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### Trading in exhaustible resources in the presence of extraction costs

Elbers, C.T.M.; Withagen, C.A.A.M.

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**TRADING IN EXHAUSTIBLE RESOURCES IN THE PRESENCE  
OF CONVERSION COSTS  
A General Equilibrium Approach\***

Chris ELBERS

*University of Amsterdam, Amsterdam, The Netherlands*

Cees WITHAGEN

*Eindhoven University of Technology, Eindhoven, The Netherlands*

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We survey the literature on the general equilibrium approach to trade in exhaustible resources and present a model in which conversion costs are incorporated and in which the usual balance of payments condition is relaxed. It is found that in many cases the problem of dynamic inconsistency will arise.

## **1. Introduction**

The existing literature on the theory of exhaustible resources in open economies can be divided into two broad categories. Firstly there is an approach which is of a partial equilibrium nature. Here a distinction can be made between several cases according to the market power of the country under consideration. One branch, to which notable contributions were made by Long (1974), Vousden (1974), Kemp and Suzuki (1975), Kemp (1976), Aarrestad (1978) and Kemp and Long (1980a, b, c), is concerned with the problem of how a single economy in the possession of an exhaustible resource (of known or unknown magnitude) should exploit its resource in an optimal way in a world where prices are given to it, in the sense that the economy's policy is assumed not to influence prices. In most studies the resource price is kept constant but in some of them it is assumed to grow exponentially. It is assumed that perfect competition on the world market prevails. Another branch takes the opposite point of view, namely that the country under consideration is a monopolist [see e.g. Dasgupta, Eastwood and Heal (1978)]. Here the economy is facing a given world demand schedule, having the price of the resource good as the single argument. Finally, also the intermediate cases

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have been studied. For the oligopoly case the work of Dasgupta and Heal (1979) can be mentioned. Newbery (1981) has elaborated on the situation where given world demand is met by a resource-rich cartel and a resource-poor fringe, in the presence of a backstop technology. Common to all contributions in this category is that they take world market prices, or the world demand schedule, given to the economy. The second category follows the general equilibrium methodology by studying equilibrium in a setting of, mostly, two trading economies or groups of countries, where demand schedules are derived within the model. Without wishing to deny the relevance of the first approach, we prefer the latter because it may give better insight into the actual working of world markets for exhaustible resources and because, obviously, the former line of thought is covered by the latter.

The objective of the present paper is twofold. Firstly we provide a brief survey of the literature on this general equilibrium approach (section 2). Subsequently, we discuss a small and simple model in this line of thought, which takes into account extraction costs. In section 3 the model is presented as well as the conditions necessary for an optimal solution. In section 4 we elaborate on these conditions and describe the optimal solution. Existence is proved in section 5. Welfare properties are presented in section 6. Section 7 gives an example. The final section 8 contains the conclusion.

## 2. A survey of equilibrium models with trade in exhaustible resources

Only recently theoretical economists have turned their attention to general equilibrium models where explicit allowance is made for exhaustible resources. Of course, natural resources were discussed earlier in international economics [see e.g. Singer (1950) and Kemp and Ohyama (1978)], but the exhaustibility of resources was not taken into account. To our knowledge Kemp and Long (1980d) and Chiarella (1980) were the first to incorporate this feature.

Kemp and Long introduce two countries: a resource-rich country (labeled 1) and a resource-poor country (labeled 2). The first country must import all its consumption and pays for this by exporting its natural resource. The initial size of this resource is  $S_0$ .  $S(t)$  denotes what is left at time  $t$ . Extraction is costless and is denoted by  $E(t)$ . Country 1 is maximizing (omitting time indices when possible)

