On the concept of green national income

Withagen, C.A.A.M.; Vellinga, N.

Published in:
Oxford Economic Papers

Publication date:
1996

ON THE CONCEPT OF GREEN NATIONAL INCOME

By NICO VELLINGA and CEES WITHAGEN

*Department of Mathematics and Computing Science,
Eindhoven University of Technology, PO Box 513, 5600 MB Eindhoven,
The Netherlands*

The present paper generalizes the Weitzman/Hartwick approach to national income accounting. We first establish a close general relation between the current value Hamiltonian of an optimal control problem and the optimal value of the objective integral if the time argument does not enter the constraints and enters the objective function only as a discount factor. This result is applied to a simple economic model covering most models found in the literature on national income accounting involving pollution and non-renewable resources. We critically review the usefulness of net national product as a welfare indicator and as an indicator for sustainability.

1. Introduction

MUCH of the current debate in the literature on the question of the suitability of net national product (NNP) as an indicator of social welfare or as an indicator of sustainability goes back to the seminal work of Hicks (1946), Samuelson (1961), and Weitzman (1976). According to Hicks an individual's income is 'the maximum value which he can consume during a week and still expect to be as well off at the end of the week as he was in the beginning' (*op.cit.*, p. 172). If this concept is extended and applied to an economy as a whole, income would be a number representing the amount of welfare which can be enjoyed over a period of time and leave the economy with the capacity to enjoy that same amount of welfare for the next period of time. Clearly, the development of the economy over a period of time is then sustainable if income, in the sense of the definition, is constant over that period of time. To illustrate these issues in a simple example, let us consider the well-known one-sector Solow (1956) model, without population growth, technical progress, and depreciation. Let F be the production function with capital (K) as input and giving consumption (C) and investments ($\dot{K} = dK/dt$) as output. NNP is then $C + \dot{K} = F(K)$. In the case at hand welfare can be identified with consumption. Clearly, the higher NNP the larger is potential welfare (because K is greater). Moreover, if we compare two economies, both aiming at sustainable development, which is defined in this context as maximal constant consumption, then the economy with the higher NNP is able to sustain the higher constant rate of consumption. Hence, NNP is a good measure for both welfare and sustainability. When multiple capital goods are involved a natural extension of NNP would include the total value of net investments: $NNP := C + p \cdot \dot{K}$, with p a vector of prices of capital. The economy would then display sustainability

if NNP is constant. This is reflected in e.g. Hartwick's rule, saying that 'society should invest the current returns from the utilization of flows from the stocks of exhaustible resources' (Hartwick, 1977, p. 975) in order to have intergenerational equity. This idea is also found in Hartwick (1990, 1991, 1994a, 1994b). Hartwick (1978) and Mäler (1991, p. 12) generalize Hartwick's rule to the case of renewable resources. These findings have also had an impact on statistical practice in that attempts are made to correct conventional NNP in order to have a measure for sustainability. See e.g. Hueting *et al.* (1992); Weale (1992); Bartelmus *et al.* (1991). The latter authors argue that in the context of national accounting there are 'doubts about the usefulness of (conventional) national account data for the measurement of long-run sustainable growth. . . . ', and they want to develop 'modified indicators' (*op.cit.*, p. 111). These modifications then consist of correcting conventional NNP for the exploitation of raw material from exhaustible resources and the value of increases or decreases of environmental quality. In a recent study Bartelmus (1994) introduces the concept of Environmentally-adjusted net Domestic Product (EDP) which is domestic product, conventionally measured, minus enviromental costs. He puts forward that 'the long run increase or decrease of EDP can be taken as a measure of, or a proxy for, a sustainable or non-sustainable path of economic growth' (*op. cit.*, p. 70). On the other hand, there are a number of authors that question the capability of (even corrected) NNP to be a measure of sustainability. In this paper we shall embed the critique of Asheim (1994), Aaheim and Nyborg (1992), and Pezzey (1994) in a rather general framework.

In terms of social welfare, sustainability is defined as constant instantaneous welfare over time. However, this might not be something the economy is aiming at. Alternatively, the economy's objective might be utilitarian in the sense of maximizing the total discounted utility flow over time, rather than Rawlsian. It was pointed out by Weitzman (1976) in the context of a neoclassical growth model and after 'heroically abstracting' that 'the welfare justification of NNP is just the idea that in theory it is a proxy for the present discounted value of future consumption' (*op. cit.*, p. 156). We will show that this is also true in a fairly generalized setting if NNP is identified with the current value Hamiltonian of the underlying optimization problem. This Hamiltonian however does not in general coincide with conventional NNP. That NNP should be corrected for, for example, the exhaustion of natural resources and the disutility of pollution. This is done e.g. by Sefton and Weale (1992, 1994) and Mäler (1991). So far, most of the work done on NNP as a welfare indicator has focused on stationary economies. The question arises what corrections should be made when there is technical progress, varying world market prices for raw materials or varying world market interest rates. Another issue that will be addressed is NNP as an instrument in cost-benefit analysis. The question is if the welfare consequences of new projects are adequately measured by the impact on instantaneous NNP. This use of NNP is advocated by Dasupta and Mäler (1991) when they argue that 'choosing projects that increase NNP increase the current-value Hamiltonian as well and, therefore, should be regarded desirable'

(*op. cit.*, p. 63). As another example, Bartelmus (1994) argues that 'national accounts... are used... in the assessment of the economic counterpart of social welfare' (*op. cit.*, p. 34) and should therefore be corrected for e.g. environmental protection expenditures. Another reference is Chichilnisky (1994) saying that 'if a politician's re-election depends on the measure of national economic growth, and it often does, green accounting could be helpful in reorienting environmental policy'. We also wish to mention Solow (1992).

