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Mathematical Economics, an Introduction

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In many ways Physics functions as an example for other sciences. When economics as a science was developed, it was modelled after physics also and so there are economic theories on 'equilibrium', not to mention the idea to model the national economy of a country as an electrical circuit.

There has always been a mutual influence between mathematics and physics. There is also a great influence of mathematics on economics. The best known examples of mathematics applied to economics in the Netherlands are of course 'Het Centraal Planbureau' and 'Het Centraal Bureau voor de Statistiek', but those applications we do not want to consider as 'theoretical' economics.

In the last decades mathematical economics has assumed large proportions, at least partly due to the development of mathematical tools which are well-suited for economic modelling and the growing close cooperation between mathematicians and economics. Twice before, Nieuw Archief voor Wiskunde published a special issue on mathematics in relation with other fields (i.e. in March 1984 on biomathematics and in July 1987 on industrial mathematics). This special issue contains ten essays and five book reviews by mathematical economists.

In this introduction we present a brief description of the field and put the contemporaneous state of the art in an historical perspective. We also give a short overview of the papers in this issue.

ARROW and INTRILIGATOR (1981) define mathematical economics as 'application of mathematical concepts and techniques to economics, particularly economic theory'. There are several important things to note here. Firstly, mathematical economics is not a branch of economics in that it relates to a specific area of the economic system such as international economics, labour economics or micro economics. So it would be erroneous to identify it, for example, with general equilibrium theory, as is sometimes done. Secondly, the definition does not exclude any mathematical technique. It will not be too difficult to ascertain that in the subsequent series of articles a large variety of techniques and concepts is employed, ranging from classical analysis to measure theory and discrete mathematics. Thirdly, the definition is restrictive in so far as applications in the field of the estimation of economic relations and statistical inference are not included. Hence, the reader will not enjoy an overview of all applications of mathematics to economics, however interesting and important econometrics is. One single issue of Nieuw Archief cannot do justice to both mathematical economics and econometrics.

There is common agreement among economists that Augustin Cournot is to be considered as the founder of mathematical economics.[1] In his Mathematical Principles of the Theory of Wealth (1838), Cournot puts forward as his explicit objective to 'apply . . . the form and symbols of mathematical analysis'. In the preface of his book however, Cournot recognises that this program is likely 'to draw on me at the outset the condemnation of theorists of repute', because in his opinion theorists had a prejudice against the use of mathematics. This prejudice was enhanced by the idea that mathematics could only contribute to numerical calculations (and not to theory) and by the fact that former attempts were 'radically at fault' and 'erroneous'. In Cournot's view not only economists would be inclined to dislike his work but he is also pessimistic about the attention his work (using the 'first principles of differential and integral calculus') would receive from mathematicians 'except as they may discover in it the germ of questions more worthy of their powers'. Several decades later LÉON WALRAS (1874), in his seminal work on general economic equilibrium, also finds it necessary to defend his mathematical approach: 'As to mathematical language, why should we persist in using everyday language to explain things in the most cumbrous and incorrect way, as Ricardo has often done and as John Stuart Mill does repeatedly . . . when the same things can be stated far more succinctly, precisely and clearly in the language of mathematics?'

Recently, the discussion on the use of mathematics in economics has been revived by KLAMER et al. (1988) and VAN DUIJN (1990). Walras' remark is still proper as an answer, be it that of course mathematical economics without an economic content is just mathematics and should only be studied as such.

In the first half of this century the use of mathematics, in particular analysis and Langrangean optimization, became broadly accepted in economics. Samuelson's Foundations of economic analysis (1947) may be seen as a point of culmination in this respect. New impulses came in the fifties from the development of game theory (VON NEUMANN and MORGENSTERN (1944)), fixed-point theorems (KAKUTANI (1941)), topology and control theory. The extensive use of these tools again gave rise to a dispute on their contributions. See KOOPMANS (1957) for a survey of the discussion. In summary, there is

1. There are some predecessors. See THEOCHARIS (1961) for a survey of the prehistory of the profession.

agreement on the efficiency and conciseness of mathematical reasoning as well as on the point that 'the mathematical method . . . forces the investigator to give a complete statement of assuredly non-contradictory assumptions' (KOOPMANS, pp. 172-173), but there are also 'warnings against overemphasis on formal reasoning as an apparent end in itself' (KOOPMANS, p. 173) and 'mathematical economists . . . should do their utmost to communicate the assumptions and the conclusions in verbal form' (KOOPMANS p. 173).

