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Published in:

Journal of International Economics

Publication date:

1993

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):

van Geldrop, J., & Withagen, C. A. A. M. (1993). General equilibrium and international trade with exhaustible resources. *Journal of International Economics*, 34(3-4), 341-357.

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General equilibrium and international trade with exhaustible resources

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Received November 1990, revised version received February 1992

The pricing of raw materials from an exhaustible resource is predominantly analysed within the context of a partial equilibrium. It is argued here that there are good reasons for a general equilibrium analysis. In this paper we present an international trade model with exhaustible resources and characterize the general equilibrium properties. The general perspective adopted leads to new insights compared with a partial equilibrium approach.

1. Introduction

The pricing of raw materials from natural exhaustible resources is an important real world issue, as has intermittently and painfully become apparent during the oil crises, the collapse of the tin cartel, etc. Not surprisingly economic theory has put considerable effort into developing models to explain price movements on the markets for raw materials, taking into account a variety of possible market structures. The seminal work by Hotelling (1931) considers the cases of perfect competition, monopoly and duopoly. The duopoly case has been extended to the more general case of an arbitrary number of oligopolists by Lewis and Schmalensee (1980a, b). Salant (1976) studies the cartel versus fringe model using the Nash equilibrium concept, whereas Gilbert (1978), Newbery (1980) and Ulph (1982) employ the open-loop von Stackelberg equilibrium concept. Recently, Groot (1990) has solved the cartel versus fringe model for the feedback von Stackelberg equilibrium concept. This list is not exhaustive but serves to indicate that considerable work has been done in this area. For recent surveys the reader is referred to Karp and Newbery (1989) and Withagen (1990). Typically these theories depart from a given demand schedule and investigate the features of a partial market equilibrium, assuming that each trader maximizes his total

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*The authors wish to thank Casper de Vries and two anonymous referees for their useful remarks. Any errors remain the authors' sole responsibility.

discounted profits subject to the conditions prevailing on the market and the physical constraints imposed by the exhaustibility of his resource. Here discounting takes place at a given and constant discount rate, which equals the capital gains on the resource according to Hotelling's rule or an analogue. Clearly the partial equilibrium analysis suffers from some weaknesses. By definition it is not able to capture possible spill-overs to other markets and differences in endowments between countries cannot be analyzed. A crucial drawback is that the rate of discount is exogenous; it is hard to believe that the interest rate is moving independently of the market price of the raw material. In atemporal models of price formation this drawback can be deemed innocuous, but the horizon agents face in resource economics is generally indeterminate or infinite and this makes the above-mentioned interaction non-negligible. Finally, a general equilibrium analysis is preferable, if only from a methodological point of view.

In spite of these arguments, attention paid to a general equilibrium analysis of trade in raw materials is rather modest, at least at the micro-economic level, in line with the overview given above [see also Withagen (1990)].¹ Let us summarize. In view of real world phenomena all the work in this field is concerned with international trade. Kemp and Long (1980a) analyze the interaction between resource-rich and resource-poor countries. They model a two-country world. One of the countries is resource-rich. Its resource can be exploited at no cost. The raw material is exported to the resource-poor country, which possesses the technology to convert it into a consumer good which is used for own consumption and which is traded with the resource-rich country. The analysis is carried out for a Cobb–Douglas type specification of the functions involved and this yields a complete characterization of the equilibrium price trajectory. Elbers and Withagen (1984) introduce bilateral ownership of the resource and extraction costs. Finally, Chiarella (1980) studies a two-country model with unilateral ownership of the resource and the non-resource technology and with physical capital and labor as means of production. For Cobb–Douglas specifications he is able to characterize the general equilibrium for the case where there exists a perfect world market for 'financial' capital as well as the case where such a market does not exist. Clearly these models can and should be generalized in several respects and this is the aim of the present paper. Our generalization lies in the fact that the number of agents on the consumption side as well as on the production side is arbitrary and, more importantly, we are able to derive interesting results without imposing severe restrictions on the functional forms. It will turn out that the equilibrium interest rate can be constant eventually or from the outset (this occurs when the resources are in

¹At the 'macro' level there are the well-known studies of, among others, Marion and Svensson (1984) and Van Wijnbergen (1985).

some sense abundant) and that otherwise the interest rate decreases monotonically. It will furthermore be shown that the capital intensity is non-decreasing and that consumption will eventually decrease.

The plan of the paper is as follows. In section 2 the model is presented and the assumptions are introduced and discussed. Section 3 goes into the problem of the existence of a general equilibrium. Section 4 derives the features of the general equilibrium. Finally, section 5 concludes.

2. The model

The model is presented here as describing international trade between a given number of countries, although this is by no means the only possible interpretation. One can also think of a closed economic system with several resources and production sectors. Let there be n countries. The initial endowments of country i ($i=1, 2, \dots, n$) consist of a stock of an exhaustible natural resource, denoted by S_{i0} , and a stock of a composite commodity, denoted by $K_{i0} > 0$. The resource stocks can be distinguished according to the costs that have to be made in order to extract the raw material. However, the raw material once extracted is homogeneous, so that there is only one price. Also the composite commodity is homogeneous. As a stock it is a factor of production and will be called capital. As a flow it can be used for consumption purposes as well as for investment. Finally, the composite commodity is a store of value. This will be clarified below.