Summarizing, the objective of the present paper is twofold. First we wish to show that the equality of future welfare and *NNP*, identified with the current value Hamiltonian, can easily be established in a general setting, if the economy is stationary (in the sense that there is no population growth, no exogenously varying prices etc.). We shall also derive the corrections that must be made in a non-stationary economy (Section 2). These results are clarified in an environmental/resource model that is presented in Section 3. Sections 4 and 5 deal with *NNP* as a welfare indicator. In Section 4 we go into some difficulties that occur when the economy is non-stationary. In Section 5 we argue that the usefulness of *NNP* in the conventional setting as an instrument in cost-benefit analysis along the lines set out by Dasgupta and Mäler (1991) is very limited because that works only for short run evaluation of projects. On the other hand, in this approach the non-stationarity does not pose a problem. A second objective of this paper is to emphasize, by means of our general framework, that Asheim (1994) and Pezzey (1994) are right in their rejection of *NNP* as a measure for sustainability. This is done in Section 6. Section 7 concludes. One important *caveat* applies. Uncertainty, although pertinent to many, if not all, environmental/resource issues, is not treated here.

2. Current value Hamiltonian and social welfare

We investigate the relation between the current value Hamiltonian of an optimal control problem and the value of the objective functional of the problem. We start from l parameters denoted by $\alpha := (\alpha_1, \alpha_2, \ldots, \alpha_l)$, m instruments denoted by $u := (u_1, u_2, \ldots, u_m)$ and n state variables given by $x := (x_1, x_2, \ldots, x_n)$. The problem is to find piece-wise continuous $u: [0, \infty) \to \mathbb{R}^m$ and piece-wise differentiable $x: [0, \infty) \to \mathbb{R}^n$ which for all $t \in [0, \infty)$ satisfy

$$\dot{x}(t) = f(x(t), u(t), t; \alpha), \quad x(0) = x_0 \tag{1}$$

given

$$g(x(t), u(t), t; \alpha) \geqslant 0 \tag{2}$$

and maximize

$$\int_0^\infty e^{-\rho t} f_0(x(t), u(t); \alpha) \, dt \tag{3}$$

Here $(f_0, f, g) = (f_0, f_1, f_2, \ldots, f_n, g_1, g_2, \ldots, g_s)$ are given functions, obeying certain regularity conditions, such as continuity or, in some cases, continuous differentiability. See e.g. Cesari (1983) or Seierstad and Sydsaeter (1987) for a

formal treatment of these conditions. We will assume throughout that all of them are satisfied. Note that (2) comprises s constraints and that α is indeed treated as a vector of parameters, which is given for the moment. Note also that the formulation of the objective (3) is special in the sense that f_0 itself is time-independent and that time only occurs in the form of an exponential function.

One can think of $x(t)$ as the state of the economy at instant of time t, given for example by the stocks of man-made capital, exhaustible resources, human capital, pollution, and renewable resources, and $u(t)$ as the value of the policy instruments at instant of time t. The parameter vector α may refer to investment opportunities. The set of differential equations (1) describes the motion of the economy ($\dot{x} := dx/dt$) and (2) involves technological and other constraints. In the objective function ρ can be interpreted as the constant rate of pure time preference. The existence of a solution to the problem stated above is by no means trivial. See e.g. Toman (1985) for existence theorems. Again, we will merely assume here that a solution exists.

Define the Hamiltonian $\mathcal{H}$ and the Lagrangean $\mathcal{L}$ in current value terms

$$\mathcal{H}(x, u, t, \lambda; \alpha) := f_0(x, u; \alpha) + \lambda \cdot f(x, u, t; \alpha) \tag{4}$$

$$\mathcal{L}(x, u, t, \lambda, \mu; \alpha) := \mathcal{H}(x, u, t, \lambda; \alpha) + \mu \cdot g(x, u, t; \alpha) \tag{5}$$

where $\lambda := (\lambda_1, \lambda_2, \ldots, \lambda_n)$, $\mu := (\mu_1, \mu_2, \ldots, \mu_s)$ and a dot between two vectors denotes the inner-product.

As necessary conditions for an optimum we have

$$\partial \mathcal{L}(x, u, t, \lambda, \mu; \alpha)/\partial u_j = 0, \quad j = 1, 2, \ldots, m \tag{6}$$

$$\mu(t) \geqslant 0, \quad \mu(t) \cdot g(x(t), u(t), t; \alpha) = 0 \tag{7}$$

$$\dot{\lambda}_i - \rho \lambda_i = -\partial \mathcal{L}(x, u, t, \lambda, \mu; \alpha)/\partial x_i, \quad i = 1, 2, \ldots, n \tag{8}$$

It follows from (5) that

$$\dot{\mathcal{L}} = (\partial \mathcal{L}/\partial x) \cdot \dot{x} + (\partial \mathcal{L}/\partial u) \cdot \dot{u} + \partial \mathcal{L}/\partial t + (\partial \mathcal{L}/\partial \lambda) \cdot \dot{\lambda} + (\partial \mathcal{L}/\partial \mu) \cdot \dot{\mu}$$

Now use (8), (6), (7), and (1) to get

$$\dot{\mathcal{L}} = -(\dot{\lambda} - \rho \lambda) \cdot f + 0 + \lambda \cdot \frac{\partial f}{\partial t} + \mu \cdot \frac{\partial g}{\partial t} + f \cdot \dot{\lambda} + 0$$

$$= \rho(\mathcal{L} - f_0) + \lambda \cdot \frac{\partial f}{\partial t} + \mu \cdot \frac{\partial g}{\partial t} \tag{9}$$

where function arguments have been omitted. However, it should be clear that everything is evaluated in the optimum. Michel (1982) shows that

$$e^{-\rho t} \mathcal{L}(t) \to 0 \quad \text{as} \quad t \to \infty \tag{10}$$

is a necessary condition in the case at hand (with t appearing in the welfare functional as $e^{-\rho t}$ only) if the welfare functional converges.

