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A game theoretic approach to analyse cooperation between rural households in Northern Nigeria

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Abstract:
Much research focuses on development of new agricultural technologies to reduce poverty levels of the large population of smallholder farms in Sub Saharan Africa. In this paper we argue that smallholders can also increase their production in a different way, namely by using their resources more efficiently through cooperation. This is obtained by grouping their (heterogeneous) resources and making joint decisions based on the aggregate resources. Afterwards, the gains of the joint production are divided, such that each farmer remains independent.

This type of cooperation is modeled using linear programming and cooperative game theory. While linear programming establishes insight in optimal farm plans for farmers that cooperate, game theory is used to generate fair divisions of the extra gain that is established by cooperation.

The model is applied to a village in Northern Nigeria. Households are clustered based on socio-economic parameters, and we explore cooperation. The optimal farm plan of the cooperative (i.e., farmers cooperate) contains more crops with high market and nutritional value, such as cowpea and sugarcane. We show that the gross margin of the cooperative is 12% higher than the sum of the individual gross margins. To divide these gains, we consider four established solution concepts from game theory that divide these extra gains: the Owen value, Shapley value, compromise value and nucleolus. An interesting result is that all farmers gain from cooperation and that the four solution concepts give similar results. Finally, we show how the provision of micro-credit can be used to stimulate cooperation in practice, benefiting the least-endowed farmers as well.

Keywords: Linear Programming, Agriculture, Household models, Cooperative Game Theory, Nigeria

JEL: C61, Q12, C71

1. Introduction

In Sub Saharan Africa approximately 64 % of the population lives in rural areas (FAO STAT, 2007), primarily earning their income from farming and related activities (Manyong *et al.*, 2005). The average farm size is small, while labour at the farm is mostly provided by the family. These small-scale farmers mainly use their income for purchasing food and other primary necessities of life. Their development is constrained by weak infrastructure, such as bad roads leading to poor access to agricultural input and output markets, and lack of basic amenities such as clean water and electricity.

Many development programmes and research institutes strive to reduce poverty levels of this large population of farmers (IITA, 2006). A considerable part of their research and activities focuses on development of new agricultural technologies, such as new varieties of crops and improved agricultural systems, with the aim to increase local production and generate wealth (World Bank 2008).

In this research we propose a different approach to increase local production. The keyword in this approach is cooperation, which enables the farmers to use available on-farm resources more efficiently. Small-scale farmers are heterogeneous with respect to their resources (Ruben and Pender, 2004). For example, some farmers have an excess of land, given their labour and capital resources, while other farmers may have excess of other resources. However, these resources are not commonly traded, and the excess resources remain unused, usually as a result of weak or absent markets. Common reasons of such market imperfections include high transaction costs due to weak infrastructure, absence of a credit market facilitating trades, risk and uncertainty and information asymmetry (De Janvry *et al*, 1991; Binswanger and Rosenzweig, 1986). Note that we do not exclude the need for developing new and improved technologies, which can easily be incorporated in this approach.

In this paper we model a farm cooperative[1] in which farmers group their resources, i.e., land, labour and capital, and jointly make decisions based on the aggregate resources after which they divide the gains of the joint production. Hence, (in the absence of market clearing of excess resources,) such cooperation between farmers, in which they group complementary resources, leads to more (allocatively) efficient production.

The innovation in the framework presented in this paper is the application of linear production games (Owen, 1975) to model farm cooperative in a rural African setting. Linear production games combine linear programming (LP) and cooperative game theory. While a suitable LP model determines an optimal farm plan for each possible collection of farmers, cooperative game theory is used to divide the gain in a farm cooperative.

First, we develop a farm household model, using LP. This model is based on the commonly applied agricultural farm household model, e.g. described by Schweigman (1985) and Hazell & Norton (1986). An extensive body of research uses such models to analyse farm household decisions. See Hazell and Norton (1986) for an overview, more recent applications include amongst others, Abdoulaye and Sanders (2006), Dorward (2006), and Woelcke (2006). These LP models represent the main decisions in a farm household, namely, production, market and consumption decisions. The constraints

[1] Note that a farm cooperative is different from a collective farm, which is operated and owned by a group of people, i.e., like the Kibbuts in Israel. This is contrary to our approach, where farmers remain independent.

reflect the major resources used in farming: land, labour, and capital. The solution of the LP model leads to an optimal farm plan.

As cooperation possibly leads to more efficient use of resources, total gain from farm production is likely to increase. Indeed, in such a case a problem arises on how to divide this extra gain among the cooperating farmers. Clearly, a farmer is only inclined to join a cooperative if the expected return is higher than when producing alone. This equally applies to every other farmer. Moreover a farmer is likely to remain part of the cooperative in the long-run, if his return is representative to his contribution, and if there is no distrust about the fairness of the division. Hence, a fair division rule needs to be established for long-run stability, such that distrust amongst farmers is prevented.

For the construction of such a division rule we turn to cooperative game theory. Cooperative game theory is a mathematical framework to analyse cooperation. Its main focus lies on the study of divisions of joint revenues in cooperatives. For a theoretical overview on cooperative games we refer to Tijs (2003). Empirical applications of cooperative game theory in agriculture and land use include Aadland and Kolpin (1998), Suzuki and Nakayama (1976) and Lejano and Davos (1999), though applications in a rural African setting are currently unknown to the authors.

We apply four game theoretical concepts in our framework: the Owen value (Owen, 1975), Shapley value (Shapley, 1953), nucleolus (Schmeidler, 1969), and compromise value (Tijs, 1981). Each of these concepts proposes a division of the extra gains that are established by cooperation from a different point of view.

We illustrate the applicability of the framework by presenting a case study of rural households in Northern Nigeria. The objectives are to show how farm plans change when farmers cooperate, to measure the magnitude of gains in cooperation compared to without cooperation, to compare outcomes from different concepts, and to suggest a strategy to stimulate cooperation.

This paper is organised as follows. In Section 2 we provide some preliminaries on LP and game theory. We describe the cooperative model in Section 3. Thereafter, in Section 4, we discuss the data and area of study and in Section 5 the results of the applied model are described. In Section 6 we provide an example of a micro-credit system to stimulate cooperation and we conclude.

2. Preliminaries

In this section we introduce some preliminaries on LP problems and cooperative game theory.