The preferences of each country are given by a utilitarian social welfare function having only the consumption profile as an argument; no direct welfare is derived from holding stocks or having raw materials. Welfare of economy i is given by

$$U_i(C_i) := \int_0^\infty e^{-\rho_i s} u_i(C_i(s)) ds, \tag{2.1}$$

where ρ_i is the positive constant rate of pure time preference, $C_i(t)$ is the rate of consumption at time t and u_i is the instantaneous utility function. The following assumptions are made about u_i :

Assumption U.1. u_i is continuous on \mathbb{R}_+ and continuously differentiable on \mathbb{R}_{++} , $u_i(0) = 0$.

Assumption U.2. u_i is strictly increasing and strictly concave.

Assumption U.3. $u_i'(0) = \infty$; $0 < \eta_i \leq -cu_i''(c)/u_i'(c) = \eta_i(c) < \infty$ for all $c > 0$ and some constant η_i .

These assumptions are standard in the growth literature and they facilitate the analysis to a large extent.

The technological capabilities of the economies can be described as follows. The resources are not replenishable. So, if $E_i(t)$ is the rate of exploitation of resource stock i at time t , it is required that

$$\int_0^{\infty} E_i(s) ds \leq S_{i0}, \quad (2.2)$$

$$E_i(t) \geq 0 \quad \text{for all } t \geq 0. \quad (2.3)$$

Exploitation is not costless. In order to exploit one has to use capital as an input. Following Kay and Mirrlees (1975), Heal (1976) and Kemp and Long (1980b) we postulate an extraction technology of the Ricardian fixed proportions type:

$$K_i^e(t) = a_i E_i(t), \quad (2.4)$$

where a_i is a positive constant and $K_i^e(t)$ denotes the amount of capital used at t by economy i for extraction purposes.

Capital can, together with the homogeneous raw material, also be allocated to the production of the composite commodity. Let $R_i(t)$, $K_i^y(t)$, $Y_i(t)$ and F_i denote the rate of use of the raw material at t , capital input at t , composite commodity output at t and the production function, respectively, then

$$Y_i(t) = F_i(K_i^y(t), R_i(t)). \quad (2.5)$$

The following standard assumptions are made about F_i :

Assumption F.1. F_i is continuous on \mathbb{R}_+^2 , continuously differentiable on \mathbb{R}_{++}^2 , concave and homogeneous of order 1.

Assumption F.2. F_i is strictly increasing on \mathbb{R}_{++}^2 .

Assumption F.3. $F_i(K, 0) = F_i(0, R) = 0$ for all $K \in \mathbb{R}_+$ and all $R \in \mathbb{R}_+$.

Assumption F.4. $\lim_{R \rightarrow 0} F_i(K, R)/R = \infty$ for all $K > 0$; $\lim_{K \rightarrow 0} F_i(K, R)/K = \infty$ for all $R > 0$.

Some comments are in order. For the characterization of the equilibrium the constant returns to scale (CRS) assumption is important and standard in

resource economics [see, for example, Dasgupta and Heal (1974)]. It implies that in a competitive environment profits will be zero, from which nicely behaving factor price frontiers can be obtained. If one does not make the CRS assumption, this is impossible. Necessity of both inputs is a plausible assumption. In Assumption F.4 one recognizes elements of the well-known Inada conditions. The first part of Assumption F.4 says that the raw materials are essential.

There is a world market for raw materials. The prevailing spot price in terms of the composite commodity will be denoted by $p(t)$ at time t . It is also assumed that there exists a perfect world market for capital services with spot price $r(t)$ (also in terms of the composite commodity). This implies that capital is perfectly mobile and can be traded instantaneously and costlessly. In the context of an infinite horizon model this assumption makes sense as a first approximation. We finally postulate the existence of a perfect world market for ‘financial’ capital. So that no permanent equilibrium on the current accounts is required: an economy can borrow and lend at will provided its total discounted expenditures do not exceed its total discounted income. Define the compounded discount factor as

$$\pi(t) := \exp\left(-\int_0^t r(\tau) d\tau\right). \tag{2.6}$$

Then the budget constraint of economy i can be written as

$$\begin{aligned} & \int_0^\infty \pi(s) \{p(s)E_i(s) + Y_i(s)\} ds + K_{i0} \\ & \geq \int_0^\infty \pi(s) \{p(s)R_i(s) + C_i(s) + r(s)(K_i^y(s) + K_i^e(s))\} ds. \end{aligned} \tag{2.7}$$

The interpretation of this condition is straightforward. $E - R$ are the net exports of the raw material. $Y - C$ gives net exports of the produced composite commodity. $r(K_i^y + K_i^e)$ are the capital costs of production and extraction, respectively.