Then it is straightforward to show that

$$\mathscr{H}(t) = \mathscr{L}(t)$$

$$= e^{\rho t} \int_t^\infty \rho e^{-\rho s} f_0(x, u) \, ds - e^{\rho t} \int_t^\infty e^{-\rho s} \left[\lambda \cdot \frac{\partial f(x, u, s)}{\partial s} + \mu \cdot \frac{\partial g(x, u, s)}{\partial s} \right] ds$$

(11)

The first equality in (11) uses (7) and the second equality gives the solution of the differential eq. (9).

In the sequel $\mathscr{W}(t)$ is defined as follows

$$\mathscr{W}(t) := e^{\rho t} \int_t^\infty \rho e^{-\rho s} f_0(x, u) \, ds$$

Thus $\mathscr{W}(t)$ gives total future welfare from t on, discounted to t. Hence

$$\mathscr{H}(t) = \mathscr{W}(t) - e^{\rho t} \int_t^\infty e^{-\rho s} \left[\lambda \cdot \frac{\partial f(x, u, s)}{\partial s} + \mu \cdot \frac{\partial g(x, u, s)}{\partial s} \right] ds \qquad (12)$$

So, the optimal current value Hamiltonian at time t equals the value of the objective functional from time t on times the rate of time preference and corrected for non-autonomous parts in f and g. Much, if not all, of the literature mentioned in the Introduction is based on this observation. If the rate of time preference ρ is allowed to vary over time we get (12) again, be it that $e^{\rho t}$ should be replaced everywhere by the discount factor

$$\exp \left(\int_0^t \rho(\tau) \, d\tau \right)$$

and ρ by $\rho(s)$ in the definition of $\mathscr{W}$ right after the integration operator.

With regard to the cost-benefit interpretation of national income the following result is important. Under some regularity conditions (see e.g. Malanowski, 1984) the Hamiltonian, parametrized with respect to the parameters α, is differentiable and

$$\frac{\partial \mathscr{W}(t; \alpha)}{\partial \alpha} = e^{\rho t} \int_t^\infty \rho e^{-\rho s} \frac{\partial \mathscr{H}(x, u, s, \lambda; \alpha)}{\partial \alpha} \, ds \qquad (13)$$

In the sequel these results will be applied to the discussion on national accounting but first we shall construct a model that will serve as a concrete example.

3. A prototype resource/environment model

In order to illustrate our main points we shall employ the model outlined below. Our particular choice is motivated by the requirement that it more or less covers the models used in the literature that addresses the issue of income and welfare, but is at the same time as simple as possible.

There are three physically distinguishable commodities: an exhaustible natural resource, pollution, and a so-called composite commodity. The stock of the exhaustible natural resource owned by the economy at instant of time t is denoted by $S(t)$. The stock of pollution is $P(t)$, the stock of the composite commodity (capital) owned by the economy is $H(t)$, and the stock of the composite commodity used is $K(t)$.

The rate of exploitation of the resource is $E(t)$. Exploitation is costless. The use of the raw material in production is $R(t)$. So

$$\dot{S}(t) = -E(t), \quad S(t) \geqslant 0, \quad E(t) \geqslant 0, \quad S(0) = S_0 \tag{14}$$

given

$$E(t) = R(t) + X_e(t) \tag{15}$$

where X_e is the net exports of the raw material.

The production of the flow of the composite commodity takes place according to a production function F with the stock of the composite commodity and the raw material as inputs. We also allow for exogenous technical progress. Output is used for domestic consumption (C), net exports (X_y), or for abatement purposes (A)

$$F(K(t), R(t), t) = C(t) + X_y(t) + A(t) \tag{16}$$

Pollution is assumed to accumulate proportionally to production. Decay is proportional to the stock of pollution where the factor of proportionality μ depends on abatement

$$\dot{P}(t) = \varphi F(K(t), R(t), t) - \mu(A(t))P(t), \quad P(0) = P_0 \quad \text{given} \tag{17}$$

The world market for the raw material is competitive and the price ruling at instant of time t is $p(t)$. The flow of the composite commodity is taken as the *numeraire*. The internationally ruling rate of interest is $r(t)$ and the capital market is perfect. The accumulation of capital owned by the economy can now be described as follows

$$\dot{H}(t) = r(t)H(t) + p(t)X_e(t) + X_y(t) - r(t)K(t), \quad H(0) = H_0 \quad \text{given} \tag{18}$$

In order to have a meaningful budget constraint we add

$$\liminf_{t \to \infty} \exp\left(-\int_0^t r(\tau)\,d\tau\right) H(t) \geqslant 0 \tag{19}$$

which says that in the long run total discounted expenditures should not exceed total discounted income.

The economy's welfare functional is given by

$$U(C, P) := \int_0^\infty e^{-\rho t} u(C(t), P(t))\,dt \tag{20}$$

where ρ denotes the rate of time preference and u is the instantaneous utility function which is increasing in C and decreasing in P.

In the sequel we study this model in alternating forms. Cases with 'no physical capital' are mathematically identified with $K \equiv H \equiv 0$ and $F(0, R) \equiv R$. We have 'no international trade' if $K \equiv H$ and $p \equiv 0$. The well-known Dasgupta and Heal (1974) model arises when P is absent from the instantaneous utility function and when there is no international trade. The classical Ramsey (1928) model occurs if, in addition, the natural resource plays no role.