$$I_1 = \int_0^{\infty} e^{-\rho_1 t} u_1(C_1) dt,$$

subject to

$$\int_0^{\infty} E(t) dt \leq S_0, \quad E(t) \geq 0, \quad (1)$$

$$C_1 = pE. \quad (2)$$

Here  $\rho_1$  is the rate of time preference ( $\rho_1 \geq 0$ ),  $u_1$  is the instantaneous utility function,  $C_1$  is the rate of consumption.  $u_1$  is assumed to be increasing, strictly concave and satisfying  $u'(0) = \infty$ .  $p$  is the price of the resource good in terms of the consumption. Constraint (1) is self-evident, and (2) expresses that the balance of payments is always in equilibrium, or alternatively, that no consumption goods can be stored and consumed later. The resource-poor country has no resources, but it can convert the resource good into consumer goods by means of a technology described by a production function  $F(E)$ .  $F$  is strictly increasing, concave and satisfies  $F'(0) = \infty$ . Country 2 maximizes

$$I_2 = \int_0^{\infty} e^{-\rho_2 t} u_2(C_2) dt,$$

subject to

$$C_2 = F(E) - pE, \quad E \geq 0,$$

where  $u_2$  satisfies the same conditions as  $u_1$ . The meaning of  $\rho_2$  and  $C_2$  is clear.

For the moment we shall assume that  $p(t)$  is for both countries a known and given function of  $t$  so that both countries act as price-takers. It follows that country 2's problem is reduced to the maximization (at each moment of time) of

$$F(E) - pE, \quad E \geq 0,$$

making demand for the resource independent of the second country's preferences. Since the assumptions made so far make certain that  $\hat{E}(t)$  (the solution of the problem presented above) is strictly positive for all  $t$ , we must have  $F'(\hat{E}) = p$ . No other general conclusions can be drawn: the signs of  $\dot{C}_i$ ,  $\dot{p}$  and  $\dot{E}$  (dots refer to time derivatives, hats are omitted when possible) depend on  $u_i''C_i/u_i'$  and  $F''E/F'$ . If these quantities are bounded from below by  $-1$ ,  $\dot{C}_i < 0$ ,  $\dot{E} < 0$  and  $\dot{p} > 0$ . An example of this is given by the 'Cobb-Douglas' case:

$$u_i(C_i) = C_i^{\nu_i}, \quad 0 < \nu_i < 1, \quad i = 1, 2,$$

$$F(E) = E^\alpha, \quad 0 < \alpha < 1.$$

In this special case, which can be solved explicitly, exploitation is decreasing exponentially.

Now, drop the assumption that  $p(t)$  is taken as given by both countries and instead assume that the resource-rich country takes into account the demand of the (still passive) resource-poor country,  $F'(E) = p$ . In general nothing more

can be said than that the solution pattern will differ from the competitive case. If the production function is Cobb–Douglas, the two solution paths however coincide, that is: the resource-rich country has no effective monopoly power. A similar result has been derived by Stiglitz (1976) in the context of a closed economy.

If the resource-poor country is aggressive, it offers arbitrarily low prices for the resource. This must be accepted by the resource-rich country, whether it is active or not.

Chiarella (1980) works along the same lines as Kemp and Long. The differences between the models are given below:

- In both countries the fully employed labour force ( $L_i$ ) grows exponentially at an exogeneously given rate  $n_i$ .
- The utility functions are  $L_i \ln C_i/L_i$ .
- The model allows for capital accumulation and technical progress in the following way:

$$\dot{K} = e^{\lambda t} K^{\alpha_1} E^{\alpha_2} L_2^{\alpha_3} - pE - C_2.$$

Both countries are assumed to behave passively. After a complicated analysis the following results are obtained:

- (a) Due to the particular function  $u_1$  the supply of the resource good is price-inelastic.
- (b) For the share of each country in total consumption and for the growth rate of the resource price there exist asymptotic values which are monotonically approached. Whether optimal paths tending to these values are increasing or decreasing depends on the initial values  $S_0, K_0, L_{i0}$ .
- (c) As long as the equilibrium values are not reached the optimal path does not satisfy the Solow–Stiglitz efficiency condition and hence the solution is not Pareto-efficient.

If an additional market is introduced where the resource-poor country can borrow the non-resource good at an interest rate  $r$ , the main results are:

- (a)  $\dot{p}/p = r$ ; the Solow–Stiglitz efficiency condition is always met.
- (b) The share in total consumption is growing for the country that has the smaller rate of time preference.