From the sixties on, the use of mathematics has increased tremendously: nowadays there are textbooks and journals on mathematical economics; there are chairs in this field in almost every economics faculty as well as the opportunity for students in mathematics to study mathematical economics. In our opinion all this is to be considered as a positive development. However it cannot be denied that the above-mentioned warnings have not always been taken into account to a sufficient degree. And in that sense of course the critique of Klamer and Van Duijn mentioned above is justified. The growing diversity of the tools employed is associated with a level of specialisation which does not facilitate the communication among the practitioners of mathematical economics. In our opinion this presents a permanent challenge for the profession.

Not only mathematical economists use many different mathematical tools, but also some mathematicians have worked in economics. We do not mean those economists (like Tinbergen) who have been trained as mathematicians but who contributed very little to mathematics and extensively to economics, but those who have contributed important work in both fields. The first that comes to one's mind is of course Von Neumann, but one may also think of Aumann, Rockafellar, Smale, Dantzig and others. The Eastern European mathematical economists have a special place. In Eastern Europe the Arrow-Debreu model of a 'private-ownership' economy was out of the question before 1990. The work of Eastern European mathematical economists sometimes has more of an engineer approach. Of course we should mention Kantorovich but also Bródy (from Hungary), Loś and Loś (from Poland), and Makarov and Rubinov (from Russia).

There are a few mathematicians, who although not active in economics, have an open eye for the developments in that field. For instance, in all the books of I. Ekeland on mathematics, we know of, there are economic examples and illustrations.

In two fields, there are contributions from both mathematicians and economists. Firstly, the theory of optimization (including duality, convex analysis and optimal control) and secondly in game theory. Nowadays game theory in general and differential game theory in particular is being used extensively in explaining economic phenomena with great success and also Dutch scientists have considerable influence (e.g. VAN DAMME (1987), OLSDER (see BAŞAR and OLSDER (1982)), TIJS (see PETERS and VRIEZE (1987)), and DE ZEEUW (1984)).

In mathematics there are fields, like stability theory, that owe a lot to physics and physicists. There is very little in mathematics that finds its roots in economics. Actually we only know one important example. In search for methods to compute economic equilibria, SCARF (1973), found a constructive proof of Brouwer's fixed point theorem. This was the starting point for the development of many simplicial algorithms. In The Netherlands Van der Laan and Talman (both present with a paper in this volume) are specialists in this field.

We now turn to a brief description of the contributions in this issue. We do not intend to provide a survey of the complete field; this would be impossible for reasons of space. The articles should be looked upon as examples of the topics mathematical economists are nowadays concerned with and of the tools that are employed. This implies that some important areas such as game theory do not get a chance.

Furth studies a model of a duopoly (two sellers) where the duopolist may (and will) offer differentiated products. The problem is to derive the demand schedule. The paper by Van Geldrop and Withagen deals with the existence of a general equilibrium in economies with exhaustible resources, where the commodity space is of infinite dimension. Gielens and De Vries use the theory of stochastic processes in the analysis of the movements of foreign exchange rates. The article by Gilles and Ruys goes into the question how the communication structure between economic agents affects the functioning of an economy. Van der Laan presents an international trade model and analyses several policy measures in a general equilibrium context. In his contribution Nijmeijer gives a system-theory analysis of an economy and goes into controllability and input-output decoupling issues. Ten Raa also takes input-output analysis as a point of departure and discusses practical and theoretical aspects. In Talman's contribution there is a survey of equilibrium programming algorithms and an example of how they can be used not only for exchange economies but also for economies with production. Vorst's paper is on the use of probability theory in financial economics with emphasis on the pricing of options. Finally, Weddepohl considers an economy with increasing returns in production and selling costs.

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