If it is assumed that F, μ, and u are well-behaved, the model presented here is an optimal control model of the type described in the previous section. In the terminology of that section, S, P, and H are the state variables, E, R, X_e, C, X_y, and K are the policy instruments and, if we assume, just for expository purposes, that abatement is to be set at a constant level indefinitely, A is a parameter.

The Hamiltonian reads

$$
\begin{aligned}
\mathscr{H}(S, P, H, R, X_e, C, K, \lambda, t; A) &= u(C, P) + \lambda_1[-R - X_e] \\
&\quad + \lambda_2[\varphi F(K, R, t) - \mu(A)P] \\
&\quad + \lambda_3[rH + pX_e + F(K, R, t) - C - A - rK] \\
&= u(C, P) + \lambda_1\dot{S} + \lambda_2\dot{P} + \lambda_3\dot{H}
\end{aligned}
\tag{21}
$$

We have eliminated E and X_y using (15) and (16). Here λ_1, λ_2, and λ_3 are the shadow prices (in terms of utility) of the resource, pollution ($\lambda_2 < 0$), and man-made capital respectively. The equivalent of expression (11) is

$$
\mathscr{H}(t) = e^{\rho t} \int_t^\infty e^{-\rho s}\{\rho u(C, P) - \lambda_2 \varphi F_s - \lambda_3[\dot{r}(H - K) + \dot{p}X_e + F_s]\}\, ds
\tag{22}
$$

where F_s is the partial derivative of F with respect to time. In the sequel it will be assumed that the economy is indeed maximizing welfare as given by (20), either in a planning setting or as the outcome of a decentralized general equilibrium where the consumers have preferences which are described as in (20). Net national product (in utility terms) is defined as utility plus net changes in the value of capital (man-made, non-renewable and pollution). So

$$
NNP = u(C, P) + \lambda_1\dot{S} + \lambda_2\dot{P} + \lambda_3\dot{H} = \mathscr{H}
$$

Sometimes we are interested in NNP in money terms. Instantaneous utility can be linearized as a first order approximation: $u(C, P) \approx u_c C + u_p P = \lambda_3 C + u_p P$, because the shadow price of consumption is equal to marginal utility.

4. NNP as a welfare measure

It is seen from (21), (11), and (12) that if r, p and F are time independent, we have

$$
\mathscr{H}(t) = \rho e^{\rho t} \int_t^\infty e^{-\rho s} u(C, P)\, ds = \mathscr{W}(t)
$$

saying that the Hamiltonian at instant of time t is proportional to total future welfare. If national income (in utility terms) is to reflect welfare, it should therefore include instantaneous utility and should be corrected for the decrease of the exhaustible resource, the increase of pollution and the increase of national non-resource wealth, all of these evaluated at the optimal shadow prices. If we linearize utility we find

$$\mathcal{H}(t) \simeq \lambda_3 C + u_p P + \lambda_1 \dot{S} + \lambda_2 \dot{P} + \lambda_3 \dot{H}$$

So, as a first approximation the value of pollution should be included. For practical purposes the great difficulty is of course to find the right shadow prices, especially in the absence of consensus in the economy on the aggregate social welfare function (particularly with respect to P in the instantaneous utility function).

In the sequel of this section we shall concentrate on several cases where the motion of the state variables is non-autonomous.

4.1. Technical progress

Aronsson and Löfgren (1993) deal with technical progress in the classical Ramsey model (no trade, no exhaustible resources, no pollution). They correctly put forward that with anticipated technical progress ($F_t > 0$), the current value Hamiltonian underestimates welfare. This also holds in the case at hand, if there is no pollution (see eq. (22) with $\dot{p} = \dot{r} = 0$). In the presence of pollution this positive effect is mitigated by the fact that, *ceteris paribus*, our type of technical progress will enhance production and thereby pollution (note that $\lambda_2 < 0$). However, the overall impact of technical progress here is still that conventional national income should be corrected in a positive way, because $\lambda_2 \varphi + \lambda_3 > 0$ (otherwise production would be set equal to zero).

4.2. Non-constant world market prices

The case of varying world market prices of the raw material is analyzed by Sefton and Weale (1992). Their model does not take pollution into account. They assume $\dot{p}/p = r = \rho$ and a linear instantaneous utility function. It is shown that for this situation welfare equals

$$\mathcal{H}(t) = \mathcal{W}(t) - rp(t) \int_t^\infty (E(s) - R(s))\, ds \tag{23}$$

This result follows from eq. (22) by putting $F_t = \dot{r} = 0$, $\dot{p} = rp$ and $\lambda_3 = 1$ (since utility is linear: $u(C, P) = C$). Hence, the Hamiltonian under- or overestimates social welfare depending on the country being a future net exporter or importer of the raw material. Now, there is a case for the assumption $\dot{p}/p = r$. With perfect competition on the raw material market and a perfect capital market this is just Hotelling's rule. But the choice $r = \rho$ is more difficult to justify.