In a general competitive equilibrium with perfect foresight each economy maximizes its social welfare (2.1) subject to the technological constraints (2.2)–(2.5) and its budget condition (2.7), taking the price trajectories p and r as given. This yields demand and supply schedules for the composite commodity and the raw material. In equilibrium supply meets demand:

$$\sum_i R_i(t) \leq \sum_i E_i(t) \quad \text{for all } t, \tag{2.8}$$

$$\sum_i K_i^y(t) + \sum_i K_i^e(t) \leq \sum_i K_i(t) \quad \text{for all } t, \tag{2.9}$$

where $K_i(t)$ denotes the capital holdings of economy i at time t . Formally, $K_i(t)$ is the solution of

$$\dot{K}_i(t) = p(t)(E_i(t) - R_i(t)) + Y_i(t) - C_i(t) + r(t)(K_i(t) - K_i^e(t) - K_i^y(t)). \quad (2.10)$$

In addition, clearly $p(t)=0$ if (2.8) holds with strict inequality and $r(t)=0$ if (2.9) holds with strict inequality.

Note that in comparison with the existing literature discussed above, a number of generalizations are introduced: extraction costs, arbitrary composite commodity production functions and an arbitrary number of consumers and producers.

3. Existence of a general equilibrium

The model outlined in the previous section does not allow for the standard Arrow/Debreu approach to establish the existence of a general equilibrium. In this section we go into the causes of this problem and discuss how existence can nevertheless be dealt with. The treatment here will be rather informal because a thorough analysis is not the aim of the present paper. The interested reader is referred to van Geldrop and Withagen (1990a, b) and van Geldrop, Jilin and Withagen (1991).

The Arrow/Debreu approach fails for the following reason. It would require working with dated commodities because commodities are now to be distinguished according to the moment at which they become available. In view of the infinite horizon this necessitates working in an infinite-dimensional commodity space. At this point the traditional approach breaks down, since there the dimension of the commodity space is finite. So one has to look for an alternative. Evidently the literature on infinite-dimensional commodity spaces, developed by Bewley (1972), and elaborated upon by Mas-Colell (1986), Richard (1986), Zame (1987) and others, comes to mind. However, their methods are not conclusive in the model at hand either. This has to do with the fact that the initial endowments of the agents do not lie in the interior of their consumption sets, and more importantly, that the economies' production possibilities need not be bounded. However, owing to the structure of our model, the *method* originating from Bewley can be pursued, at least to some extent. The procedure amounts to showing the existence of a general equilibrium for the truncated (finite-horizon) economy, proving the uniform boundedness (uniform with respect to the horizon) of the equilibrium allocations, applying a theorem of Alaoglu yielding limits and, finally, ascertaining that the limiting allocations constitute an equilibrium in the infinite-horizon economy. Crucial in our approach is that it enables us to say a good deal about the mathematical properties of the functions describing the equilibrium. Even if the Bewley approach were to

work, prices for example would lie in a function space which, by itself, would have rather unattractive properties. Our way of tackling the problem employs optimal control theorems (in the step where the existence of a finite-horizon equilibrium is proven), from which it is relatively easy to see that prices and the equilibrium allocations of consumption are continuous functions. The results are summarized in the following theorem.

Theorem 3.1 [van Geldrop and Withagen (1990b)]. An economy satisfying Assumptions U.1–U.3 and F.1–F.4 possesses a general competitive equilibrium. In the equilibrium the interest rate r , the price of the raw material p and the rates of consumption C_i are continuous over time. Moreover, there is always (i.e. for all t) production of the composite commodity.

It is instructive to give the flavor of the underlying economic arguments that yield the continuity results and positive production of the composite commodity. This gives some insight into how the assumptions are used. It can be shown that the first theorem of welfare economics applies so that a general equilibrium is Pareto efficient. This is mainly due to non-satiation in consumption. It follows from $u'_i(0) = \infty$ that consumption is always positive. Continuity of the consumption patterns is then arrived at by strict concavity of the instantaneous utility functions. Since the marginal product of raw materials is infinite at zero input, it is not efficient to consume only by eating-up the capital stock, even in the presence of exhaustible resources. Therefore there will always be production of the composite commodity. Continuity of the price trajectories then follows from the absence, in equilibrium, of arbitrage opportunities. Plausible as these results may seem, they are not trivial at all. Again, the interested reader is referred to van Geldrop and Withagen (1990b).

4. General competitive equilibrium: Characterization

In this section a characterization of the general equilibrium is given. This is greatly facilitated by two special features of the model, which are discussed first.

The existence of a perfect world market for financial capital allows for the application of Fisher's separation theorem, so that the maximization of social welfare requires the maximization of total discounted profits from productive activities. Therefore, in equilibrium, we have for all i , all t and all $(K, R) \in \mathbb{R}_+^2$:

$$F_i(K_i^y(t), R_i(t)) - r(t)K_i^y(t) - p(t)R_i(t) \geq F_i(K, R) - r(t)K - p(t)R, \quad (4.1)$$

the production of the composite commodity. Furthermore there exist non-negative constants λ_i such that

$$\pi(t)\{p(t) - a_i r(t)\} \leq \lambda_i, \quad E_i(t)[\pi(t)\{p(t) - a_i r(t)\} - \lambda_i] = 0. \quad (4.2)$$

This is the Hotelling rule, which essentially says that in equilibrium there are no arbitrage possibilities. If a country is supplying the raw material during some interval of time, it should be indifferent with respect to the timing of its supply. So the instantaneous discounted per unit profits ($\pi(p - ar)$) must be constant along that interval.