The mathematical formulation of an LP problem is $max \{c^T x \mid A x \leq b; x \geq 0\}$, where, $x \in \Re^k$ represents the decision variables, $c \in \Re^k$ represents the objective function coefficients, which reflect in many cases the prices, $b \in \Re^m$ is the right hand side, which is often identified with the resource bundle, and $A \in \Re^{k*m}$ is the production matrix.

We can obtain shadow prices for all resources by solving the dual LP problem corresponding to the above described problem. The dual is $min \{ y^T b \mid y A \geq c, y \geq 0\}$, where $y \in \Re^m$ represents the vector of shadow prices. A shadow price is the marginal value of a resource. Hence, a unit increase of a certain resource leads to an approximate increase of the objective function with the shadow price of this resource.

In cooperative game theory economic agents are called *players*. Let N be a finite set of players and let 2^N denote the collection of all subsets of N, which are called *coalitions*. The coalition in which all players are included, N, is called the *grand coalition*.

A *cooperative game* is a the pair (N,v), where $N=\{1, 2, \ldots, n\}$ is the set of players, and $v : 2^N \rightarrow \Re$ is a map assigning to each coalition $S \in 2^N$ a real number, such that $v(\emptyset)=0$. The function v is called the characteristic function of the game, $v(S)$ is called the *value of coalition S.*

The central question in cooperative game theory is how the value of the grand coalition should be divided amongst the players, such that all players have an incentive to cooperate. Frequently used division rules (or solution concepts), each with its own appealing characteristics, are the Shapley value, the compromise value and the nucleolus.

The Shapley value, introduced by Shapley (1953), is based on the marginal contribution of a player if he joins some coalition. This marginal contribution reflects a measure of importance of the player in the game. The nucleolus, introduced by Schmeidler (1969) looks for an allocation that minimises (lexicographically) the worst inequity, i.e., it minimises the maximum dissatisfaction of each coalition. The compromise value, introduced in Tijs (1989), is the efficient allocation that is a compromise between two non-efficient allocations of which one is the best for each player and the other one the minimum each player can claim.

For the mathematical formulation and the discussion of the appealing properties of the Shapley value, the compromise value and the nucleolus we refer to Tijs (2003).

At this point we turn to the special class of games where LP and cooperative game theory are combined. Owen (1975) introduced linear production (LP) games, which combine LP models to create cooperative games. Let $c, x \in \Re^k$ and matrix $A \in \Re^{k*m}$ as in the LP model and $b_i \in \Re^m$ denote a resource vector for each individual player $i \in N$. A *linear production game (N,v)* is a cooperative game in which the characteristic function is defined as $v(S) = max\{ c^T x \mid Ax \leq b(S), x \geq 0\}$, where $b(S) = \sum_{i \in S} b_i$ is the resource bundle owned by coalition S. Hence, the LP model determines the value of coalition S.

Moreover, Owen (1975) introduced a division rule specifically developed for LP games. This division rule is called the Owen value and is based on the duality concept of linear programming and uses the shadow prices to divide the gains of the grand coalition. Each player receives, based on the shadow prices, the marginal value of his own resource bundle.

3. The cooperative model

In this section we introduce the cooperative model that provides a framework for modelling cooperative farm decisions in developing countries. In Section 5 we apply this model to a group of farm households in Northern Nigeria. The novelty of this model is that it consists of the combination of two mathematical disciplines: linear programming and cooperative game theory. Linear programming is a well-established tool in optimising farm plans (Hazell and Norton (1986)). The advantage of using cooperative game theory in combination with LP is that an optimal farm plan of a group of farmers can be determined. Moreover, fair division rules from cooperative game theory can be used to divide the gain obtained in the farm cooperative.

We start with a description of the decision variables, the objective function and the constraints of the LP model. This LP model is inspired by a basic farm household LP model as presented in Schweigman (1985, pp. 18-30) and Hazell and Norton (1986) and the exact mathematical formulation is given in Appendix A.

The decisions a farmer has to take are manifold, though one can generally group these into decisions on production, consumption and market. The next table presents a summary of the decision variables a farmer has to take into account.

Table 1: Decision Variables

Type of decision variables	*Description decisions*
Production decisions (yearly)	Assignment of area to cropping systems
Consumption decisions (monthly)	Consumption of different crops
Market decisions (monthly)	Selling/buying of different crops Buying fertiliser Hiring labour / out hiring labour Taking / Paying off a loan

The production decisions of a farmer can be expressed by the assignment of land to different cropping systems. The farmer needs to decide which crops he wants to cultivate, which methods he wants to use to grow his crops, and when he will apply labour, fertiliser, etc. These choices are limited, because they depend on externalities, like climate, soil, etc. Therefore we introduce cropping systems, which summarise the limited choices of growing certain crops, or combinations of crops. For each cropping system it is predefined which crops are grown in the system, how much labour is required in each month and how much fertiliser needs to be applied. The farmer can divide his/her land into areas and he can assign a cropping system to each area. For example, there can be a cropping system "*Rice (high inputs)*", as opposed to "*Rice (low inputs)",* that requires a higher amount of fertiliser and labour, and as a result of the high inputs, has higher yields, than "*Rice (low inputs)"*. The farmer needs to decide if he/she wants to cultivate rice, and if so, he/she can assign a certain piece of land to the "*Rice (high inputs)"*-system or the "*Rice (low inputs)"*-system, or both. The assignment of all land to cropping systems is called a farm plan, with production quantities of crops as main outcome. Observe that leaving land fallow is also an option.

The consumption decisions depend on the nutritional value of crops, the food habits and the availability of crops. The farmer needs to decide how much his family consumes of each crop in each month.

The most important market decisions are decisions on trading of different crops, buying fertiliser, hiring (out) labour, and taking or paying off a loan. Note that all decision variables are related with each other. For example, decisions on selling and buying of a certain crop are related to the production of this crop and its chosen consumption level.

The objective in the model is to maximise the gross margin of the crop production. To calculate the gross margin, the costs of hired labour and fertiliser needs in production are subtracted from the revenue, which is the total production valued at market prices. The revenue depends on the chosen farm plan, the yields and the prices of the different crops. The cost of hired labour depends on the total number of hired labour hours and the wage rate, while the cost of the fertiliser depends on the farm plan, the required fertiliser inputs for each cropping system and the fertiliser prices.