In a closed Ramsey-type economy marginal product of capital could be identified with the interest rate and marginal product will tend to the rate of time preference. But here we are dealing with an open economy, where there is no reason to make this assumption *a priori*. Moreover, the assumption of a constant interest rate is also questionable. It has been shown by van Geldrop and Withagen (1993) in a similar model as the one employed here that, in a general equilibrium setting, the internationally-ruling interest rate is decreasing. The idea is simply that if the production function F displays constant returns to scale, equilibrium prices (r, p) should be on the factor price frontier, which is downward sloping in (r, p) space; combining this with the Hotelling rule yields that r decreases over time. For this more general framework we obtain

$$\mathcal{H}(t) = \mathcal{W}(t) - e^{\rho t} \int_t^\infty e^{-\rho s} \lambda_3 [\dot{r}(H - K) + \dot{p}(E - R)] \, ds$$

where λ_3 is the shadow price of capital. Using the fact that $\dot{\lambda}_3 = (\rho - r)\lambda_3$ (from eq. (8)) we have for $t = 0$

$$\mathcal{H}(0) = W(0) - \lambda_3(0) \int_0^\infty \exp\left(-\int_0^t r(\tau) \, d\tau\right) \dot{r}(H - K) \, dt$$

$$- \lambda_3(0) \int_0^\infty \exp\left(-\int_0^t r(\tau) \, d\tau\right) \dot{p}(E - R) \, dt$$

so that *NNP* (in utility terms) should be corrected for the net present value of additional interest revenues on capital exports and additional revenues on exports of the exhaustible resources. Both these terms can be interpreted as a kind of capital gains due to increasing prices (however, r can be decreasing).

When revising the present paper we found out that Sefton and Weale (1994) provide a generalization of their original 1992 model, which boils down to the same outcome as ours.

4.3. *The pure mining model*

This is a special case in that it is the simplest model with trade in the raw material ($K \equiv H \equiv 0$). Then we have as society's maximization problem

$$\max \int_0^\infty e^{-\rho t} u(p(t)E(t)) \, dt$$

subject to $\dot{p}/p = r$ and the resource constraint (14). Hence this economy just exports its exhaustible resource and uses the revenues to import the consumer commodity. We consider two different examples. If $u(pE) = pE$ then a necessary condition for the existence of an interior solution ($E(t) > 0$, all t) is that $\rho = r$ for all t. Then we also have

$$\mathcal{H}(0) = 0 \quad \text{and} \quad \mathcal{W}(0) = p_0 S_0 / \rho$$

If $u(pE) = (pE)^{1+\eta}/(1+\eta)$ with $\eta < 0$, then we have $\dot{E}/E = (\rho - r(1+\eta))/\eta$. Therefore, a necessary condition for not overdepleting the resource is that $\rho > r(1+\eta)$. Furthermore, tedious but straightforward calculations yield

$$\mathcal{W}(0) = \int_0^\infty \rho e^{-\rho\tau} u(p(\tau)E(\tau))\, d\tau$$

$$= \frac{\rho}{1+\eta} (p_0 S_0)^{1+\eta} \left(\frac{\rho - r(1+\eta)}{-\eta}\right)^\eta$$

$$\mathcal{H}(0) = u(pE) - pEu'(pE)$$

$$= \frac{-\eta}{1+\eta} (p_0 S_0)^{1+\eta} \left(\frac{\rho - r(1+\eta)}{-\eta}\right)^{1+\eta}$$

implying $e\mathcal{H}(0) = [\rho - r(1+\eta)]\mathcal{W}(0)$, whereas without time dependency we would get $\mathcal{H}(0) = \mathcal{W}(0)$. Therefore, the under- or overestimation depends on η being larger than or smaller than -1.

Finally, we make a critical note concerning the use of national income as a measure to compare welfare across nations. Apart from the problems mentioned above there is another difficulty which can easily be demonstrated with the aid of the classical Ramsey (1928) model. Consider two economies, indexed by 1 and 2, which have an identical technology and an identical stock of capital. They differ only in their (constant) rates of time preference. The Hamiltonian of economy $i(i = 1, 2)$ is

$$\mathcal{H}_i(t) = u(C_i) + \lambda_i(F(K_i) - C_i)$$

Along an optimum we have $\lambda_i = u'(C_i)$, so that

$$\mathcal{H}_i(t) = u(C_i) - u'(C_i)C_i + u'(C_i)F(K_i)$$

If u is strictly concave, it follows that $\mathcal{H}_i$ is strictly increasing in C_i if and, only if $F(K_i) < C_i$. It is well known that, along an optimum, $K_i(t) \to \hat{K}_i$ as $t \to \infty$, where $\hat{K}_i$ is the modified golden rule capital stock defined by $F'(\hat{K}_i) = \rho_i(i = 1, 2)$. Now suppose that $\rho_1 > \rho_2$ and $K(0) > \hat{K}_2$. Then $C_1(0) > C_2(0) > F(K_i(0))$ so that $\mathcal{H}_1(0) > \mathcal{H}_2(0)$. Therefore, the more impatient economy has the larger income. We do not think that such considerations are a good basis for comparing the performance of economies.

5. NNP as an index in cost benefit analysis

A particular application of NNP as an indicator of welfare is its use in cost benefit analysis. The most prominent advocates of this approach are Dasgupta and Mäler (1991, 1993). Their approach can be illustrated as follows.

Let (u, x) denote the optimal trajectory with λ as the corresponding co-states. Fix $t \geqslant 0$ and consider the vectors $\hat{u}(t)$ and $\bar{u}(t)$ of policy instruments at instant of time t. $\hat{u}$ and $\bar{u}$ are identified with investment projects. These vectors do

not necessarily coincide with $u(t)$, which is the optimal one. Dasgupta and Mäler now define net national product at instant of time t, given $\hat{u}(t)$ and $\bar{u}(t)$ respectively (the time argument will be dropped wherever there is no danger of confusion and f_{0u} is the vector of partial derivatives of f_0 with respect to u and f_{0x} is defined likewise)

$$\widehat{NNP} = f_{0u}(x, \hat{u}) \cdot \hat{u} + f_{0x}(x, \hat{u}) \cdot x + \lambda \cdot f(x, \hat{u}, t)$$

$$\overline{NNP} = f_{0u}(x, \bar{u}) \cdot \bar{u} + f_{0x}(x, \bar{u}) \cdot x + \lambda \cdot f(x, \bar{u}, t)$$