### 3. A model of trade with exhaustible resources

Below we present a model of trade between two countries both in the possession of a resource, a cake, say. Moreover, this type of cake is the only good in existence. The following questions arise. When will there be any

advantage of trade for both countries? What would be Pareto-optimal rates of cake-mining and cake-eating? Will free trade result in Pareto-optimal programs? What type of price-path can we expect for the resource, or alternatively what is the implicit rate of interest? This last question was the original motivation of this study. In the introduction we mentioned a number of partial equilibrium papers on optimal exploitation where a perfectly foresighted price-path was given. Such a price-path follows from our analysis below.

The question concerning the advantages of trade is problematic. If both countries have cakes of an identical kind and the only problem is to derive the rate of eating, there is no reason for trade at all. Therefore we assume that mining a cake at rate  $E_i$  ( $i = 1, 2$ ) results in a consumable rate of  $F_i(E_i)$ .  $F_i$  is to be taken increasing and strictly concave, so by Jensen's inequality there is definitely an incentive to trade [see Elbers and Withagen (1982) for the closed economy case].

There are several ways of interpreting  $F(E)$ . First we can consider  $E$  as a flow of raw material and  $F(E)$  as the production. However, we do not allow country 1 to process the second country's raw material and vice-versa. Another interpretation is to consider  $E - F(E)$  as conversion costs, as the title of this paper suggests. In that case one would, of course, expect that  $E \geq F(E)$  for all  $E \geq 0$ . However, this restriction has no influence on the further analysis. So we drop it as unnecessary.

The other features of the model are as follows. Initial resources of the cakes ( $S_{i0}$ ,  $i = 1, 2$ ) are given. Both countries maximize discounted utility over an infinite horizon. The current account of the balance of payments may be out of equilibrium but the total value of imports should not exceed the total value of exports. Formally, each country faces the following problem:

$$\max I_i = \int_0^{\infty} e^{-\rho_i t} u_i(C_i(t)) dt, \quad (3)$$

subject to

$$\int_0^{\infty} E_i(t) \leq S_{i0},$$

$$W_i(t) + C_i(t) = F_i(E_i), \quad (4)$$

$$\int_0^{\infty} p(t) W_i(t) dt \geq 0, \quad (5)$$

$$E_i(t) \geq 0. \quad (6)$$

Here, for  $i = 1, 2$ ,  $\rho_i$  denotes the rate of time preference,  $u_i$  is the utility function ( $u' > 0$ ,  $u'' < 0$ ,  $u'(\infty) = 0$ ),  $C_i$  is the rate of consumption,  $W_i$  is the

rate of net exports,  $E_i$  is the rate of exploitation,  $S_{i0}$  is the initial amount of the resource,  $F_i$  is the conversion function, increasing and strictly concave, and  $p(t)$  is the price-path of the resource good, assumed perfectly known in advance to both countries. Since the resources don't enter trade or the welfare functionals, we can rescale  $E_i$  and  $F_i$  such that the initial amounts  $S_{i0}$  are equal to unity,

$$\int_0^\infty E_i(t) dt \leq 1, \quad i = 1, 2. \quad (7)$$

The Lagrangean of the problem is (suppressing  $i$ )

$$L = e^{-\rho t} u(C) - qE + \lambda(F(E) - C - W) + \phi Wp + \mu E.$$

An optimal solution  $\hat{z} = \{\hat{C}, \hat{E}, \hat{W}\}$  satisfies (4)–(7) and, moreover, there exist non-negative constants  $\hat{q}$  and  $\hat{\phi}$  and non-negative piece-wise continuous  $\hat{\lambda}$  and  $\hat{\mu}$  such that

$$\partial L / \partial C = 0: e^{-\rho t} u'(\hat{C}) - \hat{\lambda} = 0, \quad (8)$$

$$\partial L / \partial E = 0: -\hat{q} + \hat{\lambda} F'(\hat{E}) + \hat{\mu} = 0, \quad (9)$$

$$\partial L / \partial W = 0: -\hat{\lambda} + \hat{\phi} p = 0, \quad (10)$$

$$\hat{\mu} \hat{E} = 0, \quad \hat{q} \left(1 - \int_0^\infty \hat{E} dt\right) = 0, \quad \hat{\phi} \int_0^\infty \hat{W} p dt = 0. \quad (11)$$