This objective has been used frequently in applications of farm household models in SSA (Hazell and Norton (1986)). Many authors (Upton, 1996; Abdoulaye and Sanders, 2006; Woelcke, 2006) claim that monetary objectives, like gross margin, profit, income, net revenue optimisation are suitable if a subsistence constraint is included in the model to guarantee enough consumption in the household. We show later that in our model this constraint is included. Further note that when modelling cooperation, dividing currency amongst farmers is understandable, while other objectives are less straightforward to allocate.

The constraints need to reflect the farm household situation. Table 2 gives an overview of the constraint types of the model.

Table 2: Overview Constraint types

Type of constraint[*]	*Resource parameter*
Land: - Common fields (1) - Fadama fields (2)	Available area: - Common - Fadama
Labour (monthly) (3)	Available labour
Storage balances (monthly calculation of quantity crops in store) (4-8)	Initial storage
Capital balances (monthly availability of money) (9-10)	Non-agricultural income and other expenditures, initial capital
Subsistence constraints (11-12)	Minimal nutritional needs
Loan constraints: - paying back loan (13) - maximum loan (14)	Maximum amount available loan
Time which is spent on wage labour (monthly) (15)	Maximum hours available to spend on wage labour

First, we introduce a land constraint. The total area used for the different cropping systems cannot exceed the available land. To make the model specific to the region, we include an extra restriction for the use of fadama area (low lands) for cropping systems that include crops with high demand on water, such as rice, sugarcane etc.

Second, we incorporate monthly constraints for labour supply, because the labour requirements for the chosen farm plan cannot exceed the available labour (including hired labour and excluding labour hired out). Note that we include the constraints on a monthly basis because the requirements fluctuate during the year. During the weeding and

[*] The numbers in brackets behind the constraint types refer to the inequalities in the Appendix.

harvesting period the labour requirements are high, while there are no labour requirements outside the growing season.

Moreover, we include monthly storage and capital balances. We use two extra constraints to include the assumption that purchased food crops are used for consumption and not for trade. Further a restriction is introduced for perishable crops, which cannot be stored for a long period (i.e., vegetables) and two constraints are incorporated to include the crop leftover of the previous cropping season as initial storage. The capital balance depends on the income and expenditures. Each month changes in loans, income and expenditures of trading in crops, expenses on required fertiliser and hired labour, income from labour hired out, non-agricultural income and other expenditures influence the capital balance. An extra constraint is included to initialise the capital at the start of the growing season.

Furthermore, subsistence consumption is satisfied by incorporating two constraints for the minimum nutritional intake of energy and protein needed in the family during the target year, which are the twelve months from the end of the harvest season. The first constraint guarantees that the consumption during the 5 months after the harvest contains enough energy and protein. The second constraint assures that the nutrient contents of the stored food crops at the end of the year are sufficient for the first 7 months (growing season) of the next year.

We include two constraints on lending. The loan taken during a year should be paid back before the end of the year and there is a maximum amount of money which can be borrowed.

Finally, a constraint is incorporated to set a maximum to the monthly amount of time the farmer is able to work on other farms to earn additional income against the local wage rate. This last constraint is included to reflect the non-permanent demand for agricultural wage labour, which leads to limited possibilities for getting an outside job.

Next, we introduce the LP game as defined in Section 2, by using the above-described LP model. The ingredients needed for an LP game, are the players, the objective function, the production matrix and the resource bundles. The players of the LP game are a group of farm households from a certain village. The objective function for each player is to maximise the gross margin. Furthermore, in line with the assumption of local homogeneity in production technologies, the production matrix is the same for everybody in the village, meaning that prices and production functions do not differ amongst farmers in the same village. But, as we observed before, farm households are heterogeneous in resources, hence each player has a different resource bundle.

4. Data and area of study

This section provides general information on data collection. Village level data, which is used for the objective function and the production matrix, is discussed shortly, while farm specific (resource) data is discussed in more detail.

We apply the model to a case study based on the situation in Ikuzeh village, Kajuru Local Government Area, Kaduna State, Nigeria. This village is located in the Northern Guinea Savannah. This agroecological zone is defined by a length of growing period of 151-180 days and unimodal rainfall pattern. Kadara is the major ethnic group in the village. Main crops include sorghum, maize and cowpea for upland fields. In the lowlands, sugarcane and rice are cultivated in the village. The village has low population density and the main weekly market is distant. In 2002 a baseline survey was carried out in 39 randomly selected households (IITA, 2002). From this survey we obtain information on land use strategies, yields, input use, farm sizes and social characteristics like household size, education level, age of the household head and asset and livestock ownership. During the growing season of 2005 we collected additional data on a biweekly basis. This includes budget and market data as well as data on labour requirements for different crops, and wage rates. Furthermore we collected additional information on the wage labour market during several field visits in 2006. We corrected the 2005 and 2006 price data for inflation, such that the complete set is representative for 2002.

Next, we present the village level data, which we use to determine the parameters of the objective function and the production matrix. Most important data are presented in Table 3.

Table 3: Crops and cropping systems included in the model

Cropping systems	Crops	Total required hours	Fertiliser required kg	Yields kg/ hectare	Average crop price Naira/kg	Energy MJ /kg	Protein g /kg
Sugarcane (sole) (Low Input)	Sugarcane	802	274	3720	46	0.251	10
Rice (sole) (Low Input)	Rice	1421	13	775	59	1.508	75
Rice (sole) (High Input)	Rice	1485	85	1107	59	1.508	75
Maize (sole) (Low Input)	Maize	593	27	327	32	1.492	95
Maize (sole) (High Input)	Maize	640	122	750	32	1.492	95
Sorghum (sole) (Low Input)	Sorghum	487	0	200	31	1.437	101
Sorghum (sole) (High Input)	Sorghum	571	67	400	31	1.437	101
Sorghum-Cowpea relay (Low Input)	Sorghum Cowpea	617	0 0	455 145	31 61	1.437 1.446	101 222
Late Millet (sole) (Low Input)	Late Millet	487	8	637	33	1.425	97
Hungry Rice (sole) (Low Input)	Hungry rice	253	0	335	69	1.399	122
Soybean (sole) (Low Input)	Soybean	549	0	363	32	1.404	380
Groundnut (sole) (Low Input)	Groundnut	434	0	406	27	1.626	182
Cassava (sole) (Low Input)	Cassava	977	0	1682	17	0.457	9
Cocoyam (sole) (Low Input)	Cocoyam	706	0	1158	24	0.360	15
Okra (sole) (Low Input)	Okra	253	0	400	68	0.130	16
Hot Pepper (sole) (Low Input)	Hot pepper	253	9	435	40	0.117	12

The first column of Table 3 gives the cropping systems, based on the responses in the baseline survey. The second column presents the corresponding crops. To reflect the continuous production function in this non-continuous approach we include two options for most common crops, one with high fertiliser use and labour requirements (high input use) and high yields and one with low input use and low yields. Observe that other levels can be chosen in the farm plan, by taking a mix of low and high input cropping systems. Furthermore sorghum-cowpea relay is a commonly practiced system, in which first sorghum is planted, and later in the growing season cowpea is planted in-between the sorghum.