The fact that each economy's production set is a cone makes it possible to work with factor price frontiers, so that much of the subsequent analysis of price behavior can conveniently be illustrated in (r, p) space. It is well known that if a production function is continuous and displays constant returns to scale, the unit cost function $c(r, p)$ exists, is continuous, concave and increasing. The factor price frontier of such a production function is defined as the locus of input prices which yield at most zero profits, or for which $c(r, p) = 1$. In view of the properties of the unit cost function the factor price frontier is a decreasing, convex and continuous curve in (r, p) space. Since the production functions employed in the model are strictly increasing in the interior of the positive orthant, the factor price frontiers are strictly decreasing in the interior of the positive orthant. Moreover, because both inputs are necessary for production, the factor price frontiers have no strictly positive asymptotes. To see this, suppose for example that there exists $r^* > 0$ such that for all (r, p) for which $c(r, p) = 1, r \geq r^*$. If p goes to infinity the profit-maximizing resource input must go to zero in order to keep costs at the unit level. However, to keep production at the unit level, capital input must then go to infinity, so that unit costs become infinite, contradicting the fact that they are unity.

There is a third property that is worth pointing out. The price of a unit of the capital stock equals unity because the price of the composite commodity is normalized to one. The rental price of the stock at t equals $r(t)$. Individuals should be indifferent between buying and renting the composite commodity so that it must be the case that

$$1 = \int_0^{\infty} \pi(s)r(s) ds.$$

For this identity to hold it is necessary and sufficient that $\pi(t) \rightarrow 0$ as $t \rightarrow \infty$.²

In equilibrium there will always be production of the composite com-

²This was pointed out to us by one of the referees.

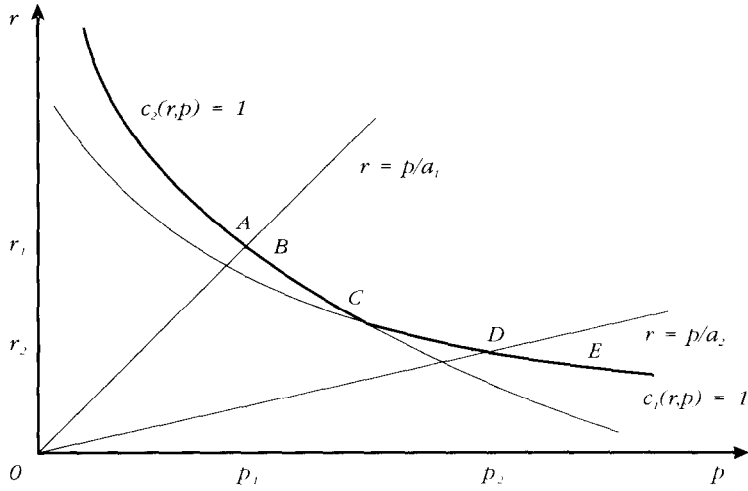


Fig. 1

dity (Theorem 3.1), so the equilibrium prices should lie on the factor price frontier of at least one country. Moreover, since the raw material is a necessary input, it must be profitable to exploit at least one of the resources. Therefore the set of prices which are candidate for an equilibrium is easily sketched; see fig. 1 above (for the case $n=2$). Here the half lines originating at O describe the prices for which there is zero instantaneous profit in extraction for each of the resources, whereas the curves display the factor price frontiers. The thick curve gives the outer envelope of the factor price frontiers. This curve contains the feasible equilibrium prices. Henceforth it will be called *world factor price frontier* for short. For the sake of clarity of exposition the following additional assumptions are made:

Assumption C.1. In the interior of the positive orthant the factor price frontiers have at most a finite number of points of intersection.

Assumption C.2. $0 < a_1 < a_2 < \dots < a_n$.

These assumptions amount to saying that essentially the technologies differ among countries and the countries are ranked according to the unit extraction costs.

One of the consequences of the fact that there is always production of the composite commodity is that it is relatively easy to show that all equilibrium prices are positive.

Theorem 4.1. $r(t) > 0$ and $p(t) > 0$ for all t .

Proof. Fix t . Since $Y_i(t) > 0$ for some i , $K_i^y(t) > 0$ and $R_i(t) > 0$ for this i . If $r(t) = 0$, then $K_i^y(t)$ is not profit maximizing because F_i is an increasing function of K_i^y . So a contradiction is obtained. The same argument applies to show that $p(t) > 0$. \square

Let (r_j, p_j) be the prices on the world factor price frontier such that extraction of resource j yields zero instantaneous profits (see fig. 1). These prices clearly exist and are unique. In fact r_1 is the maximal feasible rental rate because higher rental rates make exploitation too expensive so that there is no supply of the raw material; but this would contradict the fact that there is always production of the composite commodity because the raw material is a necessary input.