#### 4. Optimal programs in competitive equilibrium

The way the above problem is posed implies that each country takes the prices  $p(t)$  as given. As usual we call  $(\hat{z}_1, \hat{z}_2, \hat{p})$  a general equilibrium if, given  $\hat{p}$ , both countries' optimal programs  $\hat{z}_1, \hat{z}_2$  are consistent in the sense that

$$\hat{W}_1(t) + \hat{W}_2(t) = 0, \quad \forall t. \quad (12)$$

The fact that for both countries  $\hat{\lambda}_i$  is piece-wise continuous implies [from (10)] that  $\hat{p}$  must be piece-wise continuous itself. [Note that  $\hat{\phi}_i \neq 0$  from (8) and from  $u'(0) = \infty$ .] It is also immediate that  $\hat{p} > 0$ . We now state:

*Theorem 1. If  $\hat{p}$  is an equilibrium price function, then  $\hat{p}$  is continuous and strictly decreasing.*

*Proof.* Suppose there occurs a discontinuity at  $t_0 > 0$ .  $\hat{p}$  is piece-wise continuous, so the discontinuity must be an (isolated) jump. From (10)  $\hat{\lambda}_i$  also

jumps in the same direction as  $\hat{p}$ .  $\hat{C}_i$  jumps (strictly) in the opposite direction as  $\hat{p}$  [from (8)], and  $\hat{E}_i$  jumps in the same direction.

Write  $f(+)$  and  $f(-)$  for  $\lim_{t \downarrow t_0} f(t)$  and  $\lim_{t \uparrow t_0} f(t)$ .

From (9) and (10),

$$\hat{\phi}\hat{p}(-)F'(\hat{E}(-)) + \hat{\mu}(-) = \hat{\phi}\hat{p}(+)F'(\hat{E}(+)) + \hat{\mu}(+).$$

If  $\hat{E}(-) > \hat{E}(+) (\geq 0)$ ,  $\mu(-) = 0$ , and since  $F'$  is decreasing, it follows that  $\hat{p}(-) > \hat{p}(+)$ . If  $\hat{E}(-) < \hat{E}(+)$ , it follows that  $\hat{p}(-) < \hat{p}(+)$ . Equivalently,

$$\hat{p}(-) \geq \hat{p}(+) \Rightarrow \hat{E}(-) \geq \hat{E}(+),$$

$$\hat{p}(+) \geq \hat{p}(-) \Rightarrow \hat{E}(+) \geq \hat{E}(-).$$

The conclusion is that

$$C_1(t) + C_2(t) - F_1(E_1(t)) - F_2(E_2(t)) \tag{13}$$

will be positive or negative (depending on the direction of the jump in  $\hat{p}$ ) on a small interval  $(t_0, t_0 + h)$ . But from (4) and (12) expression (13) should vanish for all  $t$ , so we have a contradiction. Suppose now that  $\hat{p}(t)$  is non-decreasing on  $(t_1, t_1 + h)$ . It follows from (8) that  $\hat{C}_i$  is strictly decreasing on  $(t_1, t_1 + h)$ . On the other hand  $E_i$  is non-decreasing [eq. (9)] so we have again a contradiction.  $\square$

*Corollary.*  $\hat{E}_i$  is monotonically decreasing as long as it is positive and  $\hat{E}_i$  is continuous.

*Proof.* Note that, according to (10),  $\hat{\lambda}$  and  $\hat{p}$  only differ by a scale factor. Therefore, from Theorem 1,  $\hat{\lambda}$  is also continuous and strictly decreasing. That  $\hat{E}_i$  is non-increasing follows from eq. (9). Eq. (8) shows that  $\hat{C}_i$  is continuous. Since

$$\hat{C}_1 + \hat{C}_2 = F_1(\hat{E}_1) + F_2(\hat{E}_2),$$

the right-hand side of this expression must be continuous as well. A conceivable jump in  $\hat{E}_1$  is therefore always accompanied by a jump in  $\hat{E}_2$  in the opposite direction. One of these jumps is going to violate the condition that  $\hat{E}_i$  is non-increasing. Suppose now that  $\hat{E}_i(t_0) > 0$  for some  $t_0$ . From continuity  $\hat{E}_i(t)$  is positive in a neighbourhood of  $t_0$  and  $\hat{\mu}_i(t) = 0$  in this neighbourhood. Eq. (9) and Theorem 1 imply then that  $\hat{E}_i$  is strictly decreasing as long as it is positive.  $\square$