Columns three, four and five of Table 3 present the total labour requirements, the total required fertiliser and the yields. We estimate labour requirements from the fortnightly survey in 2005, and estimate crop yields and fertilizer inputs from the baseline survey of 2002. In the last columns we present the average crop price (KADP, 2002) and the energy and protein contents (FAO, 2007) of each crop.

The other data in the production matrix are the wage rates, which are estimated based on the fortnightly surveys of 2005 and fertiliser prices (KAFC, 2002). The average wage rate is approximately 26 Naira per hour and the average fertiliser price is 37 Naira per kg.

The baseline survey of 2002 is used to characterise the 39 households and estimate their resource parameters.

Calculations of the farm and fadama size and the value of assets follow straightforward from the baseline survey. For calculation of the livestock ownership, we convert the livestock into standard tropical livestock units (TLU) using 1 TLU = 250 kg (equals 10 goats) (Jahnke, 1982).

Computations of the labour availability and household subsistence nutritional requirements are based on the household structure, incorporating differences between children, men and women and their respective participation in agricultural production. To calculate labour availability we assume that each participating person works 30 days a month for 6 hours a day and that leisure takes place during the remaining hours of the day. Hence, we assume everybody is willing to work 6 hours a day. Furthermore we convert women and child labour into standard man-hours, using correction factors 0.67 and 0.5 respectively, based on Van Heemst *et al.* (1981). During interviews in 2006 we learned that the average time spend on agricultural wage labour is 10%. This percentage is used to calculate the maximum outgoing labour hours. The nutritional requirements are estimated based on the FAO (2007).

We estimate the maximum loan sizes based on the fortnightly surveys and on the resources, i.e., collateral, of the individual 39 farmers, such as farm size, stated value of assets and livestock.

Data on non-agricultural income and other expenses, like cooking items, health care etc, has been collected for a subset of the farmers. The village is far from the market, therefore the possibilities of earning non-agricultural wage are limited and the amount of extra income is observed to be low in all households. The amount spent on other expenses is observed to be more or less equal among all households. Therefore we assume that non-agricultural income and other expenses are equal for each household in the village, i.e., a net income of approximately 24000 Naira for each farm- household.

Before we apply the model, for both computational and presentable convenience, we use cluster analysis to classify farmers into homogeneous groups (Hazell and Norton, 1986). Based on the results of the hierarchical cluster analysis of the 39 farmers we group farmers into five clusters *A, B, C, D, E* and calculate the average characteristics in each cluster, and we construct five averages for each farmers' cluster. In the remainder of this paper we refer to them as farmer *A, B, C, D* and *E*. In the next section we analyse the attractiveness of cooperation between these five average farmers.

The 39 households are clustered using data on farm size, area for fadama, household size, livestock ownership and household stated assets, such as tools, radio, bicycle etc. In Table 4 the average characteristics of the five clusters are presented. We show characteristics used for the clustering, as well as the farm specific resource bundles used in the model, i.e., farm and fadama size, labour availability, energy and protein requirements, loan availabilities.

Table 4: Characteristics of the five clusters

Cluster	*A*	*B*	*C*	*D*	*E*
Number of farmers	23	8	4	2	2
Farmsize (ha) [ab]	4,72	6,45	18,07	13,70	6,05
Fadama (ha)[ab]	0,45	0,72	1,96	2,23	0,52
Household size (# persons) [a]	6.2	14.6	11.5	11.0	6.0
Ownership Livestock (TLU) [a]	0.5	1.7	0.9	5.7	3.2
Value of stated Assets (Naira)[a]	2900	5700	3000	2600	53700
Labour (Man hours / month)[b]	570	980	885	1104	561
Outgoing Labour (Man hours /month)[b]	57	98	88.5	110.4	56.1
Energy required (MJ) [b]	1484	2402	2778	2957	1560
Protein required (g) [b]	5664	9189	10428	11235	5898
Maximum Loan (Naira) [b]	0	2850	2850	5700	5700

Source: Baseline survey, result cluster analysis, 1 USD = 133 Naira (December 2002). [1] Cluster variable, [b] Resource parameter

From Table 4 we learn that the available (outgoing) labour and the protein and energy requirements are strongly correlated with the household size. This is not surprising as those parameters depend on the composition of the households. Furthermore we see that cluster *A* contains the largest group of farmers. The farmers in this cluster are the least endowed, since they have the smallest land and livestock holdings. Farmers from cluster *E* do not differ much from *A* with regards to household size and farm size, but both livestock ownership and stated assets are higher. Farmers from cluster *B* have similar farm size available as those from *E*, but the household size is larger and the stated assets and livestock units are smaller. Farmers from cluster *C* and *D* have both large farms and a larger than average household size. While those from cluster *C* have the largest farm size and those from cluster *D* are best endowed with livestock.