The next theorem states that if r is ever maximal, it will remain maximal.

Theorem 4.2. If, for some t_1 , $(r(t_1), p(t_1)) = (r_1, p_1)$, then $(r(t), p(t)) = (r_1, p_1)$ for all t .

Proof. $r(t) \leq r_1$ for all t . For suppose that for some $t_1 \geq 0$, $r(t_1) > r_1$. Since there exists i such that $Y_i(t_1) > 0$, we must have $p(t_1) \leq p_1$. This implies from (4.2) and Assumption C.2 that $E_i(t_1) = 0$ for all i . But then $R_i(t_1) = 0$ for all i and therefore $Y_i(t_1) = 0$ for all i , a contradiction.

So $r(t) \leq r_1$ for all t . Suppose $(r(t_1), p(t_1)) = (r_1, p_1)$ for some t_1 . Then $E_i(t_1) = 0$ for all $i > 1$ because of (4.2) (exploitation of resources more expensive than the first one gives a loss). $E_1(t_1) > 0$ because $Y_i(t_1) > 0$ for some i . Hence $\lambda_1 = 0$ and $p(t) - a_1 r(t) \leq 0$ for all t . If, for some t_2 , $p(t_2) - a_1 r(t_2) < 0$, then $E_1(t_2) = 0$. But then $Y_i(t_2) = 0$ for all i , a contradiction. \square

If in an equilibrium the rate of interest is constant at its maximal level, only the cheapest resource will be exploited along the entire equilibrium trajectory. Intuitively speaking one would expect that for a constant rate of interest to prevail it is necessary that the cheapest resource is in some sense abundant. This is true, but it is not a sufficient condition. In addition the rates of time preference need to be sufficiently large, the reason being that otherwise capital becomes negative, as is seen in the following theorem.

Theorem 4.3. $(r(t), p(t)) = (r_1, p_1)$ implies $\min_i (\rho_i) > r_1$.

Proof. The time argument will be omitted when there is no danger of confusion. K denotes the totally available stock of capital. If $(r, p) = (r_1, p_1)$, then only the cheapest resource will be exploited at any point in time. Since, therefore, $E_1 = \Sigma R_i$, we have $K = \Sigma K_i^y + a_1 \Sigma R_i$.

If $Y_i=0$, then $K_i^y=R_i=0$. If $Y_i>0$, then $K_i^y=b_iR_i$ for some number b_i (which does not depend on time), owing to constant returns to scale. Hence in view of the exhaustibility of the resources:

$$\int_0^\infty K(s) ds < \infty. \tag{*}$$

We also have

$$\begin{aligned} \dot{K} &= \Sigma F_i - \Sigma C_i = \Sigma(r_1 K_i^y + p_1 R_i) - \Sigma C_i \\ &= r_1 K - \Sigma C_i \quad (\text{since } p_1 = a_1 r_1). \end{aligned} \tag{**}$$

By definition, the following holds for all T :

$$r_1 \int_0^T K(s) ds + K_0 = K_T + \int_0^T (r_1 K(s) - \dot{K}(s)) ds.$$

Since the left-hand side is bounded uniformly in T (by (*)), so must be the right-hand side. But in view of (**) this is only the case if $r_1 < \rho_i$ for all i because, in an equilibrium, consumption in each economy satisfies the Ramsey–Keynes rule:

$$u'_i(C_i) = \phi_i e^{(\rho_i - r_1)t},$$

where ϕ_i is a positive constant, and $C_i \geq \bar{C}$ for some $\bar{C} > 0$ as $t \rightarrow \infty$ if $\rho_i < r_1$. \square

Although high rates of time preference are a necessary condition for constant prices they are not sufficient. This can easily be illustrated by means of an example which also illustrates the possibility of the occurrence of constant equilibrium prices.

Let there be one consumer with rate of time preference ρ and utility function $u(C) = 1/(1 + \eta) C^{1 + \eta}$ ($\eta < 0$); let there be one producer of the composite commodity with $F(K^y, R) = (K^y)^{1/2} R^{1/2}$; and let there be one exhaustible resource extractor with technology $K^e = aE$ ($a > 0$). It is easily seen that $(r_1, p_1) = (\frac{1}{2}\sqrt{1/a}, \frac{1}{2}\sqrt{a})$, because $p_1 = ar_1$ and $c(r, p) = 1$ is given by $4p_1 r_1 = 1$. Assume $\rho > r_1$ and define

$$Z(t) := K_0 \exp^{[(\rho - r_1)/\eta]t}.$$

The claim is: if $\int_0^\infty \frac{1}{2} Z(s)/a ds \leq S_0$, then

$$(K, C, K^y, R, K^e, E, r, p) = \left(Z, \frac{\rho - r_1(1 + \eta)}{-\eta} Z, \frac{1}{2}Z, \frac{1}{2}Z/a, \frac{1}{2}Z, \frac{1}{2}Z/a, r_1, p_1 \right)$$

constitutes a general equilibrium. This can be seen as follows:

- Given that prices are (r_1, p_1) , $(K^e, E) = (\frac{1}{2}Z, \frac{1}{2}Z/a)$ is profit maximizing in resource extraction. However, profits are zero.
- Given that prices are (r_1, p_1) , $(K^y, R) = (\frac{1}{2}Z, \frac{1}{2}Z/a)$ is profit maximizing in the production of the composite commodity. Again, profits are zero at all instants of time.
- It follows from the above that the budget constraints of the consumer, (2.7), can be written as

$$\int_0^{\infty} e^{-r_1 s} C(s) ds \leq K_0.$$

- $C = (\rho - r(1 + \eta))Z / (-\eta)$ satisfies this condition with equality and any other consumption pattern with this property yields less utility because the instantaneous utility function is strictly concave and C obeys the Keynes–Ramsey rule.
- Demand for the raw material meets supply at each instant of time by construction and total exploitation is not greater than the initial resource stock, by assumption.
- Demand for capital equals supply: $K^e + K^y = \frac{1}{2}Z + \frac{1}{2}Z = Z = K$. Furthermore, $\dot{K} = (K^y)^{1/2} R^{1/2} - C$.

So all the conditions for a general competitive equilibrium are satisfied.

There are several things to note about this example.

First, it is not difficult to see that the assumption of a single consumer, one producer and one extractor is not crucial. If there are more countries it could happen that one country is never engaged in any productive activity. Nevertheless it does not starve to death because it has been assumed that $K_{i0} > 0$.

Second, the assumption that the rate of time preference is large enough is important (as was shown in the previous theorem). If $\rho < r_1$, then with an abundant natural resource ($S_0 = \infty$), capital would go to infinity, as is well known from the Ramsey model with a linear technology, and so would total use of the resource. So, it is inconceivable that with an upper bound on the resource stock there is an equilibrium with constant prices if $\rho < r_1$.

Third, the intuition behind the example is straightforward. Because of our specification of the production function and the extraction function, $K^y/R = p/r$ and $K^e/E = K^e/R = a$. So $K/R = a + p/r$. If prices are not constant, r will go to zero and p will increase, implying that K/R is increasing. But with an initial stock of capital which is low relative to the initial size of the natural

resource, this would mean that the resource will not be exhausted, contrary to the fact that with r decreasing and p increasing, its shadow price is strictly positive. This type of argument also holds when the industrial structure is richer than in the example. For instance, the presence of other, higher cost, resource stocks will evidently not change the equilibrium. The general idea is that capital has several competing uses: extraction requires capital but capital can also serve as a consumer good. The economy has to take care that neither capital nor the exhaustible resources become negative. With high extraction costs the former constraint may prove to be binding rather than the latter.

Fourth, since in the present context a general equilibrium is Pareto efficient, it may be that when a country enters as a participant in free trade its resource stock becomes valueless but still a Pareto improvement can be realized.

Finally, it is instructive to perform a sensitivity analysis. To that end, consider the assumption underlying the previous example:

$$\int_0^{\infty} \frac{1}{2} Z(s)/a \, ds \leq S_0, \quad Z = K_0 \exp^{(\rho - r_1)/\eta t}.$$

It is straightforward to see that higher K_0 renders constant prices less likely. The same holds for lower S_0 and smaller ρ . The effect of a decrease in the efficiency of the extraction technology is to make p_1 larger and r_1 smaller. The impact on K is negative and the integral will also be negatively affected. Therefore constant prices are more likely to occur with high extraction costs, of course under the *ceteris paribus* proviso.

For obvious reasons (differences in technologies and only one commodity entering into the instantaneous utility functions) the model at hand does not fit into the standard framework of international trade theory. However, a good deal can be said about specialization and the impact of changes in initial endowments. Specialization occurs with respect to the production of the composite commodity.

Theorem 4.5. Suppose $c_i(r_1, p_1) \neq c_j(r_1, p_1)$ for all i and j , $i \neq j$. Suppose, furthermore, that there exist $t_2 \geq t_1 \geq 0$ and i and j ($j \neq i$) such that $Y_i(t) > 0$ and $Y_j(t) > 0$ for all $t \in [t_1, t_2]$, then $t_1 = t_2$.

Proof. If $(r(t), p(t)) = (r_1, p_1)$ the result is immediate. Otherwise r is decreasing and the desired result is obtained from Assumption C.1. \square

We deal next with specialization in exploitation and the order of exploitation. These issues are discussed in several textbooks [see, for example,

Dasgupta and Heal (1978)], generally in the framework of a partial equilibrium model. The phenomena mentioned above are then illustrated for the case of a common and constant interest rate with constant but differing monetary extraction costs. In the present context the situation is somewhat different because instantaneous profits are $p - a_i r$, where r is not necessarily constant, but owing to the essentially linear structure the same type of result is obtained. The proof will not be given here.

This result extends the well-known partial equilibrium outcome for competitive markets to a more general setting. But its bearing is more than a straightforward generalization if one realizes that our general equilibrium is Pareto efficient over time. Therefore, if actual markets for raw materials are supposedly competitive, statically and dynamically, then simultaneous exploitation of different resources is inefficient.