Under the assumption made so far consumption is always positive in a competitive equilibrium and hence production is also positive. However, it



could be that one of the countries' resources is exhausted in finite time. In fact, as casual inspection shows, this is the case to be expected. We shall give an example below.

The necessary conditions (4)–(11) are also sufficient for an optimum. In fact, the optimal program in the single country problem, if it exists, is unique.

*Theorem 2.* Suppose that  $\{\hat{C}, \hat{E}, \hat{W}, \hat{q}, \hat{\lambda}, \hat{\mu}, \hat{\phi}\}$  satisfies (4)–(11). Then  $\hat{C}, \hat{E}, \hat{W}$  is the unique optimal solution to (3).

*Proof.* See Elbers and Withagen (1983). The theorem is proved using standard techniques that can be found for instance in Long and Vousden (1977, theorem 6).

### 5. Existence of competitive equilibrium

The existence of an optimal program for a single country depends on the behaviour of  $p(t)$ . An (eventually) non-decreasing price is out of the question since postponing production would not harm the budget and at the same time, by producing very tiny quantities, the concavity of the production function could be evaded: there is no optimal production plan. On the other hand if prices fall too rapidly utility could be increased beyond any bound. The following (non-rigorous) argument sketches a proof of existence of a competitive equilibrium under the assumptions listed above.

Consider the following problem:

$$\max \int_0^{\infty} \{ (1 - \alpha)e^{-\rho_1 t} u_1(C_1) + \alpha e^{-\rho_2 t} u_2(C_2) \} dt, \quad \alpha > 0,$$

subject to

$$1 - \int_0^{\infty} E_i dt \geq 0, \quad i = 1, 2, \tag{14}$$

$$E_i \geq 0, \quad C_i \geq 0, \quad i = 1, 2,$$

$$F_1(E_1) + F_2(E_2) - C_1 - C_2 \geq 0.$$

The necessary and (analogous to Theorem 2) sufficient conditions for an optimum are: there exist non-negative constants  $q_1^{(\alpha)}, q_2^{(\alpha)}$  and non-negative

piece-wise continuous  $\mu_1^{(\alpha)}(t), \mu_2^{(\alpha)}(t), p^{(\alpha)}(t)$ , such that

$$\left. \begin{aligned} (1 - \alpha)e^{-\rho_1 t} u_1'(C_1^{(\alpha)}) &= p^{(\alpha)} \\ \alpha e^{-\rho_2 t} u_1'(C_2^{(\alpha)}) &= p^{(\alpha)} \\ -q_i^{(\alpha)} + \mu_i^{(\alpha)} p^{(\alpha)} F_i'(E_i^{(\alpha)}) &= 0, \quad i = 1, 2 \\ p^{(\alpha)}(F_1(E_1^{(\alpha)}) + F_2(E_2^{(\alpha)}) - C_1^{(\alpha)} - C_2^{(\alpha)}) &= 0 \\ \mu_i^{(\alpha)} E_i^{(\alpha)} &= 0, \quad i = 1, 2 \end{aligned} \right\} \quad (15)$$

Given  $q_1$  and  $q_2$  the system (15) can be solved for  $E_1, E_2, C_1, C_2$  and  $p$ .  $q_1$  and  $q_2$  can then be determined from the isoperimetric constraints (14). The solutions are continuous functions of  $t$ . As for  $\alpha$ , if  $\alpha_n \rightarrow \alpha_0$ , then  $E_i^{(\alpha_n)}, C_i^{(\alpha_n)}$  and  $p^{(\alpha_n)}$  converge to  $E_i^{(\alpha_0)}, C_i^{(\alpha_0)}$  and  $p^{(\alpha_0)}$  (almost everywhere with respect to the Lebesgue measure). Define

$$B_1(\alpha) \stackrel{\text{def}}{=} \int_0^\infty p^{(\alpha)}(F_1(E_1^{(\alpha)}) - C_1^{(\alpha)}) dt.$$