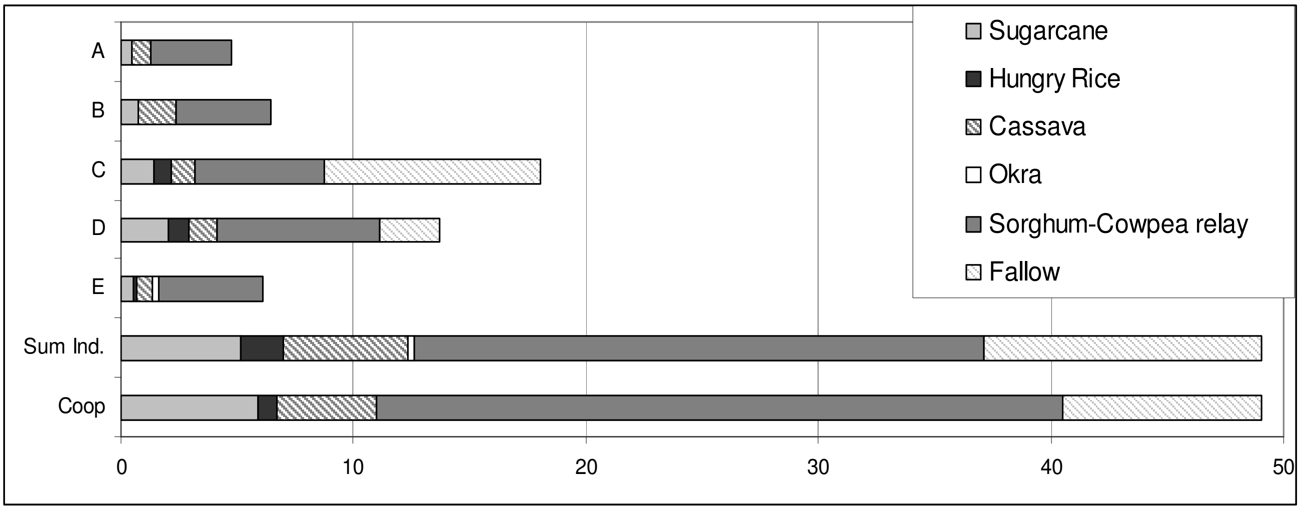
5. Case Study

In this section we apply the cooperative model, as described in Section 3, to the five average farmers described in the previous section.

5.1 LP solutions

We solve the LP model for each average farmer of cluster *A, B, C, D*, and *E.* The solution to the LP model results in an optimal farm plan for each individual farmer. Furthermore, we consider the optimal farm plan of the grand coalition or cooperative. Therefore we solve an additional LP model based on the aggregation of resources of the five farmers. For ease of comparison with the optimal solution in the cooperative, we sum the individual farm plans (i.e., the case without cooperation). In Figure 1 we present the optimal farm plans of the individuals, the sum of the individuals and the cooperative.

Figure 1: Optimal farm plans



First, observe from Figure 1, that all individual farm plans are different, which is clearly a result of the different resources. Furthermore farmers *A* and *B* use 3 different cropping systems in their optimal farm plans, while farmers *C* and *D* use 4 cropping systems and both have fallow land, and farmer *E* uses 5 cropping systems. Note that the fallow land of farmers *C* and *D* is due to their large land holdings (see Table 1) and the lack of complementary resources. Further, in all farm plans Sorghum-Cowpea relay is the dominant cropping system, with more than 50 % of the cultivated area allocated to these crops.

Next we compare the sum of the individual farm plans with the farm plan of the cooperative. We observe that all individual farmers together, fallow approximately 12 hectares, while in the cooperative fallow land reduces to 8.5 hectares. Area used for growing sorghum-cowpea relay and sugarcane is expanded, while cultivation of okra, cassava and hungry rice decrease in cooperation. Note that this shift is towards sugarcane, a high-value crop, and sorghum-cowpea, both products with high nutritional values. This shows that in the cooperative a different optimal farm plan is found, whereby resources are used differently.

Table 5 shows the resources and corresponding gross margins for the individuals, the sum of the individuals and the cooperative (grand coalition).

Table 5: Gross Margin for individuals, the sum of individuals and the grand coalition

		A	*B*	*C*	*D*	*E*	*A+...+E*	*N*
Resources	Farm (ha)	4,72	6,45	18,07	13,70	6,05	48,99	48,99
	Fadama (ha)	0,45	0,72	1,96	2,23	0,52	5,88	5,88
	Labour (hrs)	570	980	885	1104	561	4100	4100
	Outgoing Labour (hrs)	57	98	88,5	110,4	56,1	410	410
	Energy (MJ)	1484	2402	2778	2957	1560	11181	11181
	Protein (g)	5664	9189	10428	11235	5898	42414	42414
	Loan (Naira)	0	2850	2850	5700	5700	17100	17100
	Net non-agricultural income (Naira)	24361	24361	24361	24361	24361	121805	121805
	Gross Margin (Naira)	271099	396074	604609	800591	321885	2394258	2685004

Source: Result of own calculations, 1 USD = 133 Naira (December 2002)

Note that the last two columns of Table 5 are the same with respect to the resources, as by definition the resources of the grand coalition, *N*, equals the sum of all individual resources. Furthermore, the results show that the sum of the gross margin of all individual farmers (*A+B+C+D+E*) is equal to 2.394.258 Naira, while the gross margin of the cooperative is 2.685.004 Naira, an improvement of 12%. This translates to an average growth of 6000 Naira in income per head for the farm households in the cooperative.

5.2 Game theory solution concepts

Clearly, cooperation leads to substantial improvements in gross margin. But to actually form the cooperative, the farmers need to have a clear economic incentive to join. Potentially, one of the main impediments to cooperation is distrust about the fairness of the division rule (Cruijsen et al, 2005). Therefore, in this section, we apply methods from cooperative game theory to determine fair division rules, taking into account each player's impact within the cooperative.

We apply the linear production game, as described in Section 3, to analyse the farm cooperative. The farmers form the set of players, $N=\{A,B,C,D,E\}$, and we construct the game as defined in Section 2, using the LP model in Section 3 and the data of the average farmers in Section 4. Table 6 presents the complete LP-game (N,v). Hence, $v(S)$ reflects the gross margin resulting from the LP model solved for coalition S.

Table 6: Game for Gross Margin Maximisation

S	*v(S)*	*S*	*V(S)*	*S*	*v(S)*	*S*	*v(S)*	*S*	*V(S)*
A	271099	AB	667173	ABC	1507522	ABCD	2351735	N	2685004
B	396074	AC	954334	ABD	1625392	ABCE	1835246		
C	604609	AD	1137887	ABE	999496	ABDE	1955472		
D	800591	AE	600965	ACD	1754925	ACDE	2166980		
E	321885	BC	1212479	ACE	1328126	BCDE	2388637		
		BD	1349520	ADE	1471518				
		BE	728397	BCD	2050770				
		CD	1405200	BCE	1541344				
		CE	1029093	BDE	1679716				
		DE	1173810	CDE	1866015				

Source: Result of own calculations. Note: Values are given in Naira, 1USD = 133 Naira (December 2002)

First, note the intuitively straightforward interpretation of the values of the game in Table 6. Recall from Table 4 that farmers *C* and *D* are both better endowed than the other farmers, which is reflected in their gross margin (value) in Table 6. If both farmer *C* and *D* take part in larger coalitions this also leads to higher values.