What is the impact of higher initial endowments of the exhaustible resources? The first thing to note is that the world factor price frontier will not be affected. And if the exhaustible resources are already relatively abundant and the rates of time preference are high so that the equilibrium prices are constant (r_1, p_1) , nothing will change at all. So, to have an interesting case it will be assumed that we are on a trajectory with an increasing raw material price and a decreasing rental rate. Suppose that resource i is in the process of being exploited at the instant of time where the discovery of additional reserves of resource i is made. Then the shadow price (royalty) of that resource will exhibit a fall and it follows from the Hotelling rule that the raw material price will jump downwards and the rental rate will jump upwards. As a consequence, it could be that the production of the composite commodity will be specialized in another country and also that the world economy ends up with constant prices. If the discovery is made in a country whose exhaustible resource is not yet taken into exploitation the result is that the resource actually exploited will be depleted earlier and the augmented resource will be taken into exploitation sooner.

Finally we turn to a characterization of consumption and capital intensities. Initially the rates of consumption may rise, but as a consequence of the positive rate of time preference and the limited availability of the resources they eventually decrease. The decline in consumption is monotonic and consumption goes to zero as t goes to infinity. Asymptotically the relative share of consumption in total consumption depend on the rates of time preference in relation to the elasticity of intertemporal substitution. As far as the capital intensity is concerned, it is easily seen that it is increasing because r is decreasing.

These properties are proven in the final two theorems.

Theorem 4.6. (i) *There exists t_1 such that $\dot{C}_i(t) < 0$ for all i and all $t > t_1$.*
(ii) *$C_i(t) \rightarrow 0$ as $t \rightarrow \infty$, all i .*

(iii) $C_i(t)/\Sigma_i C_i(t) \rightarrow 0$ as $t \rightarrow \infty$ if and only if

$$(\rho_i - r(\infty))/(-\eta_i(0)) > \max_{j \neq i} (\rho_j - r(\infty))/(-\eta_j(0)).$$

Proof.

(i) It follows from the Ramsey–Keynes rule that

$$\dot{C}_i/C_i = (\rho_i - r)/(-\eta_i(C_i)).$$

Recall that $\eta_i(C_i) > 0$. If $r(t)$ is constant for some interval of time, then ρ_i must be larger than that constant. If r is never constant, it will decrease towards zero.

(ii) This follows immediately from (i) and the fact that η_i is bounded.

(iii) The asymptotic growth rate of C_i is $(\rho_i - r(\infty))/(-\eta_i(0))$. \square

Theorem 4.7. Suppose $r(0) < r_1$. Then $Y_i(t) > 0$ for $t \in [t_1, t_2]$ with $t_2 > t_1$ implies $\partial(K_i^Y(t)/R_i(t))/\partial t \leq 0$ for $t \in (t_1, t_2)$.

Proof. This is immediate from the fact that r is decreasing and the homogeneity of F_i . \square

Let us by way of illustration sketch the equilibrium trajectories for the situation depicted in fig. 1. If the rates of time preference are ‘high’ ($\rho_i > r_1$ for all i) and the total initial stock of capital is low relative to the resource stock of the first country, then the equilibrium is given by point *A*. The rental rate and the price of the raw material are constant over time. Only the cheapest resource will be exploited and production of the composite commodity is carried out by country 2. Production will display a constant capital intensity and the consumption of the composite commodity will monotonically decrease.

If the rates of time preference are ‘low’ ($\rho_i < r_2$ for all i), the prices will initially be given by, for example, point *B*. The cheapest resource is taken into exploitation and production of the composite commodity is carried out only by country 2. The rental rate decreases over time. When point *C* is reached, country 1 becomes more efficient in production and takes over. It will produce the composite commodity from then on. The rental rate continues to decrease. The resource of country 1 is not exhausted until a point like *E* is reached (it should be noted that in *E* exploitation of the resource of country 2 is profitable). At the instant of time where *E* is reached the resource of the first country is depleted and the second country will be the sole supplier of the raw material from that moment on. Again production

becomes less capital intensive over time and consumption eventually decreases.

There is an interesting third possibility, occurring when $r_1 > \rho_i > r_2$ for all i . In this case the equilibrium price trajectory could start at a point like B . But instead of 'crossing' (r_2, p_2) , as in the previous case, prices remain at (r_2, p_2) for some instant of time on. This type of equilibrium will occur if in the absence of the first resource the second resource is quite plentiful.

5. Conclusions

The aim of the present paper has been to analyze trade in raw materials from natural exhaustible resource in the context of a general equilibrium model. We have been able to characterize prices and allocations along the general equilibrium trajectory, under assumptions which are usually made in models of international trade. Not surprisingly many of the by now classical results from the theory of international trade are shown to remain valid in the model extended with raw materials. Apart from this modest contribution, the merits of the analysis lie in the generalization of models from resource economics. The main result in this respect is that we have identified the circumstances where constancy of the interest rate can be postulated in partial equilibrium models. In spite of the complexity of the model the analysis can be clarified in a rather simple diagram in the price space. It should be admitted that this is to a large extent due to the constant returns to scale assumption. Current research is directed towards characterization of equilibria when that assumption is relaxed.