$B_1$  is the ‘shadow value’ of country 1’s total exports. Then

$$\begin{aligned} |p^{(\alpha)}(F_1(E_1^{(\alpha)}) - C_1^{(\alpha)})| &\leq p^{(\alpha)}(C_1^{(\alpha)} + C_2^{(\alpha)}) \\ &\leq (1 - \alpha)e^{-\rho_1 t} C_1^{(\alpha)} u_1'(C_1^{(\alpha)}) + \alpha e^{-\rho_2 t} C_2^{(\alpha)} u_2'(C_2^{(\alpha)}) \\ &\leq e^{-\rho_1 t} u_1(C_1^{(\alpha)}) + e^{-\rho_2 t} u_2(C_2^{(\alpha)}) \\ &\leq e^{-\rho_1 t} u(\bar{C}_1) + e^{-\rho_2 t} u_2(\bar{C}_2). \end{aligned}$$

Hence  $B_1(\alpha) \leq M_1 + M_2$ , where

$$M_i = \max \left\{ \int_0^\infty e^{-\rho_i t} u_i(C_i) dt \mid F_1(E_1) + F_2(E_2) \geq C_i \right\} < \infty,$$

and  $\bar{C}_i$  the corresponding optimal programs. By the Lebesgue-dominated convergence theorem  $B_1(\alpha)$  is continuous in  $\alpha$ . Since obviously

$$\lim_{\alpha \downarrow 0} B_1(\alpha) < 0 \quad \text{and} \quad \lim_{\alpha \rightarrow \infty} B_1(\alpha) > 0,$$

there is by the intermediate value theorem an  $\alpha_0$  such that  $B_1(\alpha_0) = 0$ . The

corresponding system of necessary conditions are the conditions of a competitive equilibrium (4)–(12) with  $\hat{\phi}_1 = 1$  and  $\hat{\phi}_2 = 1/\alpha_0$ .

## 6. Welfare properties

In this section we show that the two classical propositions of welfare economics hold in the case at hand. Let  $(\hat{z}_1, \hat{z}_2)$  be a Pareto-optimal consumption–exploitation plan. Then

$$\int_0^\infty e^{-\rho_1 t} u_1(\hat{C}_1) dt \geq \int_0^\infty e^{-\rho_1 t} u_1(C_1) dt,$$

for all  $\{C_1\}$  that satisfy

$$C_1 = F_1 + F_2 - C_2,$$

$$\int_0^\infty E_1 dt \leq 1, \quad \int_0^\infty E_2 dt \leq 1,$$

$$\int_0^\infty e^{-\rho_2 t} u_2(C_2) dt \geq \int_0^\infty e^{-\rho_2 t} u_2(\hat{C}_2) dt.$$

Hence, there exist constants  $\hat{\phi}$ ,  $\hat{q}_1$  and  $\hat{q}_2$  and piece-wise continuous  $\hat{\mu}_1$ ,  $\hat{\mu}_2$  and  $\hat{\lambda}$  such that, for all  $t$ ,

$$e^{-\rho_1 t} u_1'(\hat{C}_1) = \hat{\lambda},$$

$$\hat{\phi} e^{-\rho_2 t} u_2'(\hat{C}_2) = \hat{\lambda},$$

$$\hat{\lambda} F_1'(\hat{E}_1) - \hat{q}_1 + \hat{\mu}_1 = 0,$$

$$\hat{\lambda} F_2'(E_2) - \hat{q}_2 + \hat{\mu}_2 = 0.$$

These conditions are equivalent to the conditions (8)–(11) by a proper choice of the co-state variables. Hence, obviously every Pareto-efficient allocation can be attained as a competitive equilibrium by an appropriate redistribution of the resources.

It is also easily seen that if a program satisfies these necessary conditions, it is Pareto-optimal. Hence, the competitive equilibrium is Pareto-optimal.