Furthermore, recall that both farmers *C* and *D* have an excess of land and a shortage of labour, hence, they do not have complementary resources, as a result of which cooperation does not lead to extra gains. This is also reflected in the game because $v(\{C,D\}) = v(\{C\}) + v(\{D\})$. Observe that the same argument, although for different resources, holds for the farmers *A* and *B*, which both have excess of labour and shortage of land. Moreover, coalitions *{A, E}* and *{B, E}* have not many complementary resources and their extra gains are low, albeit nonzero. This is reflected in the game since for coalition *{A, E}* we have $v(\{A, E\}) \approx v(\{A\}) + v(\{E\})$. This similarly holds for coalition *{B, E}*.

Finally, note that farmers *A, B* and *E* have similar excess resources, which equally applies, albeit for different resources to, farmers *C* and *D*. Intuitively, coalitions formed by two or more players from both these groups, exploit complementarities in resources, hence giving an extra gain to the players. Observe, for example $v(\{B, C\})$ is considerably higher than the sum of the individual values $v(\{B\})$ and $v(\{C\})$. In fact the extra gains between these two players are highest.

Hence, we see that forming a coalition in many cases, but not all, leads to extra gains. But, farmers' contributions to the extra gains vary. As emphasized before, the gross margin earned by the grand coalition (cooperative) needs to be divided such that every farm household has an incentive to cooperate. We address this issue in the remaining part of this section.

First we discuss the Owen value, which is a game theoretical solution concept, specifically developed for LP games. The Owen value is based on the shadow prices of the resources, which reflect their marginal value. Table 7 shows the shadow prices of the LP model for the grand coalition.

Table 7: Shadow prices for the grand coalition

	Shadow price (Naira)
Farm (ha)	0
Fadama (ha)	187906
Labour (hrs)	341
Outgoing labour (hrs)	267
Energy (MJ)	0
Protein (g)	0
Loan (Naira)	4

Source: Result of own calculations, 1 USD = 133 Naira (December 2002)

Recall from Figure 1 that not all land resources are used by the grand coalition. This implies a shadow price for farm land of zero. Furthermore the shadow price of fadama fields is high. This is expected, because fadamas, the only fields where farmers can grow the high value crop sugarcane, are scarce in the dry region of study. Moreover, note that 17% of the fields is left fallow due to a lack of labour, thus the shadow price of labour is positive. The shadow price of labour (341 Naira) is high compared to the average wage rate of 26 Naira per hour. Note that this means that if the cooperation uses one extra hour of labour (which can only be hired labour, for 26 Naira), it will result in a gross margin which is 341 Naira higher. However, a lack of capital impedes additional use of hired labour. Because of this lack of capital, we also find a positive shadow price for resources related to capital, such as loans and outgoing labour.

Further note that the nutritional requirements both have shadow prices of zero, which means that these requirements are not binding in the grand coalition. Hence, a slight increase in nutritional requirements, does not affect the gross margin and the farm plan of the cooperative.

Next we calculate the Owen value for each farmer. Therefore we value the individual resources with help of the shadow prices described above. Note that one should interpret the value of the solution concept as the share of the gross margin that a farmer (player) receives when forming the grand coalition. Table 8 presents and compares the individual gross margin and the Owen value for each farmer. Note that in this table the share which a farmer gets in cooperation is compared with his individual earnings, the increase is given in Naira and percentages.

Table 8: Owen value compared with individual gross margin

	Individual (Naira)	*Owen Value (Naira)*	*Increase (Naira)*	*Increase (%)*
A	271099	294064	22965	8
B	396074	507686	111613	28
C	604609	705772	101163	17
D	800591	849192	48601	6
E	321885	328290	6406	2

Source: Result of own calculations, 1 USD = 133 Naira (December 2002)

The results show that especially farmers *B* and *C* gain considerably with cooperation, as we expected from the game in Table 6. These improvements can be explained by the shadow prices. Farmer *B* has 150 man-hours per hectare (= 980/6.45) of labour available, which is the highest relative labour availability amongst all farmers. In his individual farm plan the shadow price of labour is zero, because not all of this resource is used, while labour has a high value in the grand coalition. The improvement of farmer *C* is mainly due to the high shadow price of fadama fields, of which farmer *C* owns relatively many. Furthermore, we observe that farmer *E* has a small increase of 2%, which means that in his individual farm plan, his resources are already used nearly efficient

In Section 2 we described other commonly used solution concepts in cooperative game theory. In Table 9 we show the results for all these solution concepts. The first row displays the individual gross margins. The next rows present the Owen value, the Shapley value, the compromise value and the nucleolus.

Table 9: Individual gross margin and all solution concepts, all values are in Naira

	A	*B*	*C*	*D*	*E*
Individual Gross Margin	271099	396074	604609	800591	321885
Owen Value	294064	507686	705772	849192	328290
Shapley value	293549	489746	702090	853221	346399
Compromise value	291838	507521	711678	845228	328739
Nucleolus	295453	507446	705537	845753	330815

Source: Result of own calculations, 1 USD = 133 Naira (December 2002)

The different solution concepts allocate values to the farmers in case of cooperation, and we observe that all allocations are individually rational, i.e., each allocation gives to each player a higher value than its individual gross margin. For example, according to the four allocations, farmer *A* gets at least 291.838 Naira when cooperating with the other farmers, which is an absolute increase of 20.739 Naira compared to its individual gross margin.

In fact, for three of the four solution concepts, it holds that for each sub-coalition the returns of the allocation of the grand coalition are higher than the returns of any smaller coalition. This means that no subgroup of the grand coalition has an incentive to split-off into a smaller coalition, i.e. these allocations are stable against coalitional split-offs. Hence, an allocation $(x_1, x_2, \ldots, x_n)$ is called *coalitional stable* if $\sum_{i\in S} x_i \geq v(S)$, for each possible coalition *S*, and $\sum_{i\in N} x_i = v(N)$, where x_i is the value allocated to farmer *i*.

Note that the Owen value, the compromise value and the nucleolus are all coalitional stable, while on the other hand the Shapley value is not coalitional stable. For example, if we take coalition {*B, C*}, we observe in Table 6 a value of this coalition of 1.212.479 Naira, whereas the table above shows that the Shapley value allocates only 1.191.836 Naira (= 489746 + 702090) to them. Hence, for this coalition it is worth to split off from the grand coalition if the Shapley value is used to allocate the gains. To the contrary the Owen value, the compromise vale and the nucleolus allocate 1.213.458 Naira, 1.219.199 Naira, and 1.212.983 Naira respectively to coalition {*B, C*}, which are all higher than 1.212.479 Naira. Nevertheless, if we rank the farmers based on the allocation of each solution concept in a decreasing order, it results for all solution concepts in the order *D-C-B-E-A*.