Assuming concavity of the Hamiltonian with respect to u we have

$$\mathcal{H}(x, \hat{u}, t, \lambda) - \mathcal{H}(x, \bar{u}, t, \lambda) = f_0(x, \hat{u}) - f_0(x, \bar{u}) + \lambda \cdot f(x, \hat{u}, t) - \lambda \cdot f(x, \bar{u}, t)$$

$$\geqslant f_{0u}(x, \hat{u})(\hat{u} - \bar{u}) + \lambda \cdot f(x, \hat{u}, t) - \lambda \cdot f(x, \bar{u}, t)$$

If it is assumed that $f_{0u}(x, u) = f_{0u}(x, \hat{u}) = f_{0u}(x, \bar{u})$ and $f_{0x}(x, u) = f_{0x}(x, \hat{u}) = f_{0x}(x, \bar{u})$ then project $\hat{u}$ should be preferred to project $\bar{u}$ if $\widehat{NNP} > \overline{NNP}$ because then $\hat{u}$ contributes more to the Hamiltonian than $\bar{u}$ does and, along an optimum, the Hamiltonian should be maximized.

Several remarks are in order:

(i) In their 1993 paper Dasgupta and Mäler omit the term f_{0x} in the definition of NNP, whereas this term is present in the 1991 paper. It can be argued that f_{0x} measures the returns (in utility) on the stocks in the economy and should therefore be included in national income. On the other hand, if national income only serves as a means to compare projects this term can be skipped;

(ii) Dasgupta and Mäler are aware of the fact that the assumption that the (shadow) prices (f_{0u}) are not affected by the projects is indeed an assumption, but they argue that the error is possibly not too large if the economy is moving to the optimum according to an efficient planning procedure;

(iii) It is important to note the instantaneous or short run character of the analysis. The analysis focuses on one instant of time and, in continuous time, this implies that stocks are not affected. No reference is made to the relation between the Hamiltonian and total future welfare. An obvious advantage is therefore that we do not have to worry about the case where the motion of the stocks is non-autonomous. But it should be stressed that the introduction of projects as entities with instantaneous impact only is rather restrictive. Many projects, once initiated, have a long-run impact, also on the state of the economy. One could even say that most projects in the context of growth and the environment are aiming at changing the stock of environmental capital in the sense of increasing environmental quality or reducing the speed of depletion of exhaustible resources. In that case the cost-benefit approach outlined above is no longer valid, as is demonstrated by, among others, Johansson and Löfgren (1994) in a particular example. They identify projects by different values of the control parameter (α). The Hamiltonian can be written as

$$\mathcal{H}(x(t; \alpha), u(t; \alpha), t, \lambda(t; \alpha)) = f_0(x(t; \alpha), u(t; \alpha)) + \lambda(t; \alpha) \cdot \dot{x}(t; \alpha)$$

where $x(t; \alpha)$ denotes the optimal value of x when project α is implemented. $u(t; \alpha)$ and $\lambda(t; \alpha)$ are defined in the same way. If f and g are autonomous it follows from (11) and (12) that

$$\mathcal{H}(t; \alpha) = \mathcal{W}(t; \alpha)$$

So α should be increased as long as it increases the Hamiltonian. Therefore we consider

$$\partial\mathcal{H}(t; \alpha)/\partial\alpha = f_{0u} \cdot (\partial u/\partial\alpha) + f_{0x} \cdot (\partial x/\partial\alpha) + \dot{x} \cdot (\partial\lambda/\partial\alpha) + \lambda \cdot (\partial\dot{x}/\partial\alpha)$$

Now, if the vector of shadow prices λ does not depend on α, the project should be carried out if

$$f_{0u} \cdot (\partial u/\partial\alpha) + f_{0x} \cdot (\partial x/\partial\alpha) + \lambda \cdot (\partial\dot{x}/\partial\alpha) > 0$$

This expression obviously has a national income interpretation and can be used for a cost benefit evaluation. However, it has been assumed that the shadow prices do not depend on the projects, which might be a more serious assumption than in the previous (short-run) case. Moreover, if time plays a role in the functions f or g so that the differential equations describing the motion of the state variables are non-autonomous, matters become even more complicated because then we have to employ (13).

In our model we can consider A as a parameter. If one follows the approach outlined here, A should be increased (marginally) if

$$\int_t^\infty e^{-\rho s}(-\lambda_2 \mu'(A)P - \lambda_3) \, ds \geqslant 0$$

(at least if we assume away technical progress and variability of p and r). This expression says that abatement should be increased as long as marginal utility of decreased pollution outweighs marginal disutility of foregone consumption.

We conclude that net national product as defined by Dasgupta and Mäler can serve as a welfare measure in cost-benefit analyses if new projects only affect current instruments. However, this measure is not appropriate if long run projects are taken into consideration. In order to evaluate such projects more knowledge about the long run optimum, the entire trajectory, is necessary. This probably requires more information but that should not be a reason to refrain from a long run approach.