References

- Bewley, T., 1972, Existence of equilibria with infinitely many commodities, *Journal of Economic Theory* 4, 514–540.
- Chiarella, C., 1980, Trade between resource-poor and resource-rich economies as a differential game, in: M. Kemp and N. Long, eds., *Exhaustible resources, optimality and trade* (North-Holland, Amsterdam) 219–246.
- Dasgupta, P. and G. Heal, 1978, *Economic theory and exhaustible resources* (James Nisbet, Welwyn).
- Dasgupta, P. and G. Heal, 1974, The optimal depletion of exhaustible resources, *Review of Economic Studies*, Symposium Issue, 3–28.
- Elbers, C. and C. Withagen, 1984, Trading in exhaustible resources in the presence of conversion costs, a general equilibrium approach, *Journal of Economic Dynamics and Control* 8, 197–209.
- Geldrop, J. van and C. Withagen, 1990a, On the Negishi-approach to dynamic economic systems, *Nieuw Archief voor Wiskunde* 4, 295–309.
- Geldrop, J. van and C. Withagen, 1990b, Existence of general equilibria in infinite horizon economies with exhaustible resources (the continuous time case), *Memorandum COSOR 90-16*, Eindhoven University of Technology.
- Geldrop, J. van, Shou Jilin and C. Withagen, 1991, Existence of general equilibria in economies with natural exhaustible resources and an infinite horizon, *Journal of Mathematical Economics* 20, 225–248.

- Gilbert, R., 1978, Dominant firm pricing policy in a market for an exhaustible resource, *The Bell Journal of Economics* 9, 385–395.
- Groot, F., 1990, The feedback Stackelberg equilibrium in the cartel-versus-fringe model, Mimeo. (Tilburg University).
- Heal, G.M., 1976, The relation between price and extraction costs for a resource with a back-stop technology, *The Bell Journal of Economics* 7, 371–378.
- Hotelling, H., 1931, The economics of exhaustible resources, *Journal of Political Economy* 39, 137–175.
- Karp, L. and D. Newbery, 1989, Intertemporal consistency issues in depletable resources, forthcoming in: J. Sweeney and A. Kneese, eds., *Handbook of natural resource and energy economics* (North-Holland, Amsterdam).
- Kay, J.A. and J.A. Mirrlees, 1975, The desirability of natural resource depletion, in: D. Pearce and J. Rose, eds., *The economics of natural resource depletion* (The Macmillan Press, London) 140–176.
- Kemp, M.C. and N.V. Long, 1980a, The interaction of resource-rich and resource-poor economies, in: M. Kemp and N. Long, eds., *Exhaustible resources, optimality and trade* (North-Holland, Amsterdam) 197–210.
- Kemp, M.C. and N.V. Long, 1980b, Exhaustible resources and the Heckscher–Ohlin theorem, in: M. Kemp and N. Long, eds., *Exhaustible resources, optimality and trade* (North-Holland, Amsterdam) 183–186.
- Lewis, T.R. and R. Schmalensee, 1980a, Cartel and oligopoly pricing of non-replenishable natural resources, in: P.T. Liu, ed., *Dynamic optimization and applications to economics* (Plenum Press, New York) 133–156.
- Lewis, T.R. and R. Schmalensee, 1980b, An oligopolistic market for non-renewable natural resources, *Quarterly Journal of Economics* 95, 475–491.
- Marion, N. and L. Svensson, 1984, World equilibrium with oil price increases: An intertemporal analysis, *Oxford Economic Papers* 36, 86–102.
- Mas-Colell, A., 1986, Valuation equilibrium and Pareto optimum revisited, in: W. Hildenbrand and A. Mas-Colell, eds., *Contributions to mathematical economics, in honor of Gerard Debreu* (North-Holland, Amsterdam) 317–331.
- Newbery, D., 1980, Oil prices, cartels, and the problem of dynamic inconsistency, *Economic Journal* 91, 617–646.
- Richard, S., 1986, Competitive equilibria in Riesz spaces, Carnegie-Mellon University Working paper.
- Salant, S., 1976, Exhaustible resources and industrial structure: A Nash–Cournot approach to the world oil market, *Journal of Political Economy* 84, 1079–1093.
- Ulph, A.M., 1982, Modelling partially cartelised markets for exhaustible resources, in: W. Eichhorn et al., eds., *Economic theory of natural resources* (Physica Verlag, Würzburg) 269–291.
- Van Wijnbergen, S., 1985, Taxation of international capital flows, the intertemporal terms of trade and the real price of oil, *Oxford Economic Papers* 37, 382–390.
- Withagen, C., 1990, Topics in resource economics, in: F. van der Ploeg, ed., *Advanced lectures in quantitative economics* (Academic Press, London) 381–419.
- Zame, W., 1987, Competitive equilibrium in production economies with an infinite-dimensional commodity space, *Econometrica* 55, 1075–1108.