Next, we explore the relative increases and observe that the four solution concepts hardly differ from each other. Table 10 shows the relative increases for each player and each solution concept.

Table 10: Relative increase in gains compared to the individual gross margin

	A	*B*	*C*	*D*	*E*
Owen Value	8%	28%	17%	6%	2%
Shapley value	8%	24%	16%	7%	8%
Compromise value	8%	28%	18%	6%	2%
Nucleolus	9%	28%	17%	6%	3%

Recall from the game (Table 6) that the extra gains are high for farmers *B* and *C*, which result in relatively large gains for both of them. Moreover, we rank the farmers based on their relative increases in a decreasing order, which results for all solution concepts, except for the Shapley value, in *B-C-A-D-E*. Note that farmer *E* has the lowest benefits, which is due to his efficient use of resources in the individual case. Finally, we see that only for farmer *B* and *E* the Shapley value is slightly different from the other solution concepts, while for the other farmers the Shapley value gives a similar outcome. For farmer *E* his marginal contributions are relatively high, even though he uses most of his resources quite efficiently in the individual case.

It is an interesting feature that three out of four different solution concepts give very similar results. This is a very remarkable result, because these solutions are based on different point of views and have all its own appealing properties.

6. Discussion and conclusions

In the previous section we showed the promising results of the cooperative model, and the next step is to develop policies for stimulating cooperation in practice. In this section we provide a suggestion for the stimulation of cooperation, after which we summarise and discuss some issues for future research.

Intuitively, both micro-credit projects targeting cooperative farmers and programs aimed at improving variable input supply, such as fertilizer, could be promising avenues. In the following we give an example to illustrate the potential benefits of a micro-credit system restricted to cooperating farmers. Such a micro-credit scheme could be beneficial for three different reasons. Firstly, a financial organisation is more likely to give credit to a cooperative than to individual farmers, as the risks of farmers defaulting on the loan are lower. Secondly, micro-credit provided to a cooperative might yield a higher pay-off compared to credit given to individual farmers. Finally, and maybe most importantly, the least endowed farmers in a cooperative would then benefit from credit, while they are not likely to have access to credit individually due to their low collateral.

That said, accurate recommendations should be based on further detailed analysis, for example through the application of a framework such as the one developed in this paper. In order to get some insight into the impact of such policies, we briefly demonstrate the consequences of introducing a micro-credit project in which a loan of

1000 Naira is provided to each cooperating farmer, which should be paid back at the end of the growing season. We discuss the impact of this policy on the Owen value, since it is especially developed for LP games.

We apply the LP model with modified resource vectors to the grand coalition, in which the loan availability is increased with 5000 Naira, as a result of an increase of 1000 Naira per farmer. In Table 11 we compare the individual gross margin without micro-credit with the individual gross margin with micro-credit and the two Owen values resulting from a cooperative without and with micro-credit respectively. We take the individual gross margin without micro-credit as baseline, and compare the results.

Table 11: The impact of a micro-credit policy compared with individual case without cooperation (baseline)

		A		*B*		*C*		*D*		*E*	
		*1000 Naira	%	Naira	%	Naira	%	Naira	%	Naira	%
Individual gross margin	No micro-credit	271	100	396	100	605	100	801	100	322	100
	Micro-credit	271	100	396	100	626	104	822	103	323	100
Owen value	No micro-credit	294	108	508	128	706	117	849	106	328	102
	Micro-credit	298	110	511	129	712	118	855	107	331	103

Table 11 shows that providing a micro-credit to individual farmers does not result in higher gross margins for the less endowed farmers *A*, *B* and *E*, whereas the well-endowed farmers *C* and *D* gain around 3 to 4 %. On the contrary, if a micro credit system is introduced which simultaneously stimulates cooperation, all farmers gain from the extra credit, given the restriction of cooperation. In case we divide according to the Owen value, the poorest farmer *A* can obtain 10 % extra gross margin and the other farmers gain as well from cooperation. Note that the largest part of the gains is a result of the cooperation, in addition, there is a small extra gain due to of the extra available credit. To summarise, individual micro-credit will not result in higher gains for the least endowed farmers, and will probably not be given to these farmers due to their low collateral and the high risks of defaulting on their loans, whereas the returns of micro-credit provided to the cooperative will lead to increases for each famer. Hence, a micro-credit system which is restricted to cooperating farmers, stimulates the cooperation and is a pro-poor mechanism, ensuring that the poorest farmers can profit from the system.

To conclude, this paper is a first step in a promising new approach in which farm household models are combined with game theory to analyse effects of cooperation on the profitability of farming. A cooperative farm household model is introduced and applied to a specific region in Northern Nigeria.

Findings from this paper clearly provide evidence that cooperation amongst farmers should be stimulated, because extra gains can be obtained. Moreover, potential for development and stability of cooperation is higher if fair division rules can be provided. We discussed four different solution concepts from game theory: the Owen value, Shapley value, compromise value, and nucleolus. They all show an individual rational outcome for the farmer cooperation, meaning that each individual farmer receives at least the same amount as when farming individually. Another interesting finding is that three solution concepts give similar results, although the rationale behind each division concept and its calculation is different.

Observe that the framework presented in this paper can be used to study cooperation in different settings, e.g. different regions, inter-household situations, etc.

Possible extensions for future research may include risk aversion, dynamic aspects or multi-objective functions.

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References

Aadland, D., Kolpin, V.,1998. Shared irrigation costs: An empirical and axiomatic analysis. Mathematical Social Science 35 (2), 203-218.

Abdoulaye, T., Sanders, J.H., 2006. New technologies, marketing strategies and public policy for traditional food crops: Milet in Niger. Agricultural Systems 90, 272-292.

Binswanger, H.P., Rosenzweig, M.R., 1986. Behavioural and material determinants of production relations in agriculture. Journal of Development Studies 22, 503 - 539

Cruijssen, F. Borm, P., Fleuren, H., Hamers, H. (2005). Insinking: a methodology to exploit synergy in transportation. CentER Discussion Paper 2005 -121.