6. NNP as an indicator for sustainability

Several authors claim that NNP can be used as a measure for sustainable development. Examples are Hartwick (1990), Hulten (1992), and Mäler (1991). Others like Pezzey (1994) and Asheim (1994), using Dixit et al. (1980), argue that these claims are incorrect. In our view the debate has come to an end because the arguments of the latter authors are convincing. So we do not go into the discussion here but restrict ourselves to summarizing the issues raised,

thereby employing the general setting of Section 2 and the model of Section 3. We shall identify sustainable development with a constant value of the instantaneous utility function, which is $f_0(x, u)$ in the framework of Section 2. Now, if $f_0(x, u)$ is constant over time then it is easy to calculate $\mathcal{W}(t)$. It equals $f_0(x, u)$ (at least if the discount factor tends to zero as time goes to infinity, which is surely the case when the rate of time preference is a positive constant). Hence, in the absence of non-autonomous elements in the system of differential equations or the constraints, it follows from (12) that a necessary condition for sustainability is that the value of investments (broadly defined) is constant over time $\lambda(t) \cdot \dot{x}(t) = 0$ (see (4)). Moreover, this is also a sufficient condition for sustainability. However, if we observe zero net investments in an economy at some particular instant of time, this does not mean that this economy finds itself in a sustainable development. For that, net investments should be zero at all instants of time. We illustrate the problem by means of an example.

Consider the model of Section 3 without pollution, without technical progress, and without international trade. We assume that $F(K, R) = K^\alpha R^\beta$ with $\alpha > \beta$. We compare two economies.

Economy 1 pursues the maximin objective, which in the case at hand seems to be an appropriate interpretation of sustainability. It is easily seen that the maximin rate of consumption in this economy is the solution of an optimal control problem of the type introduced in Section 2, with a decreasing rate of time preference. It was shown by Solow (1974) that the maximin rate of consumption is

$$C_1 = (1 - \beta)\{(\alpha - \beta)^\beta S_0^\beta K_0^{\alpha - \beta}\}^{1/1 - \beta}$$

National product in this economy is

$$NNP_1(t) = C_1 + \dot{K}_1(t) + p_1(t)\dot{S}_1(t)$$

where $p_1(t) := \partial F(K_1(t), R_1(t))/\partial R_1(t)$. We will have $NNP_1(t) = C_1$ since, as outlined above, net investments are zero ($\lambda \cdot \dot{x} = 0$ if $f_0(x, u)$ is constant).

Economy 2 pursues the utilitarian objective given in Section 3 with a constant rate of time preference. National product in economy 2 is

$$NNP_2(t) = C_2 + \dot{K}_2(t) + p_2(t)\dot{S}_2(t)$$

where $p_2(t) := \partial F(K_2(t), R_2(t))/\partial R_2(t)$. This is the value of the natural resource in terms of the consumer good.

It was shown by Pezzey and Withagen (1995) that there exists $\hat{\rho}$ such that if the pure rate of time preference ρ equals $\hat{\rho}$, Figure 1 arises. So $C_1 = C_2(0)$, $C_2 > C_1$ for an initial period of time and $C_2 < C_1$ eventually. Asheim (1994) and Pezzey (1994) show that there exists $\hat{t}$ such that $C_2(\hat{t}) > C_1$ and $\dot{K}_2(\hat{t}) + p_2(\hat{t})\dot{S}_2(\hat{t}) > 0$ implying $NNP_2(\hat{t}) > NNP_1(\hat{t})$. However, $C_2(\hat{t})$ is clearly not sustainable. Therefore, a higher NNP does not imply a higher maximin rate of consumption.

We conclude with Asheim (op.cit.) that 'it would seem impossible to develop the concept of NNP into an indicator of sustainability...'.
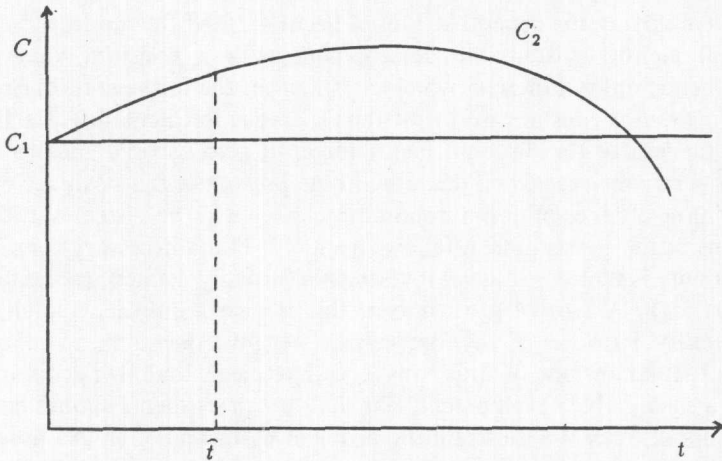
FIG. 1. Optimal consumption paths

7. Conclusions

In this paper we have studied the usefulness of net national product for several purposes: as an indicator in cost benefit analysis, as an indicator for welfare, and as an indicator for sustainability. The conclusions can be summarized as follows:

(i) NNP can serve as a measure of welfare if the necessary adjustments are made (when the system governing the economy is non-autonomous) and if actual prices are not too far off the optimal prices. The latter requirement obviously poses serious problems when unpriced commodities such as nature are involved;

(ii) If the cost benefit analysis applies to short run projects (with no long run effect) then NNP is a good indicator, at least if the prices in the economy are not too far from the long-run optimal prices. When projects have long-run effects and/or affect stocks then instantaneous NNP does not provide enough information to evaluate the projects. In that case more information on the future optimal development of the economy is necessary.

(iii) With respect to NNP as an indicator for sustainability the conclusion is negative.

Given this scepticism and given the informational requirements necessary to have a good indicator, one can seriously doubt the usefulness of the NNP concept. Indeed, it might be as easy to formulate the long run planning problem and see if the optimum or something resembling it is prevailing in reality, in order to evaluate the present state of the economy.

ACKNOWLEDGEMENTS

REFERENCES

AAHEIM, A. and NYBORG, K. (1992). 'Green National Product: Good Intentions, Poor Device?', paper presented at EAERE, Krakow.

ARONSSON, T. and LÖFGREN, K. (1993). 'Welfare consequences of technological and environmental externalities in the Ramsey growth model', *Natural Resource Modeling*, **7**, 1–14.

