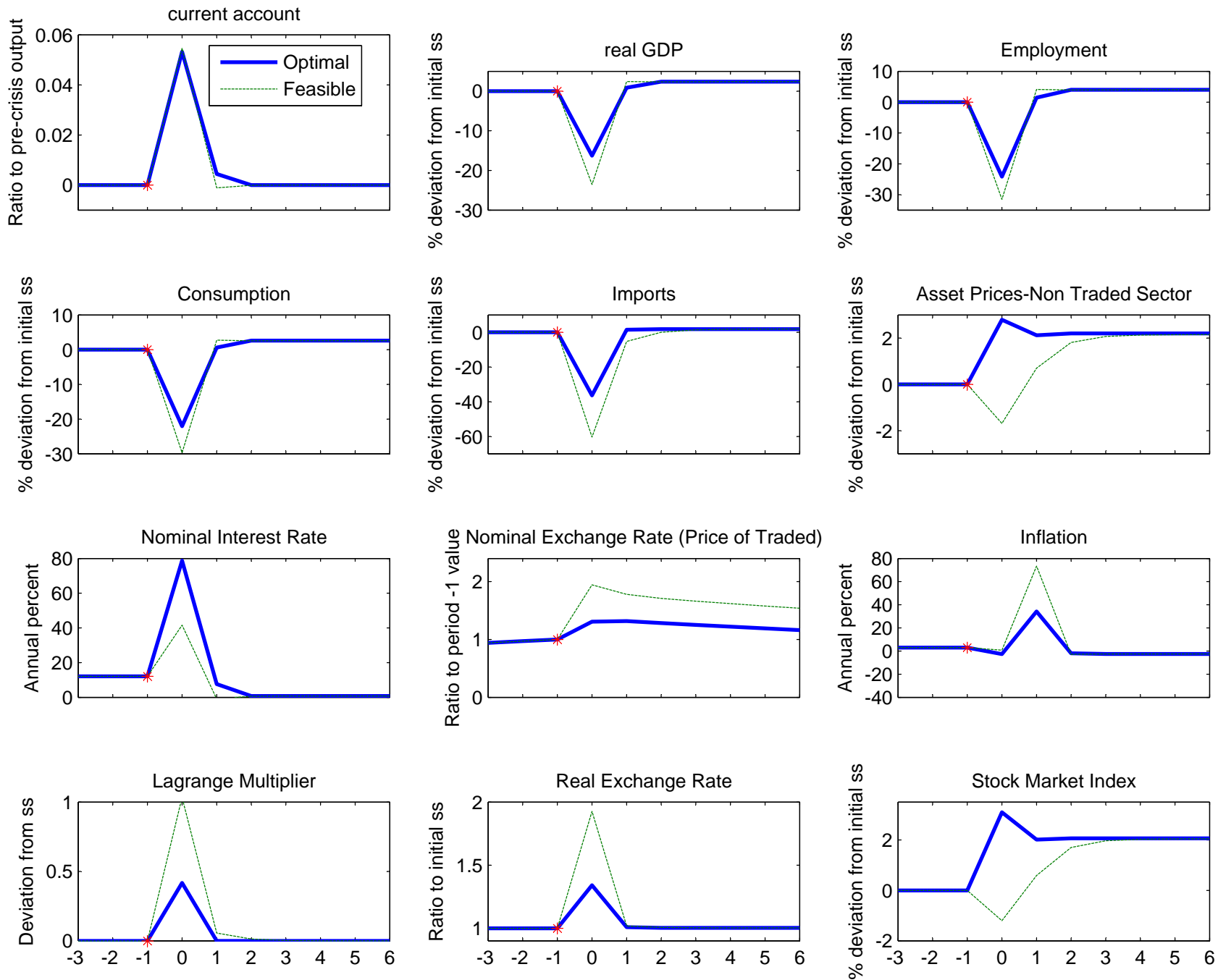
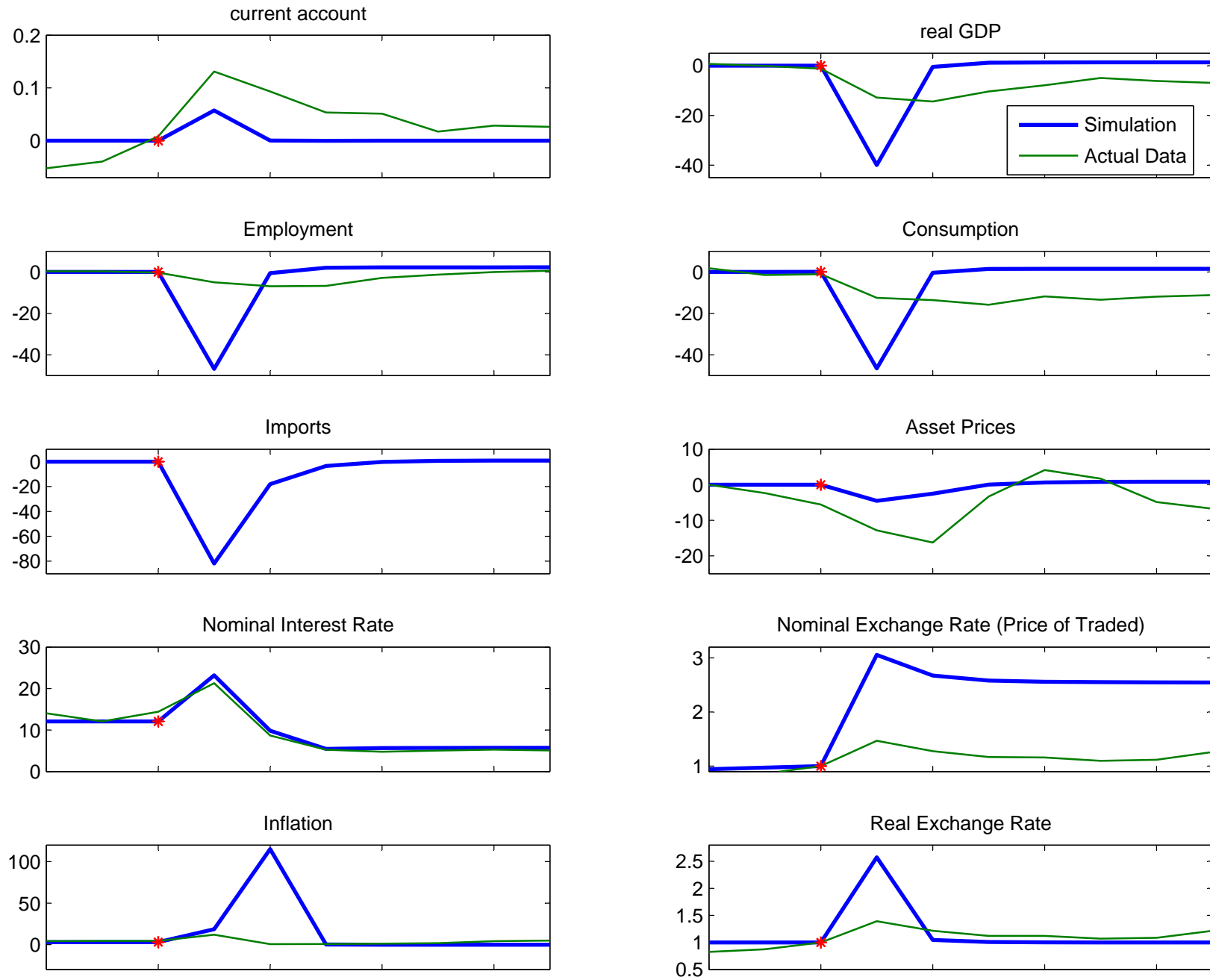


Figure 8: Model Simulation and Korean Data



Notes: (i) dimensions on vertical axes same as for Figure 7. (ii) Korean data are detrended (and seasonally adjusted where necessary) and taken from the International Monetary Fund's International Financial Statistics.