Dorward, A., 2006. Markets and pro poor agricultural growth: insights from livelihood and informal rural economy models in Malawi. Agricultural Economics 35, 157-169.

FAO STAT, 2007. FAO Corporate Statistical Database. Available from http://faostat.fao.org

FAO, 2007. “Nutritive Factors”. Available from http://www.fao.org/es/ess/xxx.asp

Hazell, P.B.R., Norton, R.D., 1986. Mathematical Programming for economic analysis in agriculture. Macmillan Publishing Company, New York, USA.

IITA, 2006. International Institute of Tropical Agriculture Annual Report 2006.

Jahnke, H.E., 1982. Livestock Production Systems and Livestock Development in Tropical Africa. Kieler Wissenschaftsverlag Vauk, Kiel, Germany.

Janvry, A. de, Fafchamps, M., Sadoulet, E., 1991. Peasant household behaviour with missing markets: some paradoxes explained. The Economic Journal 101, 1400-1417.

KADP, 2002. Unpblished data of the Kaduna State Agricultural Development Program.

KAFC, 2002. Unpblished data of the Kaduna State Fertiliser Company.

Lejano, R.P., Davos, C.A., 1999. Cooperative Solutions for Sustainable Resource Management. Environmental Management 24(2), 167-175.

Manyong, V.M., Ikpi, A. , Olayemi. K., Yusuf, J., 2005. Agriculture in Nigeria: identifying opportunities for increased commercialisation and investment. IITA, Ibandan, Nigeria.

Owen G., 1975. On the core of linear production games. Mathematical Programming 9, 358-370.

Ruben, R., Pender, J., 2004. Rural diversity and heterogeneity in less-favoured areas: the quest for policy targeting. Food Policy 29, 303-320.

Shapley, 1953. A value for n-person games. In: Kuhn, H. and Tucker, A. (Eds.), Contributions to the theory of games II. Princeton University Press, pp. 307-317.

Schmeidler, D., 1969. The nucleolus of a characteristic function game. SIAM Journal of Applied Mathematics 17, 1163-1170.

Schweigman C., 1985. Operation research problems in agriculture in developing countries. Tanzania Publishing House, Dar Es Salaam, Tanzania.

Suzuki, M., Nakayama, M., 1976. The Cost Assignment of the Cooperative Water Resource Development: A Game Theoretical Approach. Management Science 22(10), 1081-1086.

Tijs, S., 1981. Bounds for the core and the t value. In: Moesschlin, O. and Pallaschke, D. (Eds.), Game Theory and Mathematical Economics, North-Holland, Amsterdam, pp. 123-132.

Tijs, S., 2003. Introduction to Game Theory. Hindustan Book Agency, India.

Upton, M., 1996. Economics of tropical farming systems. Cambridge University Press, New York, USA.

Van Heemst, H.D.J., Merkelijn, J.J., Van Keulen, H., 1981. Labour requirements in various agricultural systems. Quarterly Journal of International Agriculture 20(2), 178-201

Woelcke, J., 2006. Technology and Policy options for sustainable agricultural intensification in eastern Uganda. Agricultural Economics 34 (2), 229-239.

World Bank, 2008. Chapter 7: Innovating through science and technology. In: World Bank Development Report 2008, Agriculture for Development.

1 Appendix A

Objective:

$$
\begin{aligned}
GrossMargin = &\sum_{i,j,t} AREA_j * yield_{ijt} * cropprice_t \\
&- \sum_{t} LABOURHIRED_t * wagerate_t \\
&- \sum_{j,t} AREA_j * fertilizerrequired_{j,t} * fertilizerprice_t
\end{aligned}
$$

Restrictions

$$\sum_{j} AREA_j \leq landavailable \tag{1}$$

$$\sum_{j \in C_{fadama}} AREA_j \leq fadamaavailable \tag{2}$$

$$
\begin{aligned}
\sum_{j} AREA_j * labourrequired_{j,t} \leq\ & labouravailable + LABOURHIRED_t \\
& - LABOUROUTHIRED_t \qquad \forall t = 1,..,T
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
STORE_{i,t} = STORE_{i,t-1} + \sum_{j} AREA_j * yield_{ijt} - CONSTORE_{i,t} - SELL_{i,t} \\
\forall i = 1,..,K, \forall t = 1,..,T
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
FOODSTORE_{i,t} = FOODSTORE_{i,t-1} + BUY_{i,t} - CONFOODSTORE_{i,t} \\
\forall i = 1,..,K, \forall t = 1,..,T
\end{aligned} \tag{5}
$$

$$STORE_{i,t} + FOODSTORE_{i,t} = 0 \text{ for } i \in K_{perishable} \forall t = 1,..,T \tag{6}$$

$$STORE_{i,0} = initialstore_i \qquad \forall i = 1,..,K \quad (7)$$

$$FOODSTORE_{i,0} = initialfoodstore_i \qquad \forall i = 1,..,K \quad (8)$$

$$\begin{aligned} CAPITAL_t = {} & CAPITAL_{t-1} + NEWLOAN_t - REPAYLOAN_t \\ & + \sum_i (SELL_{i,t} - BUY_{i,t}) * cropprice_{i,t} \\ & - \sum_j AREA_j * fertiliserrequired_{j,t} * fertiliserprice \\ & + nonagriculturalincome_t - otherexpenses_t \\ & + (LABOUROUTHIRED_t - LABOURHIRED_t) * wagerate_t \\ & \forall t = 1,..,T \end{aligned} \quad (9)$$

$$CAPITAL_0 = initialcapital \quad (10)$$

$$\begin{aligned} & \sum_i (CONSTORE_{it} + CONFOODSTORE_{it}) \quad * cropnutrients_{ih} \\ & \quad \geq nutrientrequired_h \qquad \forall t = 8,..,T \forall h = 1,..,H \end{aligned} \quad (11)$$

$$\begin{aligned} & \sum_i (STORE_{iT} + FOODSTORE_{iT}) \\ & \qquad * cropnutrients_{ih}/7 \geq nutrientrequired_h \end{aligned} \quad (12)$$

$$\sum_t REPAYLOAN_t = \sum_t NEWLOAN_t \quad (13)$$

$$\sum_t NEWLOAN_t \leq maximumloan \quad (14)$$

$$LABOUROUTHIRED_t \leq outgoinglabour \tag{15}$$

$$\text{All decision variables} \geq 0 \tag{16}$$