ASHEIM, G. (1994). 'Net National Product as an Indicator of Sustainability', *Scandinavian Journal of Economics*, **96**, 257–65.

BARTELMUS, P. (1994). *Environment, Growth and Development*, Routledge, London.

BARTELMUS, P., STAHMER, C. and VAN TONGEREN, J. (1991). 'Integrated Environmental and Economic Accounting: Framework for a SNA Satellite System', *Review of Income and Wealth*, **38**, 111–48.

CESARI, L. (1983). *Optimization—Theory and Applications*, Springer, Berlin.

CHICHILNISKY, C. (1994). 'Sustainable Development and North-South Trade', paper presented at The Rockafellar Foundation Conference on Trade and Development, Chiang-Mai, Thailand.

DASGUPTA, P. and HEAL, G. (1974). 'The Optimal Depletion of Exhaustible Resources', *Review of Economic Studies*, Symposium Issue, 3–28.

DASGUPTA, P. and MÄLER, K. (1991). 'The Environment and Emerging Development Issues?' proceedings of the World Bank Annual Conference on Development Economics.

DASGUPTA, P. and MÄLER, K. (1993). 'Poverty, Institutions and the Environmental Resource base', mimeo, Beijer International Institute of Ecological Economics.

DIXIT, A., HAMMOND, P., and HOEL, M. (1980). 'On Hartwick's Rule for Regular Maximin Paths of Capital Accumulation and Resource Depletion', *Review of Economic Studies*, **47**, 551–6.

VAN GELDROP, J. and WITHAGEN, C. (1993). 'General Equilibrium and International Trade with Exhaustible Resources', *Journal of International Economics*, **34**, 341–57.

HARTWICK, J. (1977). 'Intergenerational Equity and the Investing of Rents from Exhaustible Resources', *American Economic Review*, **67**, 972–4.

HARTWICK, J. (1990). 'Natural Resources, National Accounting and Economic Depreciation', *Journal of Public Economics*, **43**, 291–304.

HARTWICK, J. (1991). 'Degradation of Environmental Capital and National Accounting Procedures', *European Economic Review*, **35**, 642–9.

HARTWICK, J. (1994a). 'National Wealth and Net National Product', *Scandinavian Journal of Economics*, **96**, 253–6.

HARTWICK, J. (1994b). 'Sustainability and Constant Consumption Paths in Open Economies with Exhaustible Resources', mimeo, Center for Economic Studies, Munich.

HICKS, J. (1946). *Value and Capital*, Oxford University Press, Oxford.

HUETING, R., BOSCH, P., and DE BOER, B. (1992). 'Methodology for the Calculation of Sustainable National Income', Netherlands Central Bureau of Statistics, M44, The Hague.

HULTEN, C. (1992). 'Accounting for the Wealth of Nations: the Net versus Gross Controversy and its Ramifications', *Scandinavian Journal of Economics*, **94**, 9–24.

JOHANSSON, P. and LÖFGREN, K. (1994). 'On the interpretation of "Green" Measures as Cost-Benefit Rules', mimeo, Stockholm School of Economics.

MALANOWSKI, K. (1984). 'On Differentiability with Respect to Parameters of Solutions of Convex Optimal Control Problems Subject to State Space Constraints', *Applied Mathematics and Optimization*. **12**, 231–47.

MÄLER, K. (1991). 'National Accounts and Environmental Resources', *Environmental and Resource Economics*, **1**, 1–15.

MICHEL, P. (1982). 'On the Transversality Condition in Infinite Horizon Optimal Control Problems', *Econometrica*, **50**, 975–85.

PEZZEY, J. (1994). 'The Optimal Sustainable Depletion of Non-renewable Resources', paper presented at the 1994 AERE workshop, Boulder, Colorado.

PEZZEY, J. and WITHAGEN. C. (1995). 'Single-peakedness and Initial Sustainability in Capital-Resource Economies', mimeo, Eindhoven University of Technology.

RAMSEY, F. (1928). 'A Mathematical Theory of Saving', *Economic Journal*, **38**, 543–9.

SAMUELSON, P. (1961). 'The Evaluation of "Social Income": Capital Formation and Wealth', in F. Lutz and D. Hague (eds), *The Theory of Capital*, St. Martin's Press, New York.

SEFTON, J. and WEALE, M. (1992). 'The Net National Product and Exhaustible Resources: the Effects of Foreign Trade', mimeo, Cambridge University, Cambridge.

SEFTON, J. and WEALE, M. (1994). 'The Net National Product and Exhaustible Resources: the Effects of Foreign Trade', Discussion Paper no. 73, National Institute of Economic and Social Research, forthcoming in *Journal of Public Economics*.

SEIERSTAD, A. and SYDSAETER, K. (1987). *Optimal Control Theory with Economic Applications*, North-Holland, Amsterdam.

SOLOW, R. (1956): 'A contribution to the theory of economic growth', *Quarterly Journal of Economics* **70**, 65–94.

SOLOW, R. (1974). 'Intergenerational Equity and Exhaustible Resources'. *Review of Economic Studies*, Symposium, 29–45.

SOLOW, R. (1992). 'An Almost Practical Step Toward Sustainability', mimeo, Resources for the Future, Washington, DC.

TOMAN, M. (1985). 'Optimal Control with an Unbounded Horizon', *Journal of Economic Dynamics and Control*, **9**, 291–316.

WEALE, M. (1992). 'Environmental Statistics and the National Accounts', mimeo, Department of Applied Economics, Cambridge.

WEITZMAN, M. (1976). 'On the Welfare Significance of National Product in a Dynamic Economy', *Quarterly Journal of Economics*, **90**, 156–62.