## 7. An example

To depict an explicit solution we take the following translated Cobb–Douglas conversion functions:

$$F_i(E_i) = (E_i + a_i)^{\alpha_i} - a_i^{\alpha_i}, \quad 0 < \alpha_i < 1.$$

Suppose that both resources are only exhausted in infinity:

$$F_1'(E_1) = \pi_1/p \quad \text{and} \quad F_2'(E_2) = \pi_2/p,$$

where  $\pi_i$  are constants (to be determined).

Both resources will eventually be exhausted so  $E_i(t) \rightarrow 0$  as  $t \rightarrow \infty$ . Therefore

$$\begin{aligned} \pi_1/\pi_2 &= F_1'(E_1)/F_2'(E_2) = F_1'(0)/F_2'(0) \\ &= \alpha_1 a_1^{\alpha_1 - 1} / \alpha_2 a_2^{\alpha_2 - 1}, \end{aligned} \quad (16)$$

$$E_i = \left[ (\alpha_i p / a_i^{1 - \alpha_i} \pi_i)^{1/1 - \alpha_i} - 1 \right] a_i.$$

If we substitute this in the resource constraint, we get

$$\int_0^\infty E_1 dt = 1, \quad \int_0^\infty E_2 dt = 1.$$

So with (16) and after some manipulations,

$$\int_0^\infty \left[ (\alpha_1 p / a_1^{1 - \alpha_1} \pi_1)^{1/1 - \alpha_2} - 1 \right] dt = 1/a_2. \quad (17)$$

In a number of cases this condition is violated regardless of  $p$  and, hence, irrespective of the form of the utility functions. These cases are

$$\left. \begin{array}{l} \alpha_1 = \alpha_2 \quad \text{and} \quad a_1 \neq a_2 \\ \alpha_1 > \alpha_2 \quad \text{and} \quad a_1 \geq a_2 \\ \alpha_1 < \alpha_2 \quad \text{and} \quad a_1 \leq a_2 \end{array} \right\}. \quad (18)$$

We conclude that if one of the relations in (18) holds, one of the resources will be exhausted in finite time.

Note that we have a case of dynamic inconsistency here. Under the conditions postulated it is advantageous for both countries to engage in trade. In fact, the competitive equilibrium is a core allocation. On the other hand, once one country has no resources left, the other country will change its behaviour according to its new monopolist status. (It is, of course, not clear from our model what this exactly means.) There are two conceivable ways out of this problem. One is to assume the existence of a supra-national power that could force nations to behave according to their plans. The other is to admit that

when there is a danger that countries run out of their resource, strategic considerations come into the picture, so a competitive solution misspecifies behaviour.

In the cases not covered by (18), it will depend on  $p$  whether (17) is violated or not. If one finds a decreasing continuous  $p$  that satisfies (17), then from it utility functions can be derived that satisfy the conditions on the optimal consumption plan and  $C_i = F_i(E_i)$ . In this case (no trade) it is clear that both resources will only be exhausted in infinity. We believe however, that in the 'typical' case one of the resources is exhausted in finite time, and that the phenomenon can be encountered in any model of general equilibrium with exhaustible resources. It deserves to be studied more closely.

## 8. Conclusions

In the present paper we have given a simple two-country model of trade with exhaustible resources. In this model it was possible to establish existence and the familiar welfare properties of a competitive equilibrium. It is argued that typically one of the countries runs out of resources in finite time, which poses a problem of dynamic inconsistency.

It would be interesting to see exhaustible resources introduced into larger equilibrium models. Exhaustible resources represent a special type of endowments and technology that may have general implications for the equilibrium solution. As for the present model, the natural extension could be to acknowledge the fact that there are really two goods in the economy: a resource good and a consumption good. Trade should be allowed in both goods.

Finally, the literature on exhaustible resources studies production and consumption decisions by continuous time control models. Conceptually there is no difficulty in combining these decisions in a general equilibrium, although some hypothesis of price expectations must be made. However, existence of equilibrium price functions cannot be proved using conventional proofs because the dimension of the commodity space is infinite.

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